Budworm Forest Model - Analysis of the slow Forest variables

Aditya Joglekar 201401086
Rajdeep Pinge 201401103
Rushikesh Nalla 201401106
Omkar Damle 201401114

Equations for Forest Variables (E and S)

S : Average size of trees in the forest (which is an indicator of their age)

E : The level of energy reserves (this includes the condition of the foliage and health of the trees)

- Biological Significance

$$\dot{S} = r_S S\left(1 - \frac{S}{K_S}\frac{K_E}{E}\right), \qquad \dot{E} = r_E E\left(1 - \frac{E}{K_E}\right) - P\frac{B}{S},$$

- Non-dimensionalized form :

Let $\tau = t\frac{P}{K_S K_E}$, $r_{E_1} = r_E\frac{K_S K_E}{P}$ and $r_{S_1} = r_S\frac{K_S K_E}{P}$

Thus,

$$\frac{d(S_1)}{d\tau} = r_{S_1}(S_1)(1 - \frac{S_1}{E_1})$$

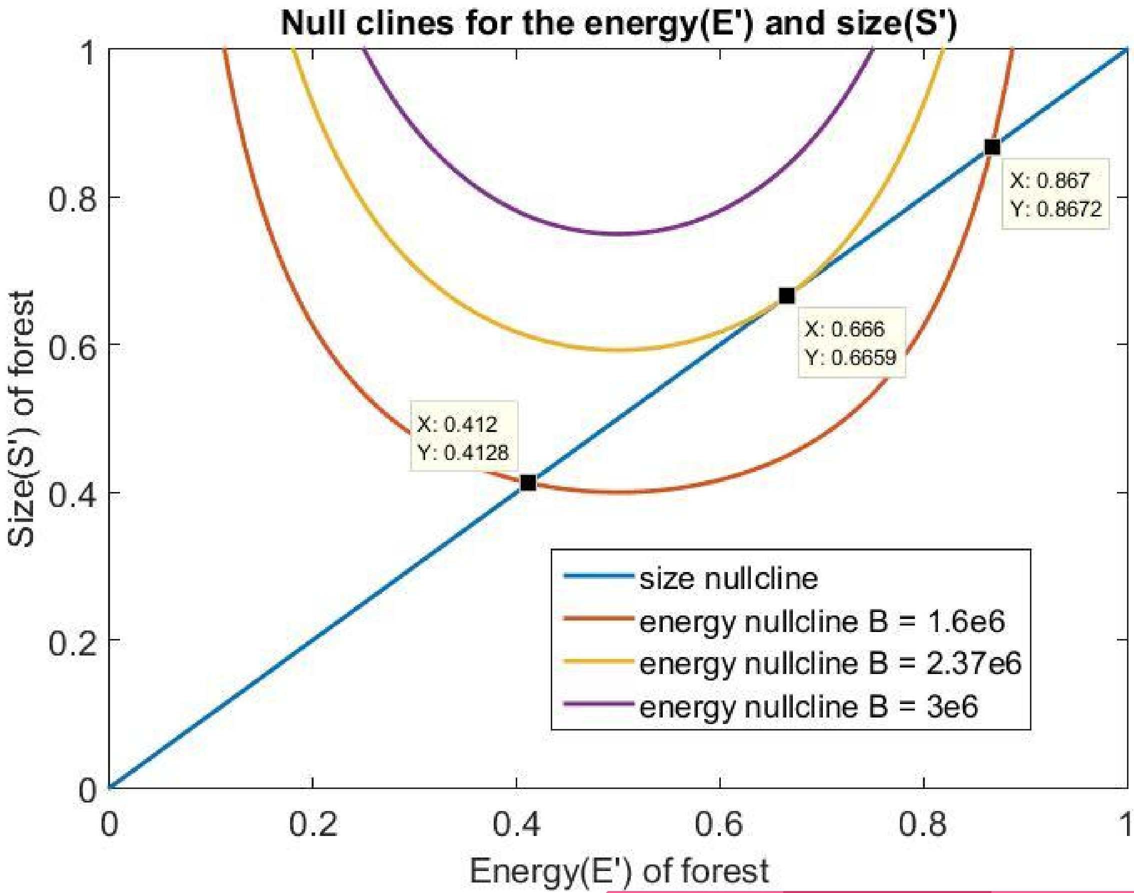
$$\frac{d(E_1)}{d\tau} = r_{E_1}(E_1)(1 - E_1) - \frac{B}{S_1}$$

Null Clines

- Equations:

$$S_1 = E_1$$

$$S_1 = \frac{B}{r_{E_1}(E_1)(1 - E_1)}$$



Analytical calculation of the critical B value

One can calculate the critical B value using two methods-

1. Equating the derivatives and the function values of the null clines.
2. Using the condition for existence of real roots of a cubic equation.

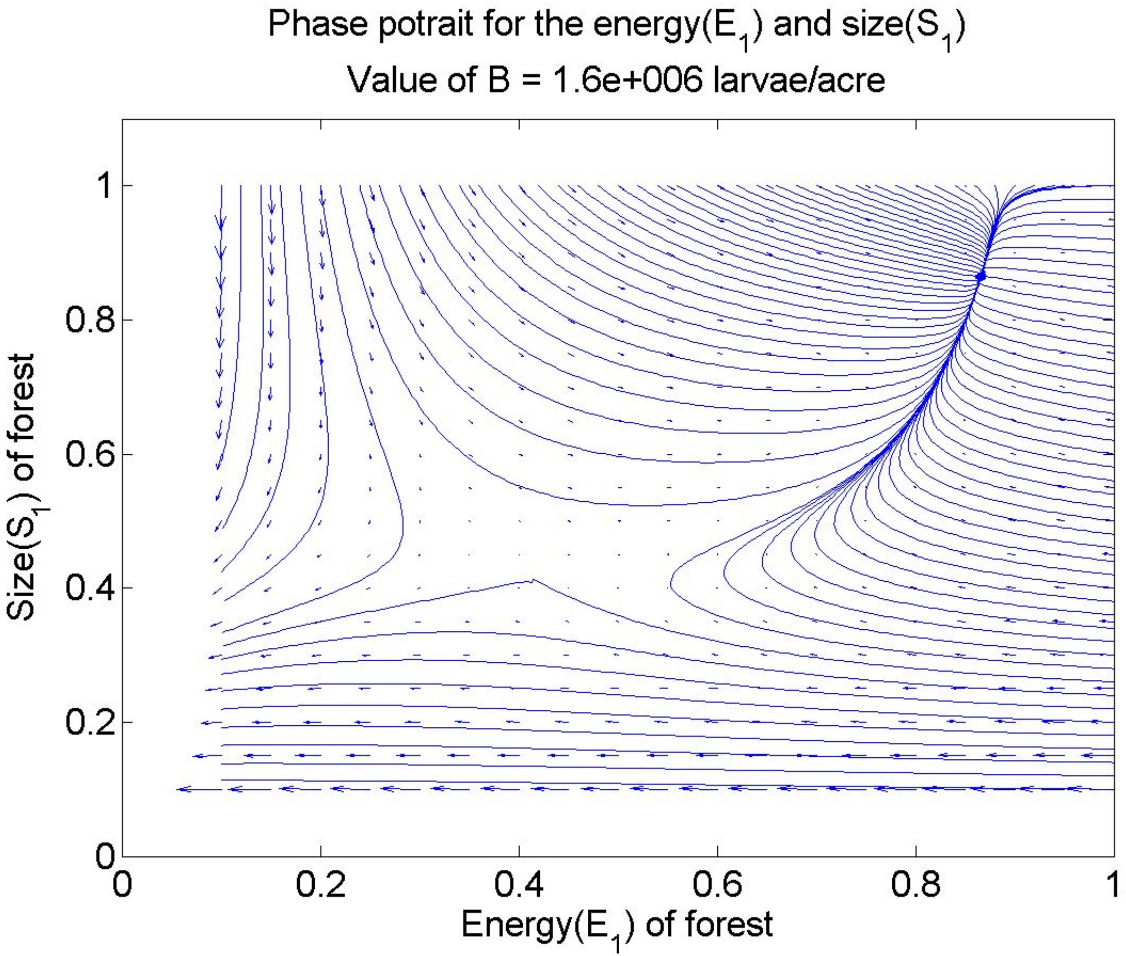
$$F(E_1) = E_1^3 r_{E_1} - E_1^2 r_{E_1} + B = 0$$

$$E_1 = 0 \qquad E_1 = 2/3$$

$$B < \frac{4 r_{E_1}}{27} \qquad \mathrm{B} = 2.37 * 10^6$$

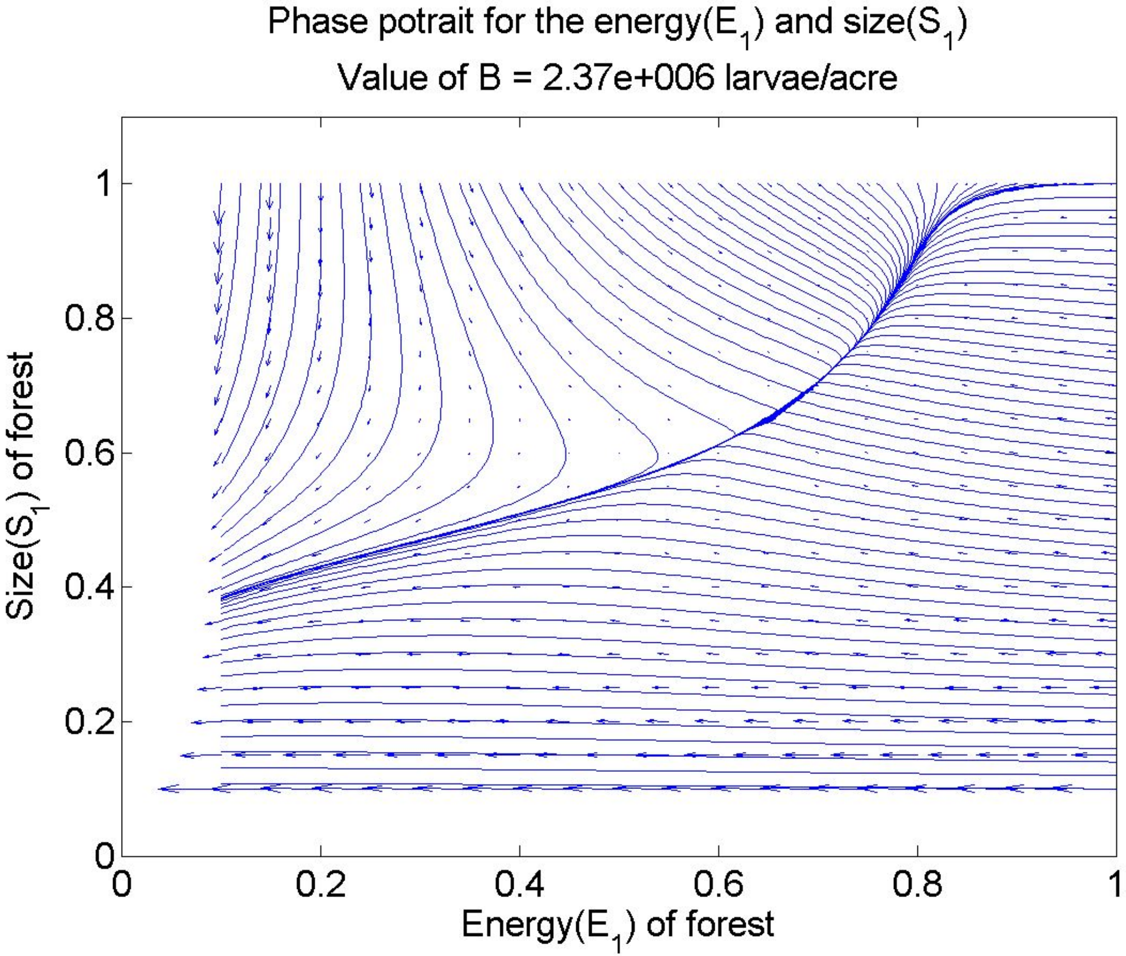
Phase Portrait 1

- $B = 1.6 \times 10^6$
- 2 Fixed Points
- Saddle Node (0.412, 0.412)
- Stable Node (0.867, 0.867)



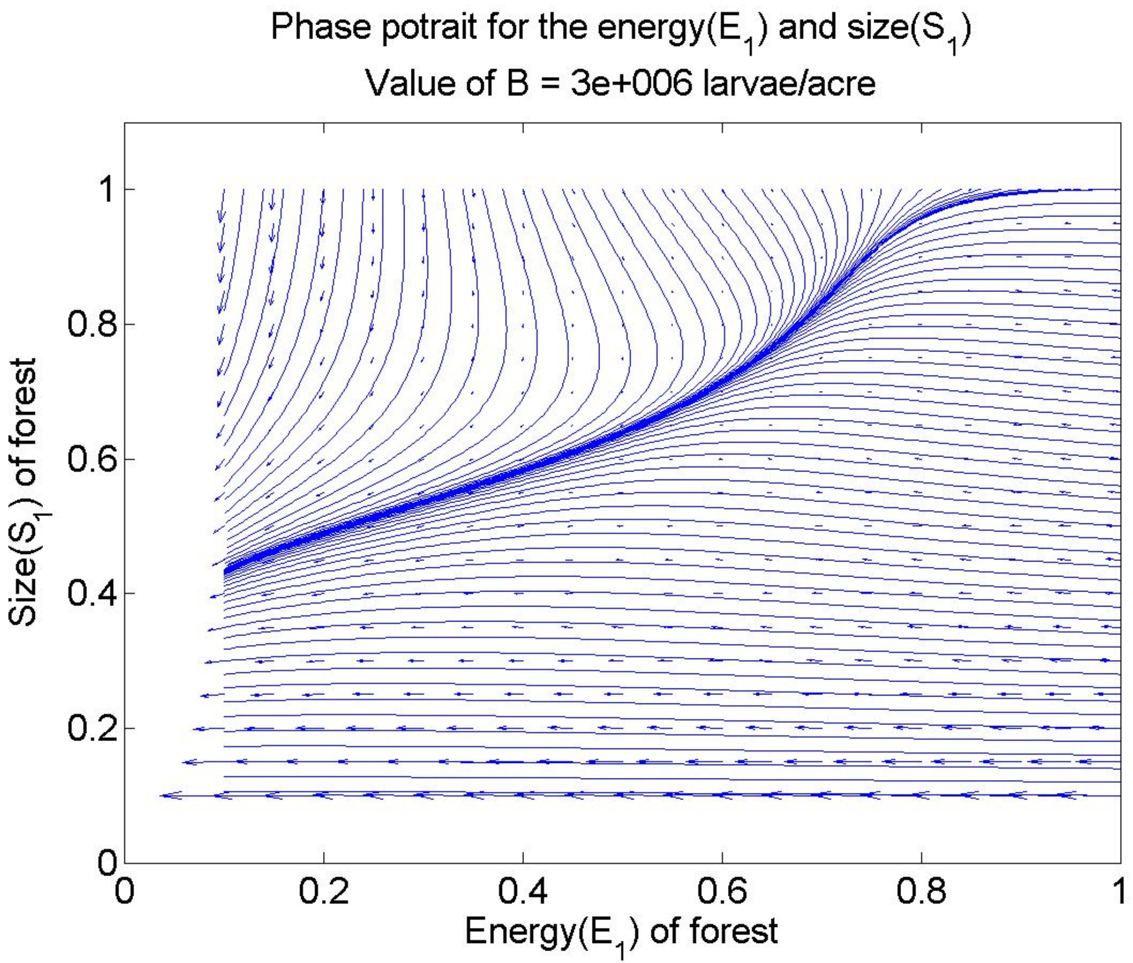
Phase Portrait 2

- $B = 2.37 \times 10^6$
- 1 Fixed Point
- Hybrid Fixed Point (0.67, 0.67)



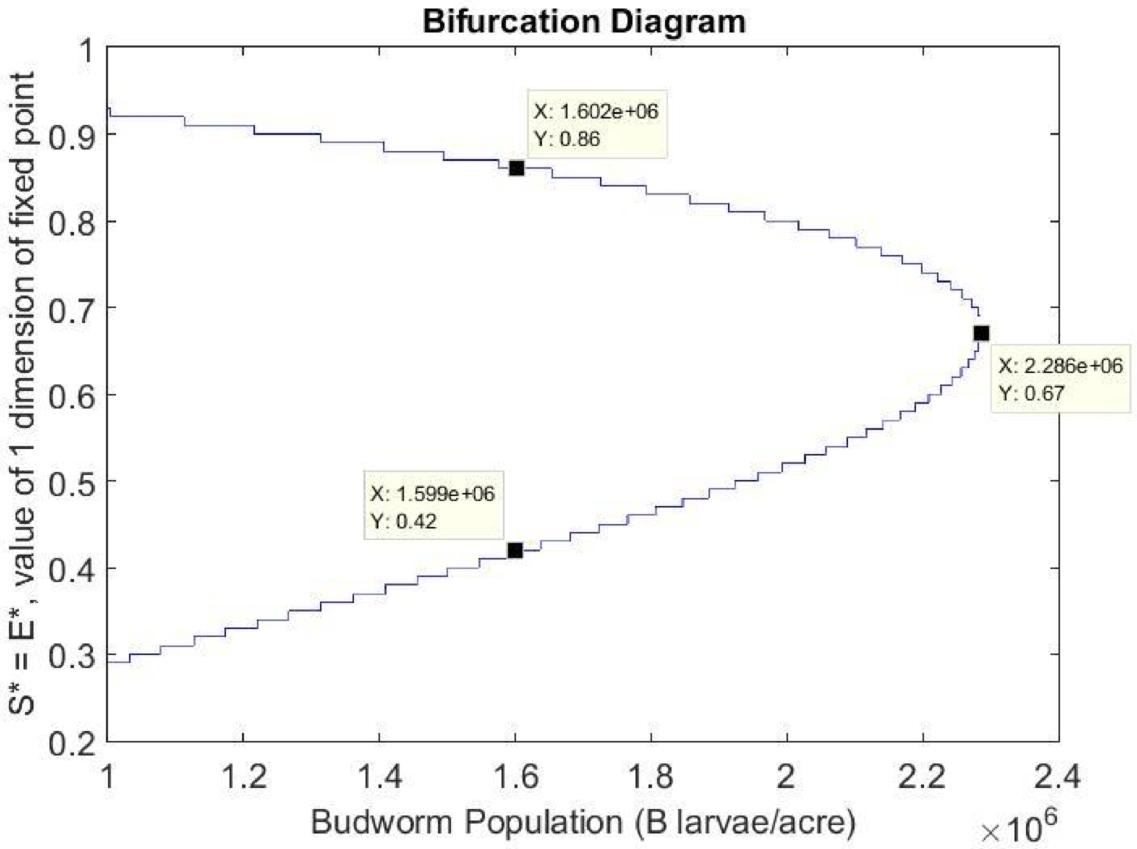
Phase Portrait 3

- $B = 3 \times 10^6$
- No Fixed Point



Bifurcation Diagram

- In 2D
- Significance
- Saddle Node Bifurcation



Results, Key Observations:

1. The system shows a saddle-node bifurcation. If Budworm population(B) is below the critical value two fixed points exist. The forest survives(the variables stabilize to the fixed point) if and only if both Size(S) and Energy(E) are large enough.
2. If Budworm population goes above the critical value, no matter what the variables are, the forest is doomed! We have to keep B below critical value to save the forest.
3. The parameters decide how much a forest can withstand. A weak forest can be snuffed out even by a small number of Budworms in the refuge while a relatively strong forest can even survive a severe outbreak!