Sociological Epidemic Behavior Modeling

Project Seminar Report

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Declaration

We hereby declare that our project report entitled "Sociological Epidemic Behavior Modeling" is our original work, submitted in partial fulfillment of the requirements for "Mathematical Modeling, Simulation, and Optimization". All sources of information have been properly cited and referenced according to academic standards. The findings and conclusions presented are based on our independent research in the field of sociological epidemic behavior modeling.

Date: August 5, 2024

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Abstract

In this project, we study the integration of opinion dynamics within SIR model. Mathematical modeling has become a dependable tool in epidemiology for effectively predicting the spread of epidemics. The SIR model is one of the simplest yet reliable models used in this field. By incorporating opinion dynamics into the SIR model, we aim to understand the impact of individual beliefs and social influence on epidemic. We compartmentalize the population based upon opinions or belief the individual has (e.g. In the case of COVID-19 epidemic, people who have taken vaccine will fall into the compartment having opinion 1 and who have not taken vaccine will fall into the compartment having opinion 2). This model allows us to examine the interactions between different belief groups and their impact on the spread and control of the disease. Through the comparative analysis with the standard SIR model, we analyse how different parameters affect the course and outcome of the epidemic.

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1 Introduction

The Susceptible-Infected-Recovered (SIR) model is a fundamental model in epidemiology, widely used to understand and predict the spread of infectious diseases. It compartmentalizes the population into three distinct groups: susceptible individuals who can contract the disease, infected individuals who are currently carrying and can transmit the disease, and recovered individuals who have overcome the infection and gained immunity. This model provides valuable insights into disease dynamics, helping to estimate the number of people in each category over time.

Diseases typically spread through interactions between individuals. Human interactions and behaviors are not solely driven by biological factors; they are profoundly influenced by beliefs and opinions. These beliefs can include a broad spectrum of topics, including politics, celebrities, sports, and more. For instance, despite the vast variety of food, music, ideologies, and entertainment available globally, our choices are guided by our personal beliefs and opinions. These preferences shape our daily lives, interactions, and even our responses to health crises. The opinions and beliefs are influenced by expert opinions, social media, television programs.

Given the significant role that opinions and beliefs play in shaping social behavior, it is worthwhile to explore their impact within the framework of the SIR model. By integrating the influence of opinions into this model, we can gain a more comprehensive understanding of how diseases spread in populations where social interactions are driven by shared or conflicting beliefs. This extended model can provide deeper insights into the interaction between disease dynamics and social behavior, ultimately leading to more effective strategies for managing public health and influencing social behavior during outbreaks.

2 SIR Model

2.1 Basic SIR Model

During the occurrence of a disease, the individuals in a population are divided into three categories: Susceptible, Infected, and Recovered. Susceptible are uninfected people who get infected when they meet an infected person. Infected are individuals affected with the disease and able to transmit it. Recovered are individuals recovered from disease and are no more susceptible or infected. [9]

$$S \to I \to R$$

S, I, and R represent the number of individuals in each compartment at a particular time and are represented by the following differential equations:

$$\frac{dS}{dt} = -\beta S I, \tag{1}$$

$$\frac{dI}{dt} = \beta S I - \gamma I, \tag{2}$$

$$\frac{dR}{dt} = \gamma I, \tag{3}$$

where:

- $S(t)$ is the number of susceptible individuals at time t,
- $I(t)$ is the number of infected individuals at time t,
- $R(t)$ is the number of recovered individuals at time t,
- β is the transmission rate,
- γ is the recovery rate.

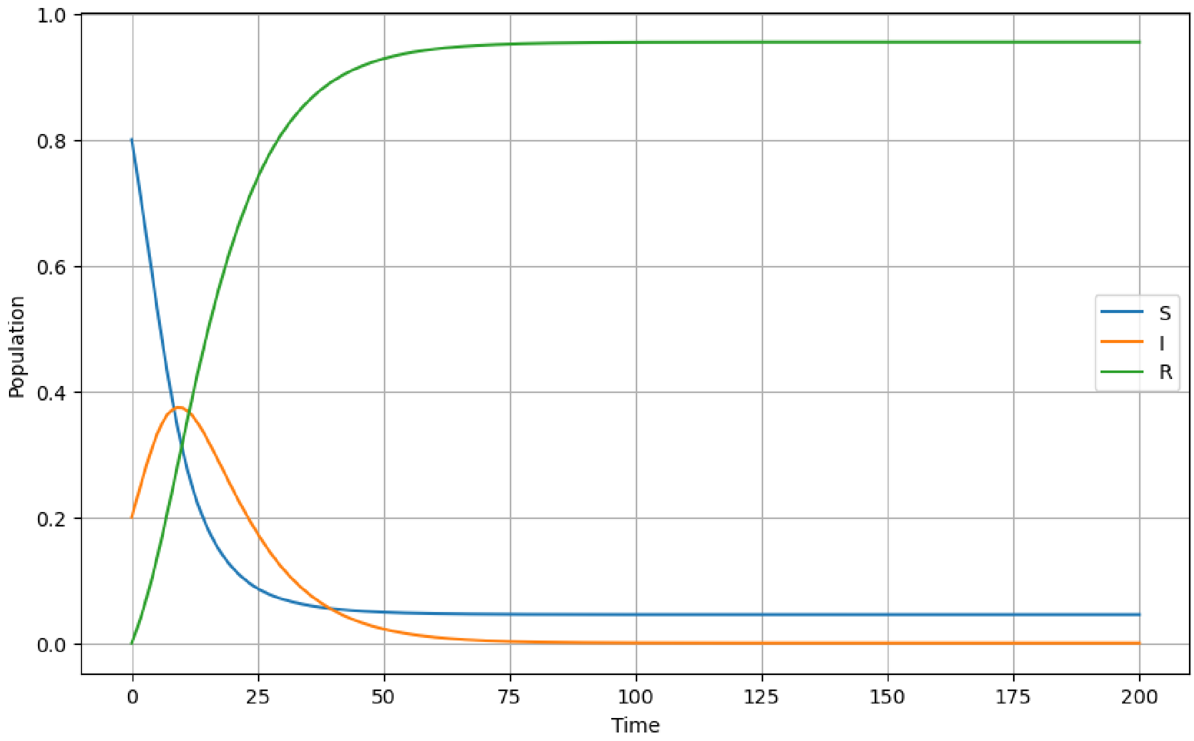

Figure 1: Standard SIR Model

Figure 1 shows the behavior of each group's population over time. The total population(N) is given by:

$$N = S(t) + I(t) + R(t)$$

The population is considered homogeneous and remains static over time, meaning that no new agents are entering or leaving the population.

3 SIR Model With Opinion Dynamics

In the SIR model with opinion Dynamics, The population of susceptible(S), infected(I), and recovered(R) individuals are further divided into two categories based on their opinions: one group with opinion 1 and the other group with opinion 2.

These opinions can be determined by various factors, such as whether individuals are vaccinated or not, or whether they have taken preventive measures or not. Depending on their opinion on these factors, individuals will be classified into either the opinion 1 group or the opinion 2 group.

3.1 Construction of Complex SIR Model

The total population is divided in following compartments: S1, S2, I1, I2, R1, R2. S1 is the compartment of susceptible having opinion 1 and S2 is the compartment of susceptible having opinion 2 likewise for I1, I2 and R1, R2.

The model describes the probability of how individuals or groups change their opinions based on interactions with others. Each group can influence or be influenced by other groups. When an individual with opinion 1 or 2 interacts with another individual with opinion 1 or 2, they may either adopt the other person's opinion or retain their own, leading to changes in the composition of the groups over time.

The probability of potential interactions between population is represented by the following probability trees:

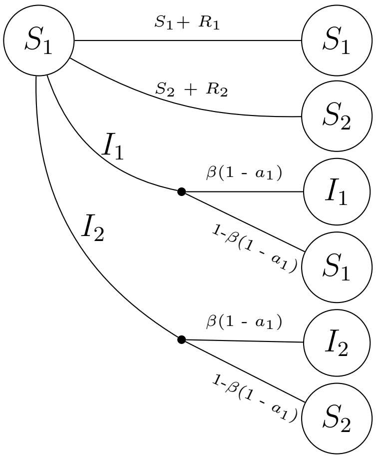

Figure 2: Susceptible with opinion 1

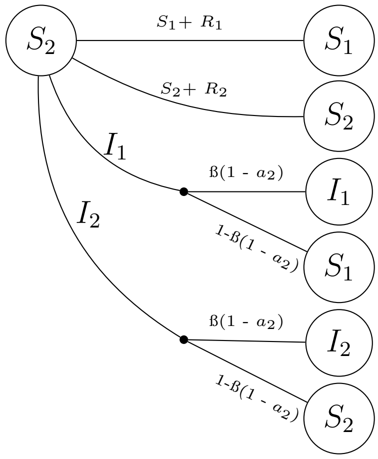

Figure 3: Susceptible with opinion 2

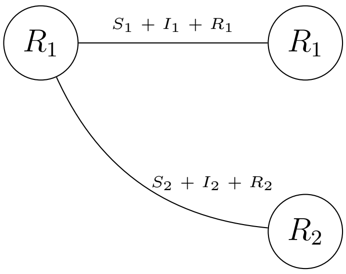

Figure 4: Recovered with opinion 1

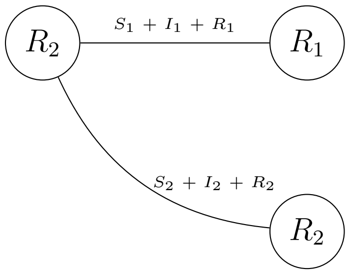

Figure 5: Recovered with opinion 2

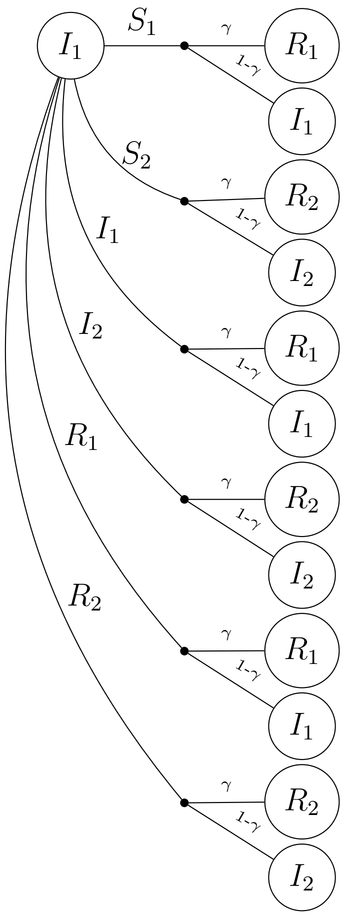

Figure 6: Infected with opinion 1

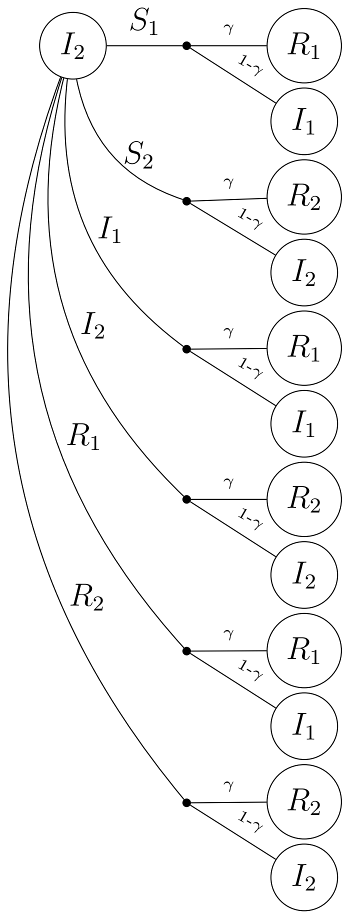

Figure 7: Infected with opinion 2

The figures above shows probability trees, each illustrating a distinct way that two populations can interact. The nodes in the trees depict various groups of people, while the branches indicate the groups with which they interact. The probability of each interaction is labeled on the branches.

3.2 Markov Graph

The probabilities of the interactions is represented by Markov graph in Figure 8. Each node represents a state. Directed edges between nodes represent the possible transitions from one state to another. Each edge is labeled with a the population transition with transition probability, which indicates the likelihood of moving from the source state to the target state.

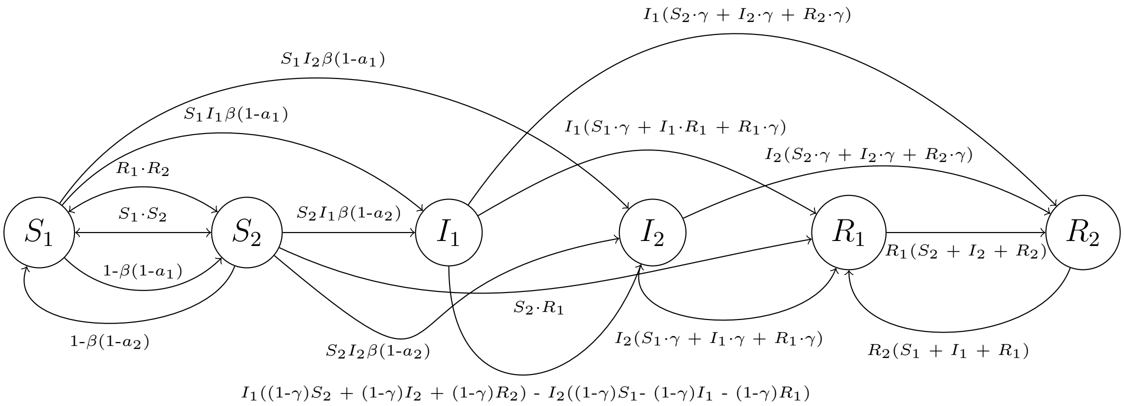

Figure 8: Markov Graph

3.3 Model Assumptions

We assume that When one group interacts with another, it can either adopt the opinion of the other group or retain its original opinion.

Each group is considered homogeneous, meaning all individuals within a group are assumed to have the same probability of changing opinions.

The model operates under a closed system assumption. This means that the total population of all groups combined remains constant. No new individuals enter the system, and no one leaves. Instead, individuals can only switch their opinions from one group to another.

Furthermore, Individuals can move between groups by changing their opinions, but they don't enter or leave the population entirely.

3.4 Mathematical Representation

To understand the construction of the model, We take the case of S1. When S1 interacts with S2, it changes its opinion to S2. When S1 interacts with I1, it either gets infected and changes to I1 or remain S1.

The probability of S1 changing its opinion to I1 is $\beta(1 - a1)$, and the probability of S1 staying the same is $1 - \beta(1 - a1)$. Therefore, Number of individuals leaving S1 after interacting with S1 is $\beta(1 - a1)I1$. Similar formulas can be derived for other interactions. The population changing its opinion to S1 will be added, and the population changing from S1 will be subtracted. This process is similar for I1, I2, R1, and R2.

Additionally, we consider the effort taken by susceptible of each opinions. a_1 and a_2 are values between 0 and 1 representing the effort taken by susceptible individuals with opinion 1 and opinion 2, respectively, to avoid infection. The effective transmission rates will be $\beta \times (1 - a_1)$ for opinion 1 and $\beta \times (1 - a_2)$ for opinion 2. Since efforts are measured on a scale from 0 to 1, multiplying the effort by the transmission probability will result in a lower effective transmission probability.

Considering all of the above, the agents aim to select the least unfavorable expected outcome in the following decision problem[1]:

$$-a_i^2 - \theta_i \cdot \beta \cdot I \cdot (1 - a_i)$$

Where:

- θ_i, $i \in \{1, 2\}$, represents Health Cost
- a_i, $i \in \{1, 2\}$, represents Level of effort

The optimal effort level is therefore:

$$a_i = \min\left\{\frac{\theta_i \beta I}{2}, 1\right\},\ i \in \{1, 2\}$$

We can describe the changes in the populations of each group over time using the following equations:

Equations for S_1 and S_2:

$$\begin{aligned}\frac{dS_1}{dt} &= -S_2S_1 - I_1\beta(1-a_1)S_1 - I_2\beta(1-a_1)S_1 - I_2S_1(1-a_1)(1-\beta) - R_2S_1 \\ &\quad + S_1S_2 + I_1S_2(1-\beta)(1-a_2) + R_1S_2 \end{aligned} \tag{4}$$

$$\begin{aligned}\frac{dS_2}{dt} &= -S_1S_2 - I_1\beta(1-a_2)S_2 - I_1\beta(1-a_2)S_2 - I_2S_2(1-a_2)\beta - R_1S_2 + S_1S_2 \\ &\quad + R_2S_1 + I_2(1-\beta)(1-a_1)S_1 \end{aligned} \tag{5}$$

Equations for I_1 and I_2:

$$\begin{aligned}\frac{dI_1}{dt} &= -S_1\gamma I_1 - S_2\gamma I_1 - S_2(1-\gamma)I_1 - I_1\gamma I_1 - I_1\gamma I_2 - I_1(1-\gamma)I_2 - R_1\gamma I_1 - \\ &\quad R_2\gamma I_1 - R_2(1-\gamma)I_1 + I_1\beta(1-a_1)S_1 + I_1\beta(1-a_2)S_2 + I_1(1-\gamma)I_2 + \\ &\quad I_2(1-\gamma)R_1 + S_1(1-\gamma)I_2 \end{aligned} \tag{6}$$

$$\begin{aligned}\frac{dI_2}{dt} &= -S_1\gamma I_2 - S_2\gamma I_2 - I_1\gamma I_2 - I_1(1-\gamma)I_2 - I_2\gamma I_2 - R_1\gamma I_2 - R_1(1-\gamma)I_2 \\ &\quad - R_2\gamma I_2 - S_1(1-\gamma)I_2 + S_2\gamma I_1 + I_2(1-\gamma)I_1 + I_1(1-\gamma)R_2 + \\ &\quad I_2\beta(1-a_2)S_2 + S_1\beta(1-a_1)I_2 \end{aligned} \tag{7}$$

Equations for R_1 and R_2:

$$\begin{aligned}\frac{dR_1}{dt} &= -S_2R_1 - I_2R_1 + R_2S_1 + R_2I_1 + S_1\gamma I_1 + I_1\gamma R_1 + I_2\gamma R_1 + I_2I_1\gamma \\ &\quad + I_1I_1\gamma + I_2S_1\gamma \end{aligned} \tag{8}$$

$$\begin{aligned}\frac{dR_2}{dt} &= -S_1R_2 + I_2R_1 + R_1S_2 - R_2I_2 + I_1S_2\gamma + I_1I_2\gamma + I_1R_2\gamma + I_2S_2\gamma + \\ &\quad I_2I_2\gamma + I_2R_2\gamma \end{aligned} \tag{9}$$

4 Implementation

These systems of ODEs can be solved using various methods. The graph of each compartments over time is as follows:

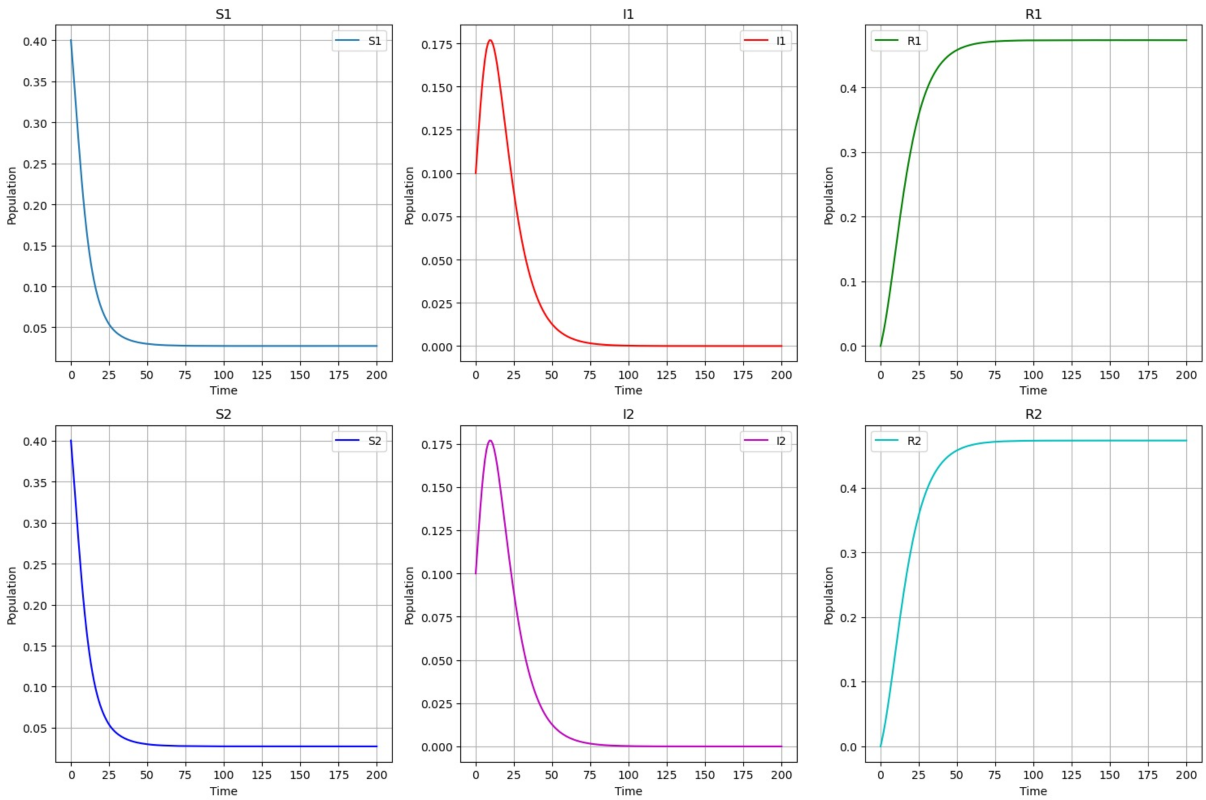

Figure 9: Graph of each compartments

The characteristics of each group's graphs are similar to those of the corresponding groups in the standard SIR model. To compare our model with the standard SIR model, we first derive the aggregated model by summing the susceptible, infected, and recovered groups. The aggregated model is as follows:

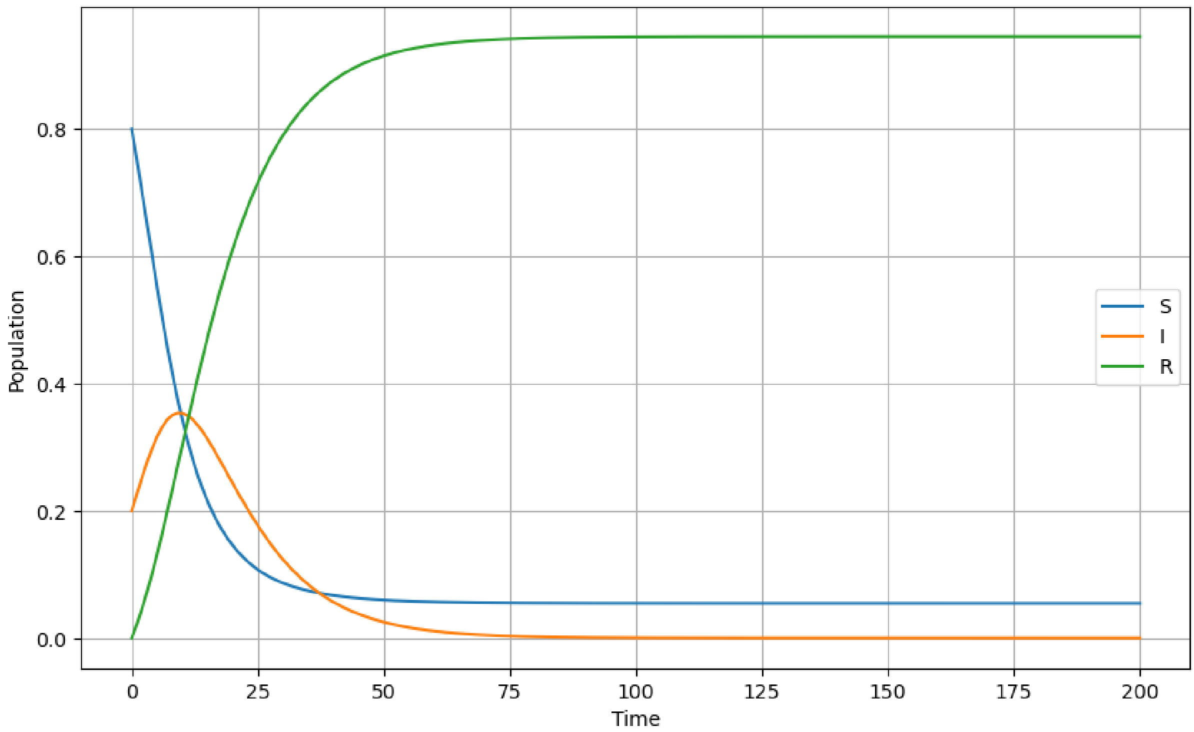

Figure 10: Graph of Aggregated SIR model with Opinion Dynamics

4.1 Fitting Process

To assess the accuracy and relevance of the models, we fit both of them. We define the transmission rate and recovery rate such that the norm distance between the two models is minimized. The compartments of the Standard SIR model are denoted by M_{standard}, and the compartments of the SIR model with opinion dynamics are denoted by M_{extended}. The mathematical representation is defined by:

$$\min \left\| \left(M_{(standard)} - M_{(extended)} \right) \right\|$$

Where the norm is defined by:

$$\sqrt{\sum_i \left(S_{(\text{standard}),i} - S_{(\text{extended}),i} \right)^2 + \sum_i \left(I_{(\text{standard}),i} - I_{(\text{extended}),i} \right)^2 + \sum_i \left(R_{(\text{standard}),i} - R_{(\text{extended}),i} \right)^2}$$

$S_{\text{standard},i}$ and $S_{\text{extended},i}$ represent the i-th components of the vectors S_{standard} and S_{extended}, respectively. Similarly, $I_{\text{standard},i}$ and $I_{\text{extended},i}$ are the i-th components of the vectors I_{standard} and I_{extended}, and $R_{\text{standard},i}$ and $R_{\text{extended},i}$ are the i-th components of the vectors R_{standard} and R_{extended}. The sum is taken over all components of the vectors.

5 Result and Discussion

After performing the fitting process, we achieved a remarkably close match with same transmission and recovery rates.

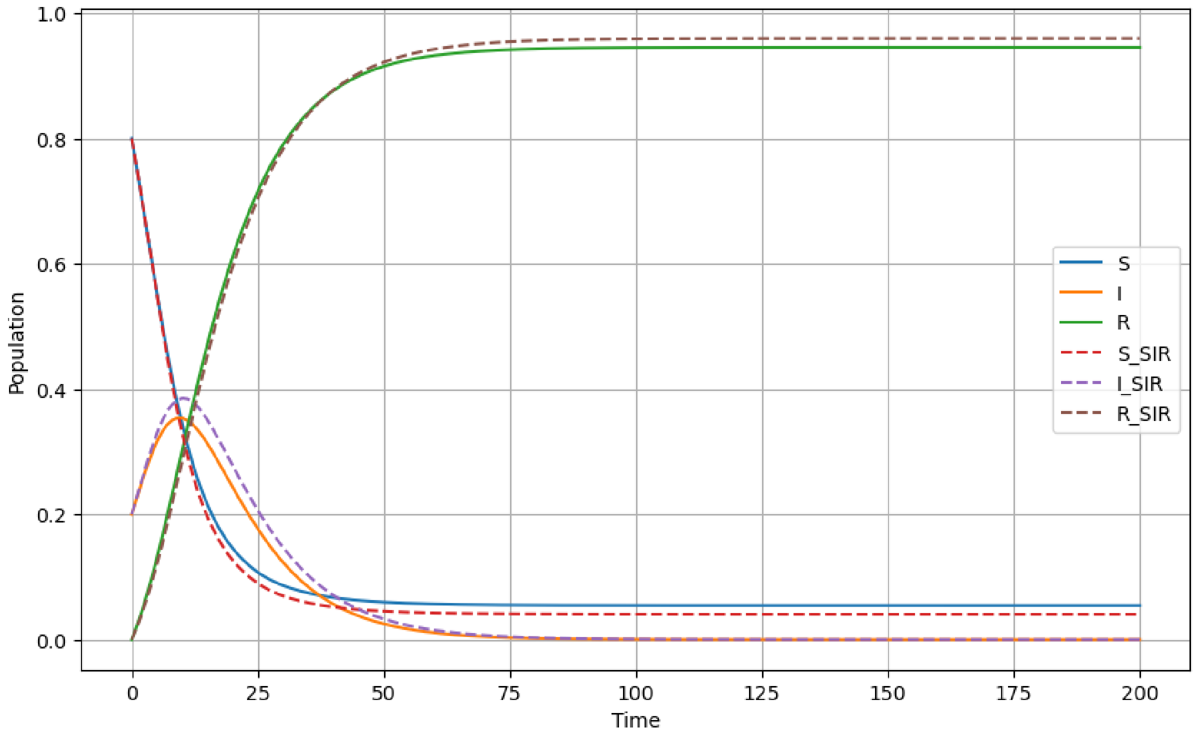

Figure 11: SIR with Opinion Dynamics vs SIR standard

In the Figure 11, the solid line depicts the model incorporating opinion dynamics, while the dashed line represents the standard SIR model. The model with opinion dynamics shows a close fit with minimal error. This suggest that opinion dynamics does not play significant role in predicting the epidemic spread within a fixed population. However, this model predicts a different disease trajectory, potentially resulting in a lower overall impact compared to the standard SIR model. Specifically, the peak number of infected individuals is lower in the opinion dynamics model. Despite this difference, both models exhibit similar overall patterns.

This similarity arises because both models are based on similar foundational principle and methods, with population dynamics influenced by factors such as transmission and recovery rates. Consequently, they both describe how populations evolve over time using analogous frameworks.

6 Conclusion

From the results, we can conclude that both models fit well with the data they are designed to predict. However, the model with opinion dynamics did not significantly improve the ability to predict how an epidemic would spread. Even though the opinion dynamics model might not be better at predicting epidemics, it can still be useful in controlling them. By understanding how opinions spread, authorities can design interventions and strategies that encourage people to take effective preventive actions.

For instance, public health campaigns can be modeled to see how they influence opinion change and compliance with health guidelines to influence public behavior and slow down the spread of a disease. This approach is more suitable for prevention and control, which are crucial in managing public health crises.

7 Further Things to do

To gain deeper insights into this model, it is possible to adjust various parameters to observe their impact. Specifically, the health cost parameters θ_1 and θ_2, as well as the effort cost parameters a_1, a_2 can be fine-tuned to evaluate their significant effects on the model's behavior.

To improve the model's precision and use, it's essential to gather real-world data. This data should cover how individuals behave, how their health opinions change, and the infection rates within various groups.

The model currently treats the population as uniform. Adding network interactions can help understand how disease and opinions spread in structured groups like communities with strong social ties or isolated groups.

Testing different public health measures like tailored messaging, vaccination campaigns, and specific social distancing rules to see how they affect people's opinions and the spread of infections.

Extending the model to include more than two opinions or continuous opinion spectra to reflect a broader range of beliefs and attitudes towards disease prevention and health behaviors.

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