Meow

meow meow

A thesis presented
by
Rushil Mallarapu
to
The Department of Statistics

in partial fulfillment of the requirements
for the degree of
Bachelor of Arts (Honors)
in the subject of
Statistics

Harvard University
Cambridge, Massachusetts
April 2025

Meow

Abstract

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Content

1 Introduction

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1.1 Hello

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In Section 1

2
Beginnings

In Section 2 we prove Equation 1.

$$a^2 + b^2 = c^2 \tag{1}$$

Hello [1].

2.1 Meow

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3
Chapter 1: Introduction

Overview of number partitioning problem.

Application: randomized control trials.

Other applications.

- Circuit design, etc.

Importance as a basic NP-complete problem.

Two questions of interest:

1. What is optimal solution.
2. How to find optimal solution.

3.1 Physical Interpretations

3.2 Statistical-to-Computational Gap

A
Hello

B
Meow meow

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