

The multi-degree of freedom system shown at right is made of masses and linear springs. Write the equations of motion for the two different systems in the form

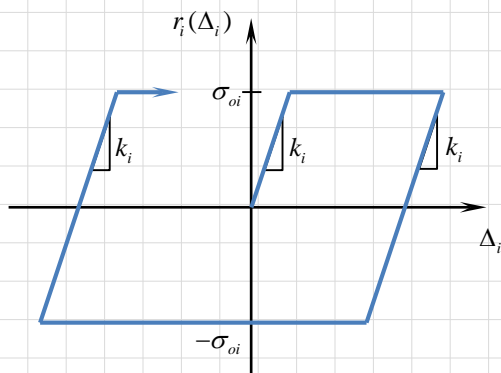
$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{r}(\mathbf{u}(t)) = \mathbf{f}(t)$$

$$\mathbf{u}(0) = \mathbf{u}_o$$

$$\dot{\mathbf{u}}(0) = \mathbf{v}_o$$

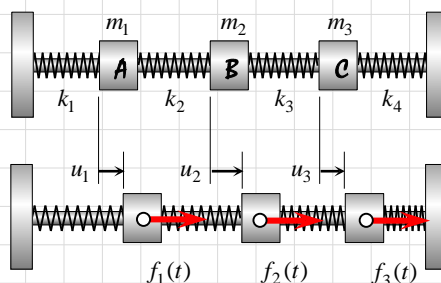
where  $\mathbf{M}$  is the mass matrix,  $\mathbf{r}(\mathbf{u})$  is the restoring force,  $\mathbf{f}$  is the external force vector,  $\mathbf{u}_o$  is the initial displacement vector and  $\mathbf{v}_o$  is the initial velocity vector. Generalize the expressions for the case of  $n$  degrees of freedom. Implement the *elasto-plastic* model for each spring element in your MATLAB code from HW8 (which uses Newmark's method for numerical integration).

Note that the “strain” in each element can be computed from the difference in displacements at its two associated ends. The force in the element, then, is consistent with that strain and the elasto-plastic response model.

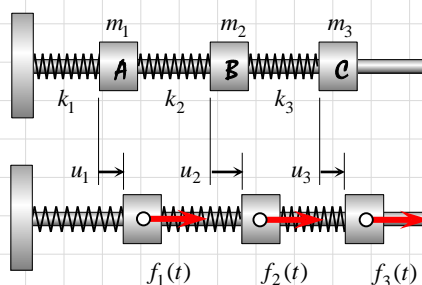


Note that the internal elasto-plastic constitutive models do not interact with each other (i.e., the force in the  $i$ th element depend only on the stretch in the  $i$ th element).

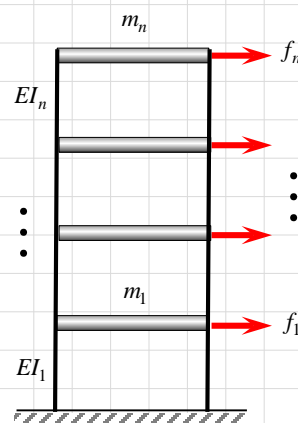
Explore the effects of yielding on the phenomenon of resonance.



System 1: “Bridge”



System 2: “Building”



System 3: “Shear Building”