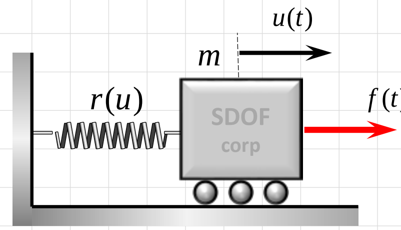
The single degree of freedom system shown at right has a nonlinear restoring force $r(u)$ and it subjected to an applied force $f(t)$. The equations of motion (and initial conditions) are



Un-damped System

$$
\begin{aligned}
m\ddot{u}(t) + r(u(t)) &= f(t) \\
u(0) &= u_o \\
\dot{u}(0) &= v_0
\end{aligned}
$$

Consider the two nonlinear elastic resisting force models as follows:

$$r(u) = ku\left(1+bu^2\right) \qquad \text{Nonlinear stiffening model}$$

$$r(u) = \frac{ku}{\sqrt{1+bu^2}} \qquad \text{Nonlinear softening model}$$

Consider also the specific loading function

$$f(t) = F_1 + F_2 \sin \Omega t$$

where F_1 and F_2, and Ω are constants that describe the forcing function (i.e., a constant part and a sinusoidally varying part.

Newmark's method satisfies the equation of motion at the discrete time points and approximates the velocity and displacement with approximations to integrals. Hence, the discrete equations are

$$
\begin{aligned}
ma_{n+1} + r(u_{n+1}) &= f(t_{n+1}) \\
v_{n+1} &= c_n + h(1-\gamma)\,a_{n+1} \qquad & c_n &= v_n + h\gamma a_n \\
u_{n+1} &= b_n + h^2(\tfrac{1}{2}-\beta)\,a_{n+1} \qquad & b_n &= u_n + hv_n + h^2\beta a_n
\end{aligned}
$$

Implement Newmark's method, including a Newton loop to solve the nonlinear equations of motion for both of the nonlinear elastic models. Explore the response of the system and the influence of nonlinearity relative to the linear elastic response.