Homework 8

The multi-degree of freedom system shown at right is made of masses and linear springs. Write the equations of motion for the two different systems in the form

$$\mathbf{M}\ddot{\mathbf{u}}(t)+\mathbf{C}\dot{\mathbf{u}}(t)+\mathbf{K}\mathbf{u}(t)=\mathbf{f}(t)$$
$$\mathbf{u}(0)=\mathbf{u}_o$$
$$\dot{\mathbf{u}}(0)=\mathbf{v}_o$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is the damping matrix, $\mathbf{K}$ is the stiffness matrix, $\mathbf{f}$ is the force vector, $\mathbf{u}_o$ is the initial displacement vector and $\mathbf{v}_o$ is the initial velocity vector. Generalize the expressions for the case of n degrees of freedom. Implement the computation of the matrices in a MATLAB code based upon the code developed in HW 7 (using Newmark's method to integrate the equations of motion).

Use an orthogonal damping matrix

$$\mathbf{C}=\mathbf{M}\boldsymbol{\Phi}\mathbb{C}\boldsymbol{\Phi}^T\mathbf{M}$$

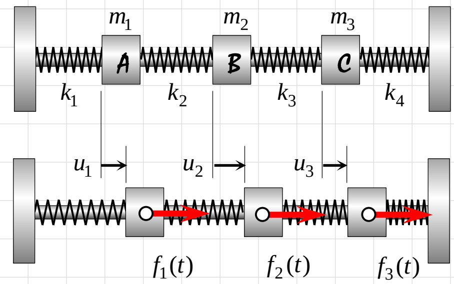
where

$$\mathbb{C}=\begin{bmatrix} 2\xi_1\omega_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & 2\xi_n\omega_n \end{bmatrix}$$

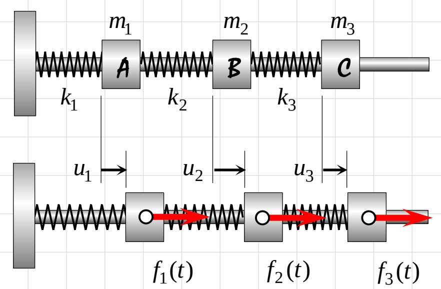
is the diagonal modal damping matrix and $\boldsymbol{\Phi}$ is the matrix whose columns are the eigenvectors from the eigenvalue problem

$$\mathbf{K}\boldsymbol{\varphi}=\omega^2\,\mathbf{M}\boldsymbol{\varphi}$$

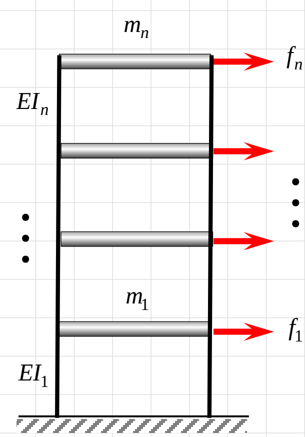
Explore the effects of damping. What are the ramifications of specifying zero damping in the higher modes of vibration?



System 1: "Bridge"



System 2: "Building"



System 3: "Shear Building"