# Finite Elements for Engineers

## Lecture 1: Welcome to the Course

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# Course Organization

- Three Modules (approx. 5 weeks each)
- Module Structure
  - Each module has several topics
  - Each topic has several lectures
- Several quizzes
- Final exam is comprehensive
- Few assignments and 2 Projects



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# Text Book

- Electronic Notes – This is what we will follow very closely.
- Reference books available in the library (see electronic notes for details)



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# Course Web Site

- Blackboard
- Announcements
- Supplemental Material
  - Electronic Notes (pdf format)
  - Sample Computer Programs
  - Slide Presentation
- Assignments
- Discussion Board (must participate)



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# Pre-Requisites

- Undergraduate Exposure to
  - Linear Algebra
  - Numerical Analysis
  - **Computer Programming**
  - Structural Analysis/Heat Transfer/Fluid Mechanics etc.



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# Module 1 (5 weeks)

- Introduction
- Direct Stiffness Method. Matrix Algebra and Programming.
- Method of Weighted Residuals
- One-dimensional Boundary Value Problems
- **1D-BVP** Computer Program



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# Module 2 (6 weeks)

- Variational Principles
- Theorem of Minimum Potential Energy
- Isoparametric Formulation
- Boundary Value Problems
- Programming Project
  - Option 1 (Solid Mechanics)
  - Option 2 (BVP or …)



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# Module 3 (4 weeks)

- Three-Dimensional Solid Mechanics and BVP Problems
- Modal Analysis
- Time-dependent Problems
  - Structural Dynamics
  - Diffusion Problems



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# Assignment 1

- Send me an e-mail stating your name, advisor's name, department, your research area and reasons why you are taking this course.



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# What is Finite Element Method?

- Numerical Method
- (Usually) Approximate Solution
- Solves algebraic, differential and integral equations
- Finally we solve a set of algebraic equations or an eigenproblem



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# FE History

- 1943: Courant, “Variational Methods for the Solution of Problems of Equilibrium and Vibrations”, *Bull Am Math Soc*.
  - Solution of torsion problem via piecewise linear polynomials over a triangular region
- Several developments in the Math and Physics communities



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# FE History

- Hrenikoff (1941) proposed the concept of discretization using rods and beams for a continuous plate
- 1950s: Langefors and Argyris
- 1956: Turner, Clough, Martin and Topp advanced a framework for modeling and analyzing aircraft panels
- 1960: Clough was the first to use the term “finite elements”
- 1960-present: Developments have mirrored advancements in computing hardware and software



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# Flavors of FEM (Step 2)

- Direct Stiffness Method
- Variational Technique
- Weighted Residuals Approach



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# Major Types of FE Problems

- Time-independent problems
- Time-dependent problems
- Linear problems
- Nonlinear problems
- Eigenproblems



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# Finite Element Terminology

- Elements
- Nodes
- Element and Material Properties
- Loads
- Boundary Conditions
- Primary and Secondary Unknowns



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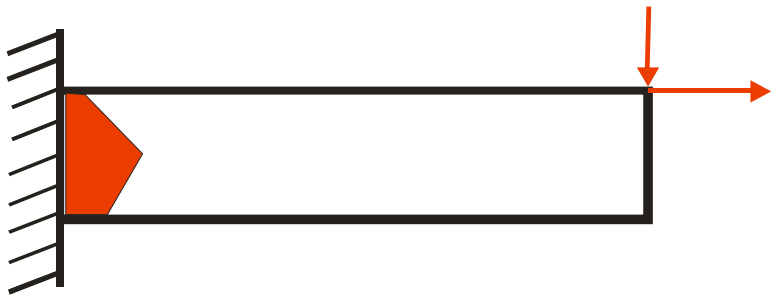
# Example

- Steel cantilever beam (10” x 2” x 0.1”)
- Loading
  - Mechanical: Concentrated load at tip of beam
  - Thermal: Temperature change
- Objective: Compute displacements, strain and stress distribution





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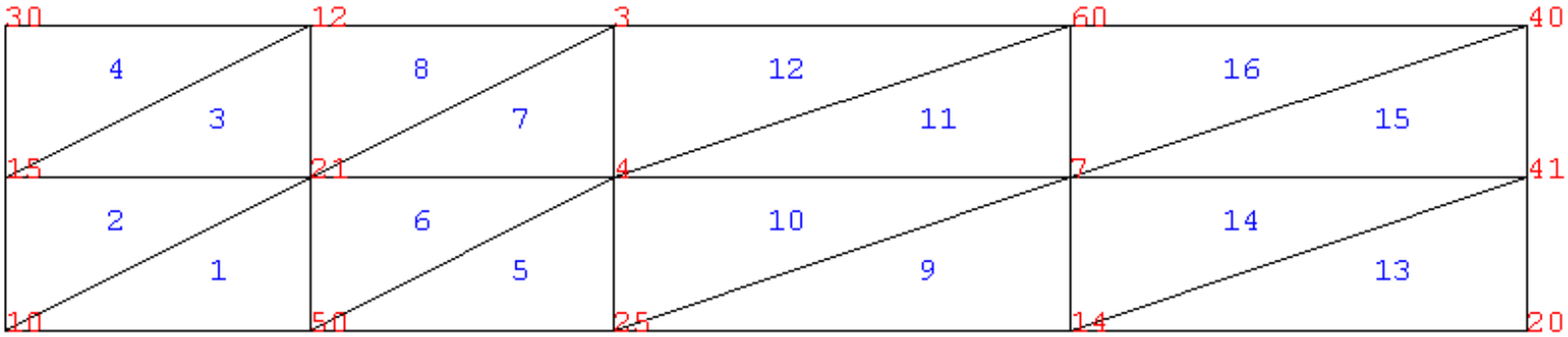
# FE Model Details: Nodes and Elements



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# FE Model Details: Loads




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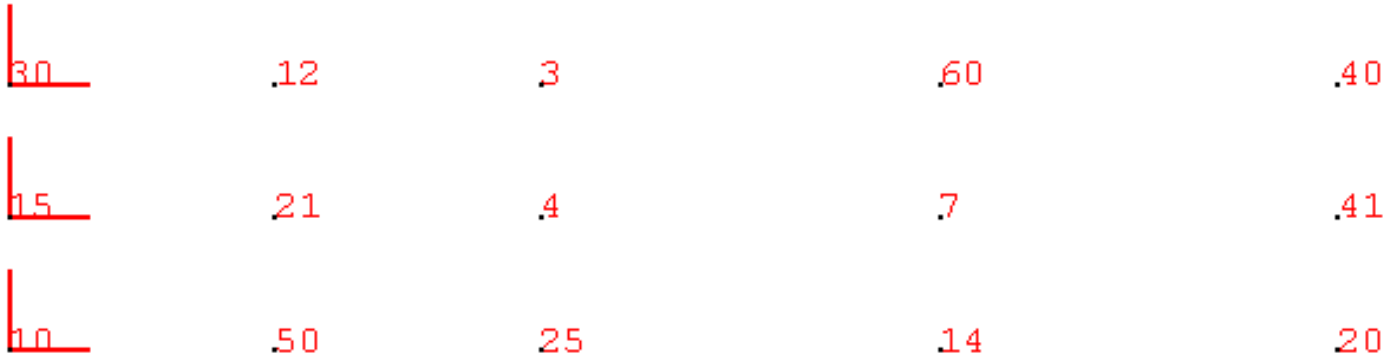
# FE Model Details: Nodal Boundary Conditions



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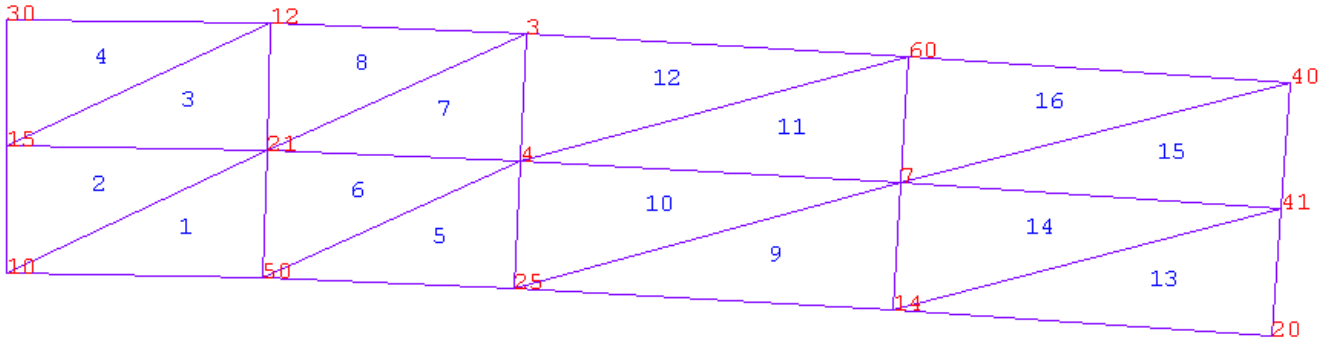
# FE Results: Deformed Shape




POST3D V 1.716
SOLID MECHANICS

Deformed Plot: STEP-1
Magnification: 62.6594
XD Min: 0
XD Max: 0.00235158
YD Min: -0.00797964
YD Max: 0

Model Limits
X Min:0
X Max:10
Y Min:0
Y Max:2
Z Min:0
Z Max:0

Project: T3-TEST
08/03/04 06:13 AM



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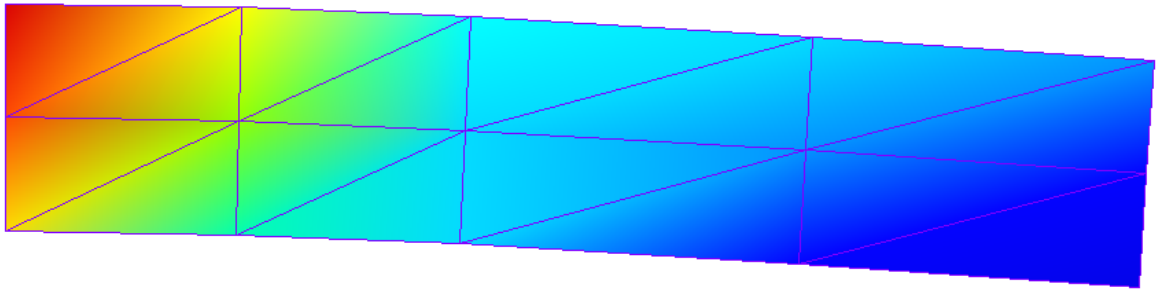
# FE Results: Stress Distribution



POST3D V 1.716
SOLID MECHANICS

Stress Plot : Mises
Equal Interval Distribution

| | |
|---|---|
| 0 | : 957.777 |
| 957.777 | : 1915.55 |
| 1915.55 | : 2873.33 |
| 2873.33 | : 3831.11 |
| 3831.11 | : 4788.88 |
| 4788.88 | : 5746.66 |
| 5746.66 | : 6704.44 |
| 6704.44 | : 7662.21 |
| 7662.21 | : 8619.99 |
| 8619.99 | : 9577.77 |
| 9577.77 | : 10535.5 |
| 10535.5 | : 11493.3 |
| 11493.3 | : 12451.1 |
| 12451.1 | : 13408.9 |
| 13408.9 | : 14366.6 |
| 14366.6 | : 15339.8 |

Model Limits
X Min:0
X Max:10
Y Min:0
Y Max:2
Z Min:0
Z Max:0

Project: T3-TEST
08/03/04 06:15 AM



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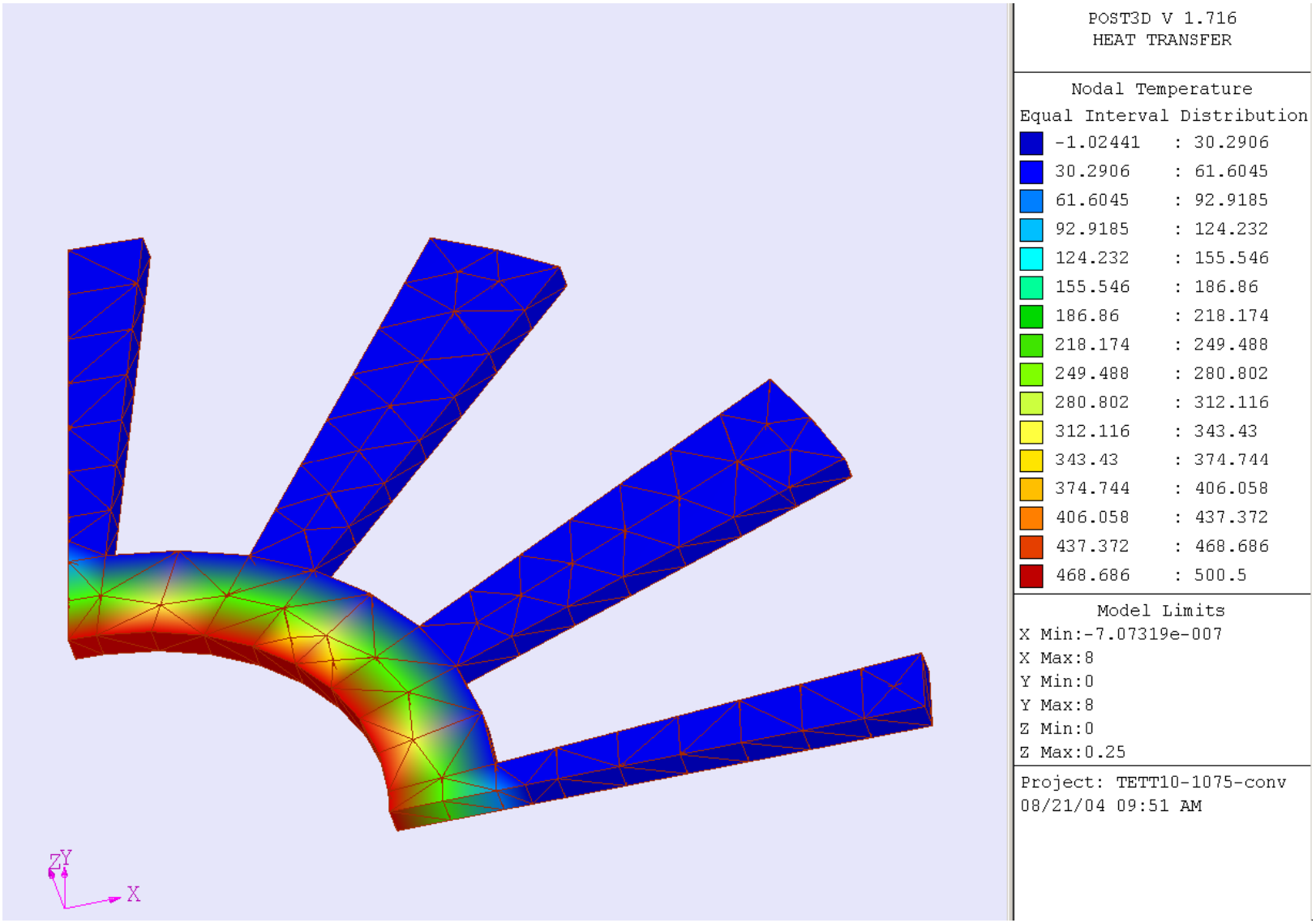
# Example: Heat Transfer Analysis



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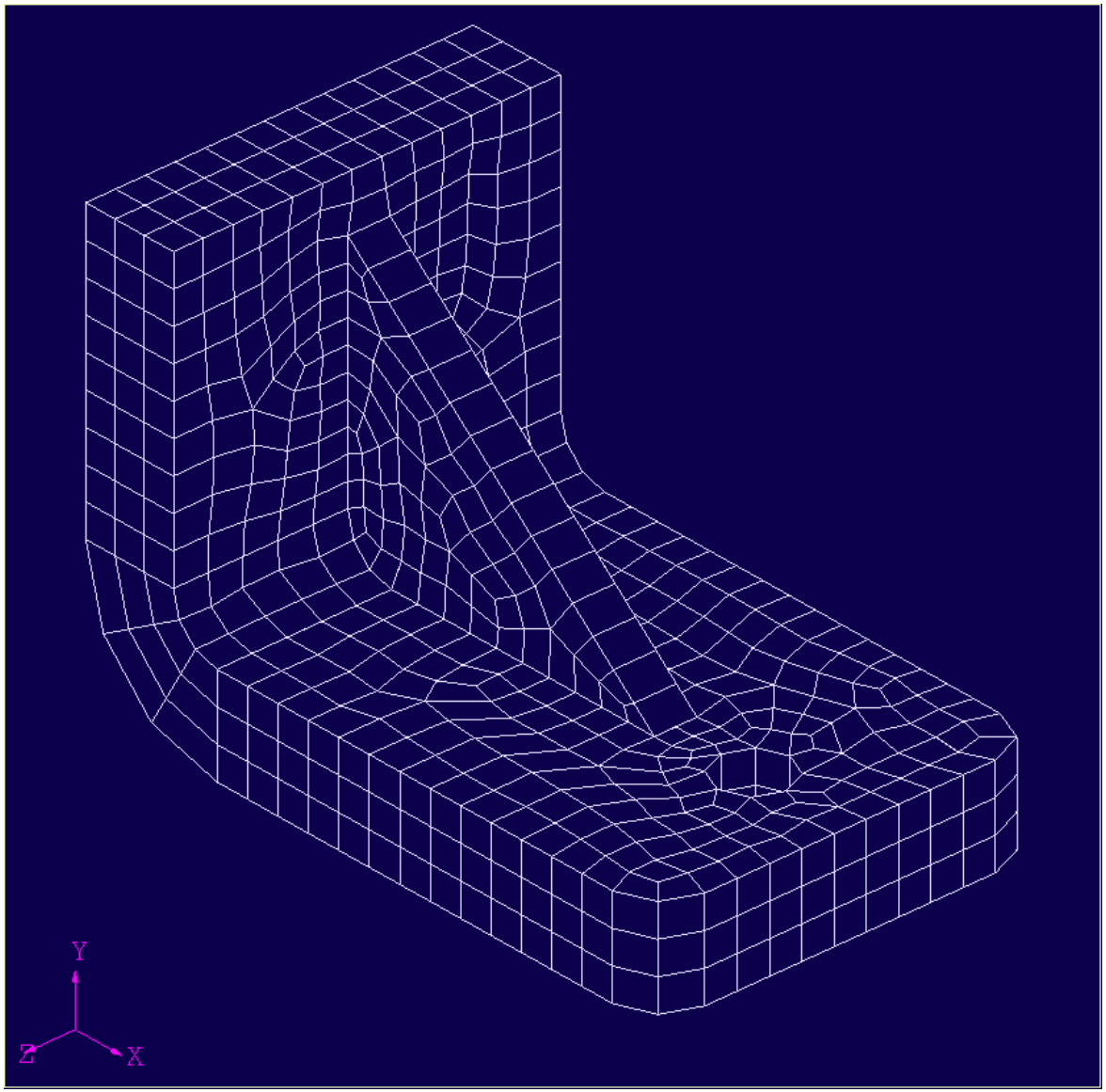
# Example: Modal Analysis



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# Finite Element Applications

- Aerospace
- Automotive
- Biomedical
- Civil Structures
- Electromagnetics
- Mechanical
- Semiconductor



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# Commercial Programs

- Learn to use one of these programs
  - ABAQUS
  - ANSYS
  - CATIA
  - LS-DYNA
  - NASTRAN (and PATRAN)
  - Unigraphics



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# Other Numerical Approaches

- Finite Difference
- Boundary Elements or Integrals
- Meshless Methods
- p-Finite Elements



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# Computer Programming

- Language
  - C++ (preferred)
  - FORTRAN
  - Matlab (?)
- OS
  - Windows (preferred)
  - Linux
- Microsoft Visual Studio



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# Helpful Tools

- Symbolic “solver” such as Maple, Mathematica, Mathcad
- Spreadsheet and graphing programs such as Microsoft Excel
- Word processing program to create reports such as Microsoft Word



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# Illustrative Example

- Estimate the area of a circle of radius R
- Assume that we
  - Do not know the formula (exact answer)
  - Do know rudimentary geometry and trigonometry



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# Solution Steps

Step 1: Discretization

Step 2: Properties of a simple triangle

Step 3: Estimate of the area of the circle

Step 4: Numerical solution



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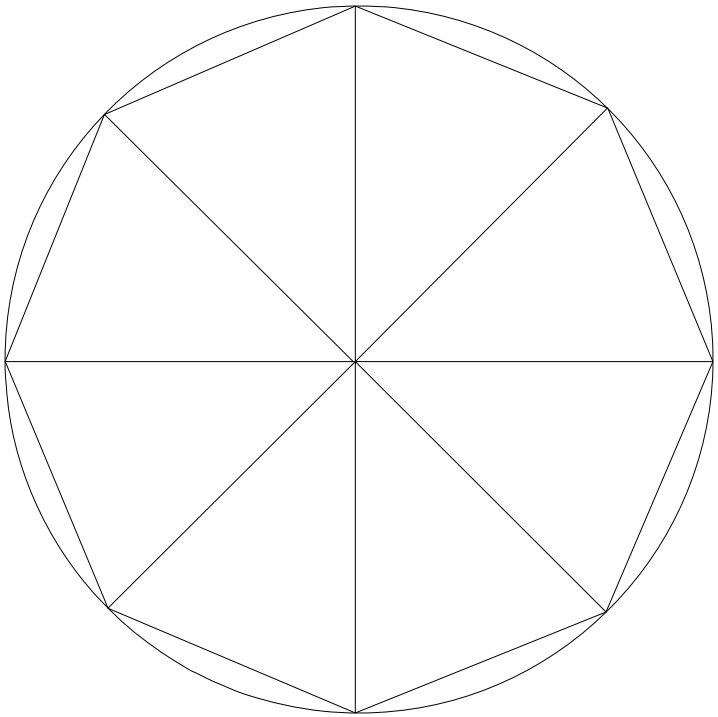
# Mesh A



Mesh A

$$a_e = \frac{R^2}{2}\sin\left(\frac{2\pi}{n}\right)$$

$$A_A^{(n)} = \frac{nR^2}{2}\sin\left(\frac{2\pi}{n}\right)$$



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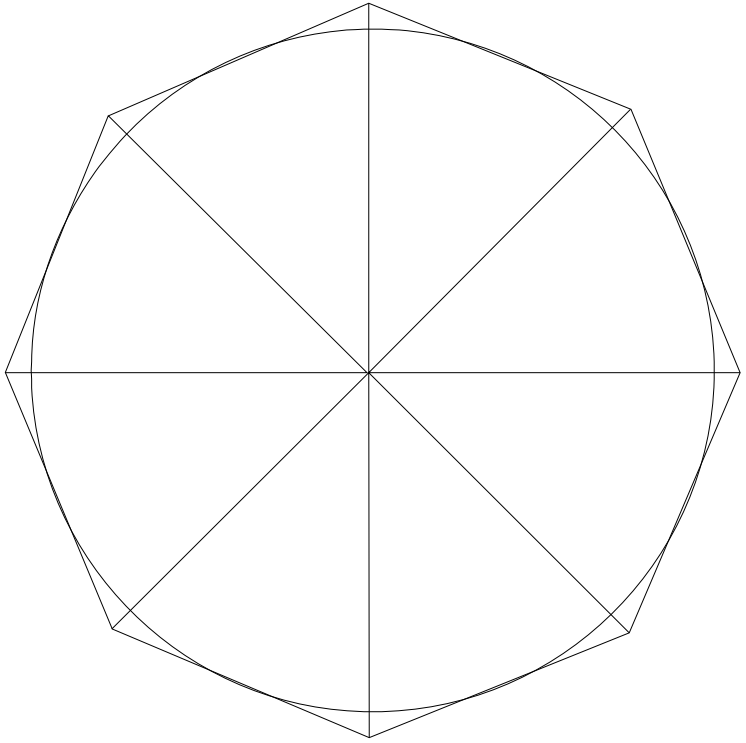
# Mesh B



Mesh B

$$A_B^{(n)} = nR^2 \tan\left(\frac{\pi}{n}\right)$$



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# Assignment 2: Computer Program

- **Problem T2L1-1 (Modified)**
- See web site for details



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# Summary

- What is Finite Element Method?
- When can it be used?
- Basic terminology – elements, nodes etc.
- Competing techniques



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# Further Reading

- There are several excellent Finite Elements texts (buy them, or check them out of the library)
- There are also excellent references on the web



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