Finite Elements For Engineers

Lecture 7A: Problem Solving Using the 1DBVP Program

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1DBVP Program Terminology

- Positive coordinate system points to the right
- FE mesh contains one or more segments
- A segment has the same properties but may contain one or more elements
- The leftmost segment (or the first segment) has the first node and the first element (or node 1 and element 1)
- You create the segment data; 1DBVP creates the nodes, elements and loads

Solid Mechanics

DE

$$-\frac{d}{dx}\left(\alpha(x)\frac{dy(x)}{dx}\right)+\beta(x)y(x)= f(x)$$

Left Mixed BC $\tau = c_a y + d_a$

Right Mixed BC $\tau = c_b y + d_b$

DE

$$-\frac{d}{dx}\left(A(x)E(x)\frac{du(x)}{dx}\right)= w(x)A(x)$$

$$\tau = F = -A(x)E(x)\frac{du(x)}{dx}$$

Solid Mechanics

$$\alpha(x) = A(x)E(x) \equiv F$$

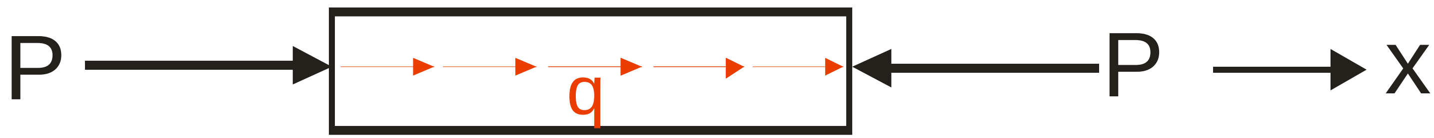
$$f(x) = w(x)A(x) \equiv F/L$$

$$\beta(x) = 0$$

$$c_a = c_b = 0$$

$$d_a = d_b \equiv F$$

Examples



$$\beta(x) = 0$$

$$\alpha(x) = A(x)E(x) \equiv F$$

$$f(x) = q \equiv F/L$$

$$\frac{\overline{AE}}{L}\left[\begin{array}{c|c} 1 & -1 \\ \hline -1 & 1 \end{array}\right]\left\{\begin{array}{c} u_1 \\ \hline u_2 \end{array}\right\} = \left\{\begin{array}{c} P \\ \hline -P \end{array}\right\} + \left\{\begin{array}{c} qL/2 \\ \hline qL/2 \end{array}\right\}$$

Heat Transfer

DE

$$-\frac{d}{dx}\left(\alpha(x)\frac{dy(x)}{dx}\right)+\beta(x)y(x)=f(x)$$

Left Mixed BC $\tau = c_a y + d_a$ Right Mixed BC $\tau = c_b y + d_b$

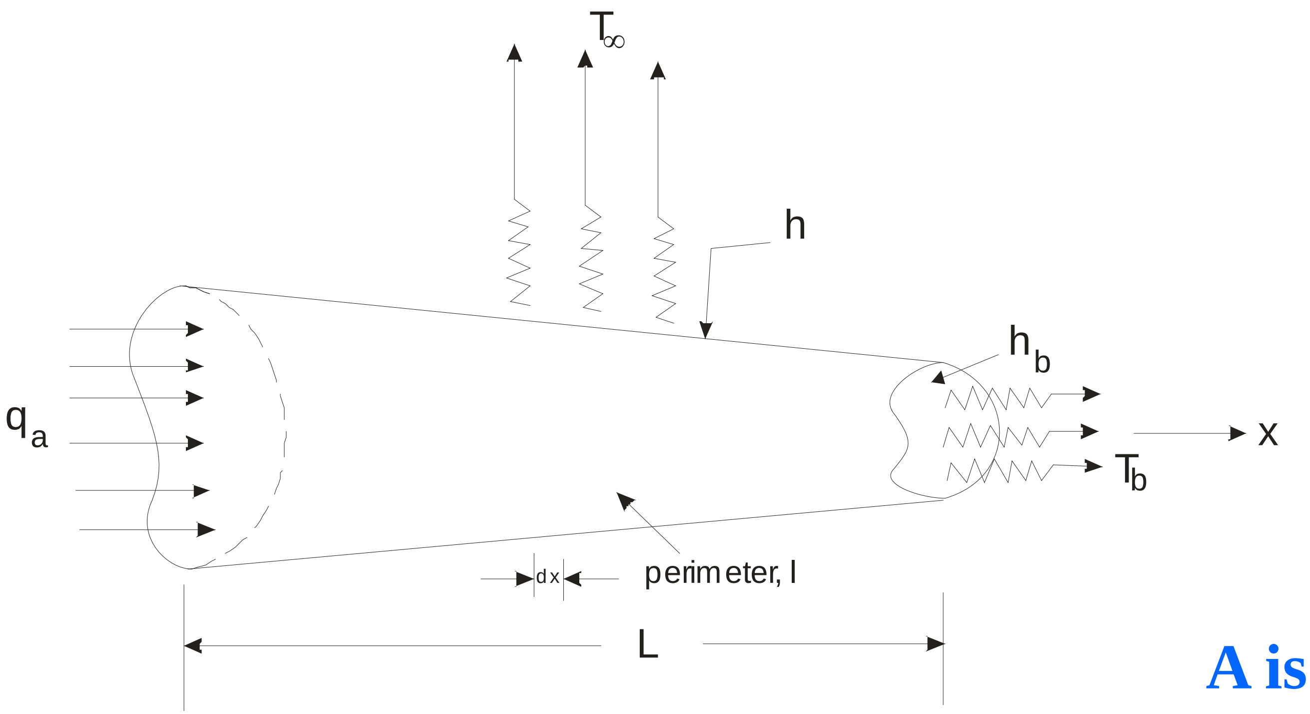
Comparing (assuming A is a constant)

$$\alpha(x)=k(x)\equiv\frac{E}{tLT}$$

$$\beta(x)=\frac{hl}{A}\equiv\frac{E}{tL^3T}$$

$$f(x)\equiv Q(x)+\frac{hl}{A}T_\infty\equiv\frac{E}{tL^3}$$

Heat Transfer

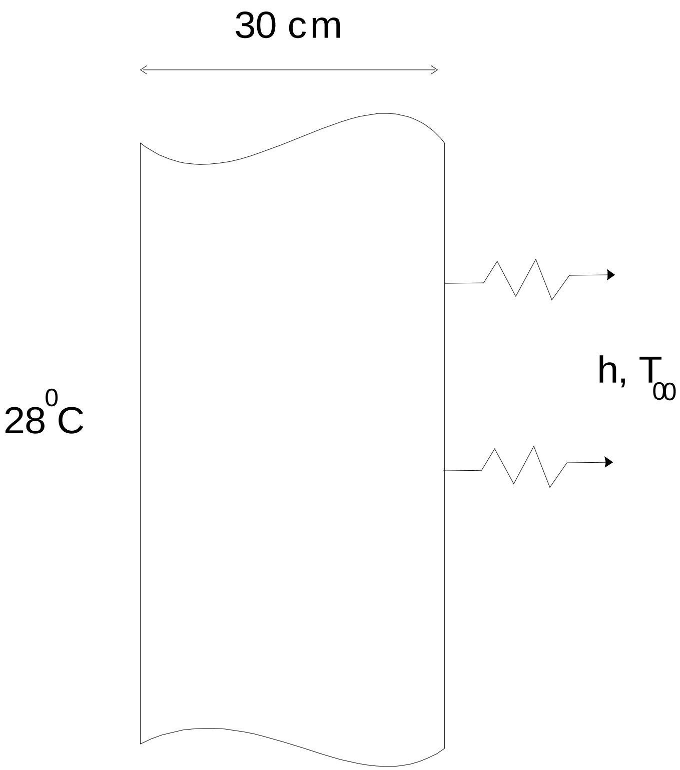


A is a NOT constant

$$\left[\left[\begin{array}{c|c} \frac{\bar{k}}{L}+\frac{\overline{hl}L}{3A} & -\frac{\bar{k}}{L}+\frac{\overline{hl}L}{6A} \\ \hline -\frac{\bar{k}}{L}+\frac{\overline{hl}L}{6A} & \frac{\bar{k}}{L}+\frac{\overline{hl}L}{3A} \end{array}\right]+h_b\left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & 1 \end{array}\right]\right]\begin{Bmatrix} T_1 \\ \hline T_2 \end{Bmatrix}=\frac{L}{2}\begin{Bmatrix} \frac{\overline{hl}}{A}T_\infty \\ \hline \frac{\overline{hl}}{A}T_\infty \end{Bmatrix}+\begin{Bmatrix} q_a \\ \hline 0 \end{Bmatrix}+\begin{Bmatrix} 0 \\ \hline h_bT_b \end{Bmatrix}$$

Example 1
(1D-C^0 Linear Element)

Problem T4L3-1



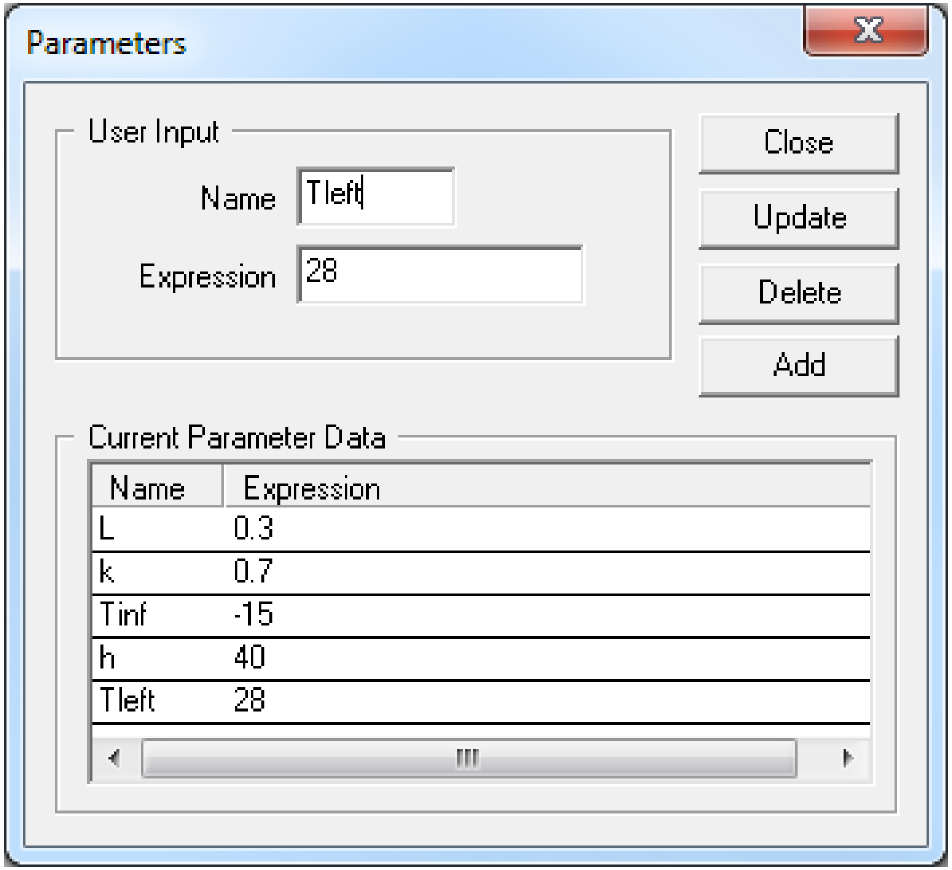
Brick wall: Determine the steady-state temperature distribution within the wall and also the heat flux through the wall.

$$k = 0.7 W/m \cdot {}^\circ C$$

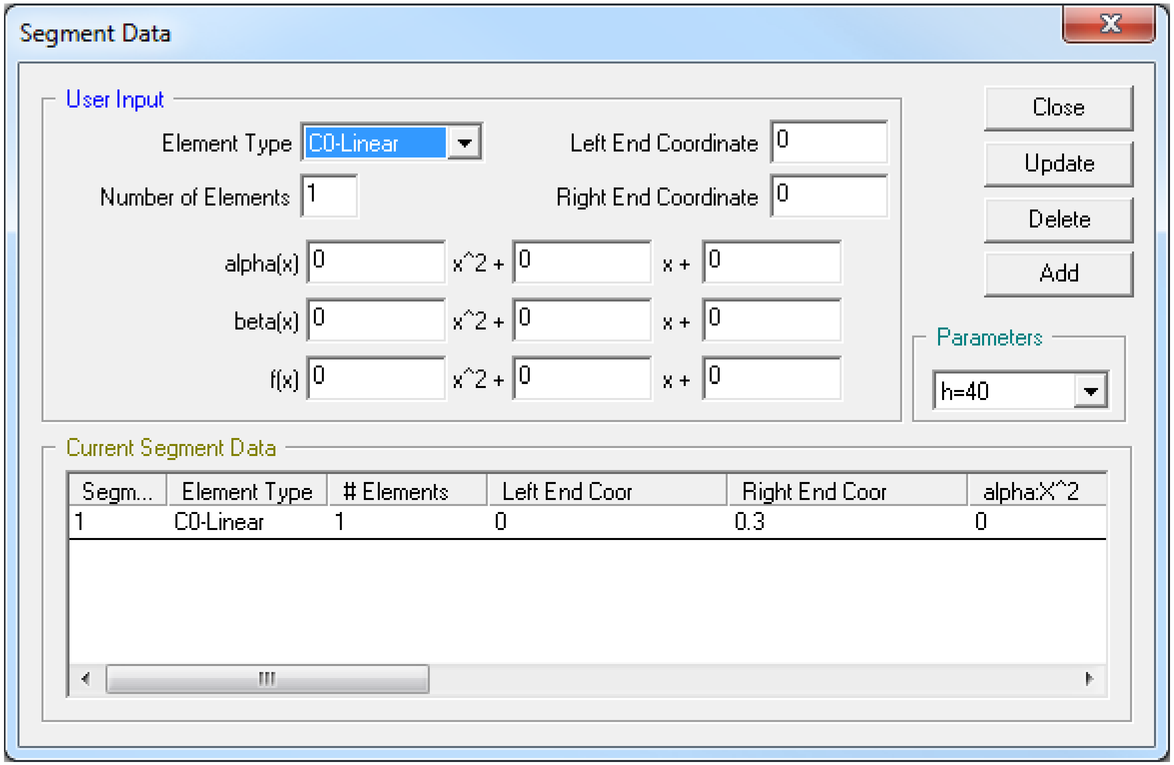
$$T_\infty = -15^\circ C$$

$$h = 40 W/m^2 \cdot {}^\circ C$$

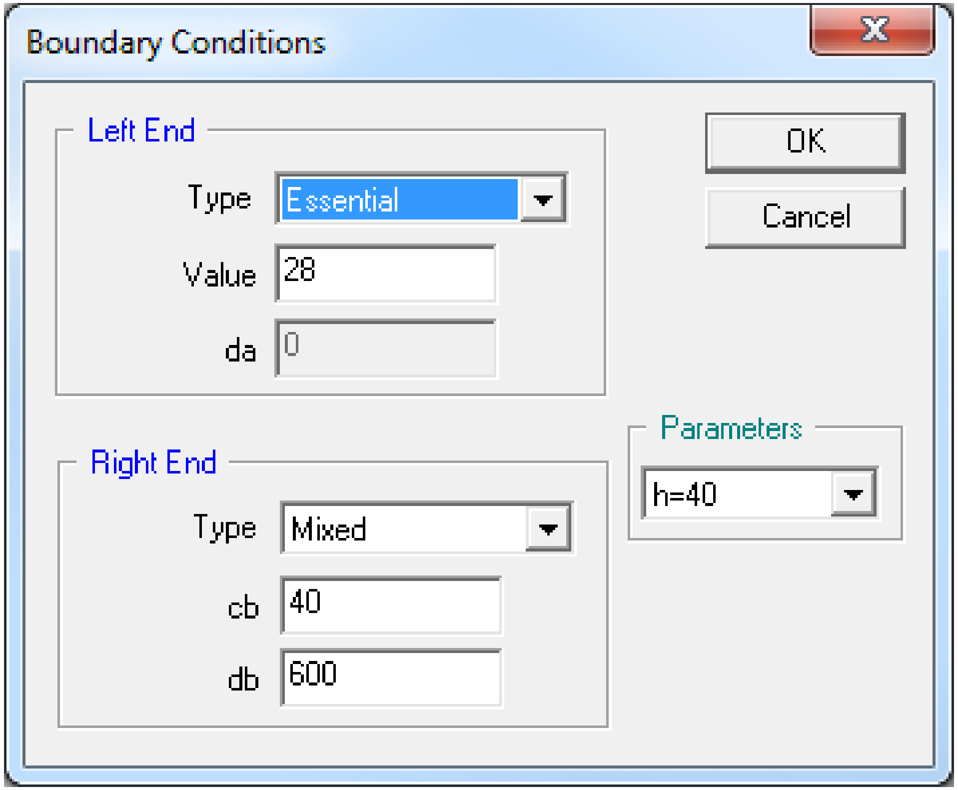
Problem T4L3-1



Problem T4L3-1



Problem T4L3-1



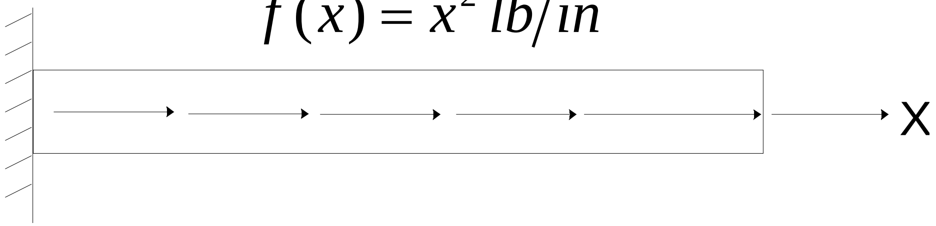
Problem T4L3-1

Mesh	Right Temp. (C)	Right Flux (W/m^2)	
1-element	-12.63	94.8	
2-elements	-12.63	94.8	
4-elements	-12.63	94.8	

Example 2A
(1D-C^0 Linear Element)

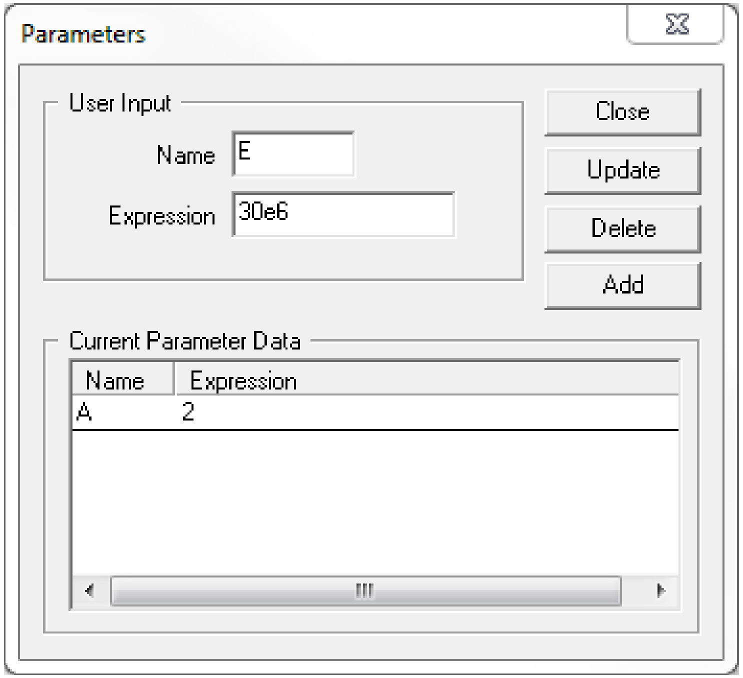
Problem T4L2-1

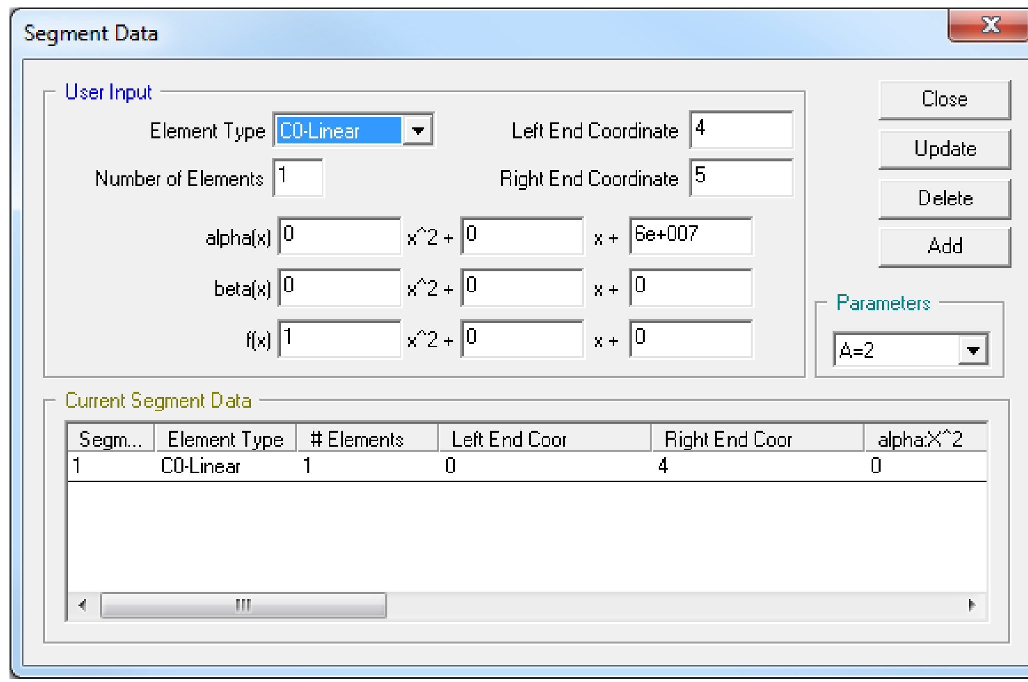
$f(x) = x^2 \; lb/in$



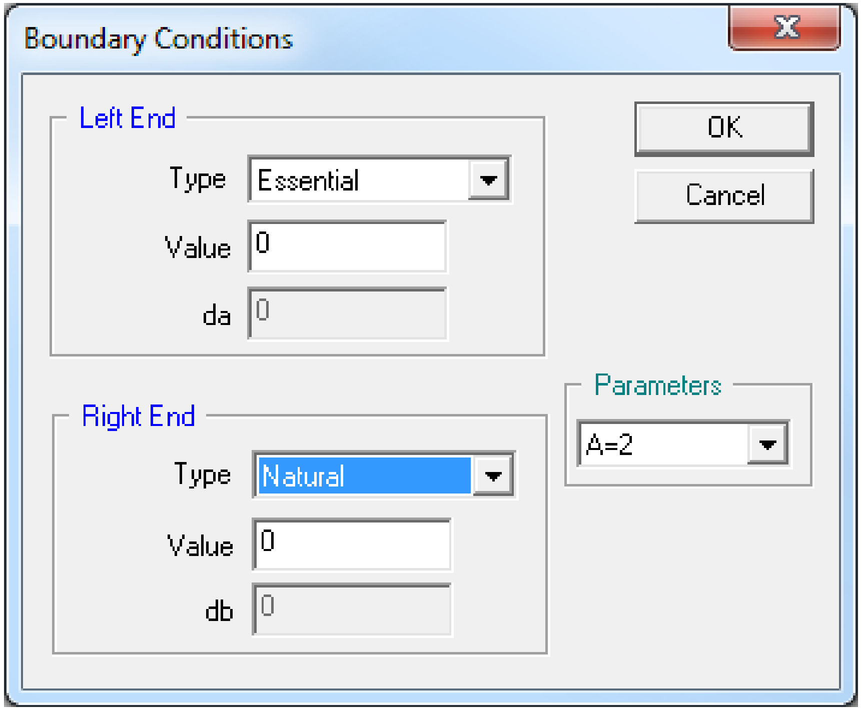
The 4" long steel bar with a 2 in^2 cross-sectional area is loaded by the given surface traction. Find the displacement and force distribution in the bar.

Problem T4L2-1





Problem T4L2-1



Problem T4L2-1

Mesh	Tip Disp. (in)	Root Force (lb)	Tip Force (lb)
1-element	$1.067(10^{-6})$	16	16
2-elements	$1.067(10^{-6})$	20.67	11.33
4-elements	$1.067(10^{-6})$	21.25	6.75
8-elements	$1.067(10^{-6})$	21.32	3.68
16-elements	$1.067(10^{-6})$	21.33	1.92

Example 2B
(1D-C^0 Higher Order Elements)

Problem T4L2-1

Quadratic Element

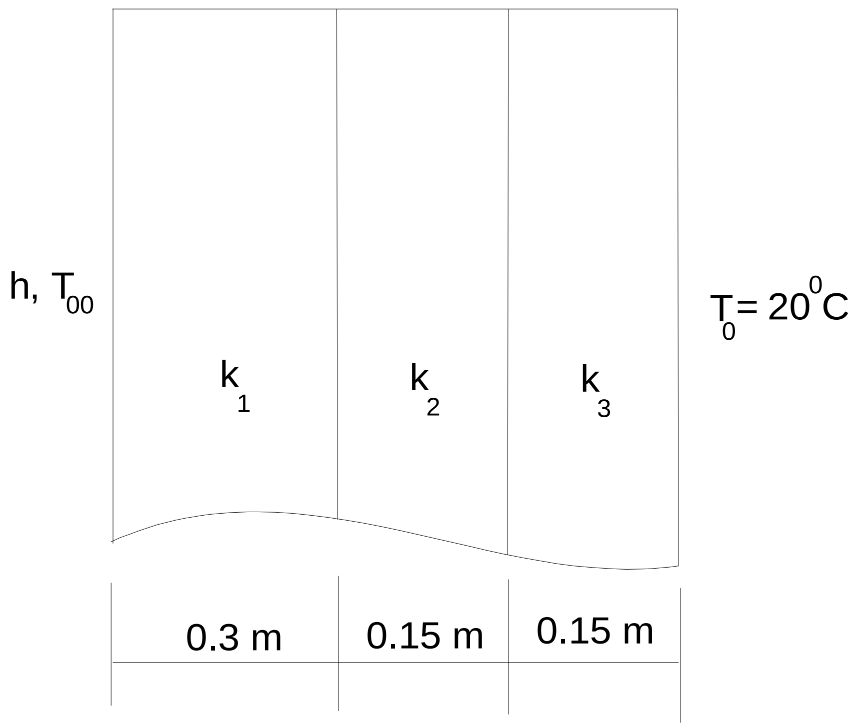
Mesh	Tip Disp. (in)	Root Force (lb)	Tip Force (lb)
1-element	$1.067(10^{-6})$	21.54	10.5
2-elements	$1.067(10^{-6})$	21.34	6.02
4-elements	$1.067(10^{-6})$	21.34	3.2
8-elements	$1.067(10^{-6})$	21.34	1.65

Cubic Element

Mesh	Tip Disp. (in)	Root Force (lb)	Tip Force (lb)
1-element	$1.067(10^{-6})$	21.3	6.4
2-elements	$1.067(10^{-6})$	21.33	3.4
4-elements	$1.067(10^{-6})$	21.33	1.75

Example 3
(1D-C^0 Linear Element)

Example T4L3-1



Mixed BC

EBC

$$c_a = -h$$

$$d_a = hT_\infty$$

$$k_1 = 20\frac{W}{m\cdot^\circ C}$$

$$k_2 = 30\frac{W}{m\cdot^\circ C}$$

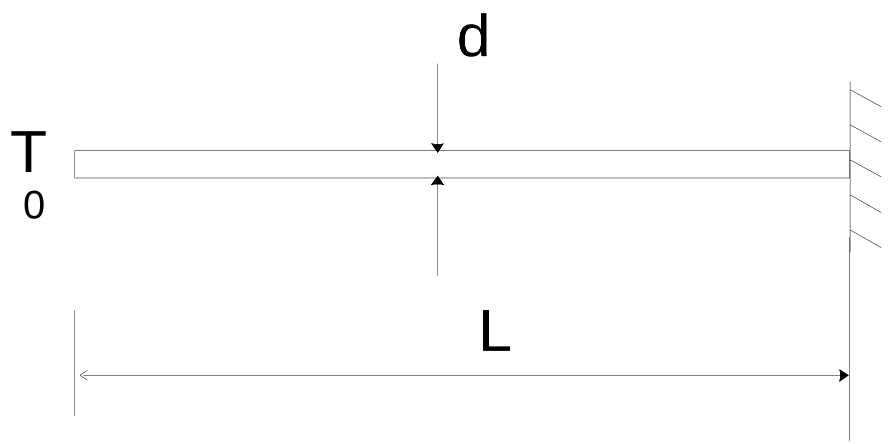
$$k_3 = 50\frac{W}{m\cdot^\circ C}$$

$$T_\infty = 800^\circ C$$

$$h = 25\frac{W}{m^2\cdot^\circ C}$$

Example 4A
(1D-C^0 Linear Element)

Example T4L3-2



$$\bar{k} = 24.8 \frac{BTU}{h \cdot ft \cdot^{\circ} F}$$

$$L = 0.516\, ft$$

$$A = \pi \frac{d^2}{4} = 5.326(10^{-4})\, ft^2$$

$$T_0 = 150^{\circ} F$$

$$T_{\infty} = 80^{\circ} F$$

$$h = 6 \frac{BTU}{h \cdot ft^2 \cdot^{\circ} F}$$

$$l = \pi d = 0.0818\, ft$$

EBC

NBC

$$\alpha = k$$

$$\beta = \frac{hl}{A}$$

$$f = \frac{hl}{A} T_{\infty}$$

Example T4L3-2

Mesh	Right Temp. (F)	Right Flux (Btu/h-ft^2)	
1-element	78.4	4268.3	
2-elements	88.97	1174.4	
4-elements	90.5	541.6	
64-elements	91.02	33.0	
256-elements	91.02	8.27	

Example 4B
(1D-C^0 Higher Order Elements)