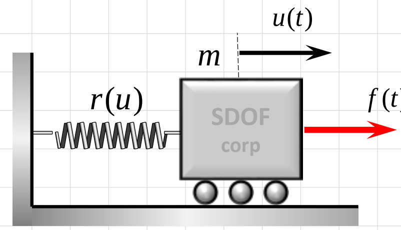
Homework 6

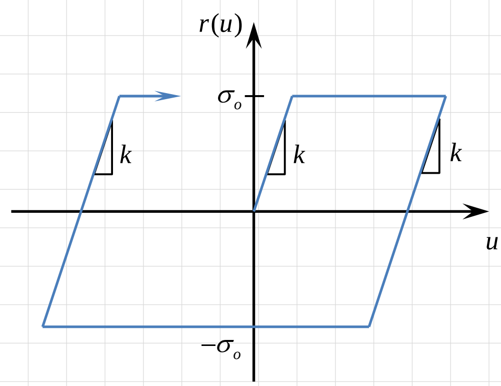
The single degree of freedom system shown at right has a nonlinear restoring force $r(u)$ and it subjected to an applied force $f(t)$. The equations of motion (and initial conditions) are

$$m\ddot{u}(t)+r(u(t))=f(t)$$
$$u(0)=u_o$$
$$\dot{u}(0)=v_0$$



Un-damped System

Consider the nonlinear response function to be elasto-plasticity with stiffness k and limiting force σ_o (symmetric in tension and compression).



Consider also the specific loading function

$$f(t)=F_1+F_2\sin\Omega t$$

where F_1 and F_2, and Ω are constants that describe the forcing function (i.e., a constant part and a sinusoidally varying part.

Newmark's method satisfies the equation of motion at the discrete time points and approximates the velocity and displacement with approximations to integrals. Hence, the discrete equations are

$$ma_{n+1}+\ r(u_{n+1})=0$$
$$v_{n+1}=c_n+h(1-\gamma)\,a_{n+1}$$
$$u_{n+1}=b_n+h^2(\tfrac{1}{2}-\beta)\,a_{n+1}$$

$$c_n=v_n+h\gamma a_n$$
$$b_n=u_n+hv_n+h^2\beta a_n$$

Implement Newmark's method, including a Newton loop to solve the nonlinear equations of motion for elasto-plastic model. Explore the response of the system and the influence of nonlinearity relative to the linear elastic response.