Homework 7

The multi-degree of freedom system shown at right is made of masses and linear springs. Write the equations of motion for the two different systems in the form

$$\mathbf{M}\ddot{\mathbf{u}}(t)+\mathbf{K}\mathbf{u}(t)=\mathbf{f}(t)$$
$$\mathbf{u}(0)=\mathbf{u}_o$$
$$\dot{\mathbf{u}}(0)=\mathbf{v}_o$$

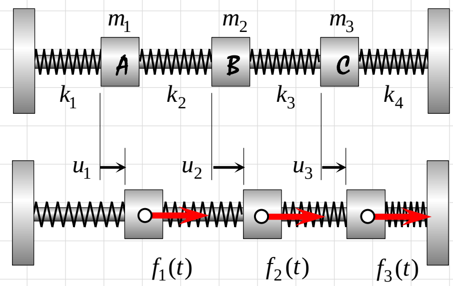
where $\mathbf{M}$ is the mass matrix, $\mathbf{K}$ is the stiffness matrix, $\mathbf{f}$ is the force vector, $\mathbf{u}_o$ is the initial displacement vector and $\mathbf{v}_o$ is the initial velocity vector. Generalize the expressions for the case of n degrees of freedom. Implement the computation of the matrices in a MATLAB code.

Implement *Newmark's method* in your MATLAB code to numerically integrate the equations of motion. Note that the Newmark equations are

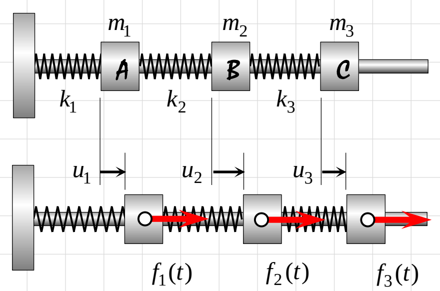
$$\mathbf{0}=\mathbf{M}\mathbf{a}_{n+1}+\mathbf{K}\mathbf{u}_{n+1}-\mathbf{f}_{n+1}$$
$$\mathbf{v}_{n+1}=\mathbf{v}_n+h\left(\gamma\,\mathbf{a}_n+(1-\gamma)\,\mathbf{a}_{n+1}\right)$$
$$\mathbf{u}_{n+1}=\mathbf{u}_n+h\mathbf{v}_n+h^2\left(\beta\,\mathbf{a}_n+(\tfrac{1}{2}-\beta)\,\mathbf{a}_{n+1}\right)$$

Show how the equations of the "shear building" are the same as the ones for System 2 if the value of the spring stiffness is replaced by the lateral stiffness of the columns (assume that the floors are rigid).

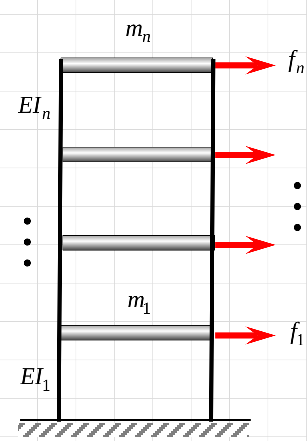
Note: It is probably best to have system 1 and system 2 be two different codes. You can try to be clever about the small change in $\mathbf{K}$ between the two cases, but when we extend to nonlinear systems the cleverness might turn out to be a headache/



System 1: "Bridge"



System 2: "Building"



System 3: "Shear Building"