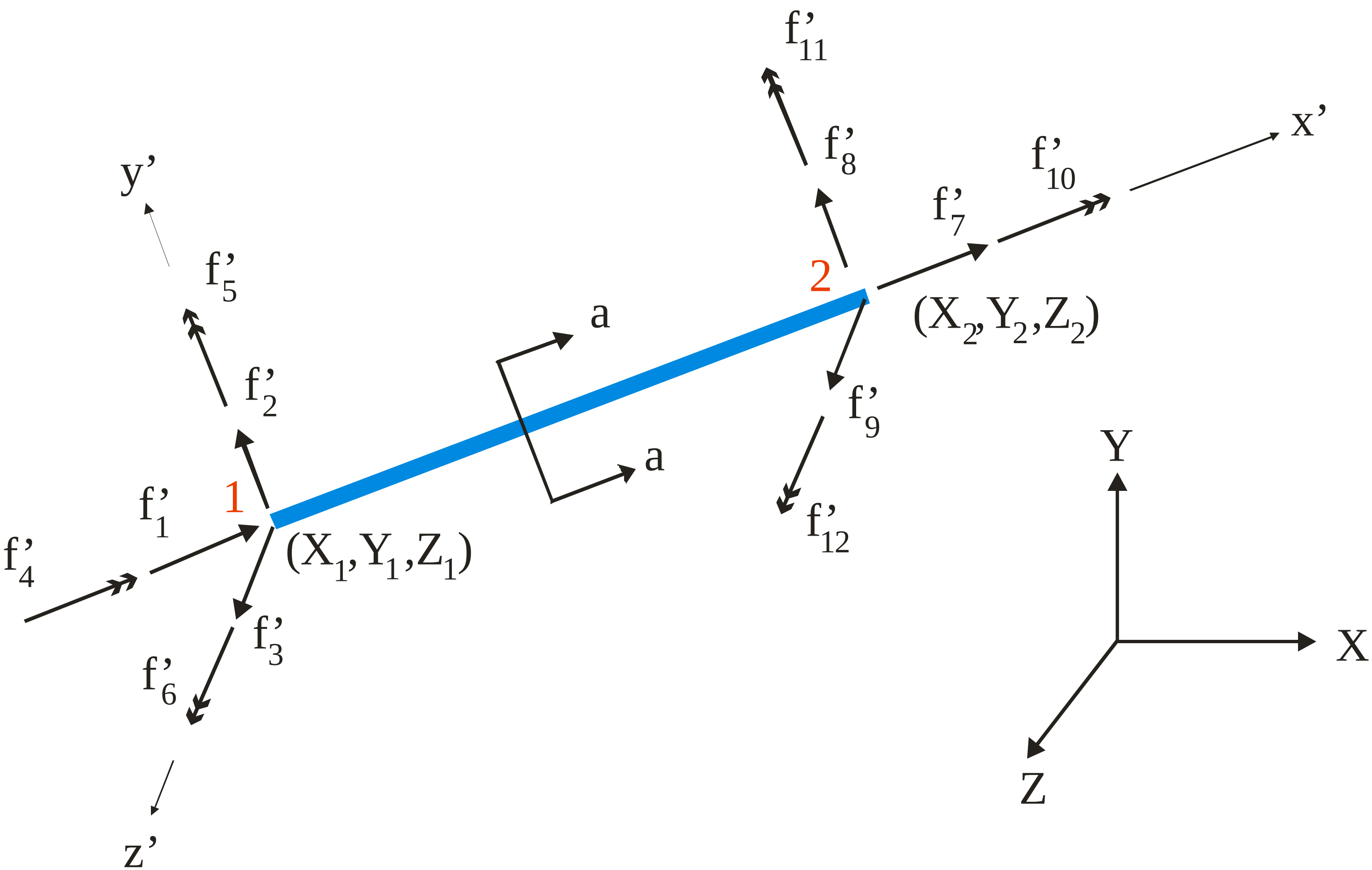
CEE432/CEE532/MAE541
Developing Software for Engineering Applications

Lecture 19: More on Frame Analysis and Implementation of Planar Frame Analysis Computer Program

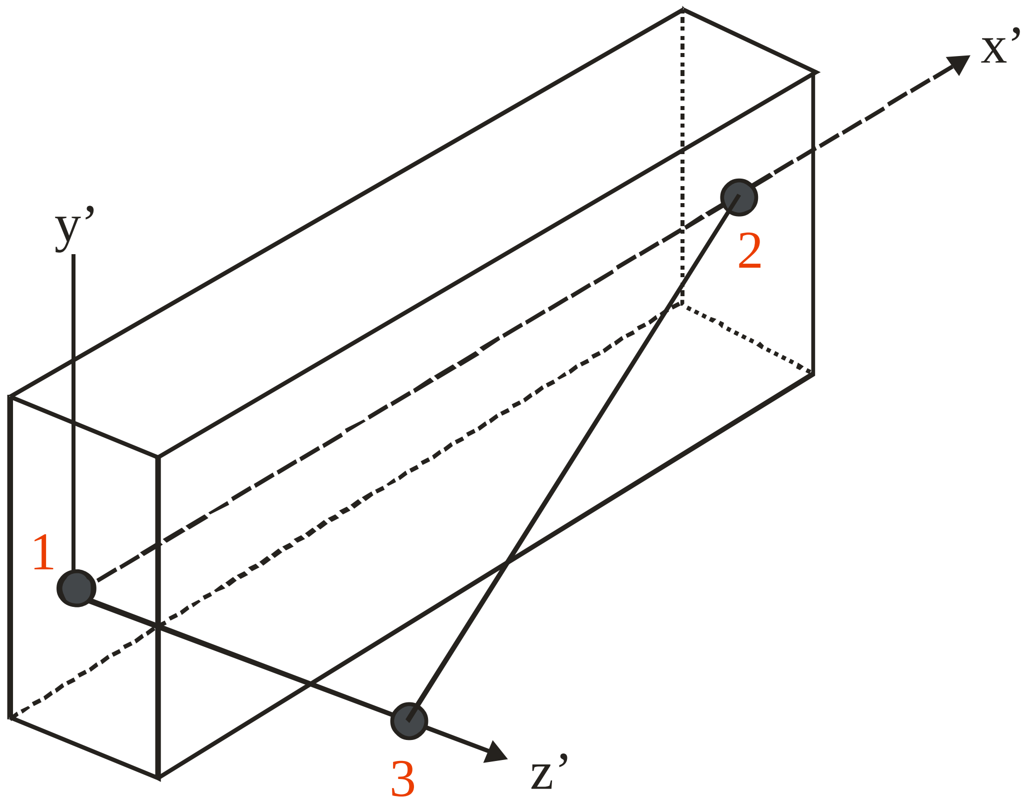
Space Frame Analysis

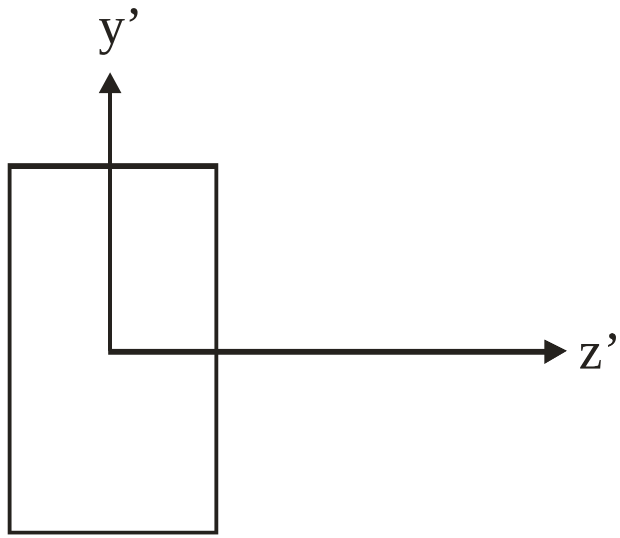


Space Beam Element

- Axial deformation along x' (2D)
- Bending about y' and z' (2D) axes
- Torsional moment about x'

Space Beam Element





View a-a

Euler-Bernoulli Beam

- Plane sections remain plane
- Small displacements and rotations
- Shear strain energy can be neglected

Element local stiffness matrix

$$\mathbf{k}'_{12\times 12} = \left[\begin{array}{c|c} \mathbf{k_{11}} & \mathbf{k_{12}} \\ \hline \mathbf{k_{21}} & \mathbf{k_{22}} \end{array}\right]_{12\times 12}$$

Element global stiffness matrix

$$\mathbf{k}_{12\times 12} = \mathbf{T}^{T}_{12\times 12}\mathbf{k}'_{12\times 12}\mathbf{T}_{12\times 12}$$

Euler-Bernoulli Beam

$$\mathbf{k}_{11} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ & \frac{12EI_z}{L^3} & 0 & 0 & 0 & \frac{6EI_z}{L^2} \\ & & \frac{12EI_y}{L^3} & 0 & -\frac{6EI_y}{L^2} & 0 \\ & & & \frac{GJ}{L} & 0 & 0 \\ & SYM & & & \frac{EI_y}{L} & 0 \\ & & & & & \frac{EI_z}{L} \end{bmatrix}$$

Euler-Bernoulli Beam

$$\mathbf{k}_{22} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ & \frac{12EI_z}{L^3} & 0 & 0 & 0 & -\frac{6EI_z}{L^2} \\ & & \frac{12EI_y}{L^3} & 0 & \frac{6EI_y}{L^2} & 0 \\ & & & \frac{GJ}{L} & 0 & 0 \\ & SYM & & & \frac{EI_y}{L} & 0 \\ & & & & & \frac{EI_z}{L} \end{bmatrix}$$

Euler-Bernoulli Beam

$$\mathbf{k}_{12} = \mathbf{k}_{21}^{\mathrm{T}} = \begin{bmatrix} -\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{12EI_z}{L^3} & 0 & 0 & 0 & \frac{6EI_z}{L^2} \\ 0 & 0 & -\frac{12EI_y}{L^3} & 0 & -\frac{6EI_y}{L^2} & 0 \\ 0 & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\ 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{EI_y}{L} & 0 \\ 0 & -\frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{EI_z}{L} \end{bmatrix}$$

Euler-Bernoulli Beam

Local-to-global transformation

$$\mathbf{T}_{12\times 12} = \left[\begin{array}{c|c|c|c} \mathbf{\Lambda} & & & \\ \hline & \mathbf{\Lambda} & & \\ \hline & & \mathbf{\Lambda} & \\ \hline & & & \mathbf{\Lambda} \end{array}\right]$$

$$\mathbf{\Lambda}_{3\times 3} = \begin{bmatrix} l_{x'} & m_{x'} & n_{x'} \\ l_{y'} & m_{y'} & n_{y'} \\ l_{z'} & m_{z'} & n_{z'} \end{bmatrix}$$

Euler-Bernoulli Beam

$$L = \sqrt{\left(X_2 - X_1\right)^2 + \left(Y_2 - Y_1\right)^2 + \left(Z_2 - Z_1\right)^2}$$

$$\mathbf{e}_{x'} = \left[l_{x'} \quad m_{x'} \quad n_{x'}\right]$$

$$l_{x'} = \frac{X_2 - X_1}{L}, \; m_{x'} = \frac{Y_2 - Y_1}{L}, \; n_{x'} = \frac{Z_2 - Z_1}{L}$$

$$\mathbf{e}_{13} = \frac{X_3 - X_1}{L_{13}} \hat{i} + \frac{Y_3 - Y_1}{L_{13}} \hat{j} + \frac{Z_3 - Z_1}{L_{13}} \hat{k}$$

$$L_{13} = \sqrt{\left(X_3 - X_1\right)^2 + \left(Y_3 - Y_1\right)^2 + \left(Z_3 - Z_1\right)^2}$$

$$\mathbf{e}_{y'} = \left[l_{y'} \quad m_{y'} \quad n_{y'}\right] \Rightarrow \mathbf{e}_{y'} = \mathbf{e}_{13} \times \mathbf{e}_{x'}$$

$$\mathbf{e}_{z'} = \left[l_{z'} \quad m_{z'} \quad n_{z'}\right] \Rightarrow \mathbf{e}_{z'} = \mathbf{e}_{13}$$

Planar Frame Analysis Program

Requirements

1. There should be no artificial restriction on the size of the problem that can be solved. In other words, use dynamically allocated arrays. Use arrays only when required.
2. The input file format (Section 2.0) must be strictly followed. Assume that the input file is created using consistent units (for length, force and temperature). Program must detect input errors and print out meaningful error messages.
3. The loading on the frame can be due to (a) nodal forces and moments, (b) element loads, and (c) nodal displacements.

Requirements

4. The nodes can be either rigid connections or an internal hinge. Cross-sectional shapes of the elements can be rectangular solid, circular solid, circular hollow (tube) or an I-section.
5. The program must compute (a) nodal displacements and rotations, (b) element nodal forces, and (c) support reactions. These computed quantities must be written in a tabular form (see Section 3.0). You must also compute the relative and absolute error norms and write them to the output file.

Requirements

6. You must use the matrix toolbox that you developed in Project 1.
7. The program must ask only for the input and output file names. It should indicate that the program has been successfully executed or display the appropriate error message. Do not display debugging statements.

Input File Format

Section 1 (2 lines)

```
*heading
appropriate comment describing the problem
```

Section 2 (1+*nN* lines)

```
*nodal coordinates
node #, x-coordinate, y-coordinate
```

Section 3 (1+*nFC* lines)

```
*nodal fixity
node #, x-fixity code, y-fixity code, z-fixity
code, x-disp value, y-disp value, z-disp value
```

Input File Format

Section 4 (1+*nLN* lines)
```
*nodal loads
node #, x-force, y-force, z-moment
….
```

Section 5 (1+*nM* lines)
```
*material data
material group #, modulus of elasticity
```

Section 6 (1+*nXS* lines)
```
*cross-sectional data
x/s group #, type, list of values
```

Input File Format

Section 7 (1+*nE* lines)

*element data

element #, start node#, end node #, material group#, x/s group number

Section 8 (1+*nEL* lines)

*element loads

element #, load type, value 1, value 2

Section 9 (1 line)

*end

Input File Format

- Fixity Codes
  - `free, specified, hinge`
- X-Section Type
  - `rects` height and width
  - `circs` radius
  - `tube` inner radius, wall thickness
  - `isection` web height, web thickness, flange width, flange thickness

Input File Format

- Element Load Types
  - `dly'` Load value start node, load value end node
  - `ploady'` Dist. from start node, load value
  - `ploadx'` Dist. from start node, load value
  - `cmoment` Dist. from start node, load value

Output File Format

- The output file must contain the details of the frame model as well as details of the nodal displacements, support reactions and **max. nodal** element strain, stress and force. Each output data should be presented in a tabular form.