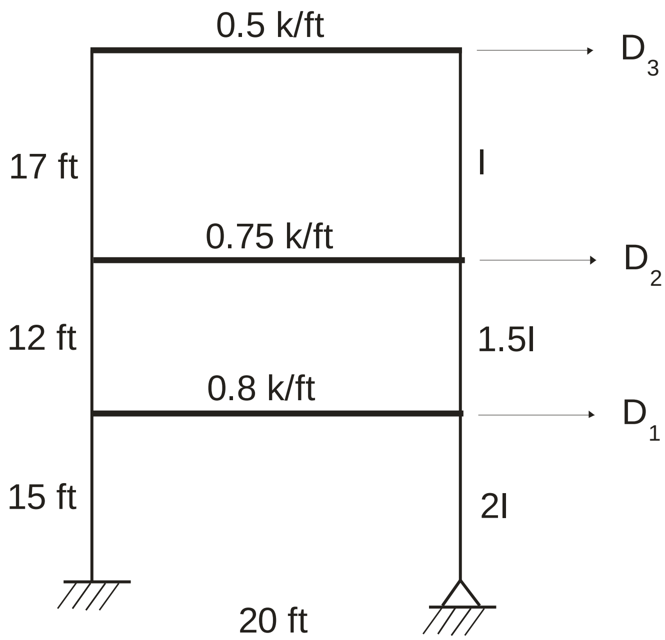
CEE526/MAE527 Finite Element for Engineers

Structural Dynamics Examples

Example 1: Modal Analysis of a Shear Frame

Compute the lowest frequency and mode shape for the frame shown in Fig. 1.1.



$E = 29000\, ksi$

$I = 600\, in^4$

Note

$F = ma$

$1lb = (1slug)(1\, ft/s^2)$

$$slug = \frac{lb \cdot s^2}{ft}$$

Fig. 1.1 Three-story shear frame building
[Example 10.3: Tedesco, McDougal and Ross, Addison Wesley]

Solution: Method 1

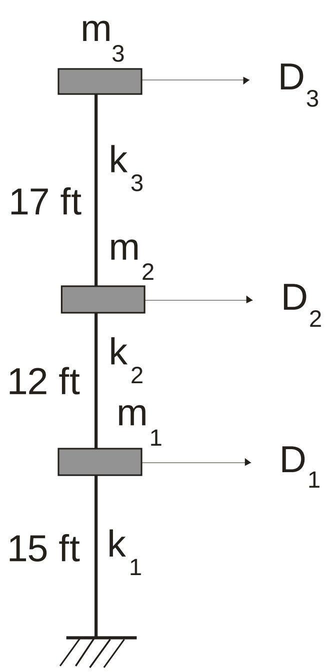


Fig. 1.2

In this method, the building is approximated as a stick structure with the mass lumped as shown in Fig. 1.2. The stiffness and mass matrices are computed as follows.

Mass

$$m_1 = 0.8\frac{k}{ft}\frac{20\ ft}{386.4\, in/s^2} = 0.04141\frac{k-s^2}{in}$$

$$m_2 = 0.75\frac{k}{ft}\frac{20\ ft}{386.4\, in/s^2} = 0.03882\frac{k-s^2}{in}$$

$$m_3 = 0.5\frac{k}{ft}\frac{20\ ft}{386.4\, in/s^2} = 0.02588\frac{k-s^2}{in}$$

Stiffness

$$k_1 = \frac{12EI}{L^3} + \frac{3EI}{L^3} = \frac{12(29000\,ksi)(2)(600)}{\left(15\times 12\,in\right)^3} + \frac{3(29000\,ksi)(2)(600)}{\left(15\times 12\,in\right)^3} = 89.506\frac{k}{in}$$

$$k_2 = \frac{12EI}{L^3} + \frac{12EI}{L^3} = 2\frac{12(29000\,ksi)(1.5)(600)}{\left(12\times 12\,in\right)^3} = 209.78\frac{k}{in}$$

$$k_1 = \frac{12EI}{L^3} + \frac{12EI}{L^3} = 2\frac{12(29000\,ksi)(600)}{\left(17\times 12\,in\right)^3} = 49.189\frac{k}{in}$$

Hence

$$\mathbf{M}_{3\times 3} = \begin{bmatrix} 0.04141 & 0 & 0 \\ 0 & 0.03882 & 0 \\ 0 & 0 & 0.02588 \end{bmatrix}$$

$$\mathbf{K}_{3\times 3} = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} = \begin{bmatrix} 299.29 & -209.78 & 0 \\ -209.78 & 258.97 & -49.189 \\ 0 & -49.189 & 49.189 \end{bmatrix}$$

Solving $\mathbf{K}_{3\times 3}\mathbf{\Phi}_{3\times 3} = \mathbf{\Lambda}_{3\times 3}\mathbf{M}_{3\times 3}\mathbf{\Phi}_{3\times 3}$ we have results from Generalized Jacobi Method[1].

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Generalized Jacobi RESULTS =================
Eigenvalue: 628.803
Eigenvector:
[1   ]2.20683         [2   ]2.87453         [3   ]4.29569

Eigenvalue: 2870.61
Eigenvector:
[1   ]-2.63866        [2   ]-2.26934        [3   ]4.44685

Eigenvalue: 12299.8
Eigenvector:
[1   ]3.50944         [2   ]-3.51384        [3   ]0.642229
```

[1] ASUTruss solution

Solution: Method 2 (Finite Element Method. Units: slg, lb, ft, s)[2]

An approximate model needs to be built since all relevant data to build the FE model are not available. Fig. 1.3 shows the FE model.

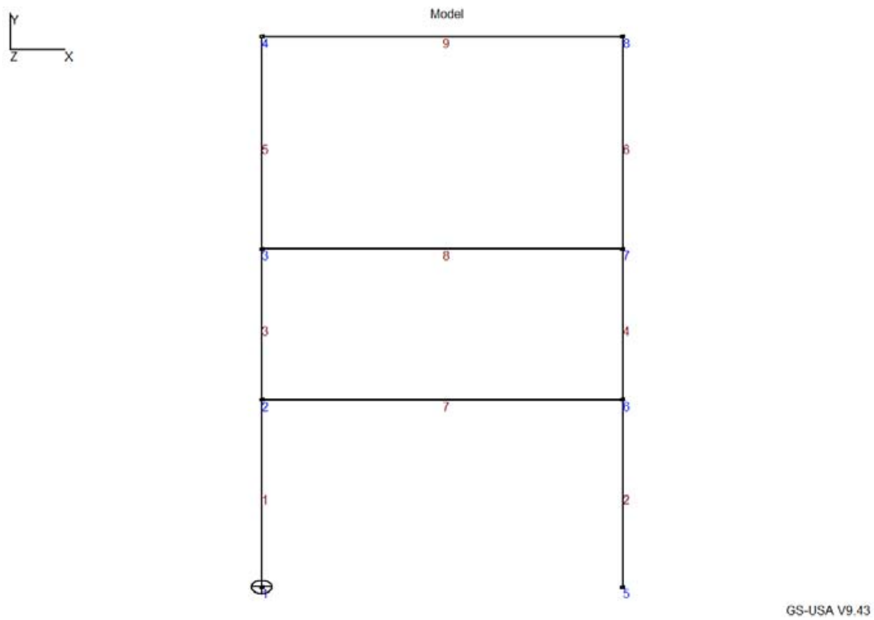


Fig. 1.3 FE model

Table 1.1 Material & Cross-sectional Properties

Material	Mass Density	Modulus of Elasticity	X/S Area	Moment of Inertia
Steel (columns)	15.2407	$4.176(10^9)$	$A = 28\,in^2$ $A = 0.19\,ft^2$	$I = 600\,in^4$ $I = 0.029\,ft^4$
			$A = 30\,in^2$ $A = 0.21\,ft^2$	$1.5I = 900\,in^4$ $1.5I = 0.043\,ft^4$
			$A = 32\,in^2$ $A = 0.22\,ft^2$	$2I = 1200\,in^4$ $2I = 0.058\,ft^4$
Concrete (beams)	4.66	$0.8352(10^9)$	$3.33'\times 1' \Rightarrow 500\dfrac{lb}{ft}$	
			$5'\times 1' \Rightarrow 750\dfrac{lb}{ft}$	
			$5.33'\times 1' \Rightarrow 800\dfrac{lb}{ft}$	

[2] GS-USA Frame solution

Eigenvalue: $\lambda_1 = 374.1$, $\lambda_2 = 2059.6$, $\lambda_3 = 9513.1$, $\lambda_4 = 18972.4$, $\lambda_5 = 27911.7$.

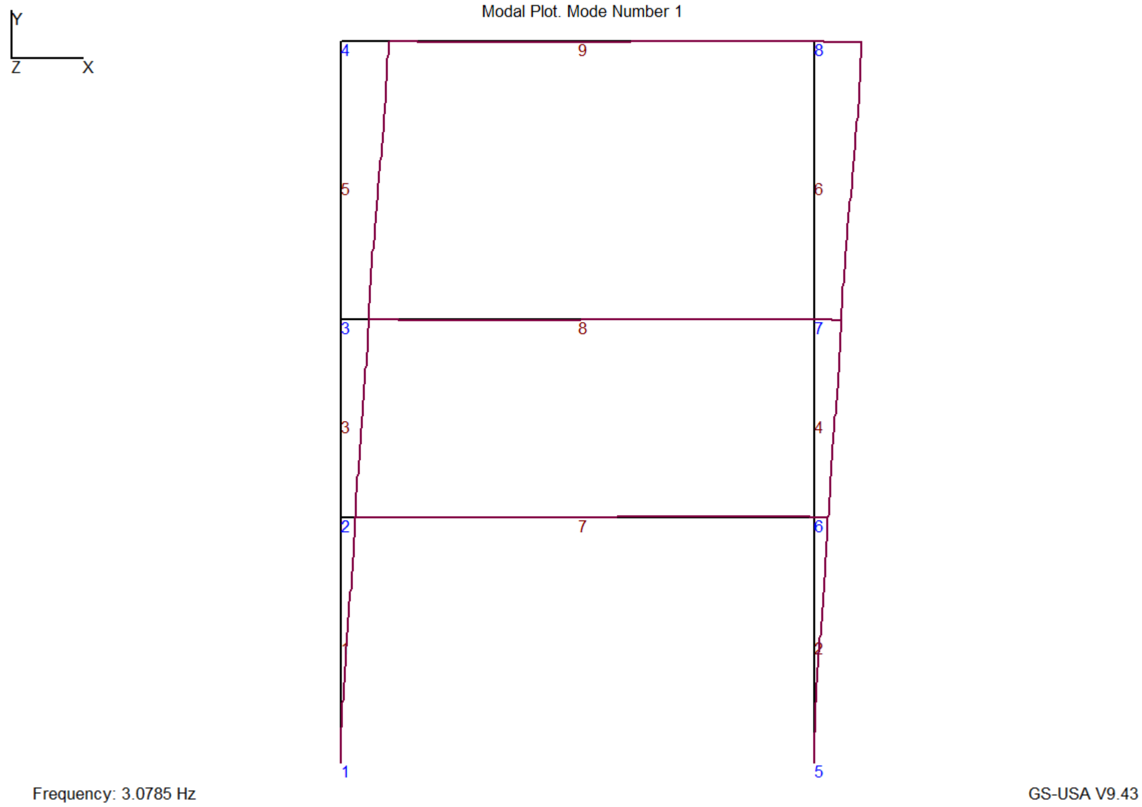


Fig. 1.4 Lowest mode shape

Note: $f_1 = \dfrac{\omega_1}{2\pi} = \dfrac{\sqrt{\lambda_1}}{2\pi} = \dfrac{\sqrt{374.1}}{2\pi} = 3.078\,Hz$

Example 3: Structural Dynamics (Wilson-Theta Method)

Use Wilson-Theta Method to evaluate the dynamic response of the frame shown in Fig. 3.1 using the loading described in Fig. 3.2 in the interval $0 \le t \le 5.0s$. Select $\theta = 1.4$, $\Delta t = 0.001s$. Take $k = 60\dfrac{k}{in}, c = 0.5\dfrac{k-s}{in}, W = 300k$.

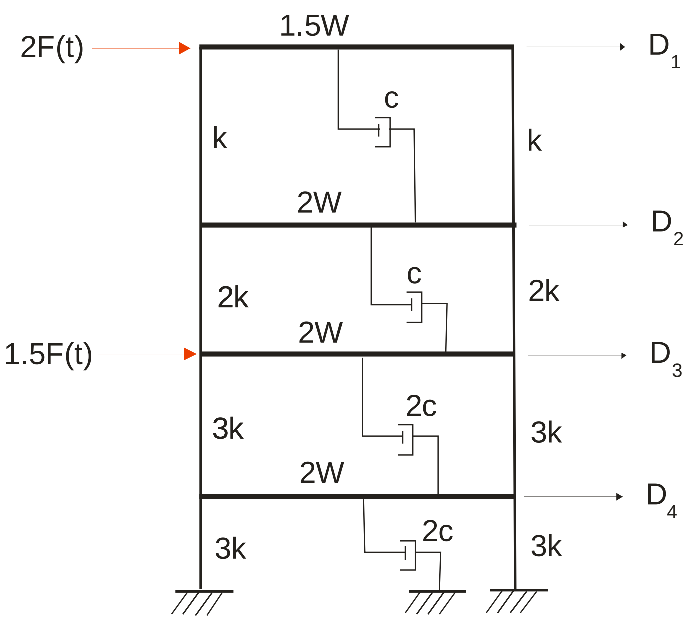


Fig. 3.1 Three-story shear frame building [Example 13.2: Tedesco, McDougal and Ross, Addison Wesley]

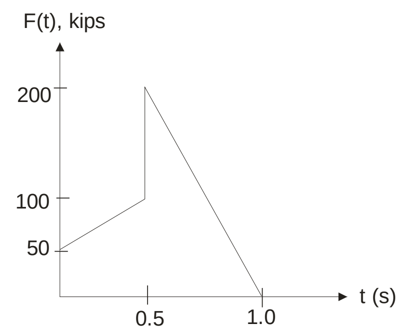


Fig. 3.2 Time-varying force

Solution[3]

The system matrices are defined below.

$$\mathbf{M}_{4\times4} = \begin{bmatrix} 1.1641 & & & \\ & 1.5528 & & \\ & & 1.5528 & \\ & & & 1.5528 \end{bmatrix} \frac{k-s^2}{in}$$

$$\mathbf{K}_{4\times4} = \begin{bmatrix} 120 & -120 & & \\ -120 & 360 & -240 & \\ & -240 & 600 & -360 \\ & & -360 & 720 \end{bmatrix} \frac{k}{in}$$

[3] ASUTruss solution

$$\mathbf{C}_{4\times4} = \begin{bmatrix} 0.5 & -0.5 & & \\ -0.5 & 1.0 & -0.5 & \\ & -0.5 & 1.5 & -1.0 \\ & & -1.0 & 2.0 \end{bmatrix} \frac{k-s}{in}$$

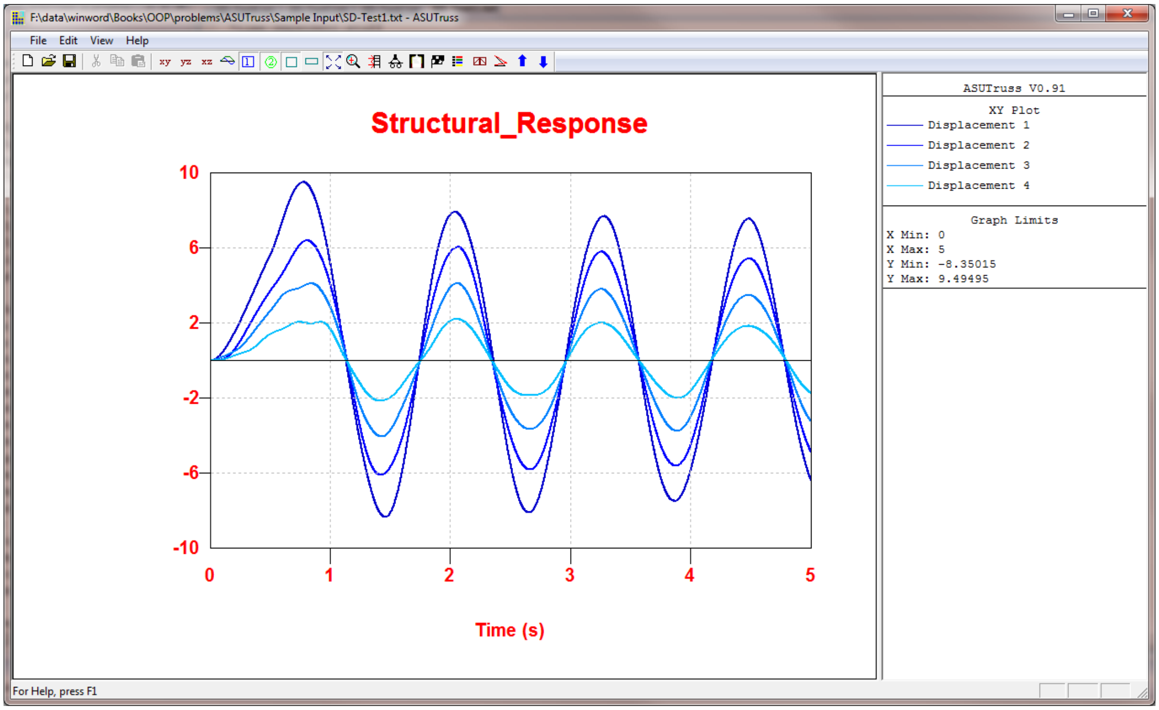


Fig. 3.3 Displacement graph

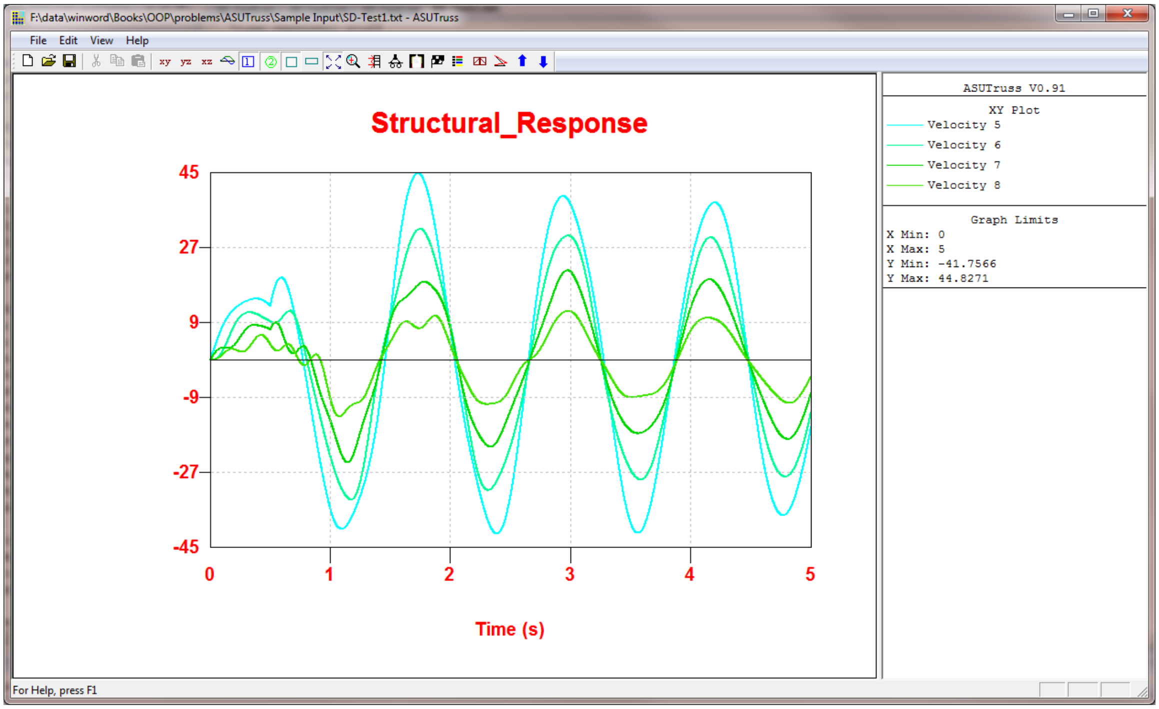


Fig. 3.4 Velocity graph

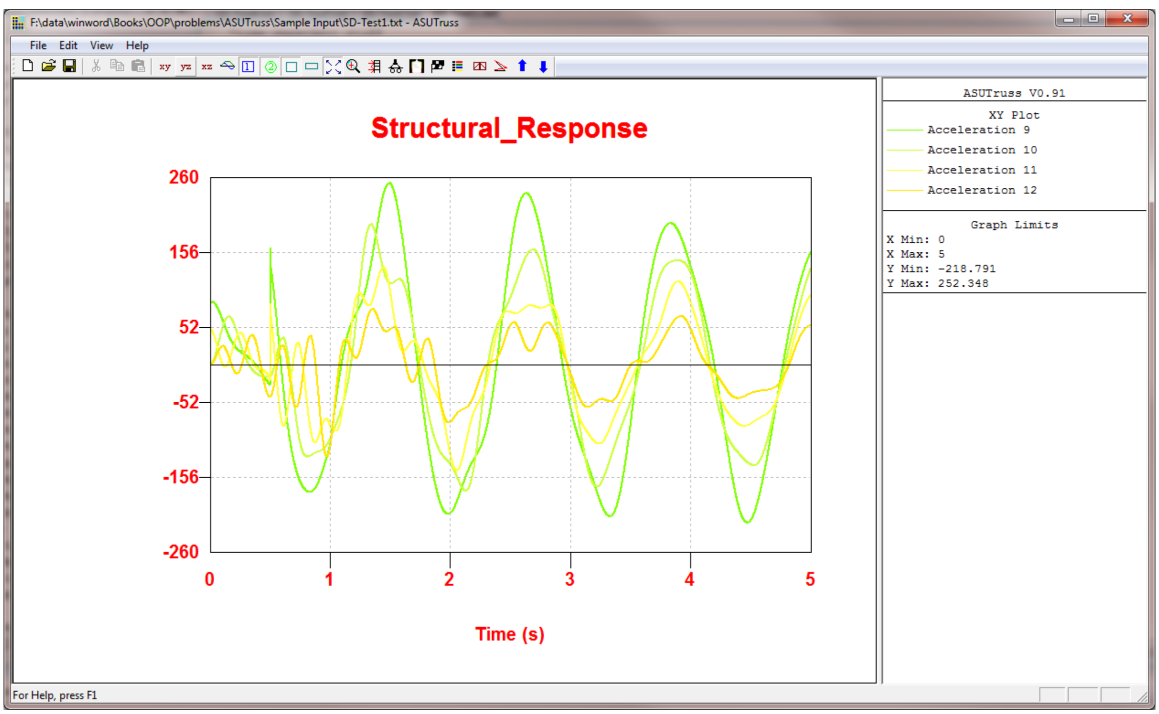


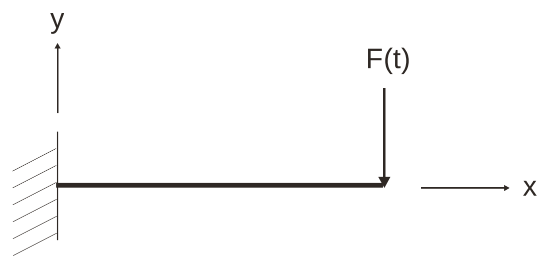
Fig. 3.5 Acceleration graph

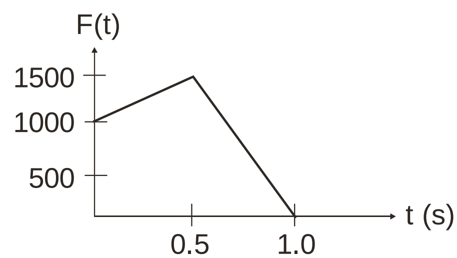
Assignment Problem 1

A planar cantilever beam of length 2 m has the following properties:

$$E = 200\,GPa, A = 0.01m^2, I = 10^{-4}\,m^4, \rho = 7850\,\frac{kg}{m^3}$$

Assume that there is no damping in the system. The loading is applied at the tip of the beam as shown in the figure below.





Use a one-element beam model, Wilson-Theta method and a time step of 0.2 s for $0 < t < 10s$ to determine the displacement, velocity and accelerations in the system.