Finite Elements for Engineers

Lecture 6: Modeling and Convergence Issues

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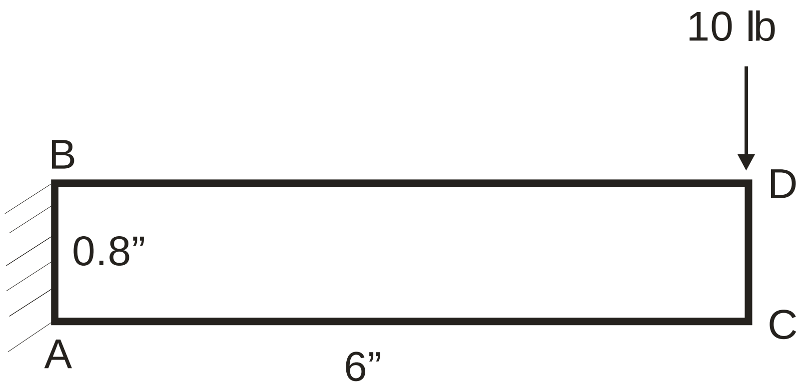
Case Study 1:
Plane Stress Analysis of a Cantilever Beam

Thin Cantilever Beam



$$t = 0.1\, in$$

$$E = 30 \times 10^6\ psi$$

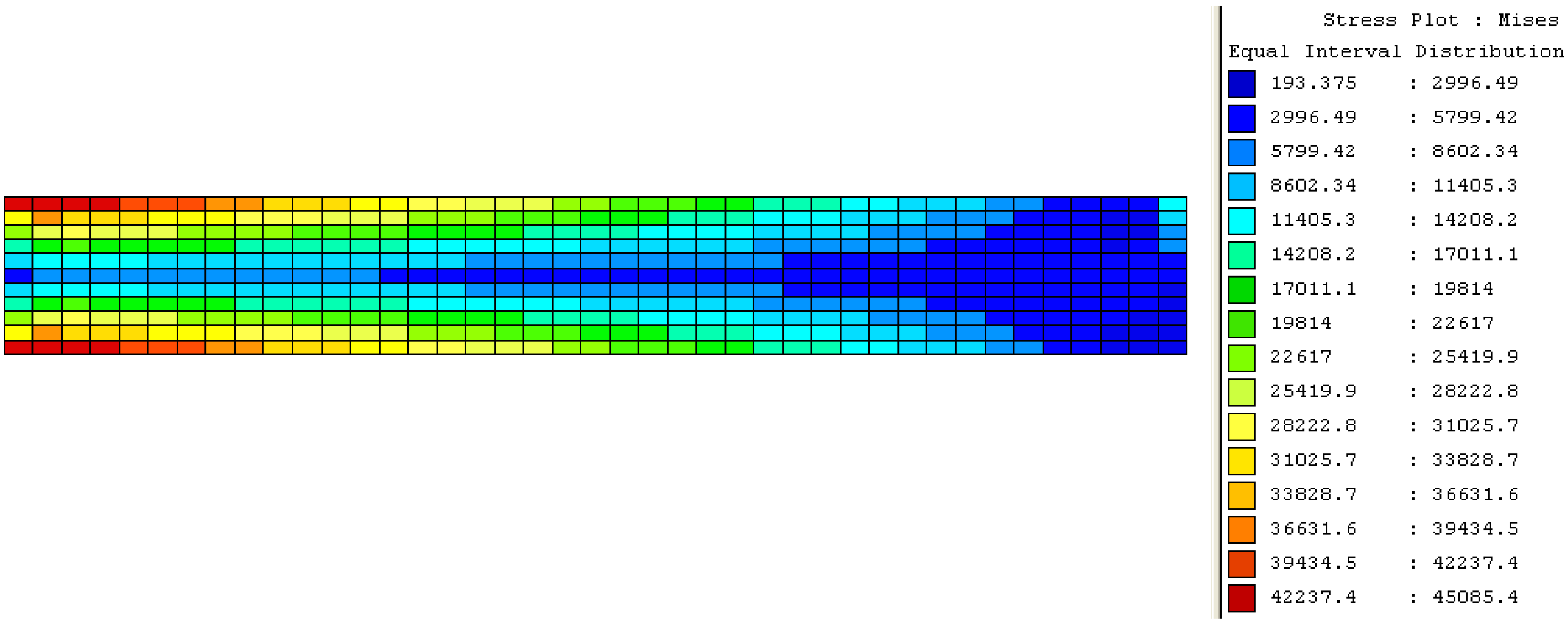
$$\nu = 0.3$$

Simple Beam Theory Results

$$\Delta = \frac{PL^3}{3EI} = \frac{(10)(6)^3}{3\left(30\times10^6\right)\left(\frac{0.1\times0.8^3}{12}\right)} = 0.005625\,\text{in}$$

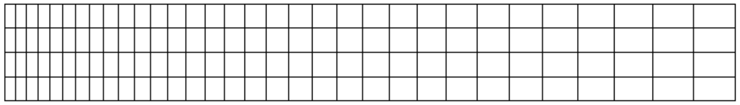
$$\sigma = \frac{Mc}{I} = \frac{(10)(6)\left(0.4\right)}{\left(\frac{0.1\times0.8^3}{12}\right)} = 5625\ psi$$

Thin Cantilever Beam: FE Model

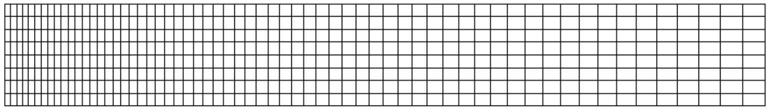


Thin Cantilever Beam: FE Model

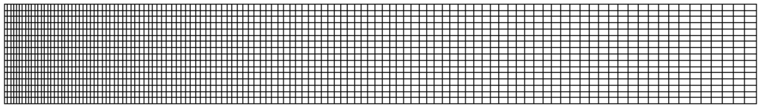
Mesh-1 (51 nodes)



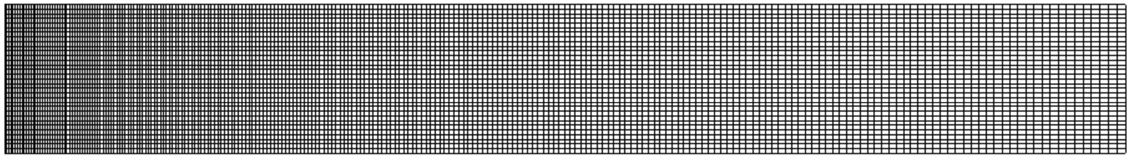
Mesh-2 (165 nodes)



Mesh-3 (567 nodes)

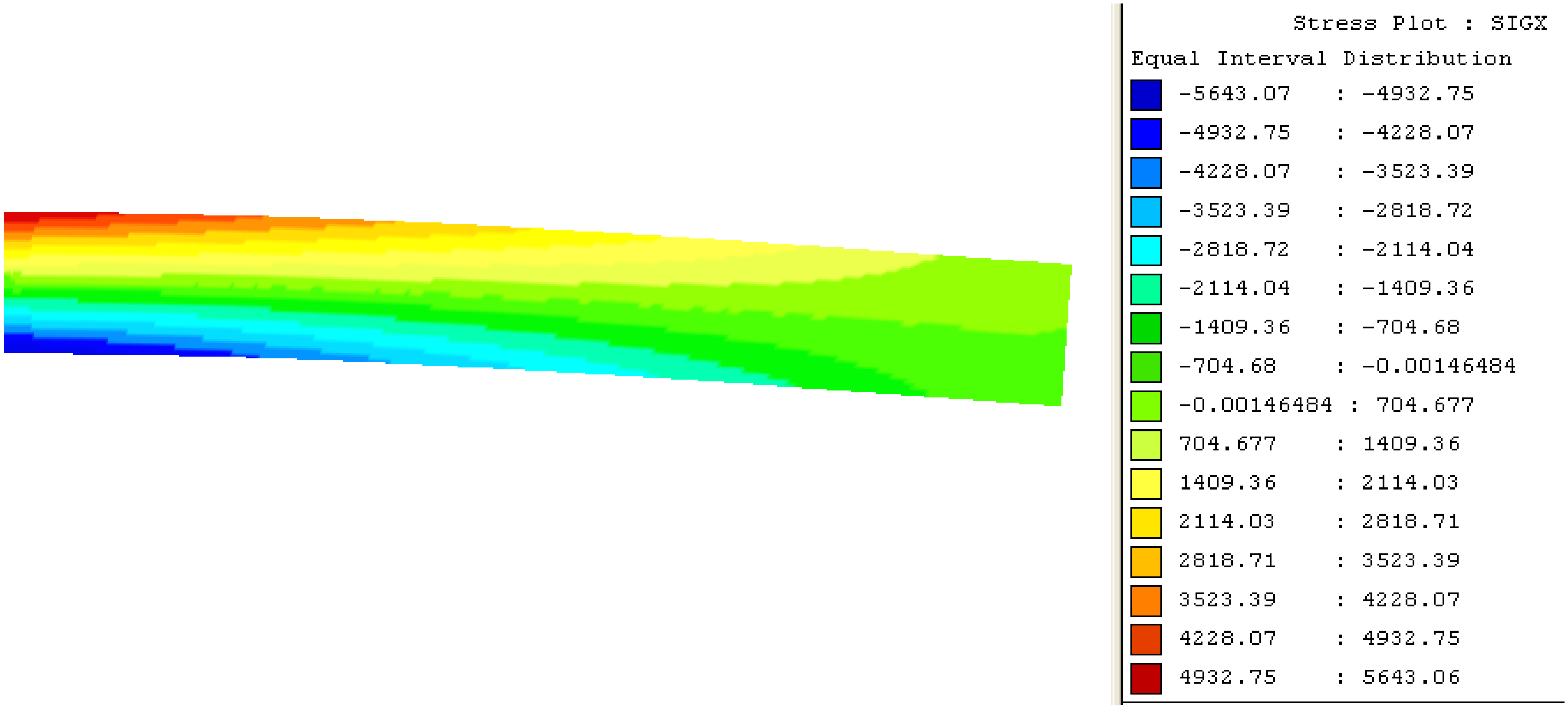


Mesh-4 (2091 nodes)

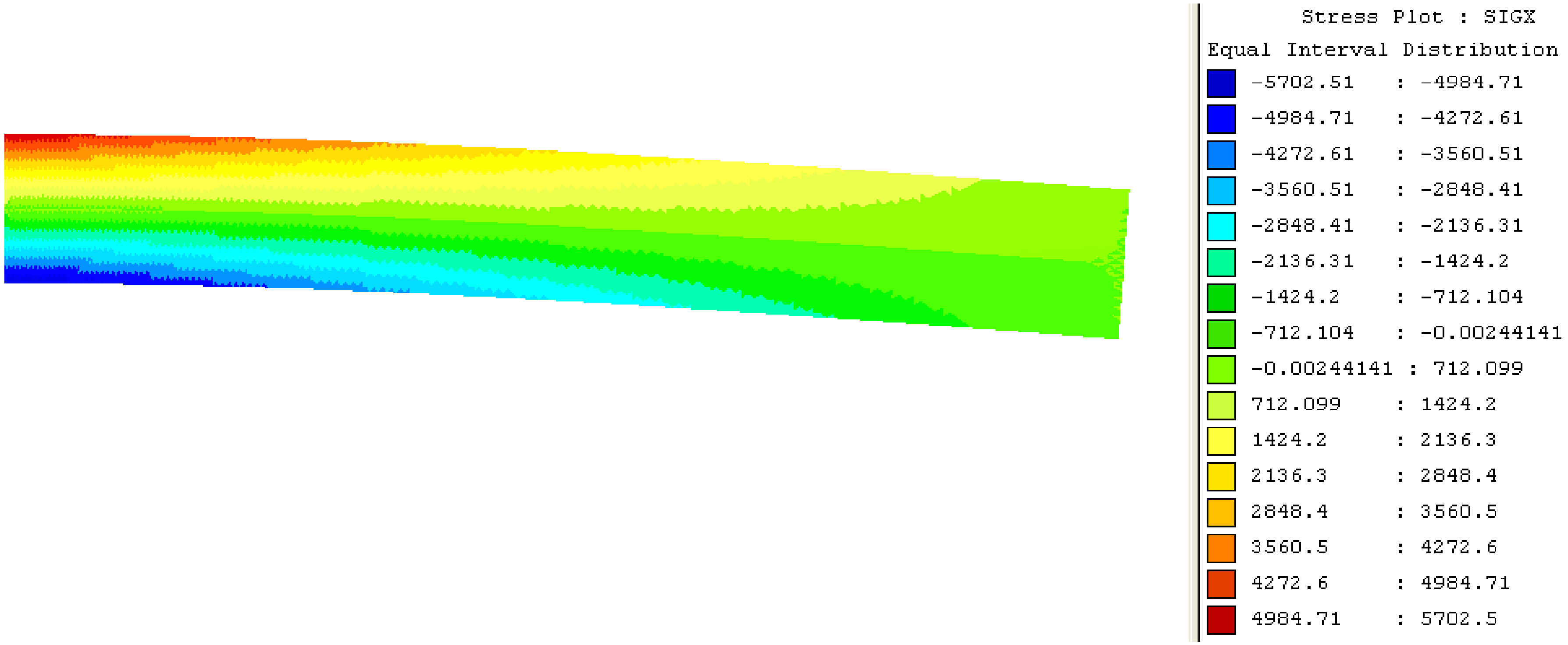


Mesh-1 (8019 nodes)

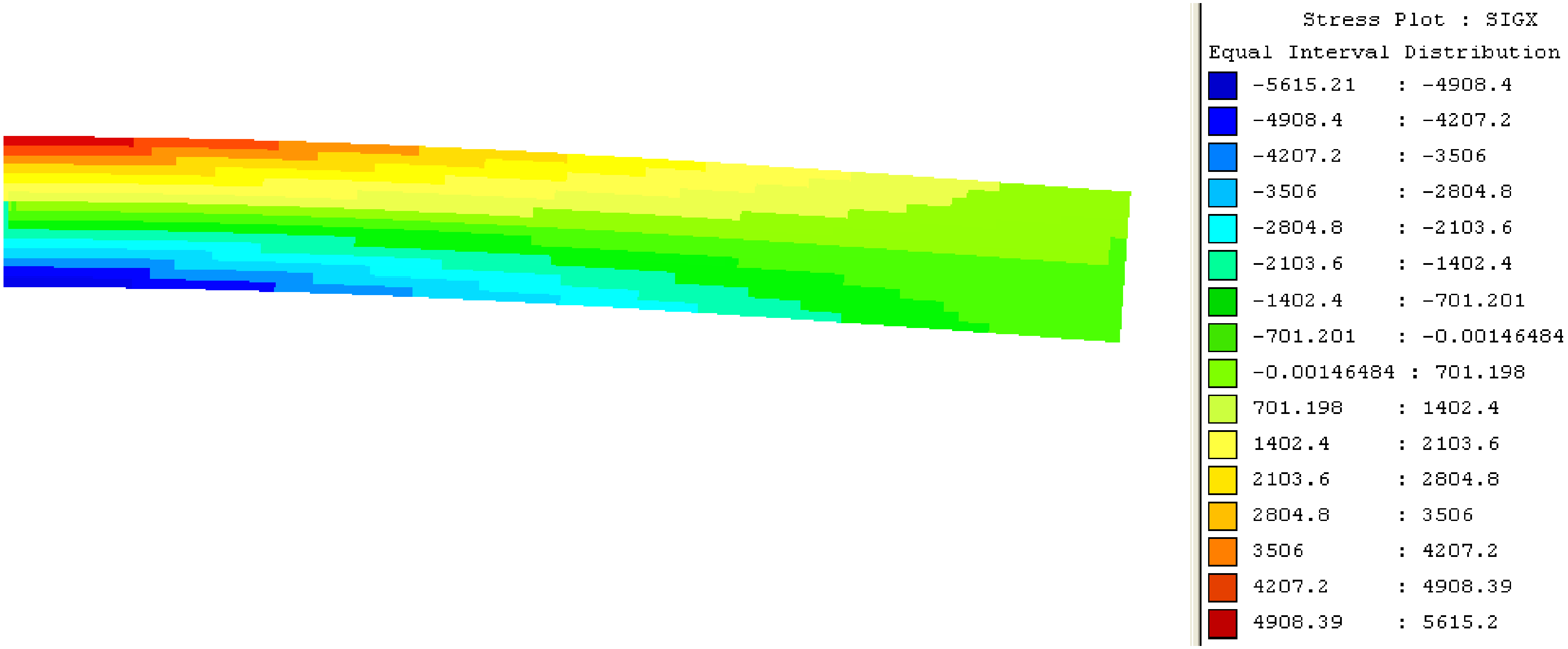
Thin Cantilever Beam: Q4 Model



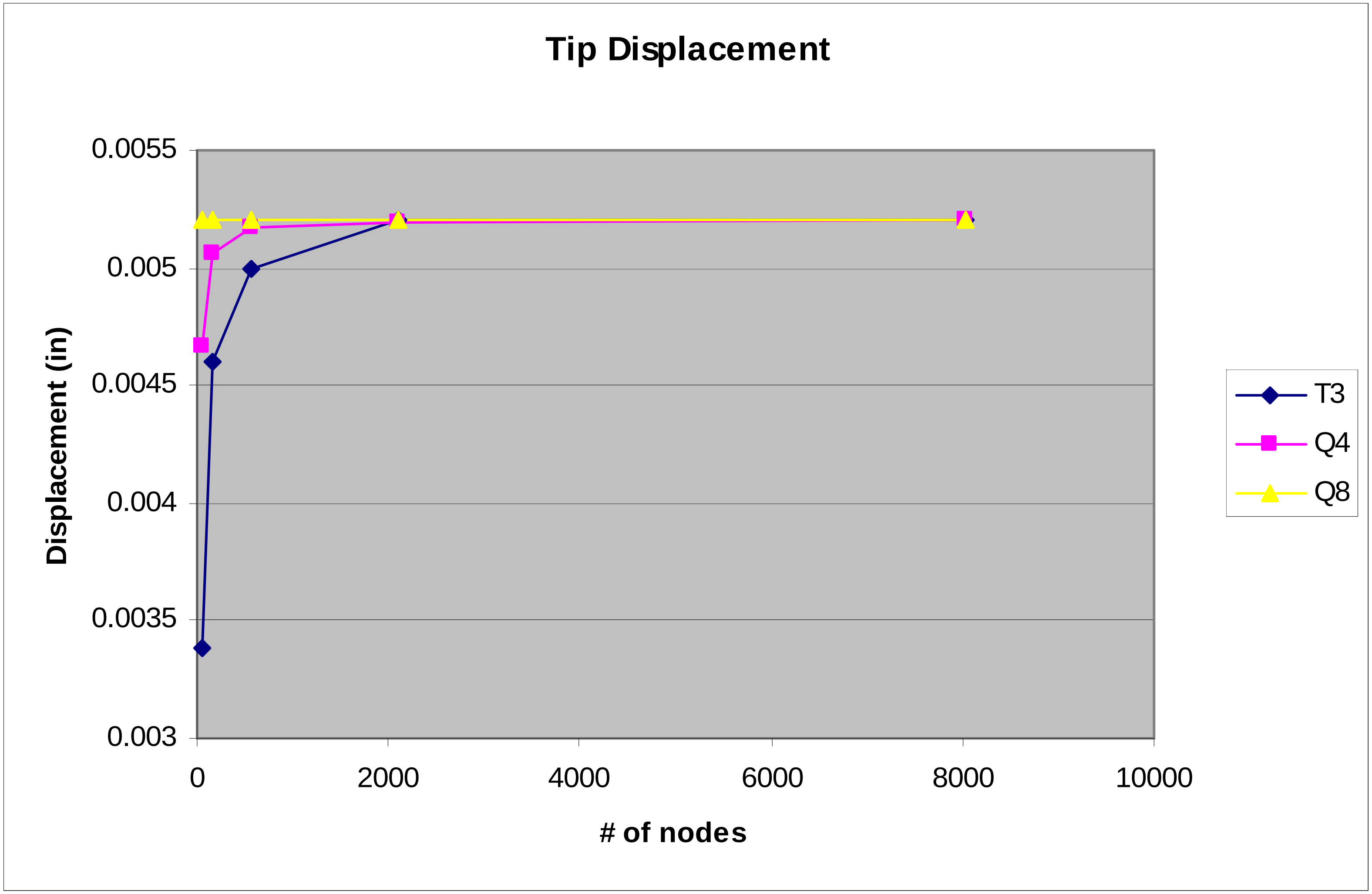
Thin Cantilever Beam: T3 Model



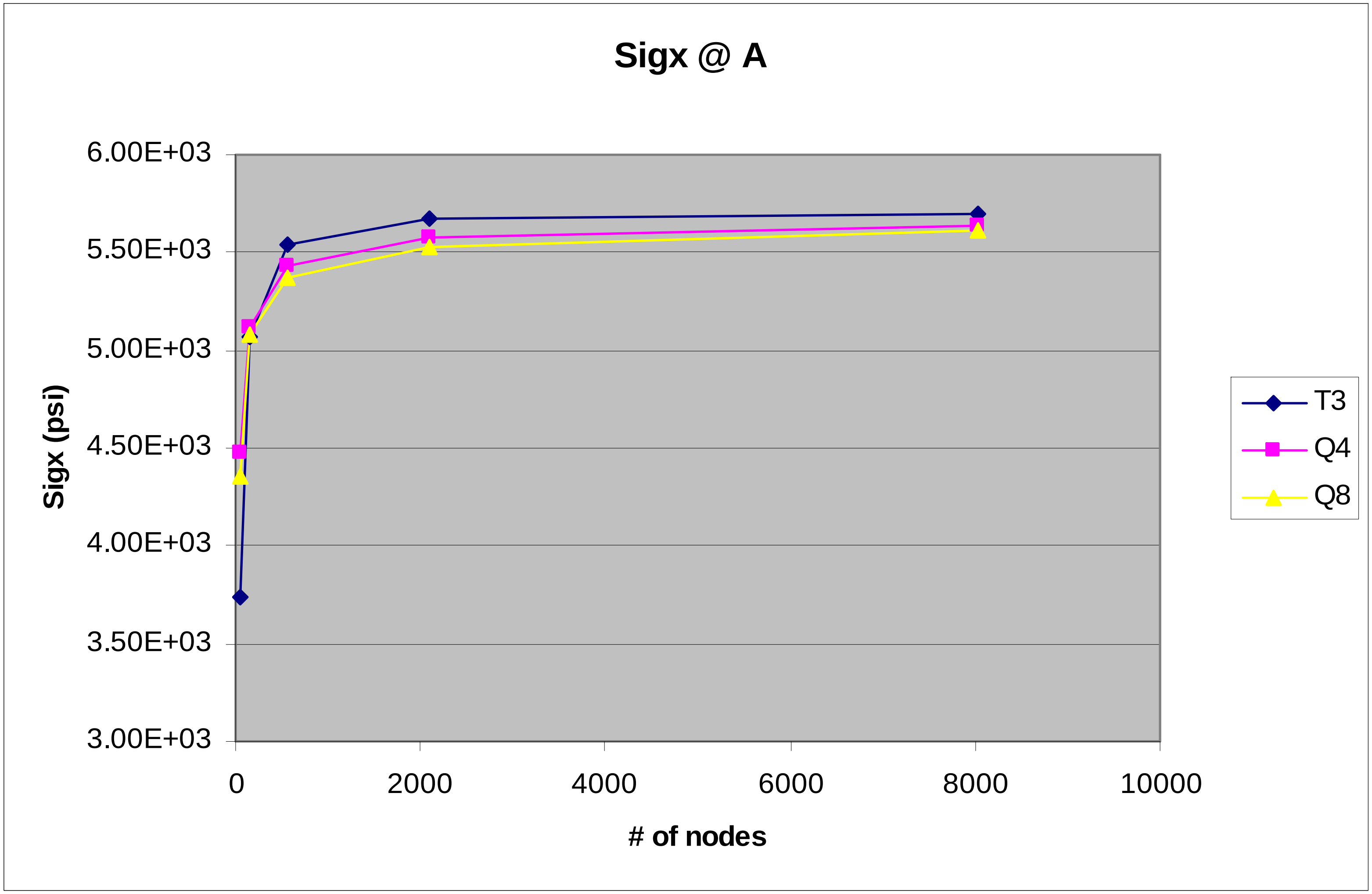
Thin Cantilever Beam: Q8 Model



Comparison of Models

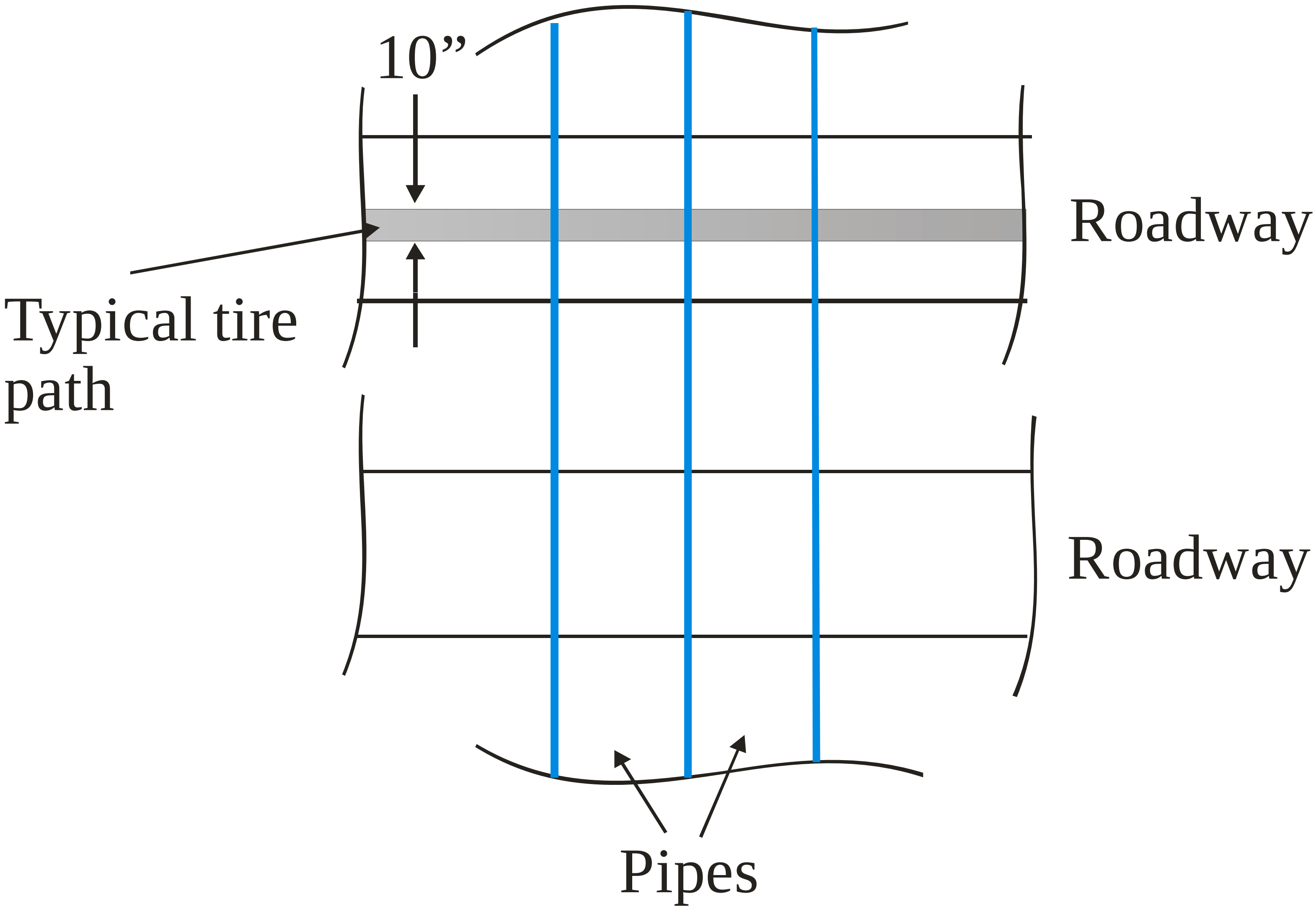


Comparison of Models

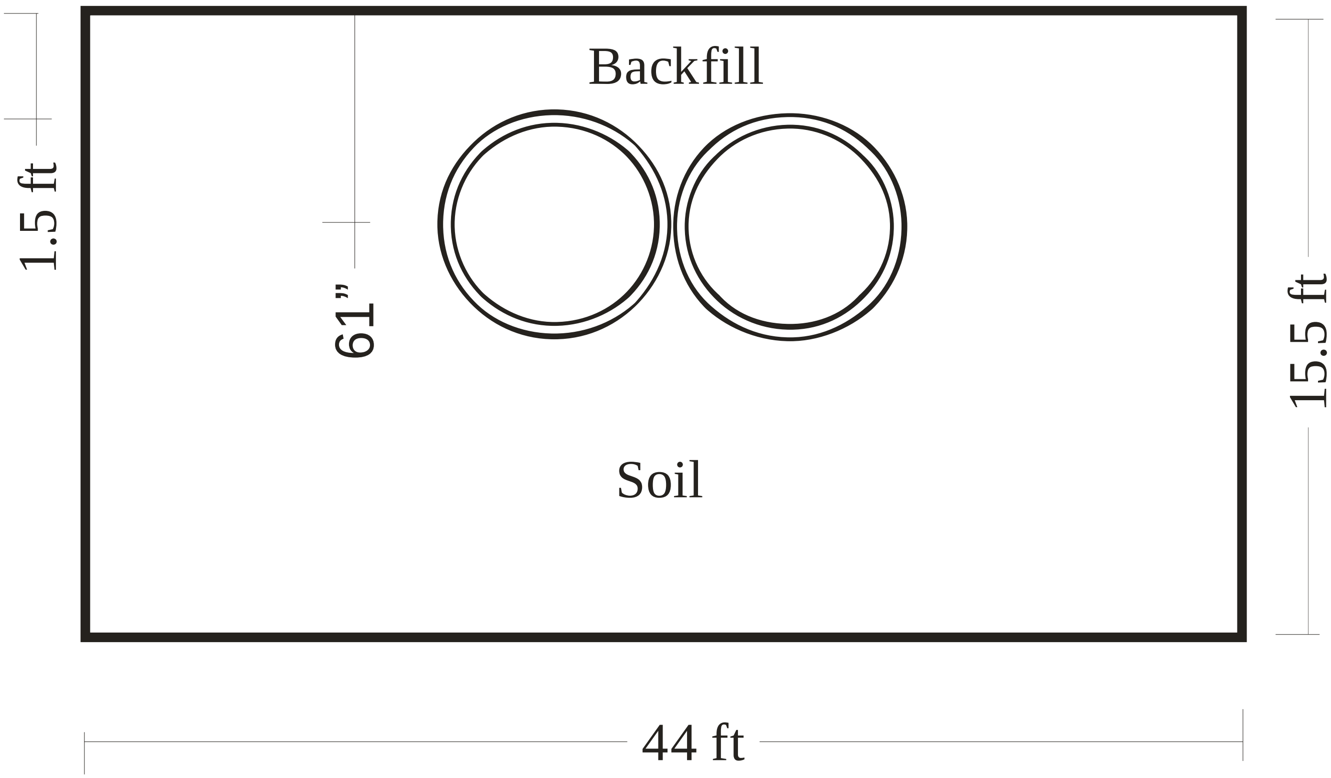


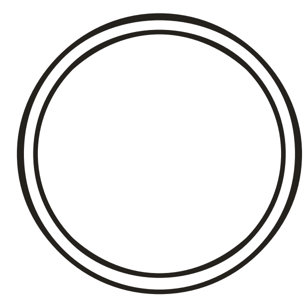
Case Study 2:
Plane Strain Model of Buried Pipes

72” Pipe Crossing



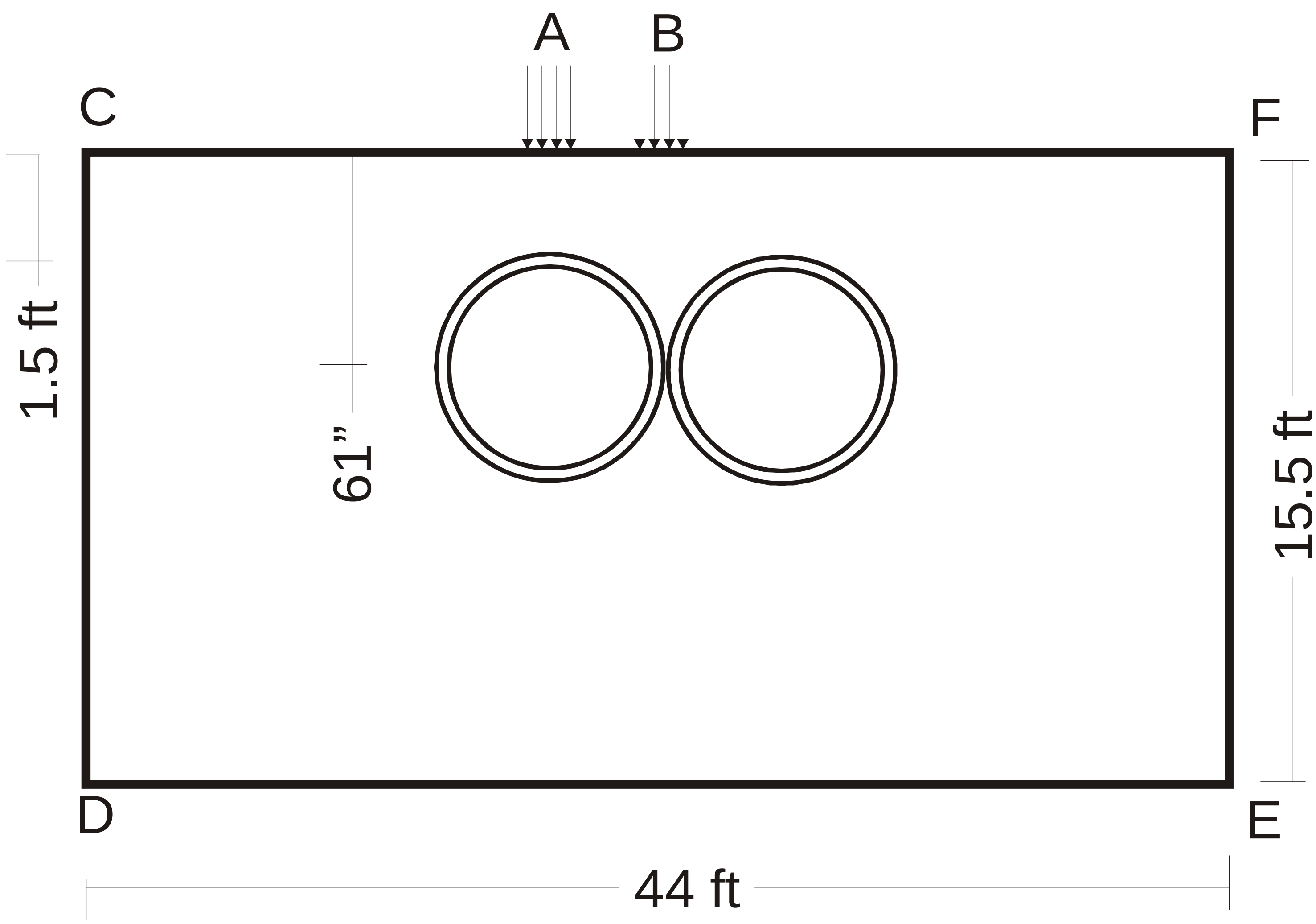
72” Pipe Crossing



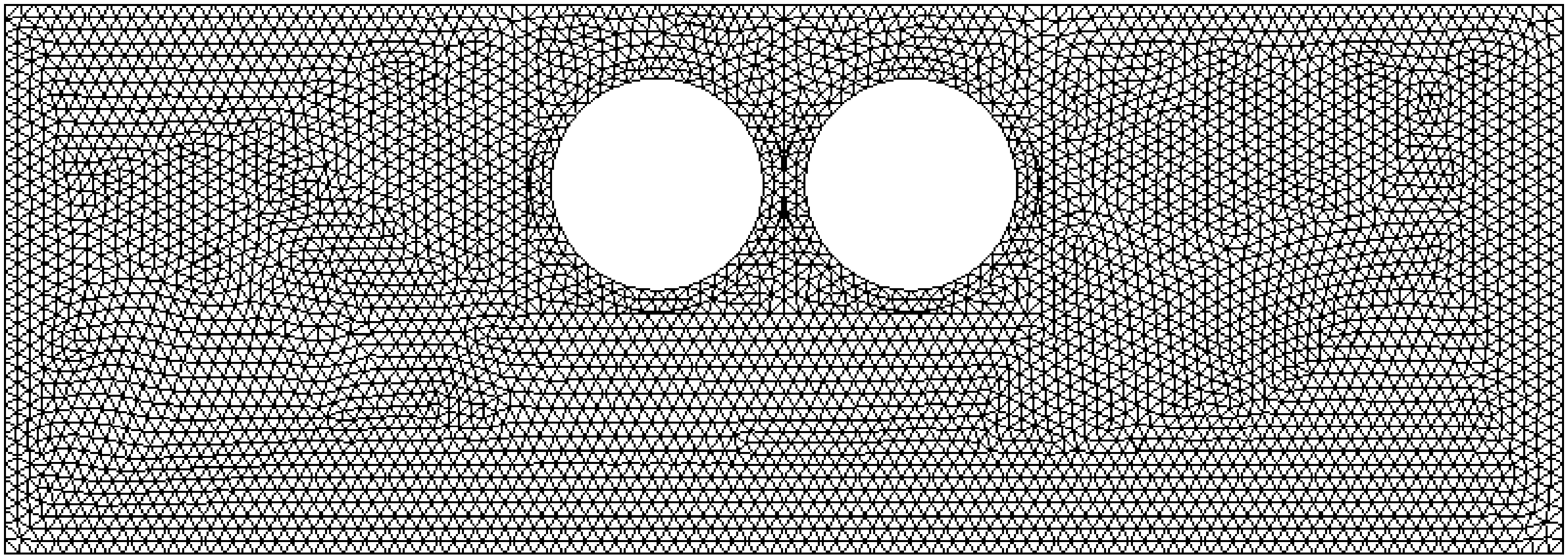


ID 72” OD 86”
Concrete Pipe

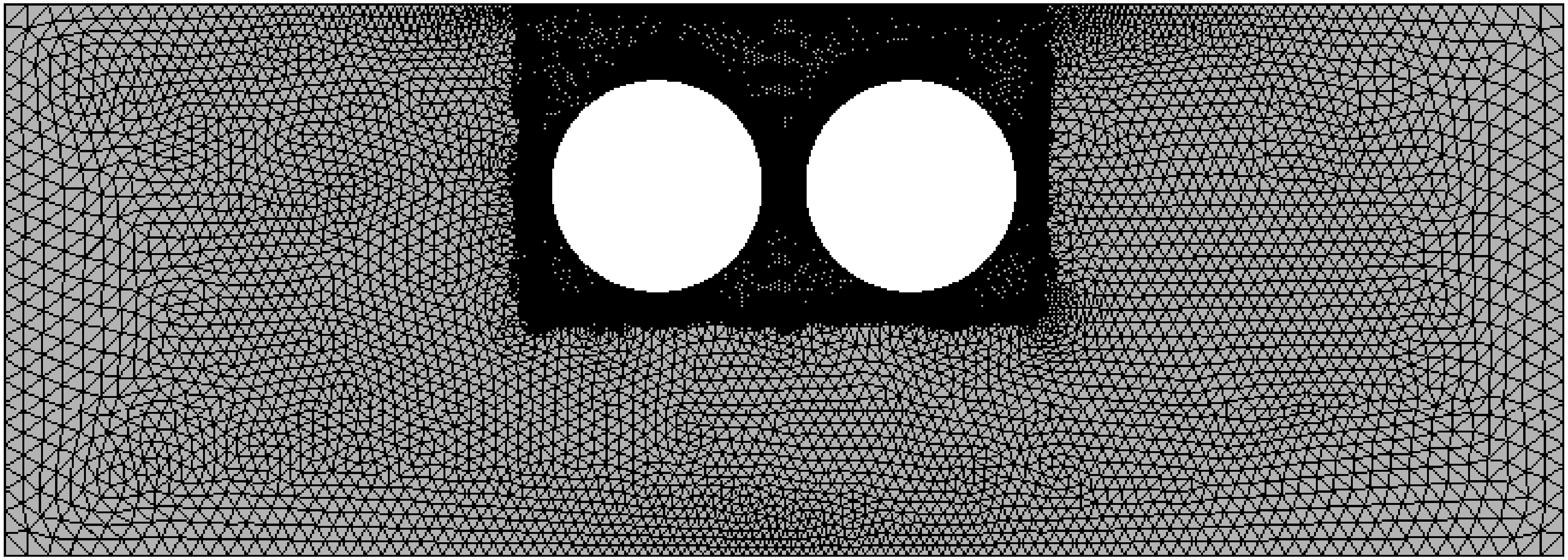
Loading Cases



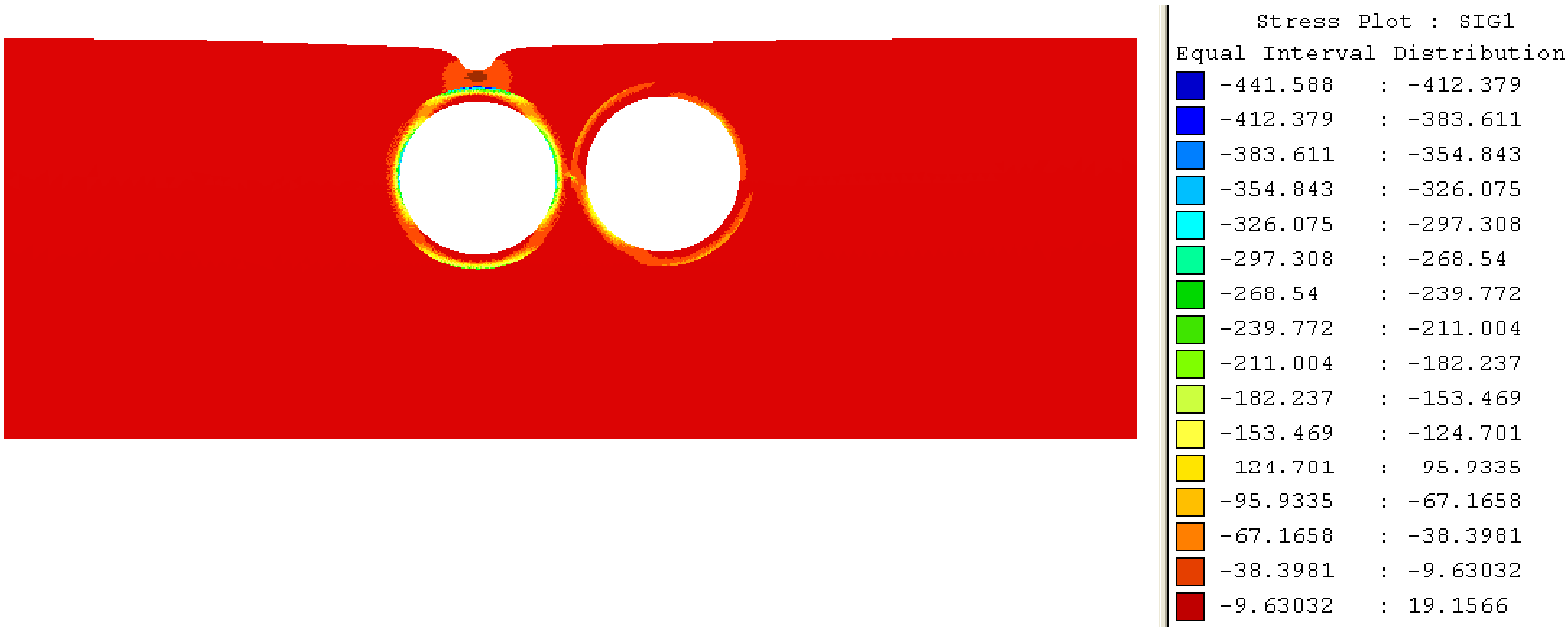
FE Model: Coarse Mesh



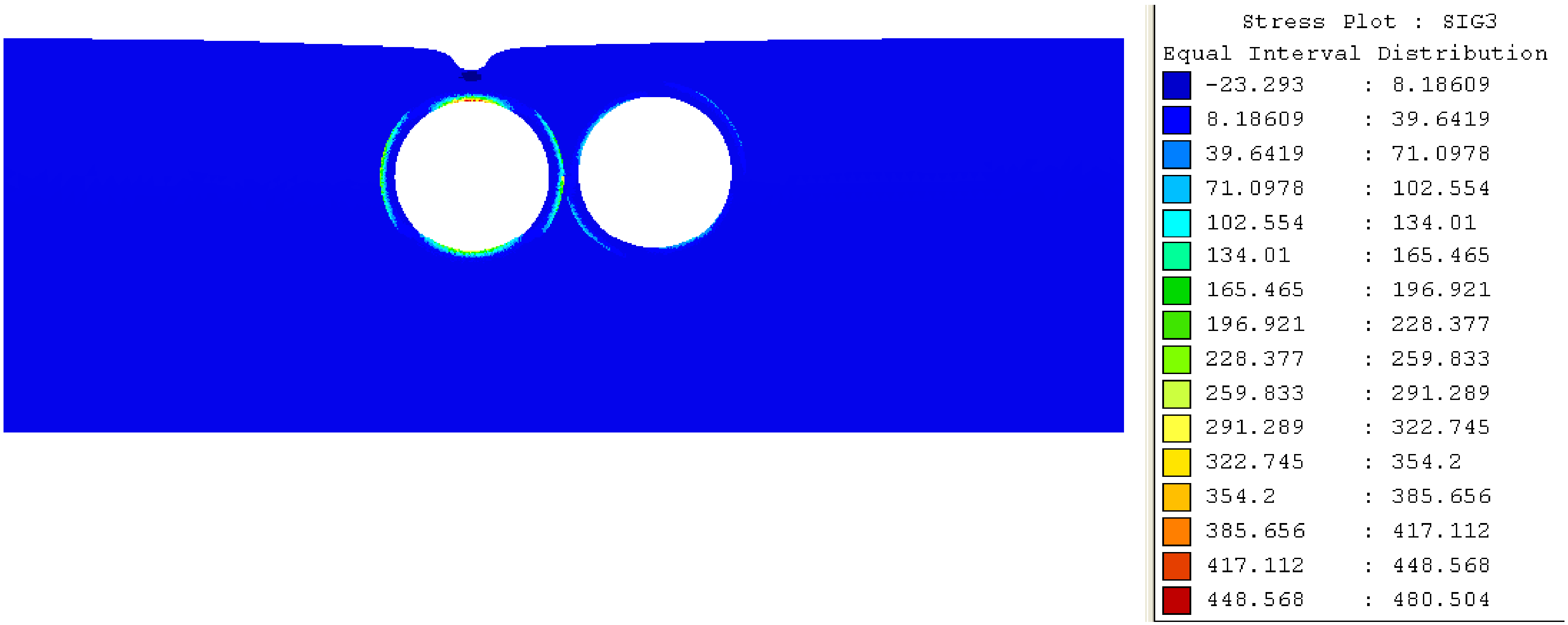
FE Model: Fine Mesh



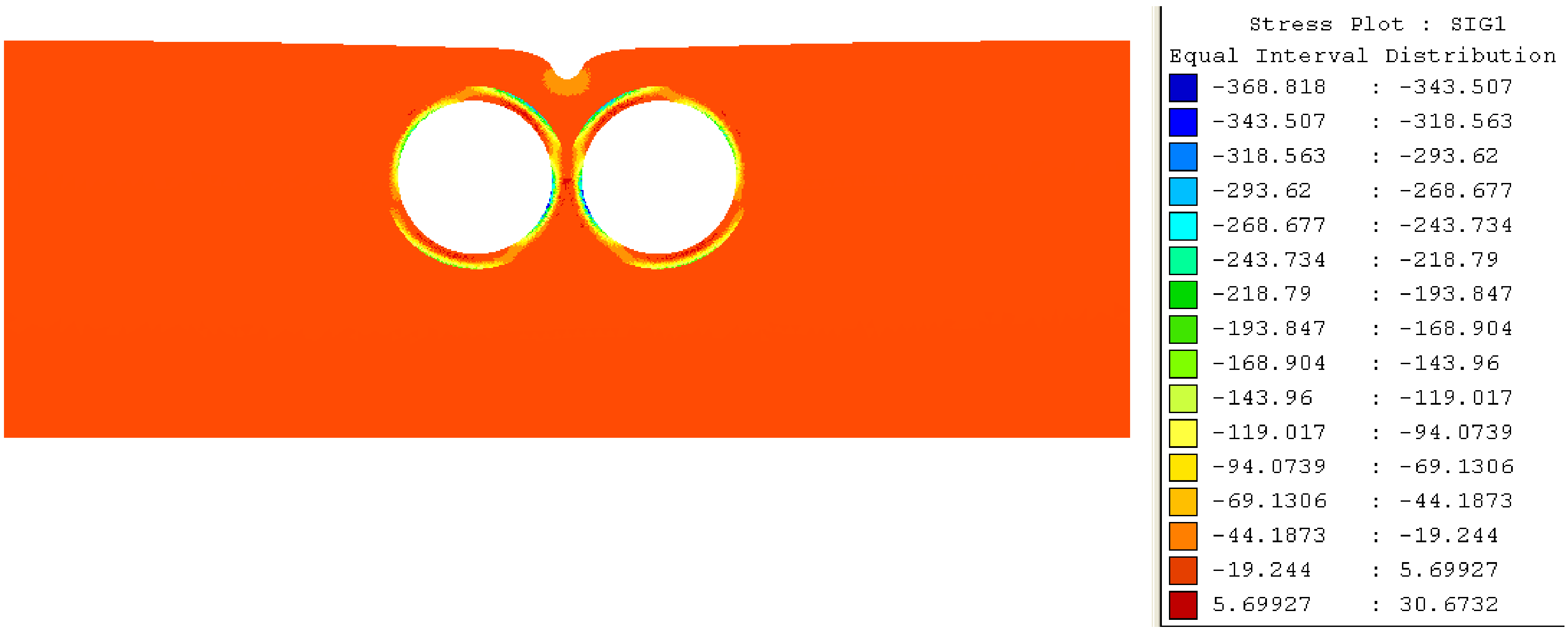
Load Case 1: Sig1



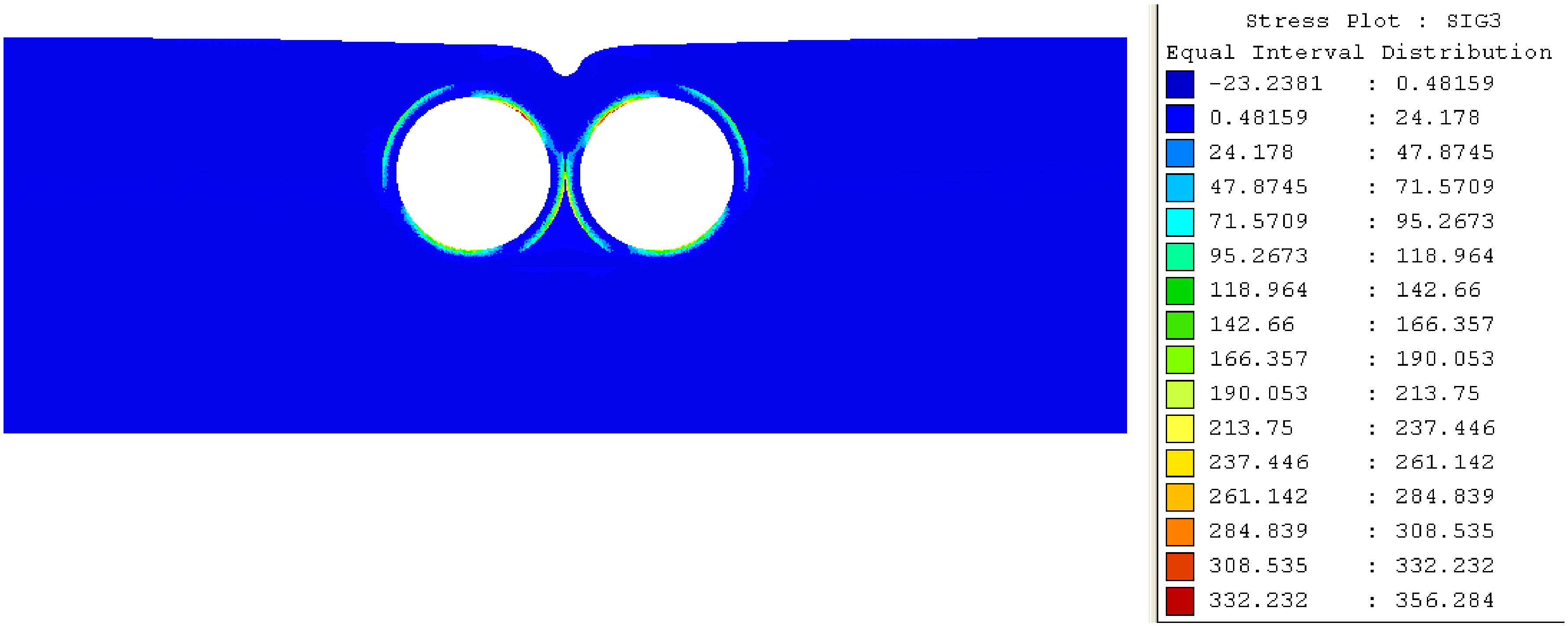
Load Case 1: Sig3



Load Case 2: Sig1



Load Case 2: Sig3



Failure Criteria

Principal Stress

$$\begin{vmatrix} \sigma_x - \sigma & \tau_{xy} & \tau_{zx} \\ \tau_{xy} & \sigma_y - \sigma & \tau_{yz} \\ \tau_{zx} & \tau_{yz} & \sigma_z - \sigma \end{vmatrix} = 0 \quad \Rightarrow \sigma_1 \le \sigma_2 \le \sigma_3$$

von Mises

$$\sigma_e = \frac{1}{\sqrt{2}} \left[\left(\sigma_x - \sigma_y \right)^2 + \left(\sigma_y - \sigma_z \right)^2 + \left(\sigma_z - \sigma_x \right)^2 + 6\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2 \right) \right]^{1/2}$$

Failure Criteria

Tresca

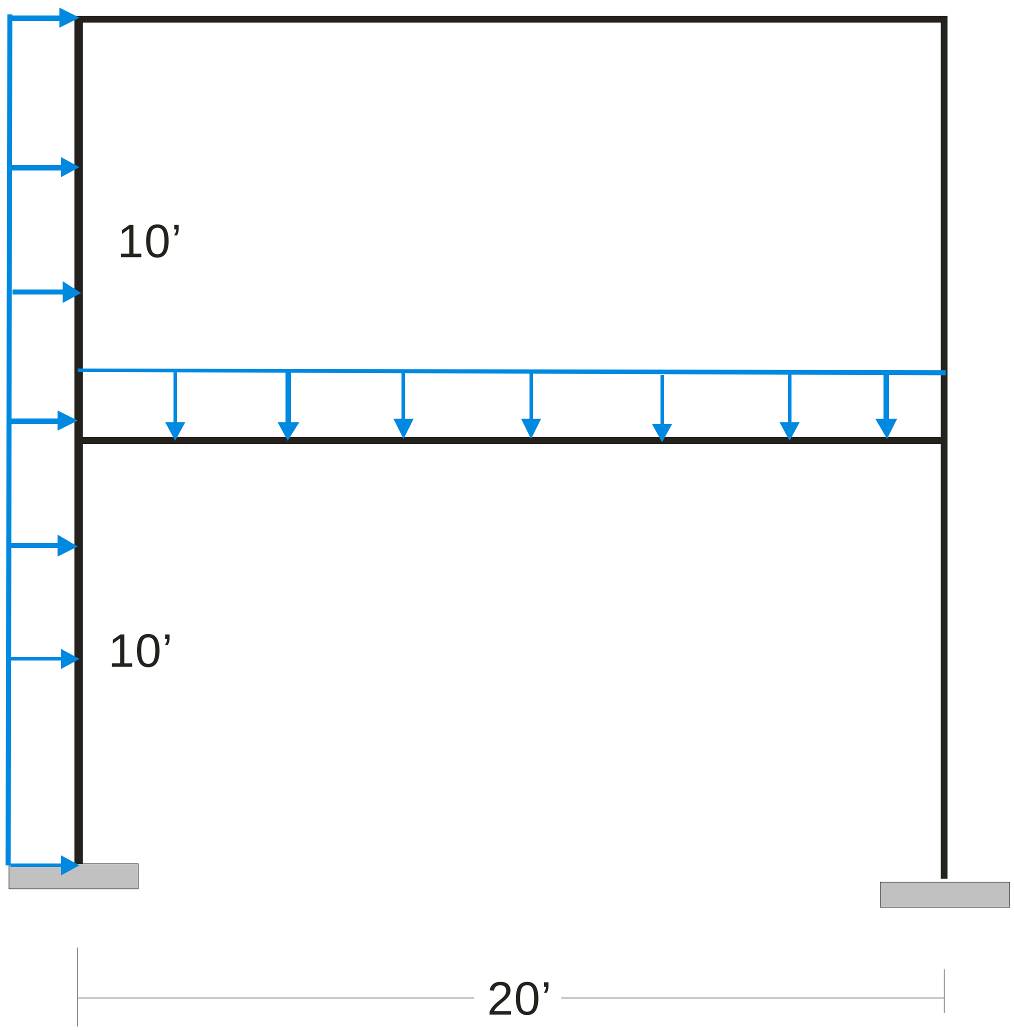
$$\tau_{\max} = \left| \sigma_3 - \sigma_1 \right|$$

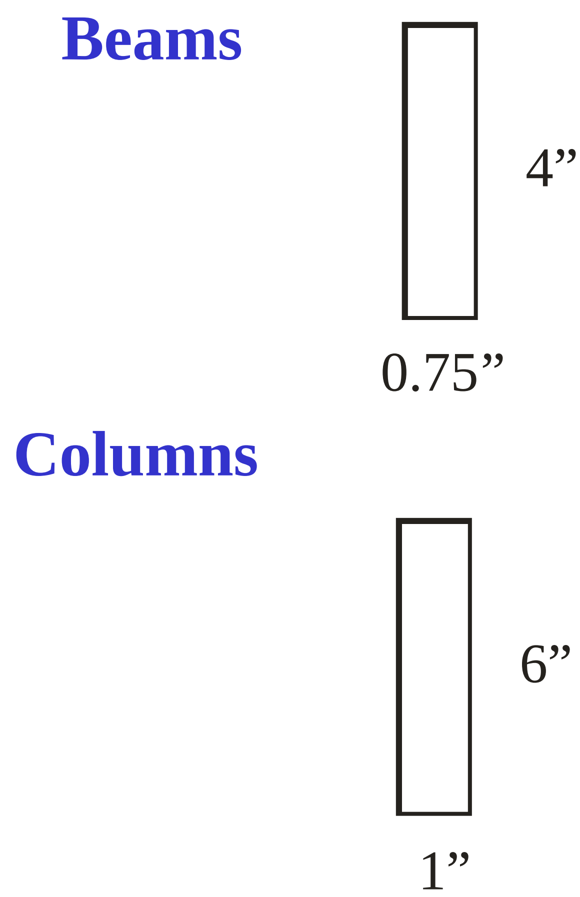
Mohr's

$$\frac{\sigma_3}{\sigma_t} - \frac{\sigma_1}{\sigma_c} \geq 1$$

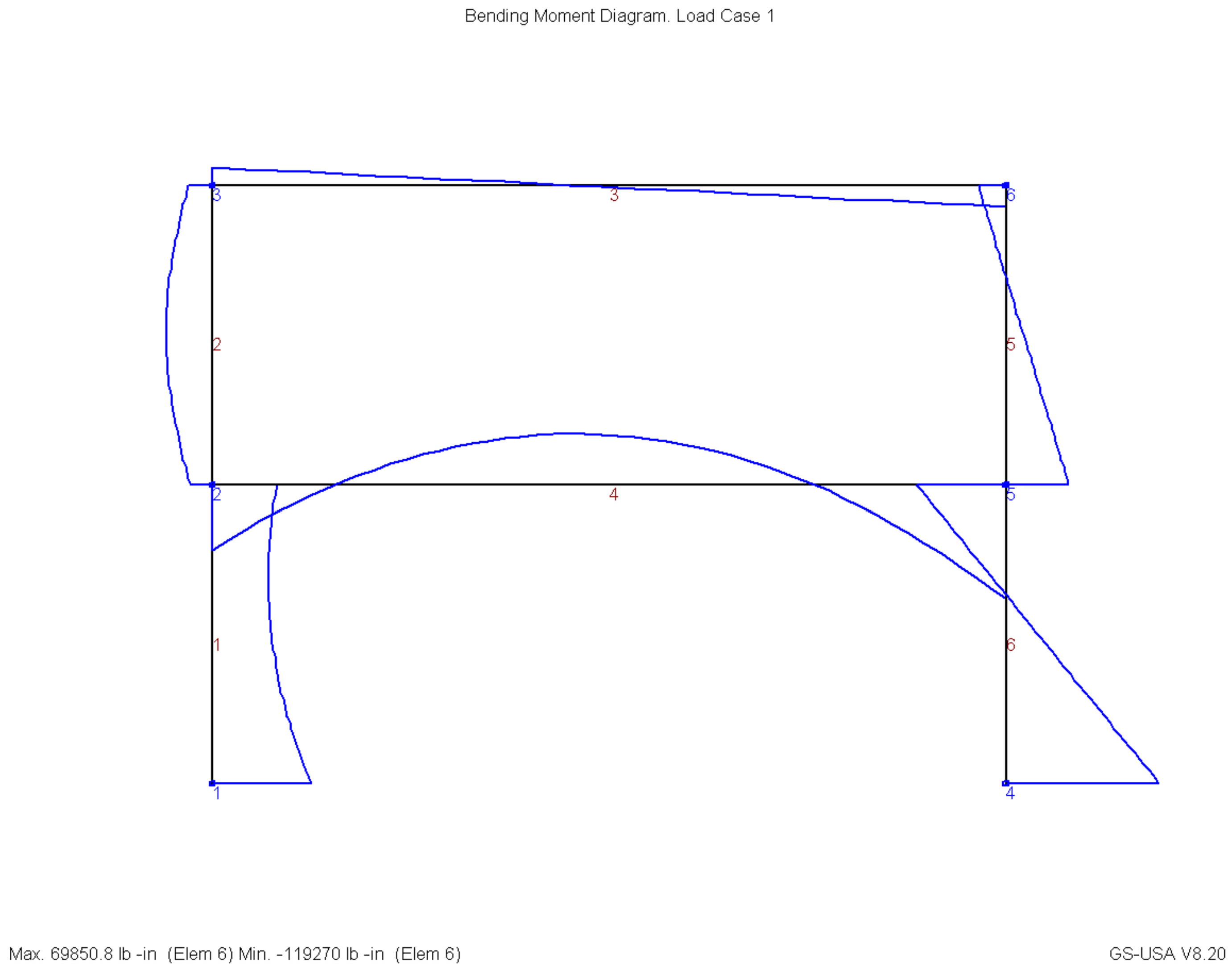
Case Study 3: Global-Local Analysis

Planar Steel Frame

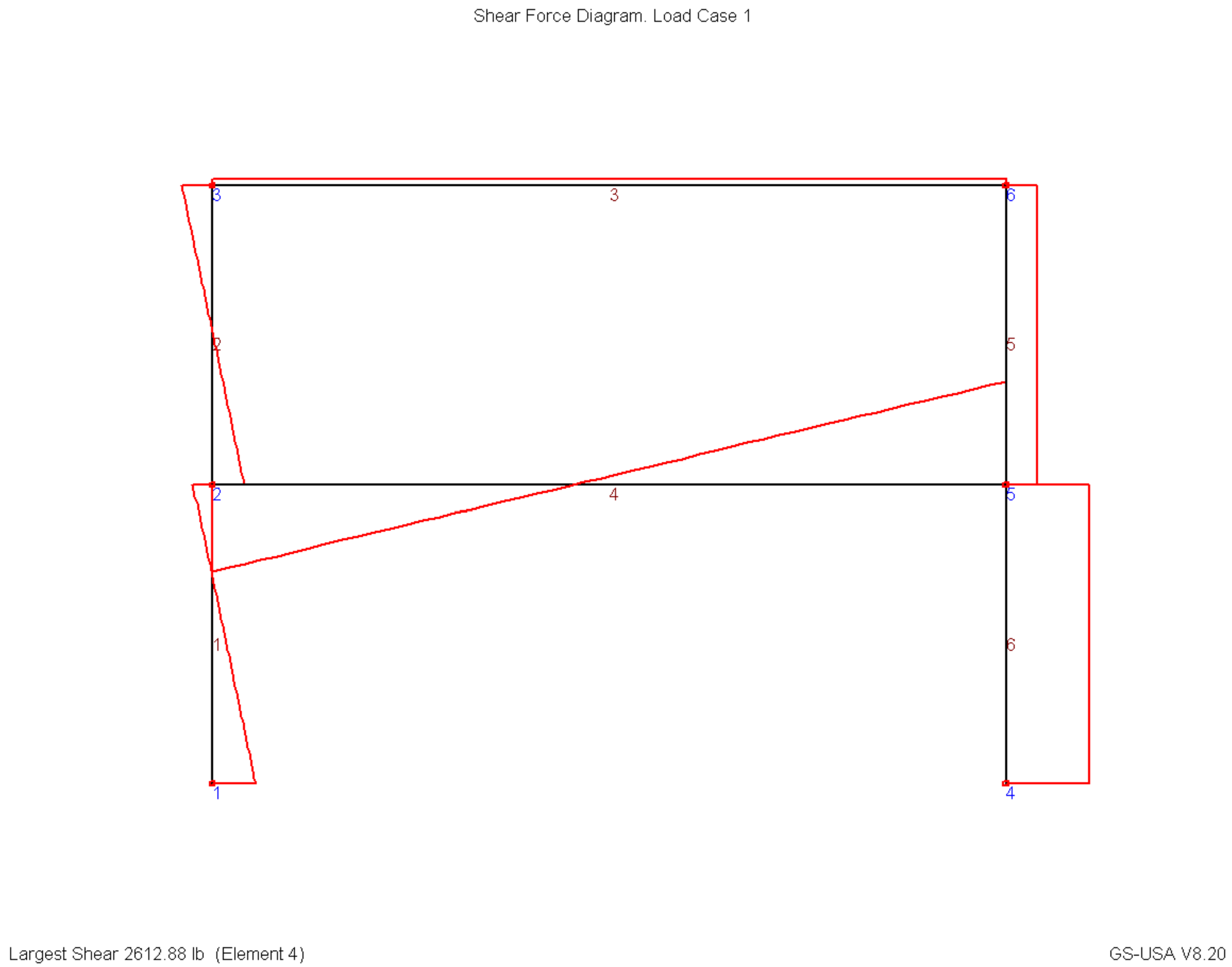




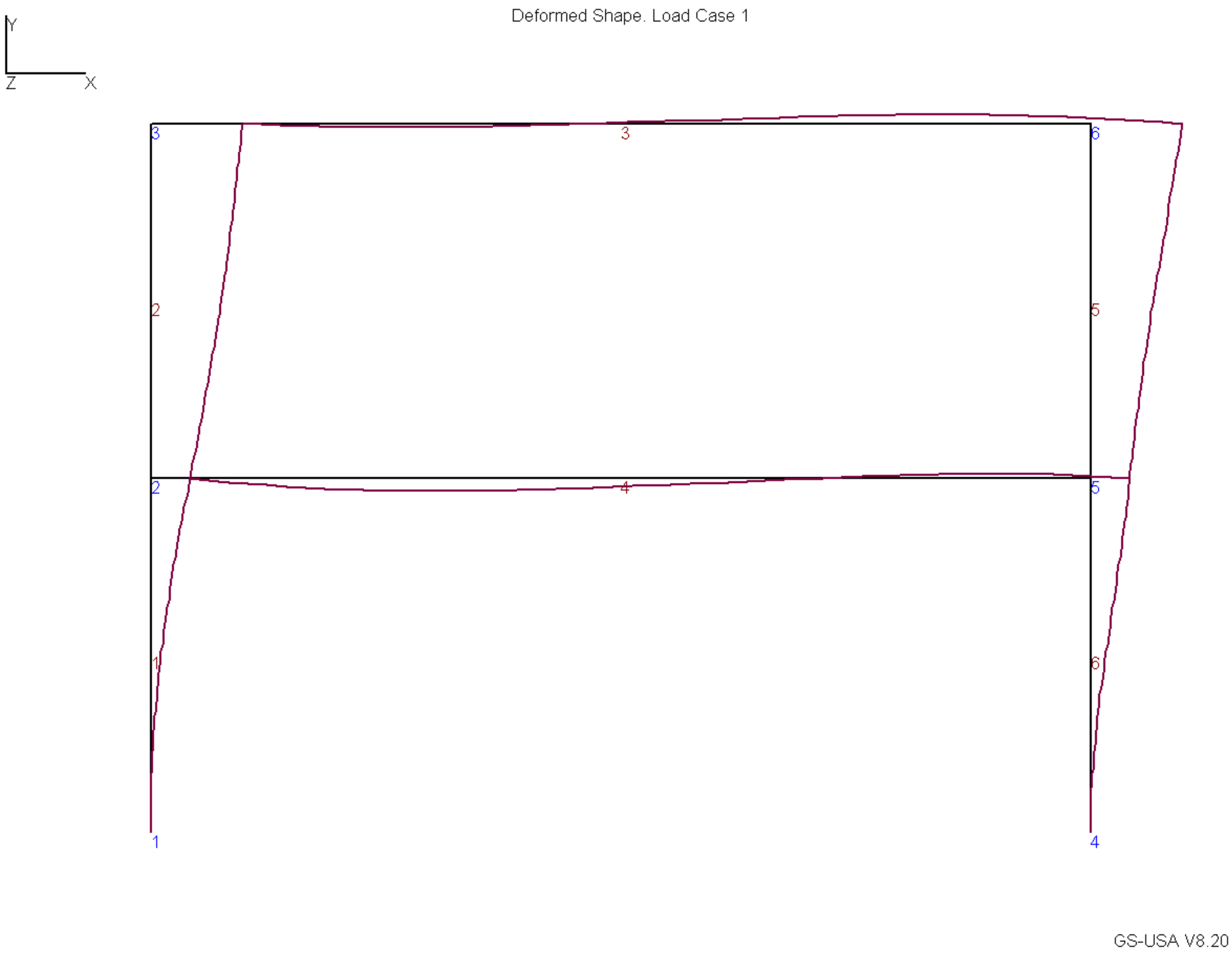
Bending Moment Diagram



Shear Force Diagram

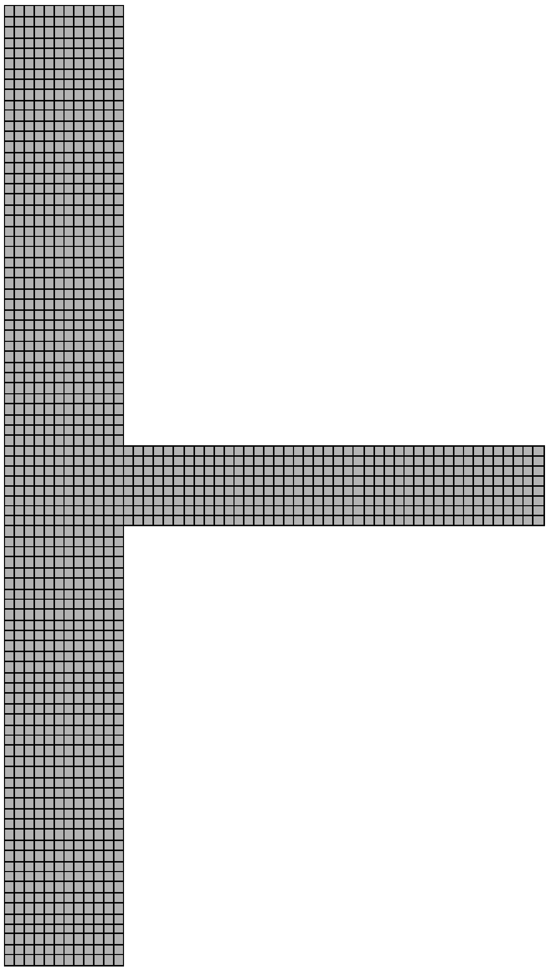


Deflected Shape

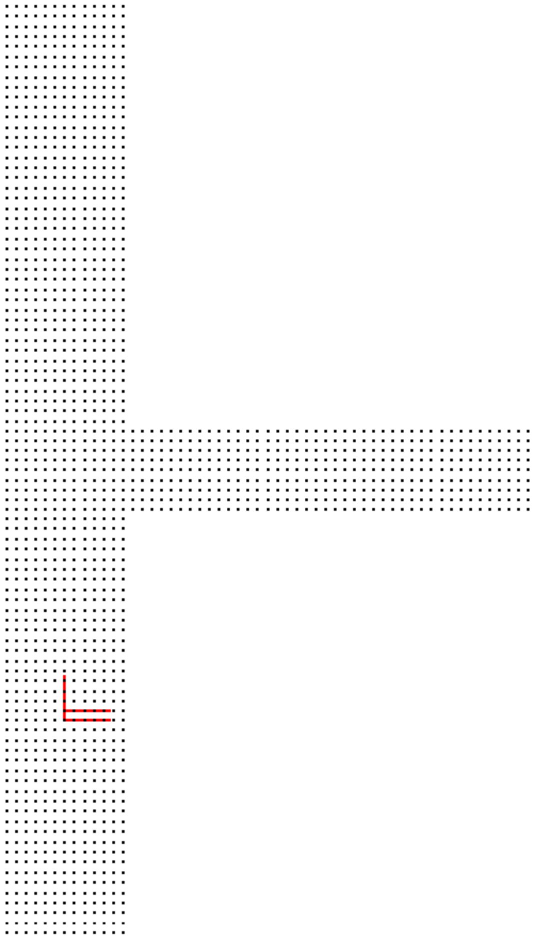


Plane Stress: Local Analysis

Mesh

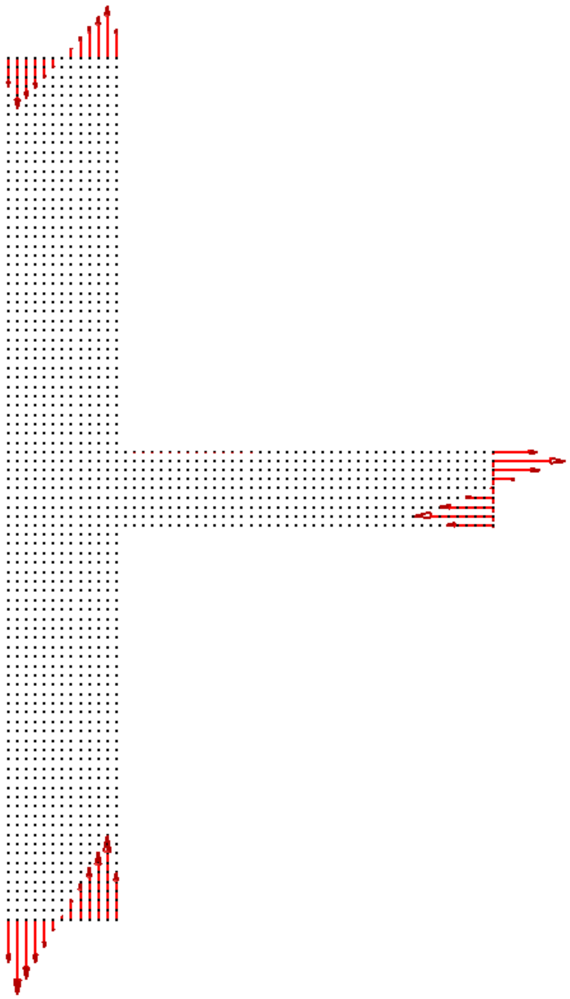


BCs

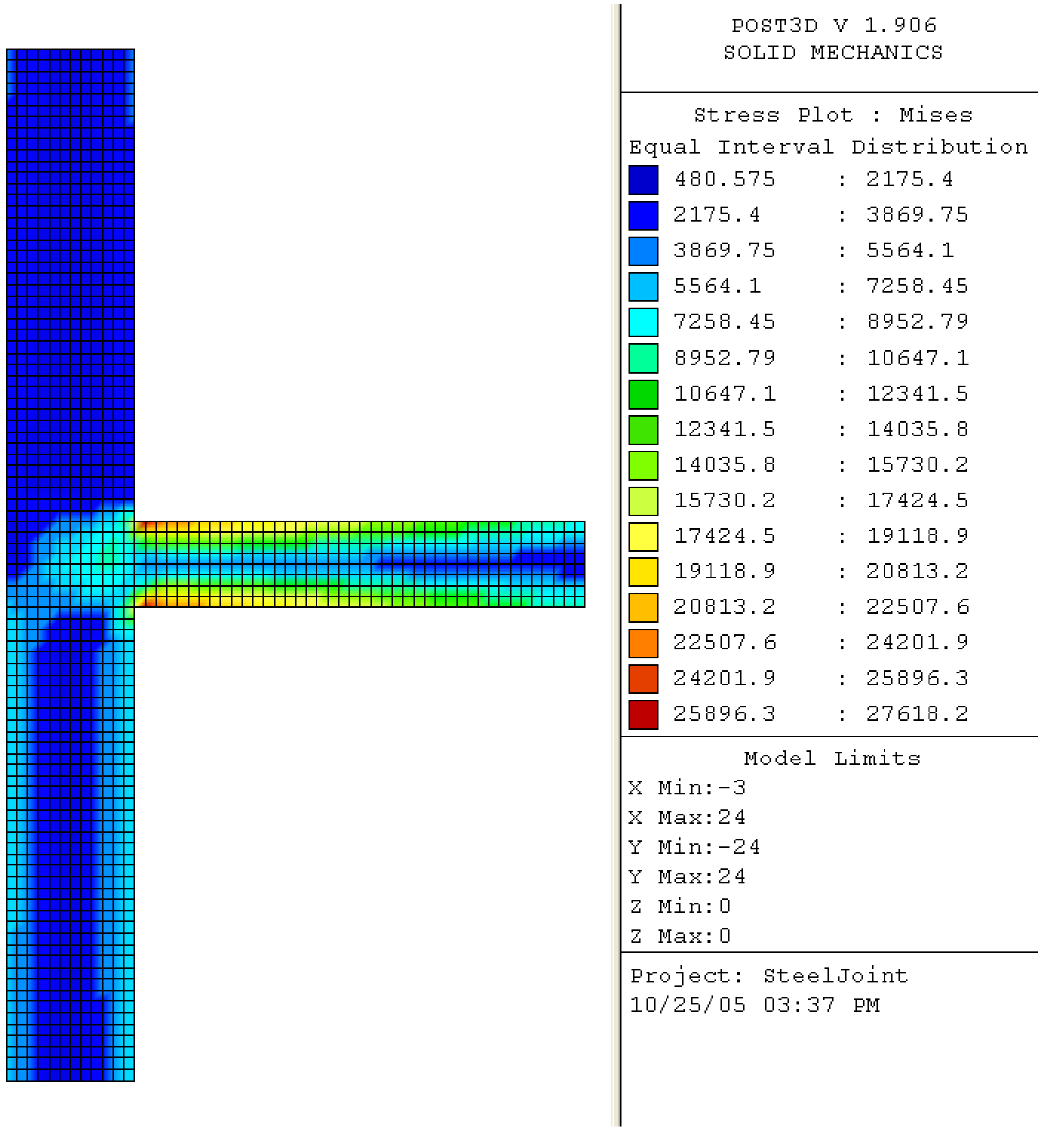


Plane Stress: Local Analysis

Loading

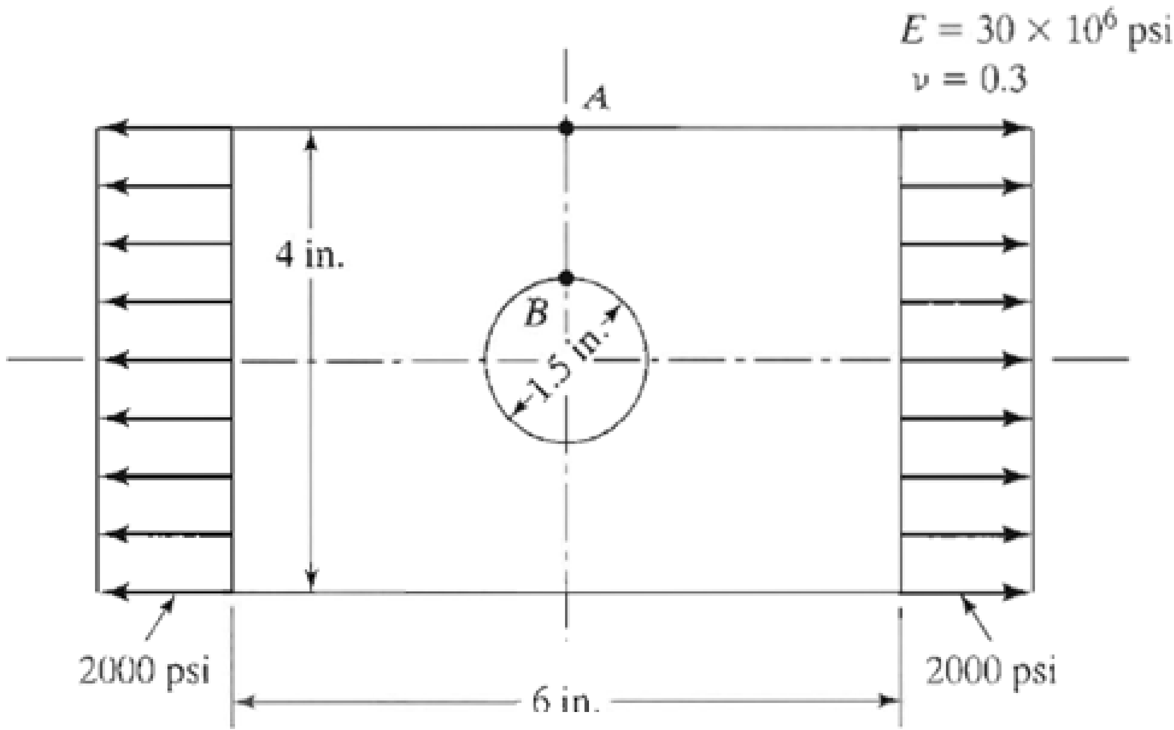


Plane Stress: Local Analysis



Modeling Project 1-1

For the steel plate with a hole shown in the figure below, determine the following:
(a) the deformed shape of the hole, and
(b) maximum von Mises stress.



Modeling Project 1-2-1

Problem 1: For the torque arm shown in Figure 1, determine the following:
(a) the maximum displacement, and
(b) maximum von Mises stress.

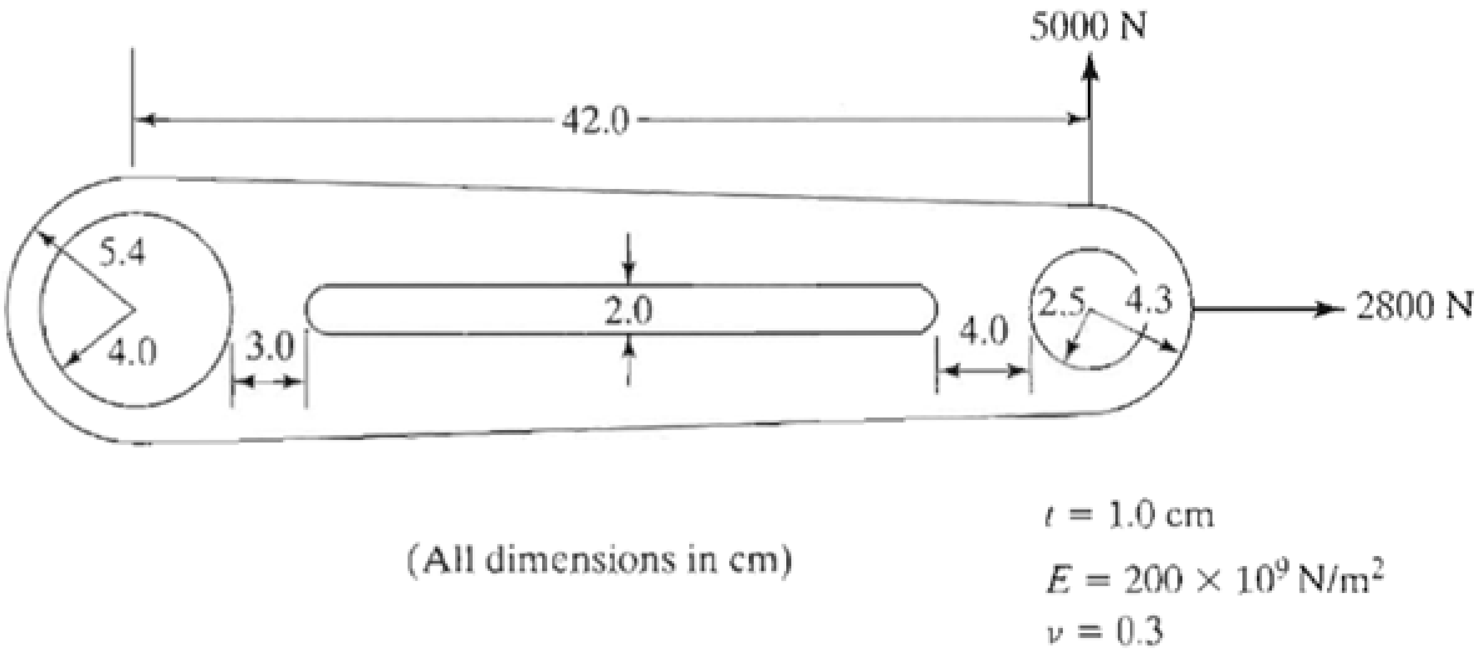


Figure 1

Modeling Project 1-2-2

Problem 2: A large, flat surface of a steel body is subjected to a line load of 100 lb/in. Consider an enclosure as shown in Fig. 2 and determine the largest deformation of the surface and the maximum von Mises stress.

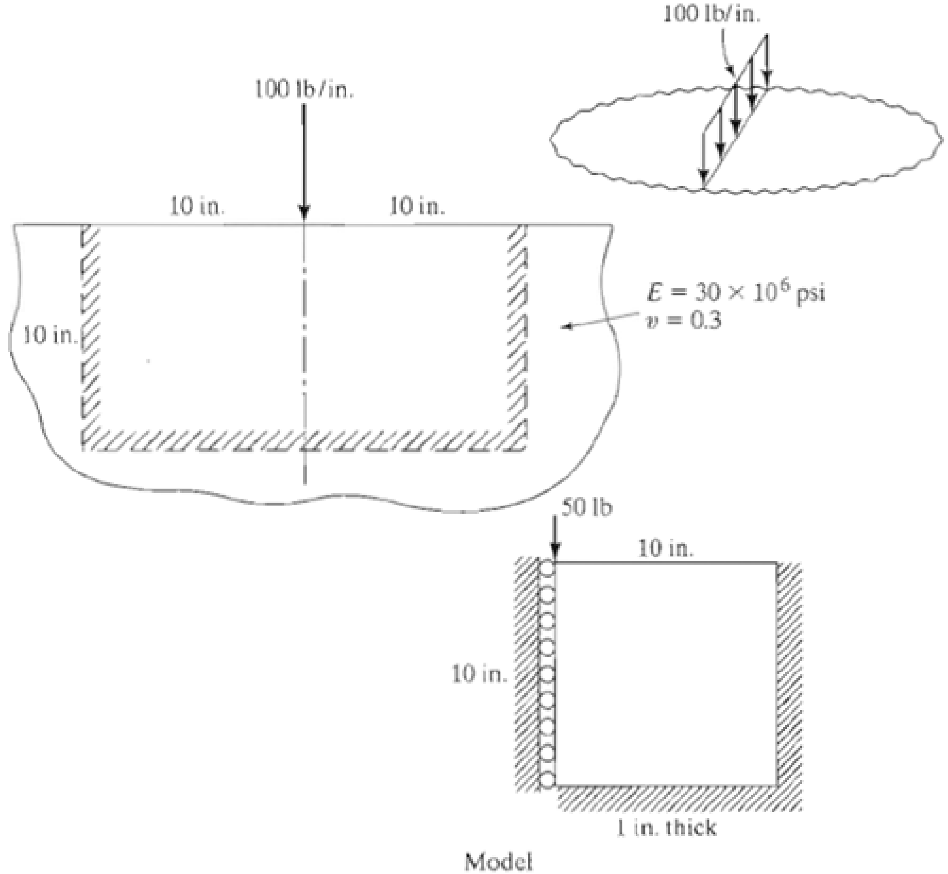


Figure 2

Modeling Project 1-3-1

Problem 1: A syringe-plunger is shown in Figure 1. Model the glass syringe assuming that the 4 mm hole end is closed under test conditions. Obtain the deformation and stresses and compare the maximum principal stress with the ultimate tensile strength of glass.

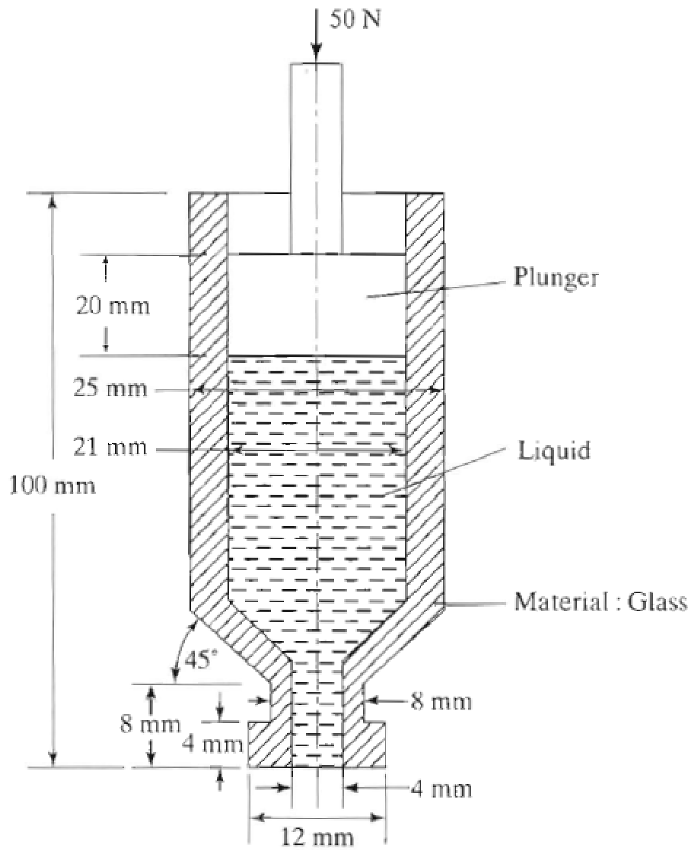


Figure 1

Modeling Project 1-3-2

Problem 2: A Belleville spring is a conical disk spring. For the spring shown in Figure 2, determine the axial load required to flatter the spring. Solve the problem using the incremental approach and plot the load-deflection curve as the spring flattens.

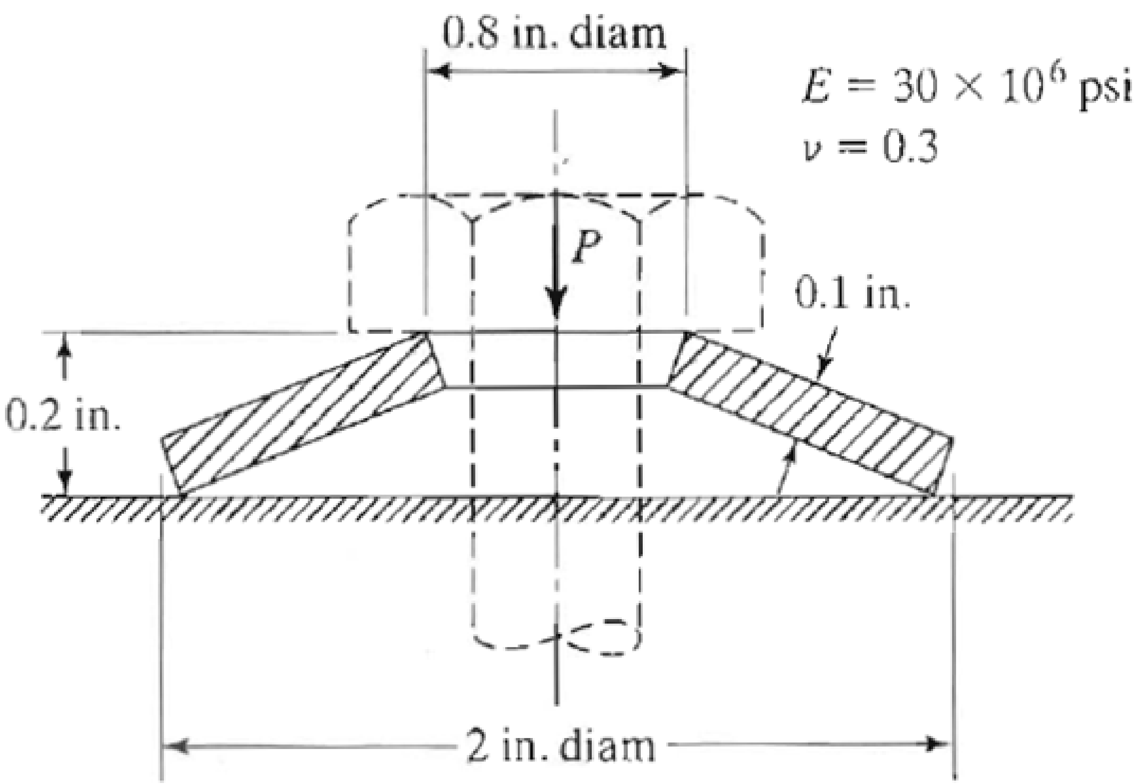


Figure 2

Modeling Project 1-3-3

Problem 3: A half-symmetric model of a plain concrete culvert (E=32 GPa, v=0.15) is shown in Figure 3. The pavement load is a uniformly distributed load of 5000 N/m^2. Determine the location and magnitude of the largest and the smallest principal stress.

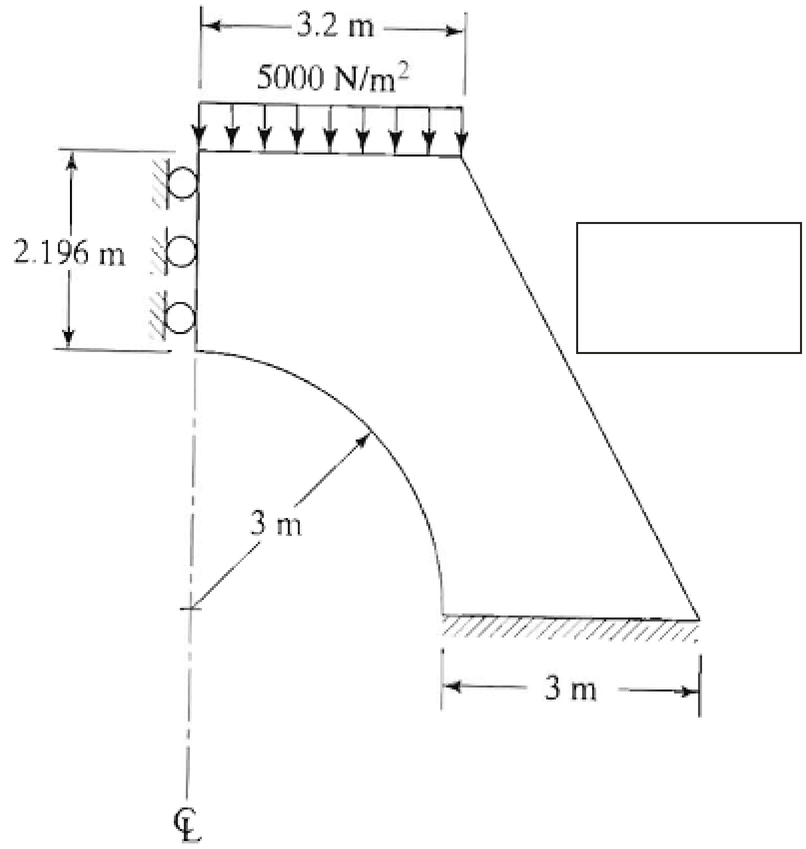


Figure 3