CEE432/CEE532/MAE541
Developing Software for Engineering Applications

Lecture 22: Finite Element Modeling of Structural Systems

Assumptions

- Linear stress-strain relationship
  - Same behavior in tension and compression
  - No damage
  - No unloading and/or reloading
- Elastic behavior
- Small displacements
- Small strains

Type of Analyses

- Static

$$\mathbf{K}_{n\times n}\mathbf{D}_{n\times m} = \mathbf{F}_{n\times m}$$

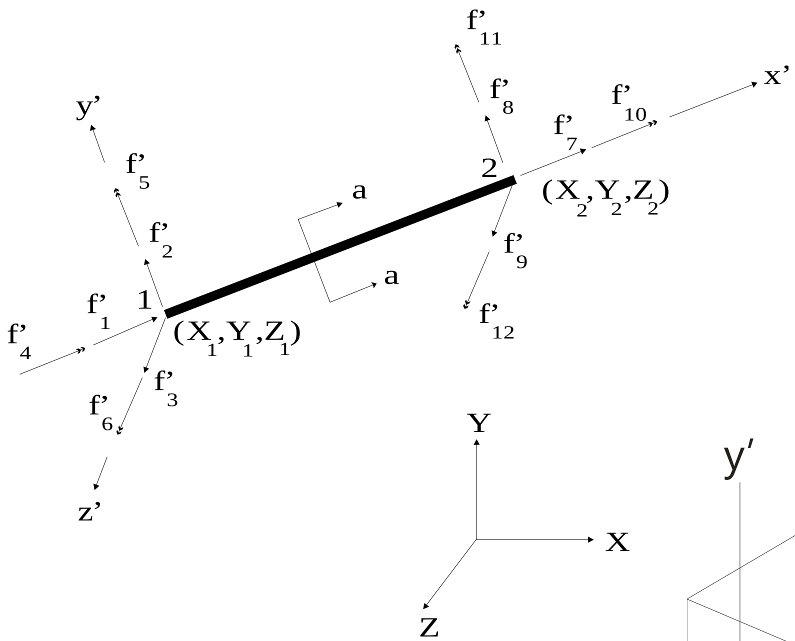
- Modal

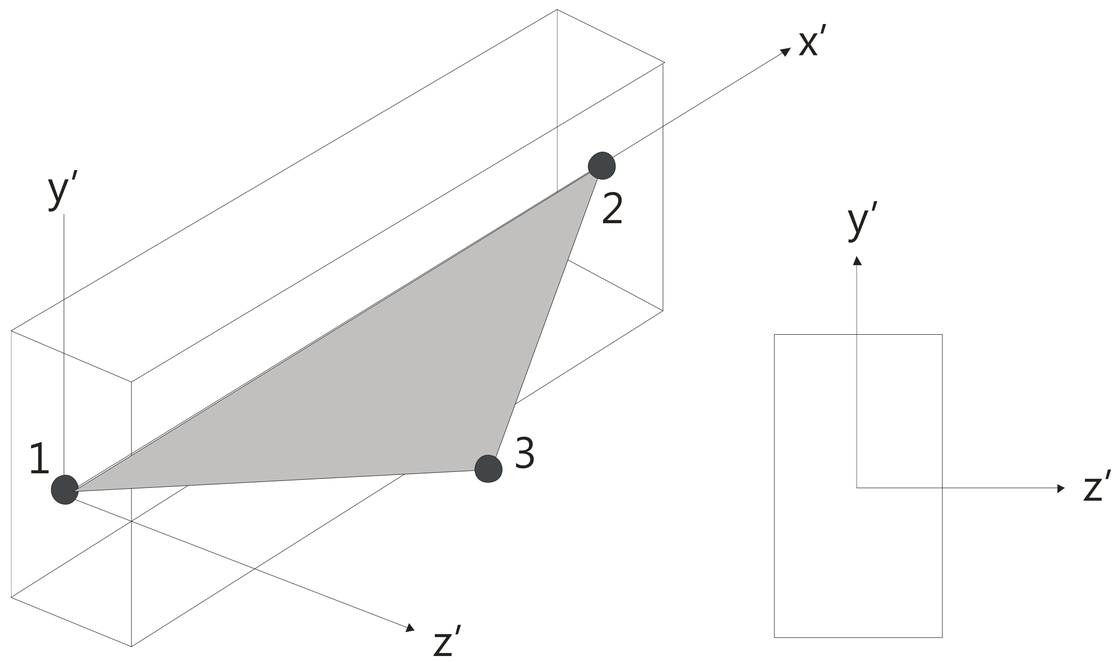
$$\mathbf{K}_{n\times n}\mathbf{\Phi}_{n\times n} = \mathbf{\Lambda}_{n\times n}\mathbf{M}_{n\times n}\mathbf{\Phi}_{n\times n}$$

- Dynamic

$$\mathbf{M}_{n\times n}\ddot{\mathbf{D}}_{n\times m} + \mathbf{C}_{n\times n}\dot{\mathbf{D}}_{n\times m} + \mathbf{K}_{n\times n}\mathbf{D}_{n\times m} = \mathbf{F}(t)_{n\times m}$$

Beam Finite Element





View a-a

Plate/Shell Finite Element

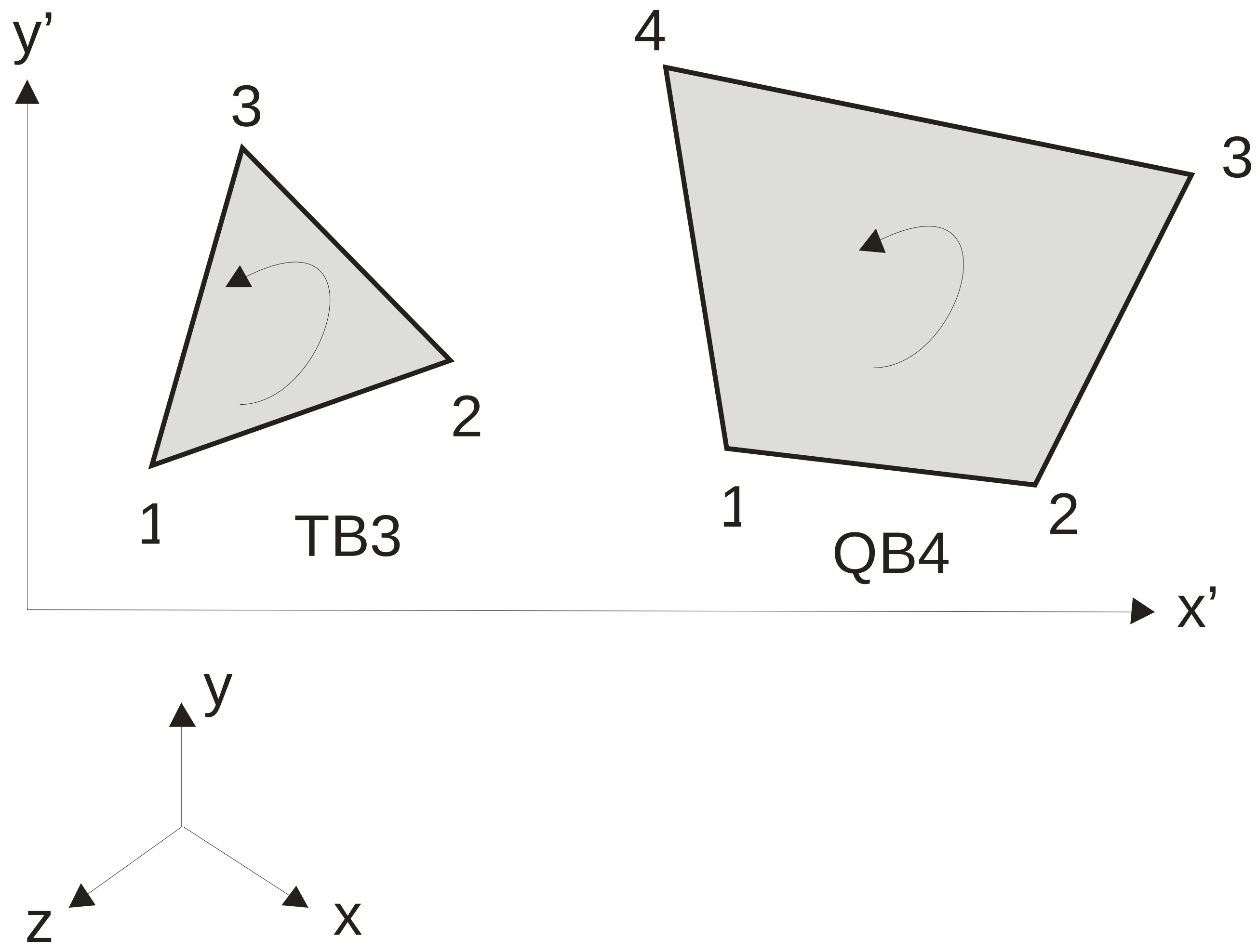
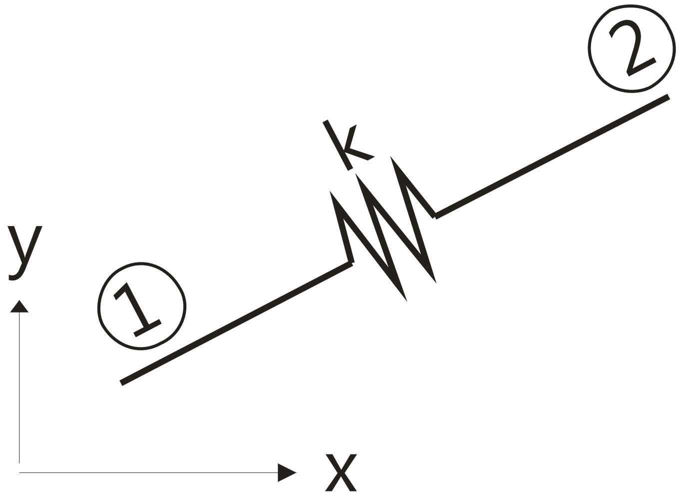
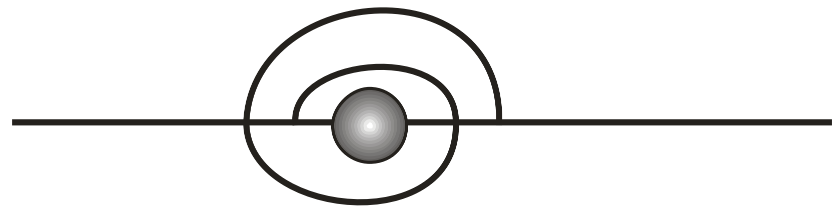


Plate vs Shell

- Shell (TB3/QB4)
  - 6 dof/node
  - Flat and thin
  - Membrane plus bending
- Plate
  - 3 dof/node
  - Flat and thin
  - Bending only
  - Special case of shell

Spring Elements





Element Response

Element	Response
Beam	Element nodal forces
Shell	$\{\sigma_x, \sigma_y, \tau_{xy}\}$
Translational Spring	Element axial force
Torsional Spring	Element torsional moment

Material Model

- Metal
  - Steel
  - Aluminum
- Brittle Composites
  - Concrete
  - Brick
- Elastic behavior
- Isotropic (E, CTE, Poisson’s Ratio)

Element Properties

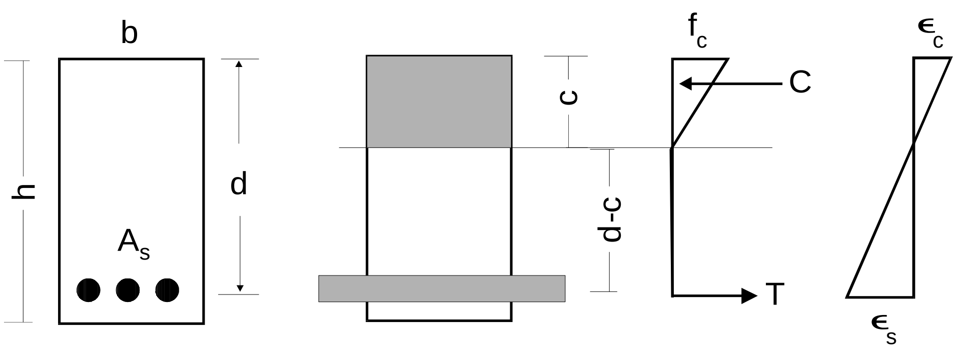
- Concrete Modulus of Elasticity

$$E_c = w_c^{1.5}\, 33\sqrt{f_c'}\ psi$$

$$E_c = 57000\sqrt{f_c'}\ psi$$

- Moment of Inertia

$$I_{gt} = \frac{bc^3}{3} + nA_s(d-c)^2$$



$$I_e = I_{gt} + \left(\frac{M_{cr}}{M_a}\right)^3 \left(I_g - I_{gt}\right)$$

Loading

- Dead Load
  - Self-weight
  - Others
- Live Load
- Wind Load
- Snow Load
- Rain Load
- Seismic Load

Connection Points

- Centroid of cross-section
- Mid-plane of shell element
- Beam and shell elements are compatible
  – Eccentricities in connections?

Modeling Approximations

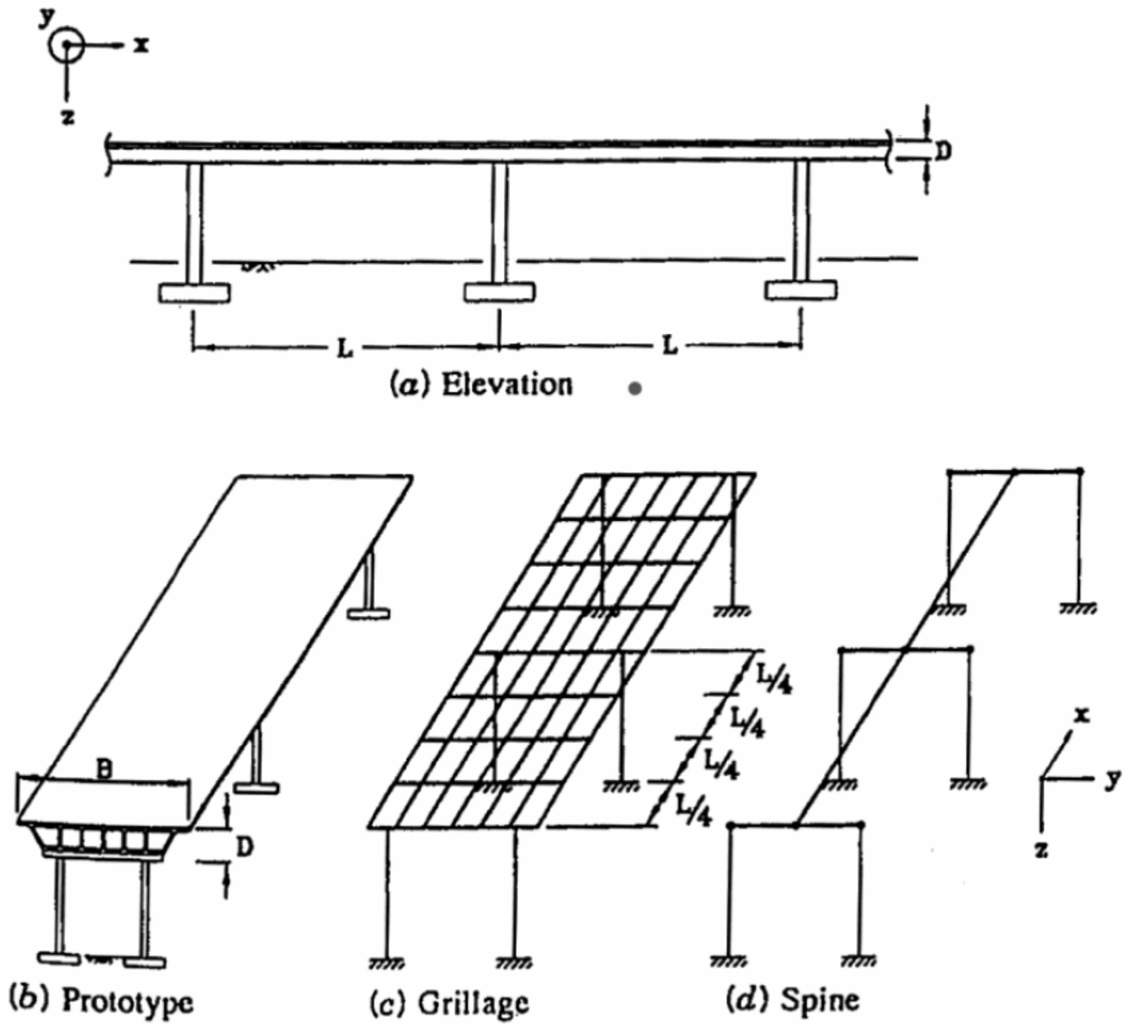


Figure 4.2-4 Superstructure Models (Priestley, et al 1996).

Modeling Approximations

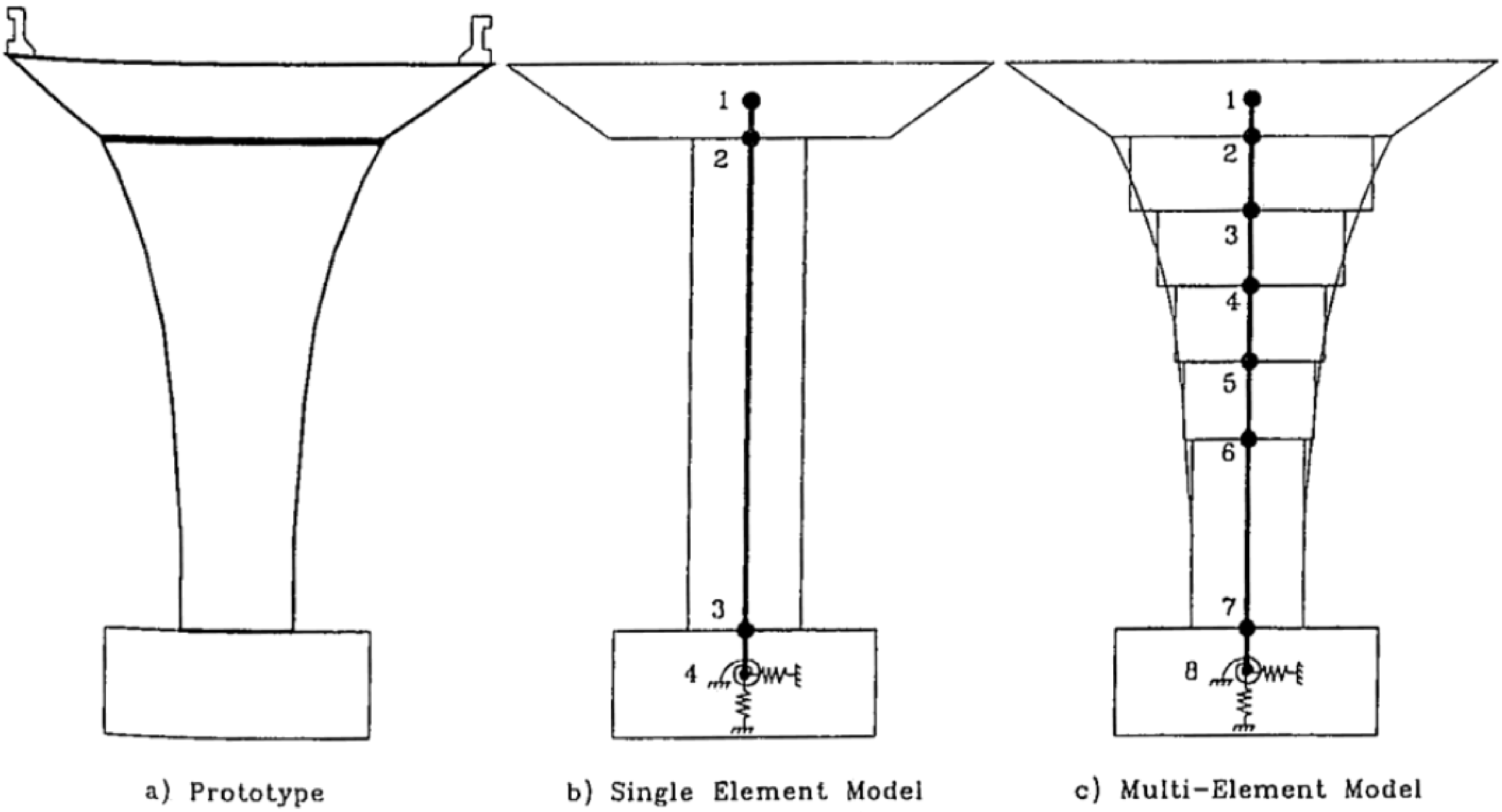
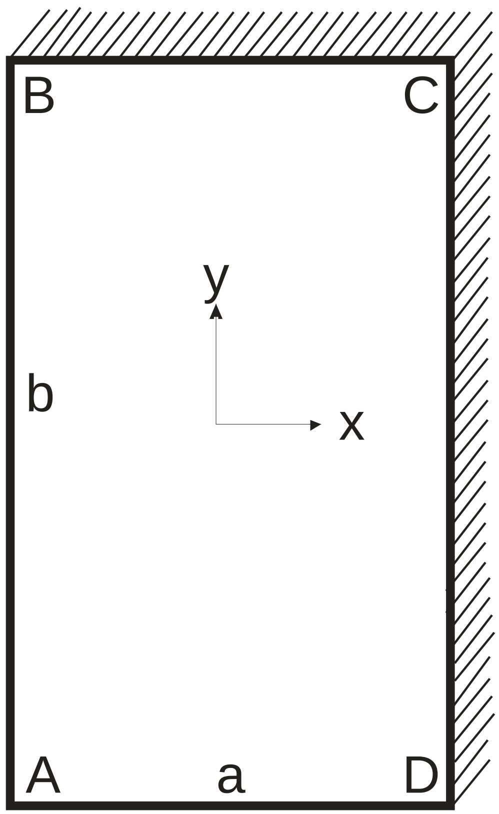


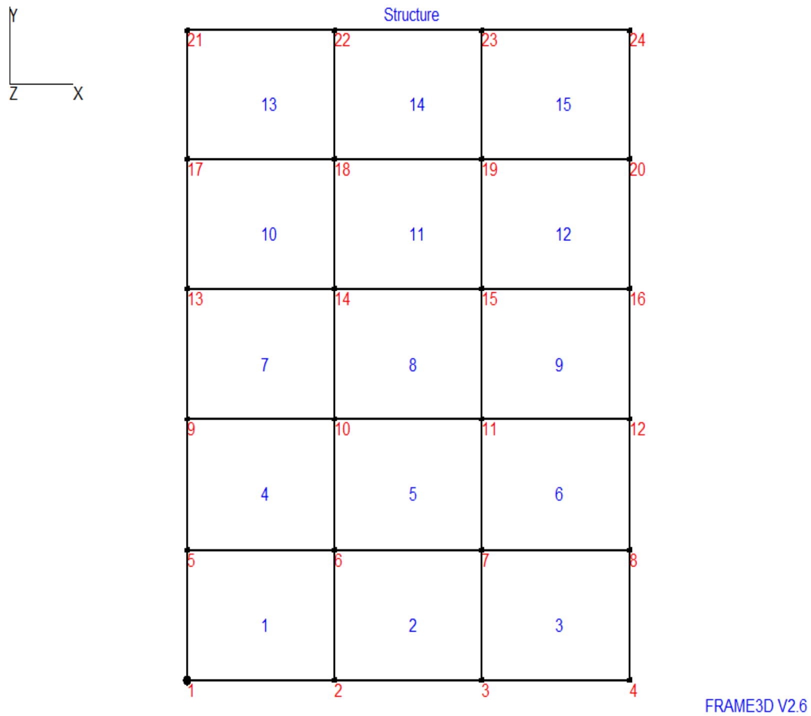
Figure 4.2-5 Single-Column Bent Models (Priestley et al, 1996).

Case Study: Thin Steel Plate

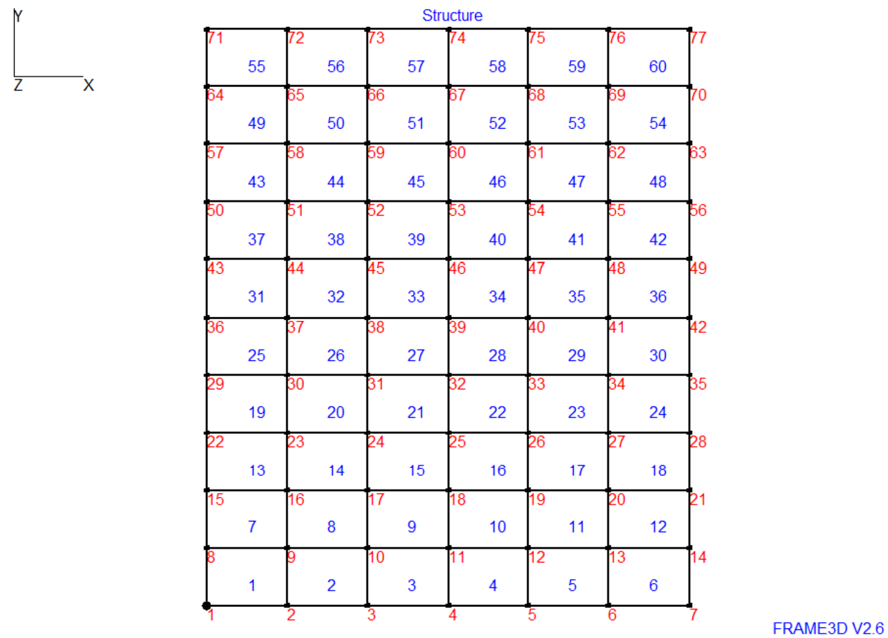


$$t = 0.015\,m$$
$$a \times b = 0.15\,m \times 0.25\,m$$
$$E = 200\,GPa$$
$$\nu = 0.3$$
$$\rho = 7850\ kg/m^3$$
$$p = 2000\ N/m^2$$

Convergence Study

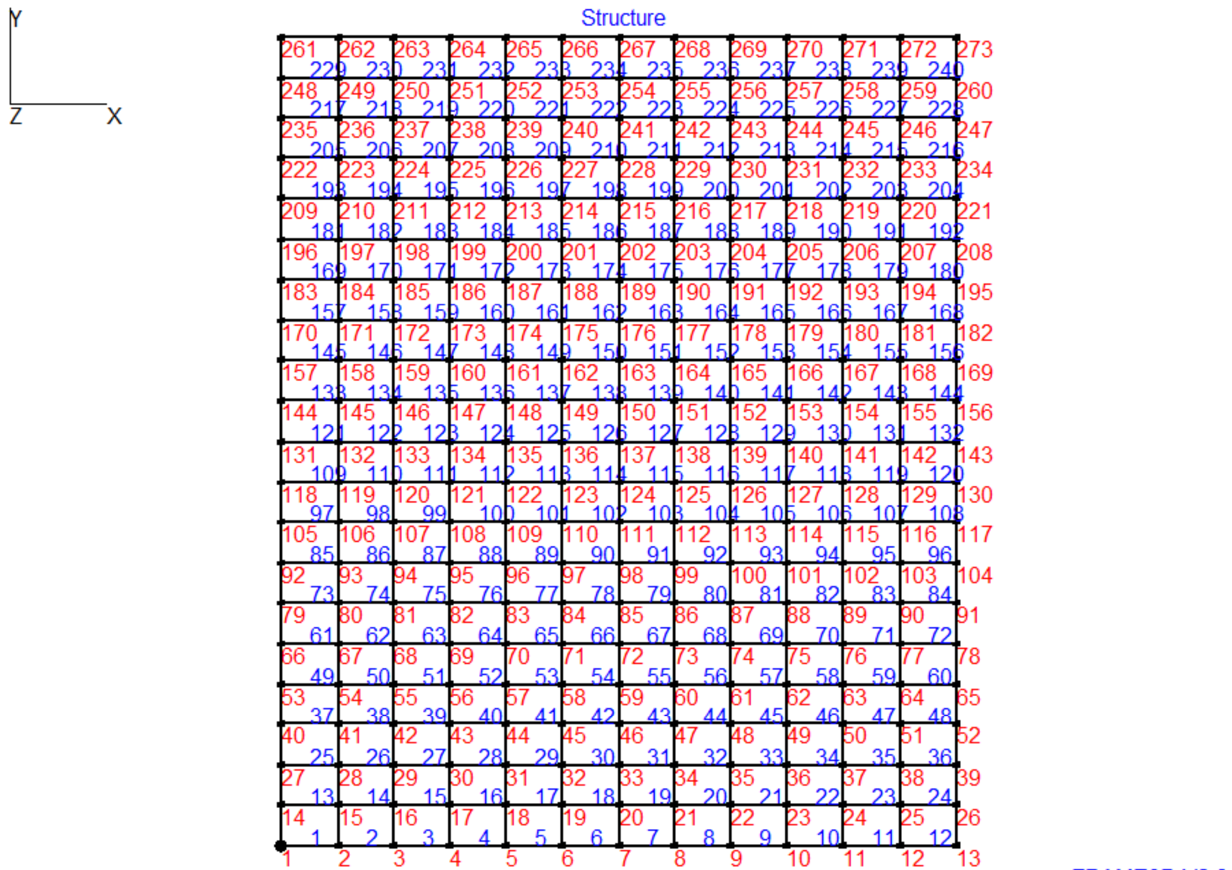


P1M1



P1M2

Convergence Study

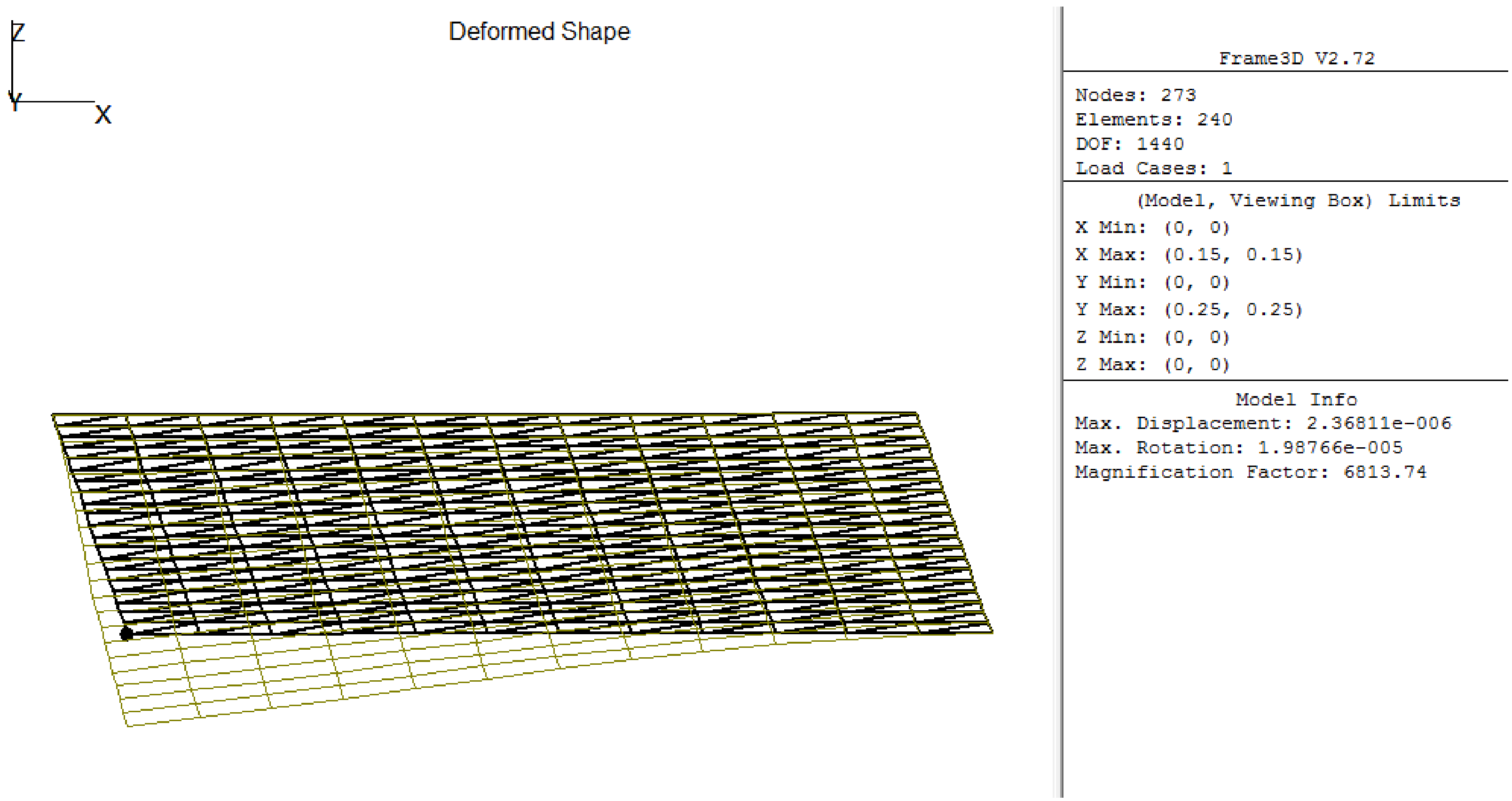


P1M3

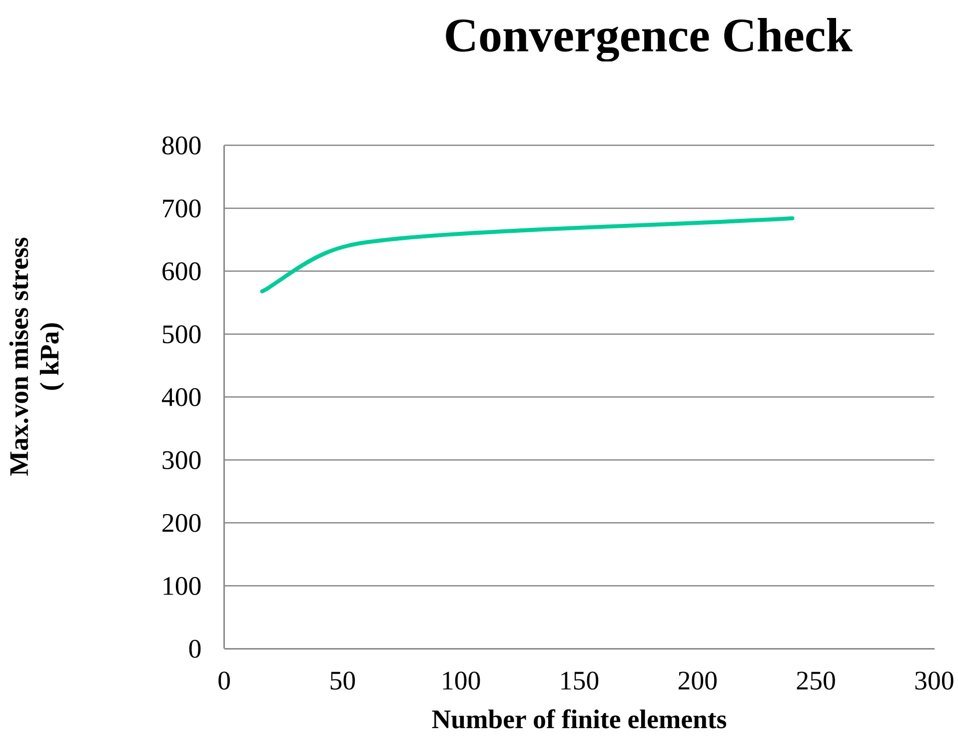
FEA Results

Model Name	Element mesh size (m)	No. of nodes	Max. von Mises stress (Pa)	Max. Z-Nodal displacement (m)
P1M1	0.05	24	568812	2.43×10^{-6}
P1M2	0.025	77	645999	2.38×10^{-6}
P1M3	0.0125	273	684336	2.37×10^{-6}

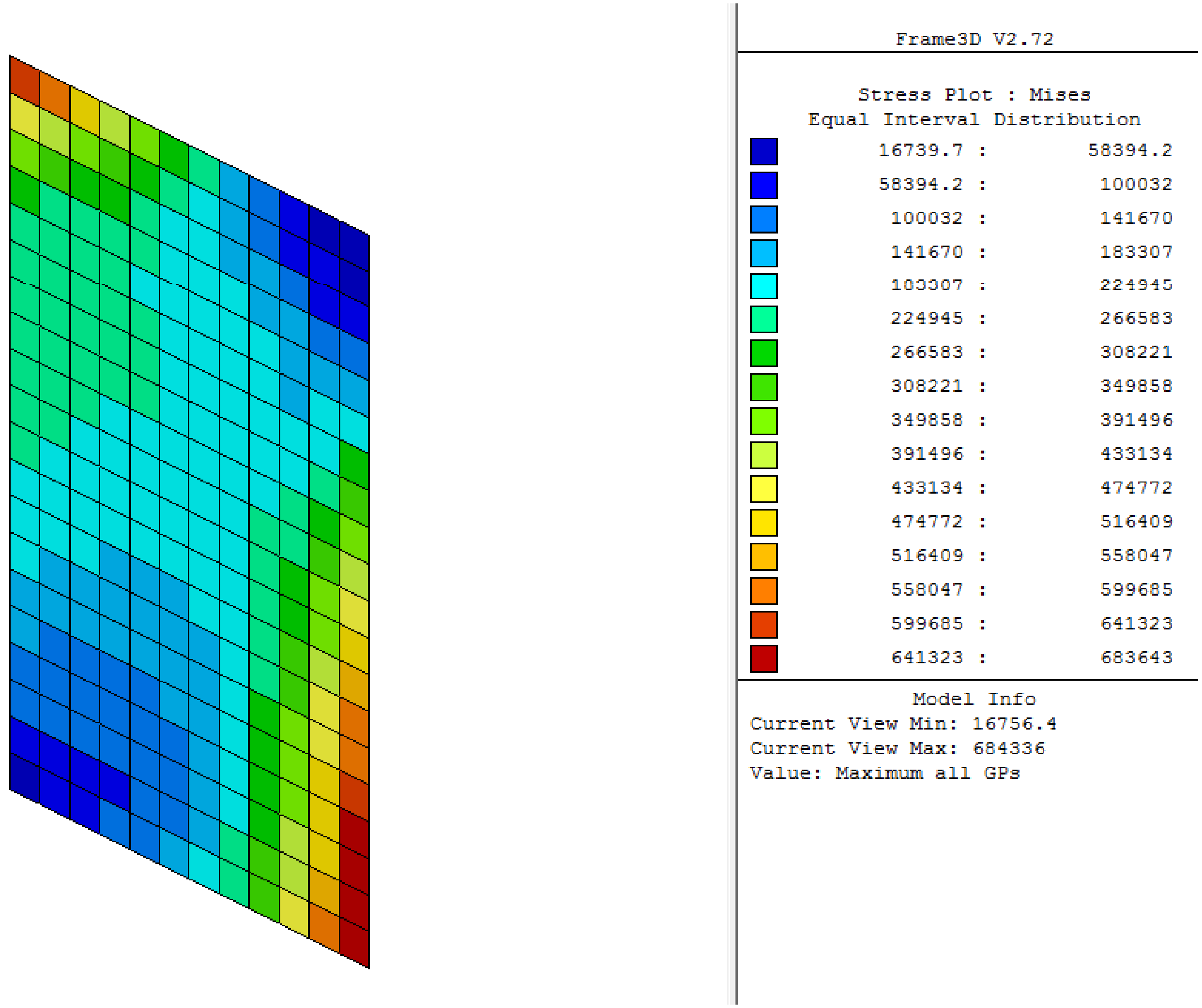
Deformed Shape Plot



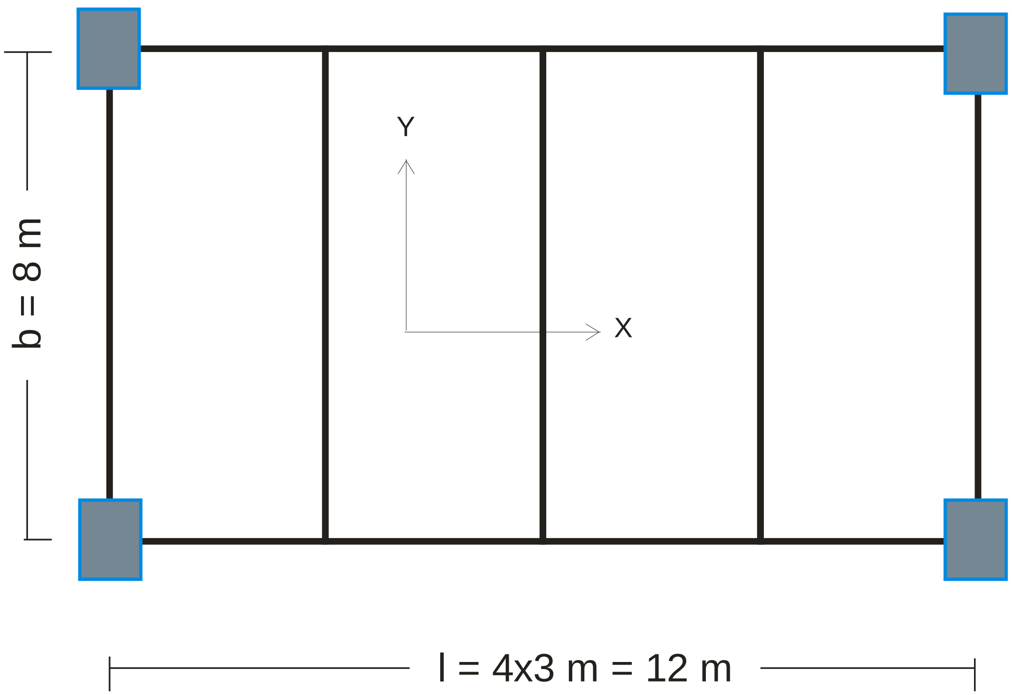
Stress Convergence



von Mises Stress Plot



Case Study: Reinforced Concrete Slab-Beam-Column System

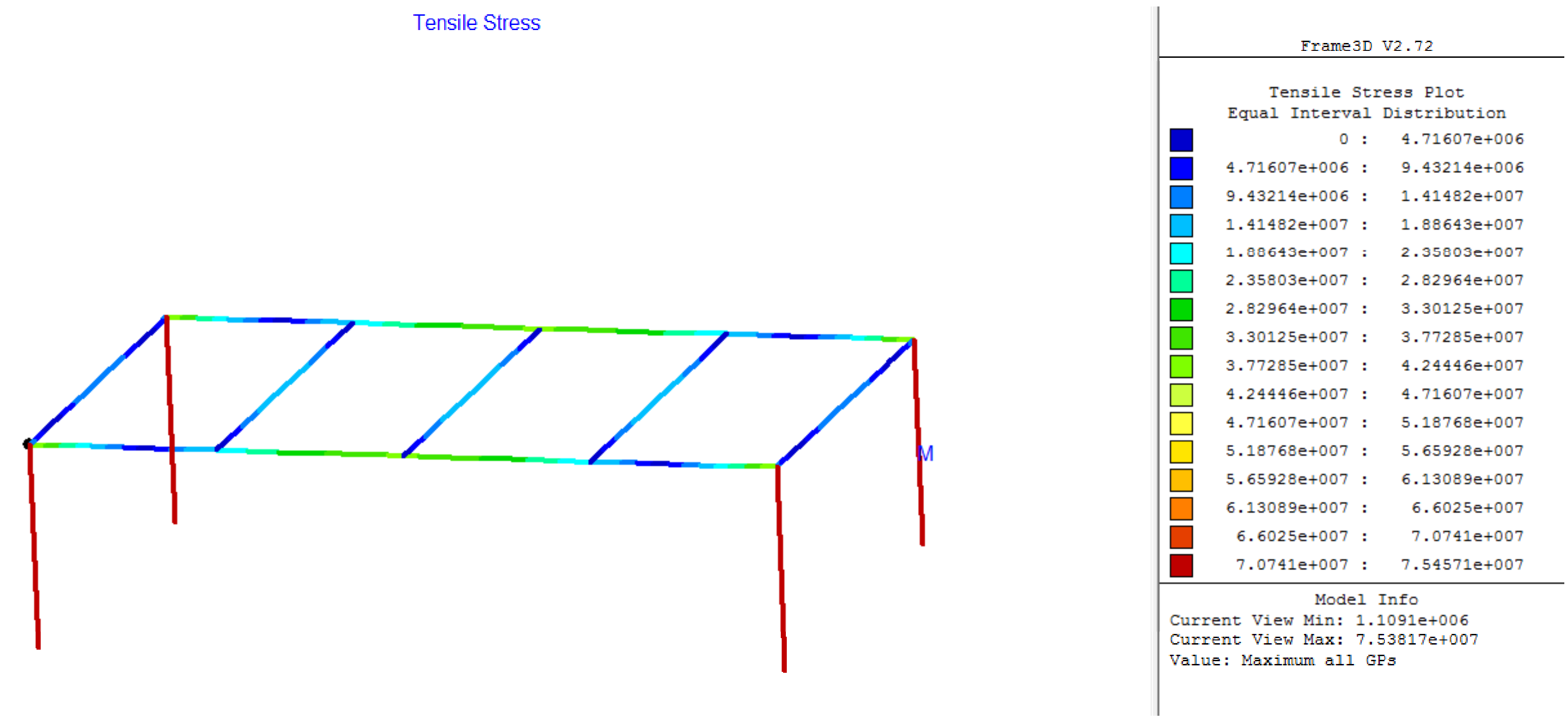


$$t = 0.1\,m$$
$$h \times w = 0.4\,m \times 0.25\,m$$
$$E = 32\,GPa$$
$$\nu = 0.15$$
$$\rho = 2400\ kg/m^3$$
$$p = 5000\ N/m^2$$

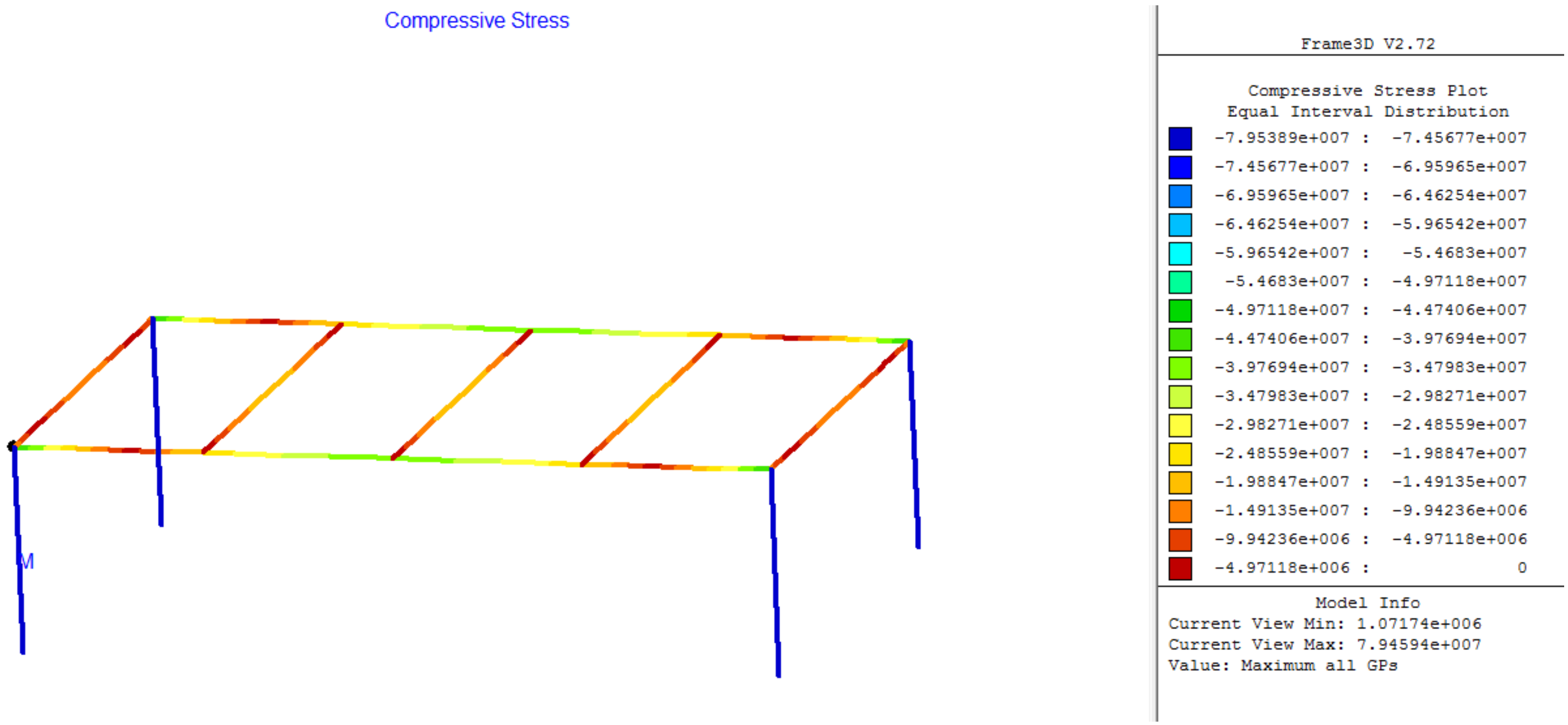
FEA Results

Model Name	Element mesh size (m)	No. of elements	Max. Slab Compressive Stress (MPa)	Max. Slab Tensile Stress (MPa)	Max. Z-Nodal displacement (m)
P3M0	1.0	164	10.7	10.1	0.08440
P3M1	0.5	384	11.9	10.2	0.08455
P3M2	0.25	1536	12.5	10.2	0.08447

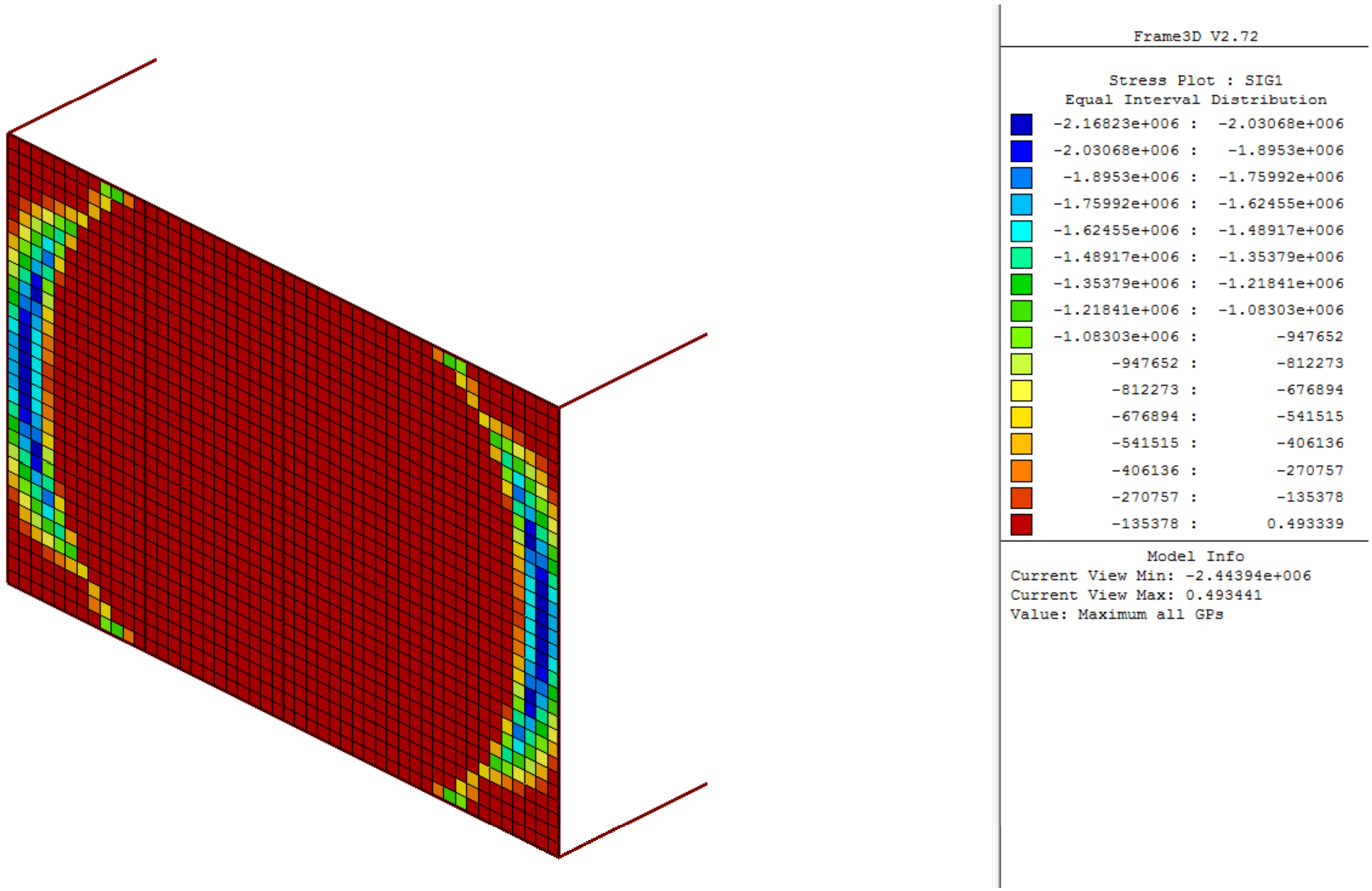
Max. Tensile Stress Plot



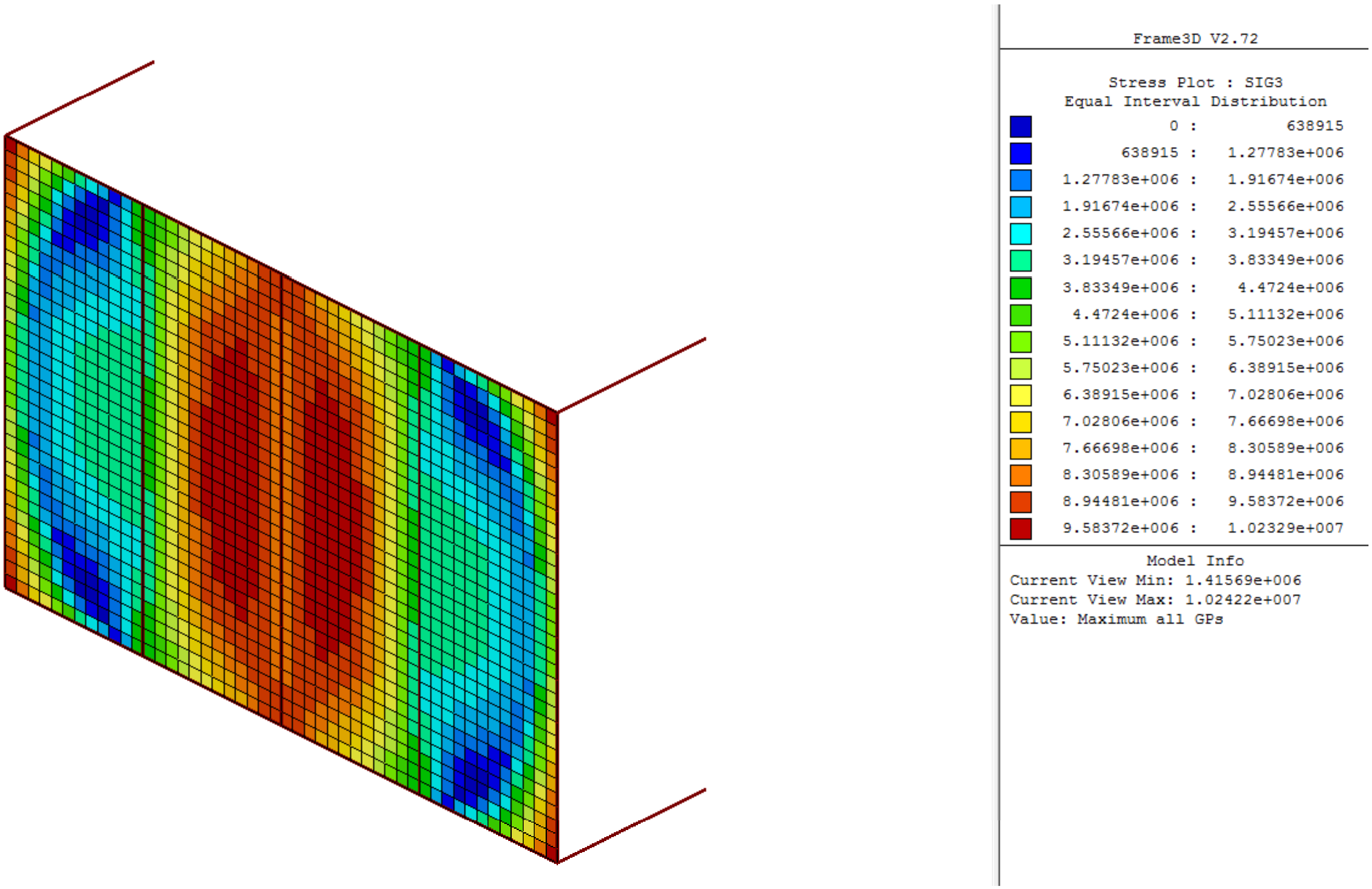
Max. Compressive Stress Plot



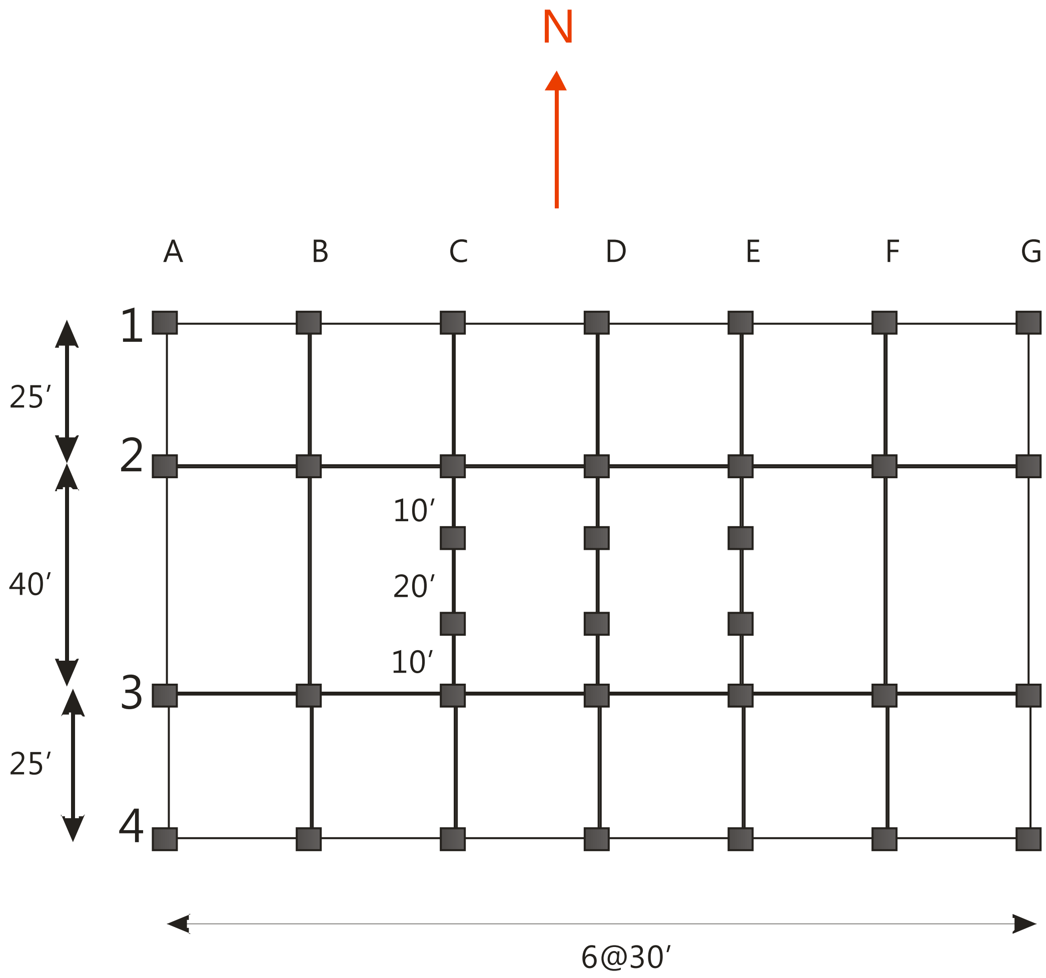
Principal Stress 1 Plot



Principal Stress 3 Plot



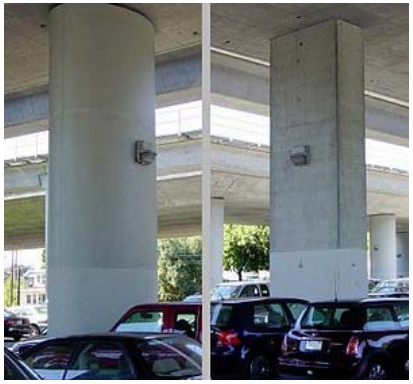
Project 4: Modeling and Analysis of a Multi-Storied Building

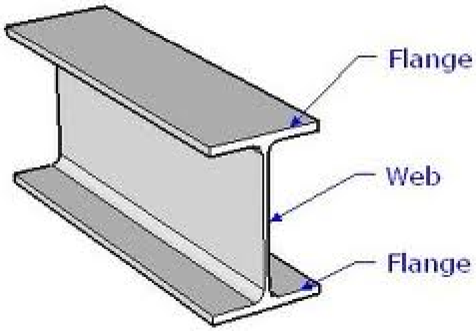


Floor	Height (ft)
Basement	16
Other floors	13

Structural Elements

- Beams and Columns
  - Wide-flange sections, welded box beam
  - High strength reinforced concrete
- Floor system
  - 3” cellular deck plus 2.5” concrete slab





Loading

- Live load
  - 100 psf (1^{st} floor), 80 psf (other floors), 20 psf (roof)
- Dead Load
  - Self-weight
  - Roof: 48 psf (felt, gravel plus decking)
- Wind Load
  - N-S direction only
- 1 load case: DL + LL + WL

Tutorial #1

- Model a 0.5” cantilever steel plate that is 60” long and 10” wide and is subjected to a uniformly distributed loading of 2 psf. Compute the largest transverse displacement and von Mises stress.

Solution Strategy

- Can you compute an analytical, approximate solution (max. displacement, max. normal stress?
- What other response parameters do you know that can be used as a check?
  - Displacements
  - Stresses
  - Reactions

Solution Steps

- Select consistent units (lbm, in, s, lb)
- Try a single element solution
  - Analyze and go through the checks
- Refine the model
  - Analyze and go through the checks
- Refine the model even more
  - Analyze and go through the checks
- Document the results and check for convergence

Tutorial #2

- Model the building shown below using only beam elements.

Tutorial #3

- Model the building shown below using beam and shell elements.