A short technical report

Units and Their Implications

Supplemental Material for
CEE321 Structural Analysis & Design
CEE526 Finite Elements for Engineers
CEE532 Developing Software for Engineering Applications

S. D. Rajan
Civil, Environmental and Sustainable Engineering Program
School of Sustainable Engineering and the Built Environment
Arizona State University
Tempe, AZ 85287-5306

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1.0 Introduction

Engineering problems in the US are typically solved using two different types of units. The first is the International System of Units abbreviated as SI abbreviated SI from French: Le Système international d'unités). SI units are sometimes referred to as metric units. The second is the US Customary System (USCS) units which is derived from English (or, British imperial) units.

Table 1.1 Base Units in the SI System of Units[1]

Base Quantity	Name	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Thermodynamic Temperature	Kelvin	K
Amount of Substance	mole	mol
Luminous Intensity	candela	cd

Sometimes it is more convenient to express the values in SI as multiples or fractions of 1000 or 10^3.

Table 1.2 SI Prefixes

Prefix	Notation	Factor	Prefix	Notation	Factor
nano-	n	10^{-9}	kilo-	k	10^3
micro-	μ	10^{-6}	mega-	M	10^6
milli-	m	10^{-3}	giga-	G	10^9

[1] http://physics.nist.gov/cuu/Units/

Table 1.3 Some Commonly Used Base Units

Unit	Remarks	Unit	Remarks
m	Meter Length. SI.	in	Inch. Length. USCS.
ft	Feet. Length. USCS.	km	Kilometer. Length. SI.
kg	Kilogram. Mass. SI.	slug	Slug. Mass. USCS.
lbm	Pound mass. Mass. USCS.	g	Gram. Mass. SI.
s	Second. Time. SI & USCS.	min	Minute. Time. USCS.
h	Hour. Time. USCS.	K	Kelvin. Temperature. SI.
C	Celsius (centigrade). Temperature. SI	F	Fahrenheit. Temperature. USCS.
A	Ampere. Current. SI & USCS.	lbm	Pound mass. Mass. USCS.

Table 1.4 Some Commonly Used Derived Units

Unit	Remarks	Unit	Remarks
N	Newton. Force. SI	lb	Pound. Force. USCS.
Pa	Pascal. Pressure. SI	psi	Pounds per square inch. Pressure. USCS.
J	Joule. Energy. SI	BTU	British Thermal Unit. Energy. USCS.
W	Watt. Power. SI	BTU/s	BTU per unit time. Power. USCS.

2.0 Computing the Correct Mass Density

Irrespective of the type of units used, the basic quantities are mass (**M**), length (**L**) and time (**T**) and all units can be expressed in terms of these basic quantities. According to Newton's Second Law

$$\text{Force} = (\text{Mass})\ (\text{Acceleration}) \qquad (1)$$

In SI Units, a force (**F**) of 1 Newton (N) is defined as the force required to accelerate a mass of 1 kilogram (kg) one meter per second per second $\left(1m/s^2\right)$. In other words

$$1N = (1kg)\left(1m/s^2\right) = 1kg - m/s^2 \qquad (2)$$

In USCS units, a force of 1 pound (lb) is defined as the force required to accelerate a mass of 1 slug, one feet per second per second $\left(1\ ft/s^2\right)$. In other words

$$1\ lb = (1slug)\left(1\ ft/s^2\right) = 1slug - ft/s^2 \qquad (3)$$

The $N-kg-m-s$ and $lb-slug-ft-s$ sets of units are consistent units. We can certainly convert from SI to USCS units and back. For example, using

$$\begin{aligned} 1slug &= 14.594\,kg \\ 1\ ft &= 0.3048\,m \end{aligned} \qquad (4)$$

we can see that

$$1lb = 1slug - ft/s^2 = 14.594\,kg\,(0.3048m)\frac{1}{s^2} = 4.44825\,N \qquad (5)$$

Problems can arise if we use non-consistent units. For example, pound-mass (lbm) is defined as the mass having a weight of one pound at sea level.

We can use non-consistent units as long as we suitably modify Eqn. (1) and write it as

$$\text{Force} = \mathbf{c}\ (\text{Mass})\ (\text{Acceleration}) \qquad (6)$$

where **c** is a conversion constant. With consistent units, using Eqn. (3) we have

$$1\ lb = c(1slug)\left(1\ ft/s^2\right) \Rightarrow c = 1.0\frac{lb-s^2}{slug-ft} \qquad (7)$$

To use the non-consistent lbm as the units of mass, we recognize that

$$1\ lb = c(1lbm)(32.2\ ft/s^2) \Rightarrow c = \frac{1}{32.2}\frac{lb-s^2}{lbm-ft} = 0.03106\frac{lb-s^2}{lbm-ft} \qquad (8)$$

To use the non-consistent lbm as the units of mass and inches (in) as the units of length, we recognize that

$$1\,lb = c(1lbm)(32.2\ ft/s^2)(12\,in/\,ft) \Rightarrow c = \frac{1}{386.4}\frac{lb-s^2}{lbm-in} \qquad (9)$$

What does this mean? Most general purpose computer programs require that consistent set of units be used and the programs do not explicitly account for the **c** factor. In particular we will look at the issues of specifying mass density and acceleration due to gravity values.

Consider a rectangular cross-section steel beam that is $5.08\,cm \times 10.16\,cm \times 3.048\,m$ long $(2" \times 4" \times 10')$. The mass density of steel is often given as $7872\frac{kg}{m^3}$ or $0.284\frac{lb}{in^3}$ [2]. The weight of the steel beam in SI units at sea level can be computed as

$$W = 0.0508m \times 0.1016m \times 3.048m \times 7872\frac{kg}{m^3} \times 9.81\frac{m}{s^2} = 1214.86\,N$$

If we use USCS units, then we need to recognize that the mass density is in fact $0.284\frac{lbm}{in^3}$ not $0.284\frac{lb}{in^3}$. The consistent mass density value is then $\boxed{0.284 \times \frac{1}{386.4}\frac{lbm}{in^3} = 0.00073499\frac{lbm}{in^3}}$ with the acceleration due to gravity as $\boxed{386.4\,in/s^2}$. To verify this, let's recompute the weight of the steel beam using Eqns. (4) and (5) as

$$W = 2\,in \times 4\,in \times 120\,in \times 0.00073499\frac{lbm}{in^3} \times 386.4\frac{in}{s^2} = 272.64\,lb \approx 1214.86N$$

Similarly, if we use ft instead of in, we need to use consistent mass density value as $\boxed{0.284 \times 12^3 \times \frac{1}{32.2}\frac{lbm}{ft^3} = 15.2407\frac{lbm}{ft^3}}$ with the acceleration due to gravity as $\boxed{32.2\ ft/s^2}$. Again let's recompute the weight of the steel beam using Eqns. (4) and (5) as

$$W = \frac{2}{12}\ ft \times \frac{4}{12}\ ft \times 10\ ft \times 15.2407\frac{lbm}{ft^3} \times 32.2\frac{ft}{s^2} = 272.64\,lb \approx 1214.86N$$

[2] http://www.matweb.com

Table 2.1 shows a list of possible sets of units that can be used to specify material properties using steel as an example.

Table 2.1 Mass Density & *g* for Steel

Units (F-M-L-T)	Mass Density Value to Use	Acceleration Due to Gravity Value to Use
N-kg-m-s	7872	9.8
kN-kg-mm-s	7.872×10^{-6}	9.8×10^{-3}
μ N-gm-mm-s	0.007872	9800
10^{-5}N-gm-cm-s	7.872	980
lb-slug-ft-s	**15.2407**	**32.2**
lb-slug-in-s	**7.3499×10^{-4}**	**386.4**
lb-lbm-ft-s	(490.751/32.2)=**15.2407**	**32.2**
lb-lbm-in-s	(0.284/386.4)=**7.3499×10^{-4}**	**386.4**
Mlb-Mlbm-in-ms*	0.73499×10^{-9}	0.0003864

It should be noted that in each row where non-consistent units are used, the mass density value is adjusted to make the values of mass density and acceleration due to gravity consistent. Hence, it should come as no surprise that the mass density values for **lb-slug-ft-s** combination is the same as **lb-lbm-ft-s** combination, or **lb-slug-in-s** combination is the same as **lb-lbm-in-s** combination, even though the mass units are different.

3.0 Some Famous Unit Conversion Errors[3]

Story 1: On September 23, 1999 NASA lost the $125 million Mars Climate Orbiter spacecraft after a 286-day journey to Mars. Miscalculations due to the use of English units instead of metric units apparently sent the craft slowly off course -- 60 miles in all. Thrusters used to help point the spacecraft had, over the course of months, been fired incorrectly because data used to control the wheels were calculated in incorrect units. Lockheed Martin, which was performing the calculations, was sending thruster data in English units (pounds) to NASA, while NASA's navigation team was expecting metric units (Newtons).

* Million lb, Million lbm, inches and millisecond

[3] http://spacemath.gsfc.nasa.gov/weekly/6Page53.pdf

Problem 1 - A solid rocket booster is ordered with the specification that it is to produce a total of 10 million pounds of thrust. If this number is mistaken for the thrust in Newtons, by how much, in pounds, will the thrust be in error? (1 pound = 4.5 Newtons)

Answer: 10,000,000 'Newtons' x (1 pound / 4.448 Newtons) = 2,200,000 pounds instead of 10 million pounds so the error is a 'missing' 7,800,000 pounds of thrust…an error that would definitely be noticed at launch!!!

Story 2: On January 26, 2004 at Tokyo Disneyland's Space Mountain, an axle broke on a roller coaster train mid-ride, causing it to derail. The cause was a part being the wrong size due to a conversion of the master plans in 1995 from English units to Metric units. In 2002, new axles were mistakenly ordered using the pre-1995 English specifications instead of the current Metric specifications.

Problem 2 - A bolt is ordered with a thread diameter of 1.25 inches. What is this diameter in millimeters? If the order was mistaken for 1.25 centimeters, by how many millimeters would the bolt be in error?

Answer: 1- inch = 25.4 millimeters so 1.25 inches x (25.4 mm/ 1 inch) = 31.75 millimeters. Since 1.25 centimeters = 12.5 millimeters, the bolt would delivered 31.75 - 12.5 = 19.25 millimeters too small!

Story 3: On 23 July 1983, Air Canada Flight 143 ran completely out of fuel about halfway through its flight from Montreal to Edmonton. Fuel loading was miscalculated through misunderstanding of the recently adopted metric system. For the trip, the pilot calculated a fuel requirement of 22,300 kilograms. There were 7,682 liters already in the tanks.

Problem 3 - If a liter of jet fuel has a mass of 0.803 kilograms, how much fuel needed to be added for the trip?

Answer: In order to calculate how much more fuel had to be added, the crew needed to convert the quantity in the tanks, 7,682 liters, to a weight, subtract that figure from 22,300 kilograms, and convert the result back into a volume (liters).

7,682 liters x (0.803 kilograms/ 1 liter)= 6,169 kg 22,300 kg – 6,169 kg = 16,131 kg 16,131 kg x (1 liter / 0.803 kilograms) = 20,088 liters of jet fuel.

Between the ground crew and flight crew, however, they arrived at an incorrect conversion factor of 1.77, the weight of a liter of jet fuel in pounds. This was the conversion factor provided on the refueller's paperwork and which had always been used for the rest of the airline's fleet. Their calculation produced:

7,682 liters x (1.77 pounds/liter) = 13,597 which they interpreted as kilograms but was actually the fuel mass in pounds! Then they continued the calculation:

22,300 kg – 13,597 'kg' = 8,703 kg 8,703 kg ÷ 1.77 = 4,916 liters ….so they were actually 15,172 liters short of fuel!