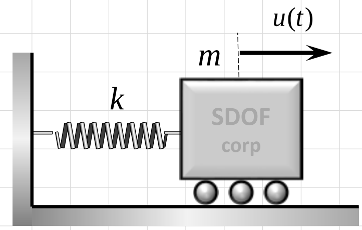
Homework 4



Un-damped System

ω	natural frequency

Analyze *Newmark's method* for solving the undamped vibration problem by finding the exact solution to the discrete equations. The equation of motion and initial conditions are

$$\ddot{u}(t) + \omega^2 u(t) = 0$$
$$u(0) = u_o$$
$$\dot{u}(0) = v_0$$

and the classical solution is

$$u(t) = u_o \cos\omega t + \frac{v_o}{\omega}\sin\omega t$$

Newmark's method satisfies the equation of motion at the discrete time points and approximates the velocity and displacement with approximations to integrals. Hence, the discrete equations are

$$\begin{aligned} a_{n+1} + \omega^2 u_{n+1} &= 0 \\ v_{n+1} - v_n - h\left[\gamma a_n + (1-\gamma)\, a_{n+1}\right] &= 0 \\ u_{n+1} - u_n - hv_n - h^2\left[\beta a_n + (\tfrac{1}{2} - \beta)\, a_{n+1}\right] &= 0 \end{aligned}$$

Derive the expressions for the exact solution to the numerical solution of the un-damped SDOF oscillator and explore the performance of Newmark's method over the range of the integration parameter values.

In particular, find the values of the parameters the assure numerical stability of the method. For what values do we get numerical damping? How much? For what values do we get growth of oscillations over time? How does growth relate to the concept of stability limit of the numerical integrator.

Write a MATLAB code to do the numerical integration to verify your observations about the performance of Newmark's method for the un-damped SDOF oscillator.

Add the damping term to the differential equation and use Newmark's method to solve the differential equation numerically (but no need to try to find the exact analysis of the numerical method).