Homework 12

Implement the algorithm for generation of Ritz vectors in a MATLAB code for an N degree of freedom model. Generate the vectors from the sequence

$$\boldsymbol{\psi}_1 = \mathbf{K}^{-1}\mathbf{M}\mathbf{1}$$
$$\tilde{\boldsymbol{\psi}}_{i+1} = \mathbf{K}^{-1}\mathbf{M}\boldsymbol{\psi}_i$$

where $\mathbf{K}$ is the stiffness matrix, $\mathbf{M}$ is the mass matrix, and $\mathbf{1}$ is a vector of ones (every element is one). Orthogonalize the Ritz vectors using the Gram-Schmidt process

$$\boldsymbol{\psi}_i = \tilde{\boldsymbol{\psi}}_i - \sum_{k=1}^{i-1}\left(\tilde{\boldsymbol{\psi}}_i^T\mathbf{M}\boldsymbol{\psi}_k\right)\boldsymbol{\psi}_k$$

Normalize the orthogonalized vectors to have unit length, i.e.,

$$\boldsymbol{\psi}_i^T\mathbf{M}\boldsymbol{\psi}_i = 1$$

Store the resulting Ritz vectors in an $N\times M$ array with the columns as the Ritz vectors

$$\boldsymbol{\Psi} = \begin{bmatrix} | & | & & | \\ \boldsymbol{\psi}_1 & \boldsymbol{\psi}_2 & \cdots & \boldsymbol{\psi}_M \\ | & | & & | \end{bmatrix}$$

Use the Ritz vectors to compute the M lowest eigenvectors of the system by the *subspace iteration* algorithm, defined as follows:

a. Initialize subspace vectors to orthogonal Ritz vectors

$$\mathbf{Q}^{old} = \boldsymbol{\Psi}^{Ritz}$$

b. Improve Ritz subspace

$$\mathbf{Q} = \mathbf{K}^{-1}\mathbf{M}\mathbf{Q}^{old}$$

c. Form reduced matrices (project onto the subspace)

$$\bar{\mathbf{M}} = \mathbf{Q}^T\mathbf{M}\mathbf{Q}$$
$$\bar{\mathbf{K}} = \mathbf{Q}^T\mathbf{K}\mathbf{Q}$$

d. Solve eigenvalue problem

$$\bar{\mathbf{K}}\boldsymbol{\varphi} = \lambda\bar{\mathbf{M}}\boldsymbol{\varphi}$$

e. Store eigenvectors in an $M\times M$ matrix

$$\boldsymbol{\Phi} = \begin{bmatrix} | & | & & | \\ \boldsymbol{\varphi}_1 & \boldsymbol{\varphi}_2 & \cdots & \boldsymbol{\varphi}_M \\ | & | & & | \end{bmatrix}$$

f. Improve subspace vectors

$$\mathbf{Q}^{new} = \mathbf{Q}\boldsymbol{\Phi}$$
$$\mathbf{Q}^{old} \leftarrow \mathbf{Q}^{new}$$

Repeat (return to b.)

Note that you project to get new reduced mass and stiffness matrices at every iteration of subspace iteration. The eigenvalue problem that you solve is only and M by M matrix eigenvalue problem.

Study the performance of the algorithm by plotting the improved Ritz vectors on top of the actual eigenvectors of the system (which you can easily compute in MATLAB from the original system matrices for small problems).