

Finite Elements For Engineers

Lecture 7A: Problem Solving Using the 1DBVP Program

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1DBVP Program Terminology

- Positive coordinate system points to the right
- FE mesh contains one or more segments
- A segment has the same properties but may contain one or more elements
- The leftmost segment (or the first segment) has the first node and the first element (or node 1 and element 1)
- You create the segment data; 1DBVP creates the nodes, elements and loads

Solid Mechanics

DE

$$-\frac{d}{dx}\left(\alpha(x)\frac{dy(x)}{dx}\right) + \beta(x)y(x) = f(x)$$

Left Mixed
BC

$$\tau = c_a y + d_a$$

Right
Mixed BC

$$\tau = c_b y + d_b$$

DE

$$-\frac{d}{dx}\left(A(x)E(x)\frac{du(x)}{dx}\right) = w(x)A(x)$$

$$\tau = F = -A(x)E(x)\frac{du(x)}{dx}$$

Solid Mechanics

$$\alpha(x) = A(x)E(x) \equiv F$$

$$\beta(x) = 0$$

$$f(x) = w(x)A(x) \equiv F/L$$

$$c_a = c_b = 0$$

$$d_a = d_b \equiv F$$

Examples



$$\beta(x) = 0$$

$$\alpha(x) = A(x)E(x) \equiv F$$

$$f(x) = q \equiv F/L$$

$$\frac{\overline{AE}}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} P \\ -P \end{Bmatrix} + \begin{Bmatrix} qL/2 \\ qL/2 \end{Bmatrix}$$

Heat Transfer

DE

$$-\frac{d}{dx} \left(\alpha(x) \frac{dy(x)}{dx} \right) + \beta(x) y(x) = f(x)$$

Left Mixed BC $\tau = c_a y + d_a$

Right Mixed BC $\tau = c_b y + d_b$

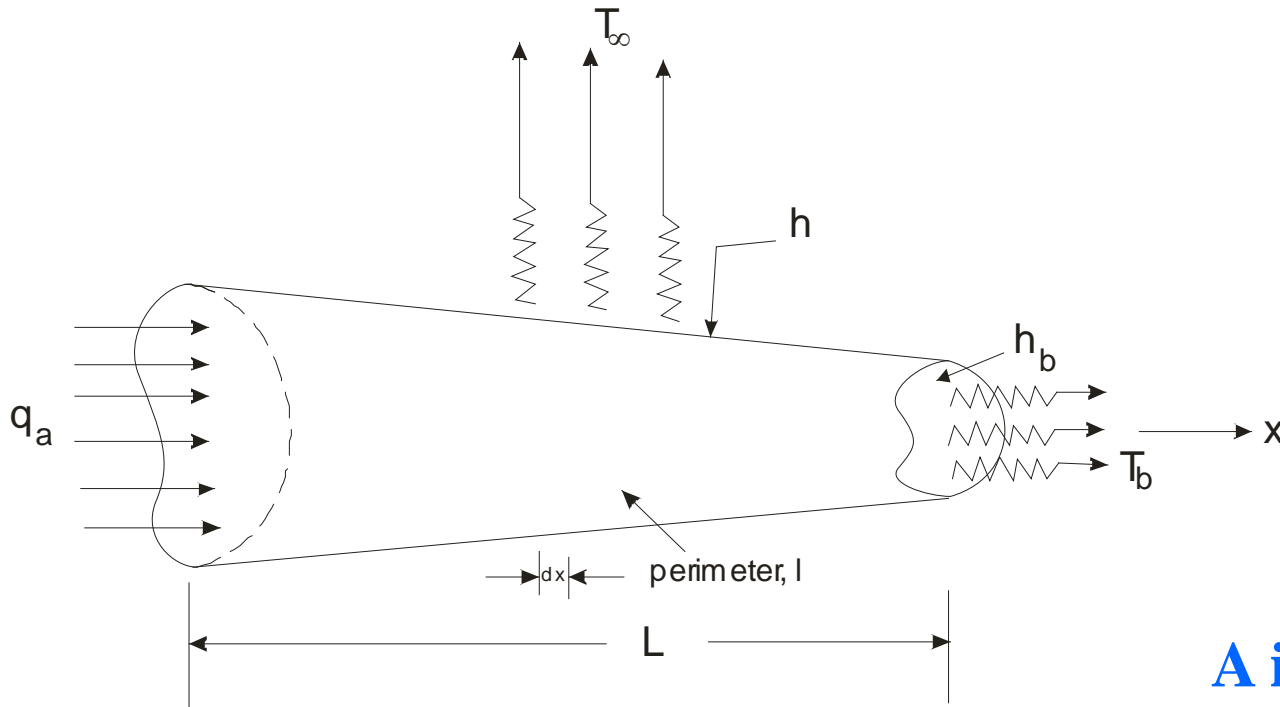
Comparing (assuming A is a constant)

$$\alpha(x) = k(x) \equiv \frac{E}{tLT}$$

$$\beta(x) = \frac{hl}{A} \equiv \frac{E}{tL^3T}$$

$$f(x) \equiv Q(x) + \frac{hl}{A} T_\infty \equiv \frac{E}{tL^3}$$

Heat Transfer



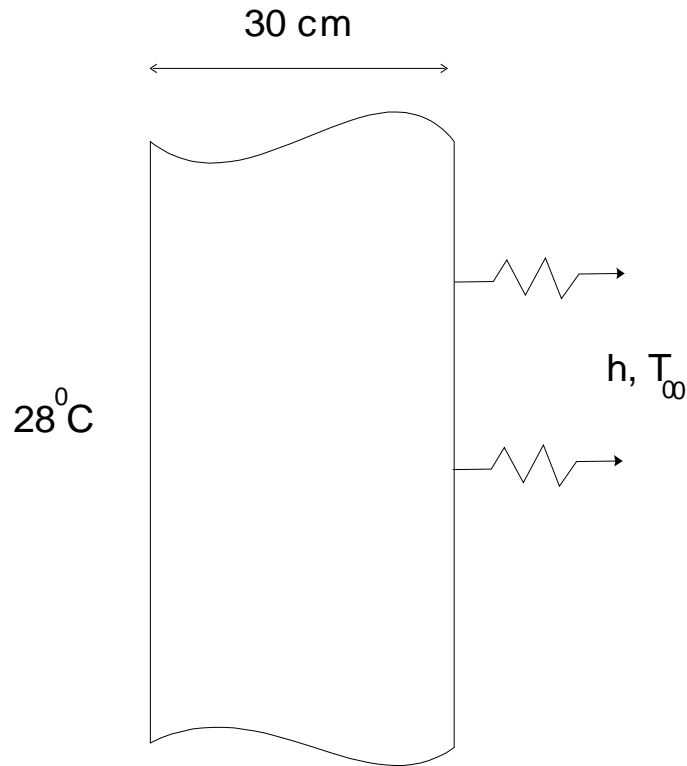
A is a NOT constant

$$\left[\left[\begin{array}{c|c} \frac{\bar{k}}{L} + \frac{\bar{h}lL}{3A} & -\frac{\bar{k}}{L} + \frac{\bar{h}lL}{6A} \\ \hline -\frac{\bar{k}}{L} + \frac{\bar{h}lL}{6A} & \frac{\bar{k}}{L} + \frac{\bar{h}lL}{3A} \end{array} \right] + h_b \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & 1 \end{array} \right] \right] \left\{ \begin{array}{c} T_1 \\ T_2 \end{array} \right\} = \frac{L}{2} \left\{ \begin{array}{c} \frac{\bar{h}l}{A} T_\infty \\ \frac{\bar{h}l}{A} T_\infty \end{array} \right\} + \left\{ \begin{array}{c} q_a \\ 0 \end{array} \right\} + \left\{ \begin{array}{c} 0 \\ h_b T_b \end{array} \right\}$$

Example 1

(1D- C^0 Linear Element)

Problem T4L3-1



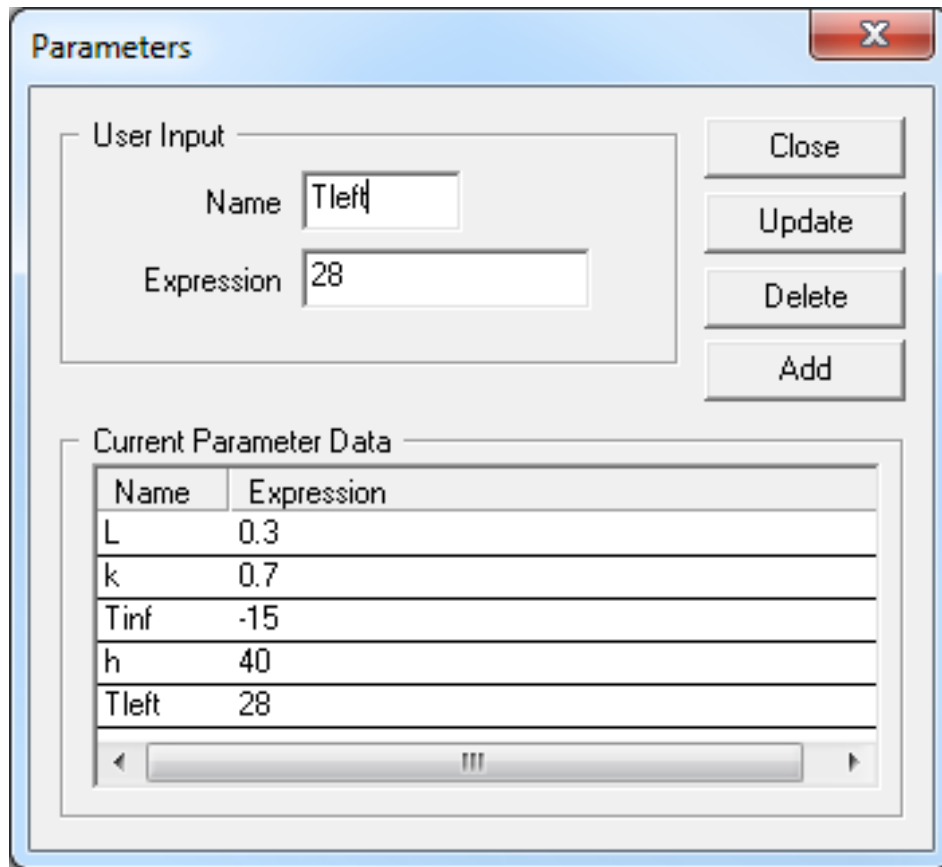
Brick wall: Determine the steady-state temperature distribution within the wall and also the heat flux through the wall.

$$k = 0.7 \text{ W/m} \cdot ^\circ\text{C}$$

$$T_{\infty} = -15^\circ\text{C}$$

$$h = 40 \text{ W/m}^2 \cdot ^\circ\text{C}$$

Problem T4L3-1



A screenshot of a software window titled "Parameters" with a standard Windows-style title bar (blue background, red close button). The window is divided into two main sections. The top section, labeled "User Input", contains two text input fields: "Name" with the text "Tleft" and "Expression" with the text "28". To the right of these fields are four buttons stacked vertically: "Close", "Update", "Delete", and "Add". The bottom section, labeled "Current Parameter Data", contains a table with two columns: "Name" and "Expression". The table lists five parameters: L (0.3), k (0.7), Tinf (-15), h (40), and Tleft (28). Below the table is a horizontal scrollbar.

Name	Expression
L	0.3
k	0.7
Tinf	-15
h	40
Tleft	28

Problem T4L3-1

Segment Data [X]

User Input

Element Type: **C0-Linear** Left End Coordinate:
Number of Elements: Right End Coordinate:

alpha(x): x² + x +
beta(x): x² + x +
f(x): x² + x +

Close
Update
Delete
Add

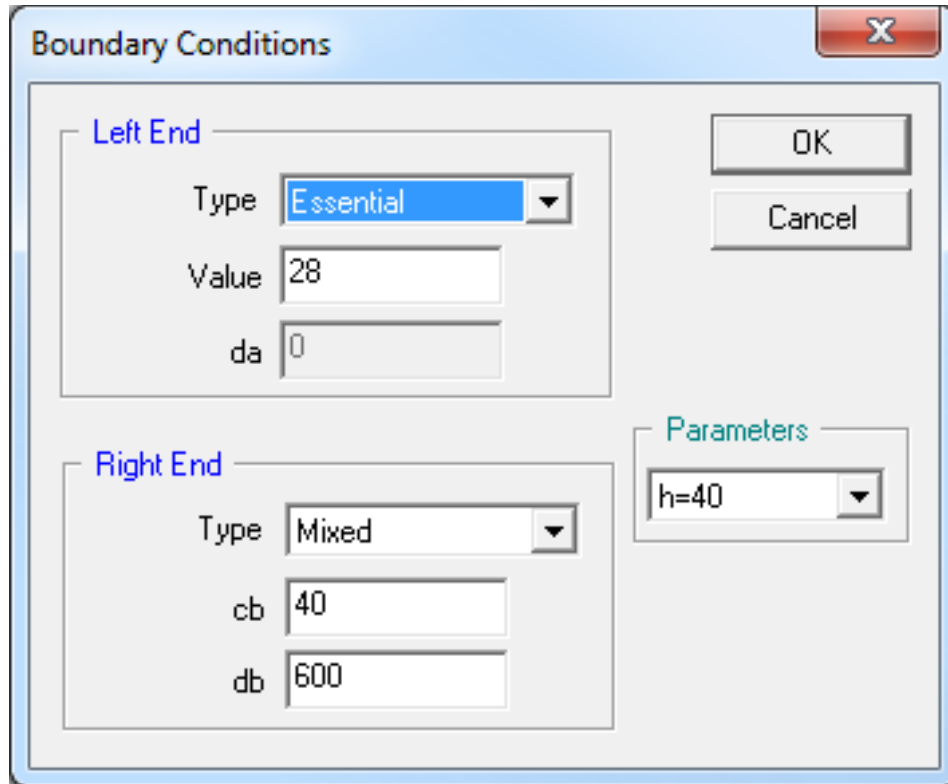
Parameters
h=40

Current Segment Data

Segm...	Element Type	# Elements	Left End Coor	Right End Coor	alpha:X ²
1	C0-Linear	1	0	0.3	0

III

Problem T4L3-1



The image shows a software dialog box titled "Boundary Conditions" with a standard Windows-style title bar (blue background, red close button). The dialog is divided into several sections. On the left, there are two main sections: "Left End" and "Right End", both labeled in blue. The "Left End" section contains a "Type" dropdown menu set to "Essential", a "Value" text box containing "28", and a "da" text box containing "0". The "Right End" section contains a "Type" dropdown menu set to "Mixed", a "cb" text box containing "40", and a "db" text box containing "600". To the right of these sections are two buttons: "OK" and "Cancel". Further to the right, there is a "Parameters" section with a dropdown menu showing "h=40".

Boundary Conditions

Left End

Type: Essential

Value: 28

da: 0

Right End

Type: Mixed

cb: 40

db: 600

Parameters

h=40

OK

Cancel

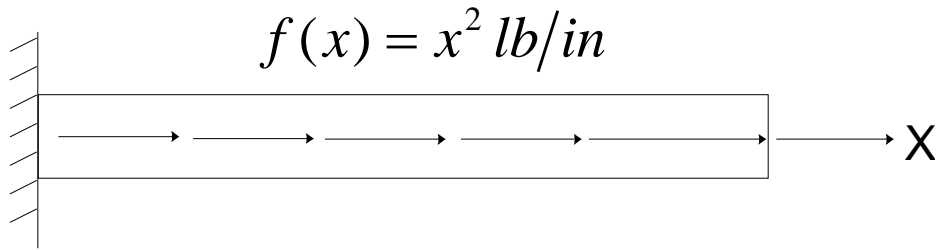
Problem T4L3-1

Mesh	Right Temp. (C)	Right Flux (W/m ²)	
1-element	-12.63	94.8	
2-elements	-12.63	94.8	
4-elements	-12.63	94.8	

Example 2A

(1D- C^0 Linear Element)

Problem T4L2-1



The 4" long steel bar with a 2 in^2 cross-sectional area is loaded by the given surface traction. Find the displacement and force distribution in the bar.

Problem T4L2-1

Parameters

User Input

Name:

Expression:

Buttons: Close, Update, Delete, Add

Current Parameter Data

Name	Expression
A	2

Segment Data

User Input

Element Type: Left End Coordinate:

Number of Elements: Right End Coordinate:

alpha(x): x² + x +

beta(x): x² + x +

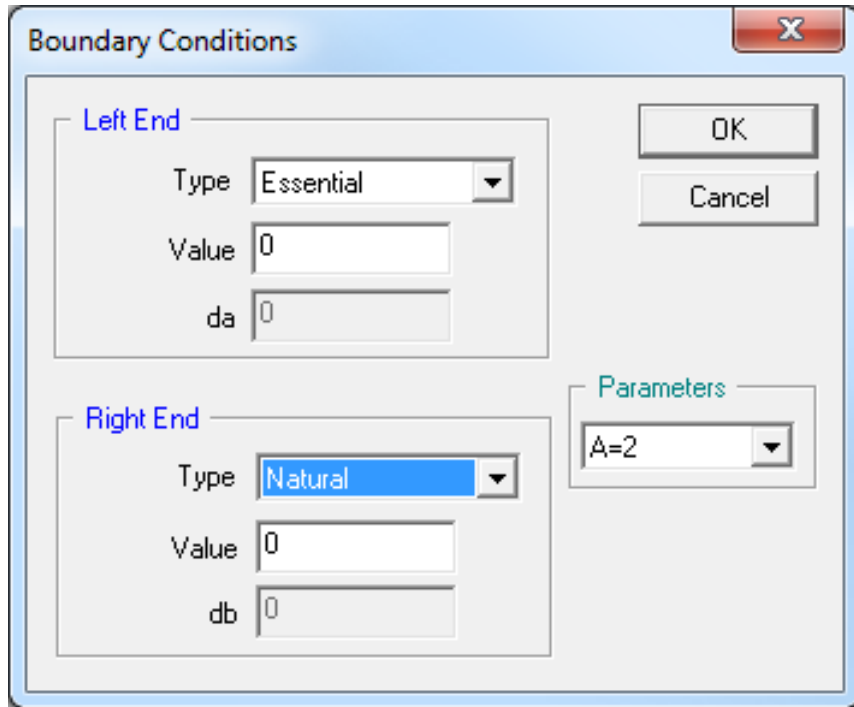
f(x): x² + x +

Parameters:

Current Segment Data

Segm...	Element Type	# Elements	Left End Coor	Right End Coor	alpha:X^2
1	CO-Linear	1	0	4	0

Problem T4L2-1



The image shows a software dialog box titled "Boundary Conditions" with a standard Windows-style title bar (blue background, red close button). The dialog is divided into two main sections for boundary settings and a "Parameters" section.

Left End (indicated by blue text):

- Type: A dropdown menu set to "Essential".
- Value: A text input field containing "0".
- da: A text input field containing "0".

Right End (indicated by blue text):

- Type: A dropdown menu set to "Natural".
- Value: A text input field containing "0".
- db: A text input field containing "0".

Parameters (indicated by green text):

- A dropdown menu set to "A=2".

At the top right of the dialog are two buttons: "OK" and "Cancel".

Problem T4L2-1

Mesh	Tip Disp. (in)	Root Force (lb)	Tip Force (lb)
1-element	1.067(10 ⁻⁶)	16	16
2-elements	1.067(10 ⁻⁶)	20.67	11.33
4-elements	1.067(10 ⁻⁶)	21.25	6.75
8-elements	1.067(10 ⁻⁶)	21.32	3.68
16-elements	1.067(10 ⁻⁶)	21.33	1.92

Example 2B

(1D- C^0 Higher Order Elements)

Problem T4L2-1

Quadratic Element

Mesh	Tip Disp. (in)	Root Force (lb)	Tip Force (lb)
1-element	1.067(10 ⁻⁶)	21.54	10.5
2-elements	1.067(10 ⁻⁶)	21.34	6.02
4-elements	1.067(10 ⁻⁶)	21.34	3.2
8-elements	1.067(10 ⁻⁶)	21.34	1.65

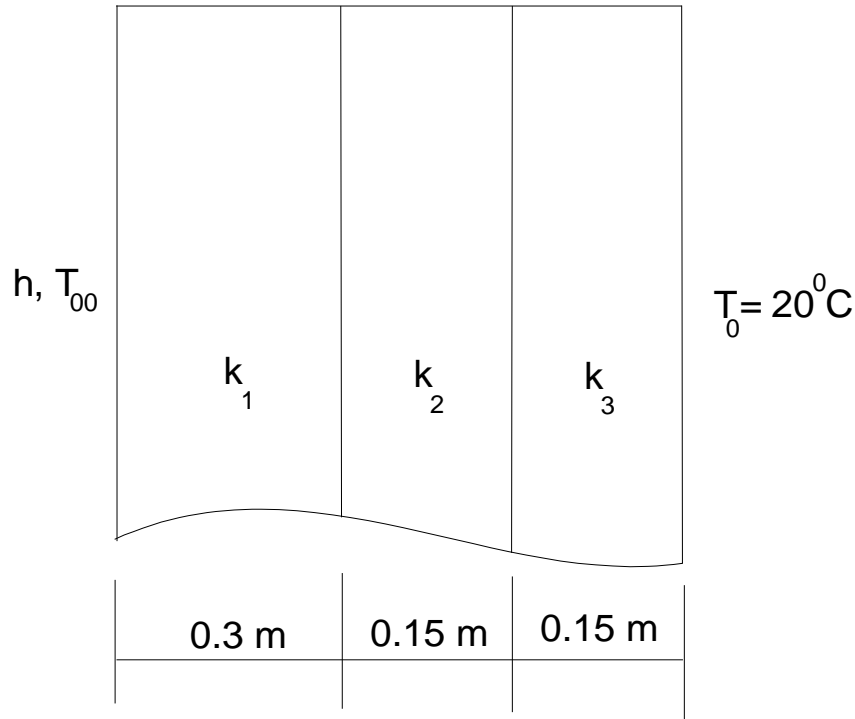
Cubic Element

Mesh	Tip Disp. (in)	Root Force (lb)	Tip Force (lb)
1-element	1.067(10 ⁻⁶)	21.3	6.4
2-elements	1.067(10 ⁻⁶)	21.33	3.4
4-elements	1.067(10 ⁻⁶)	21.33	1.75

Example 3

(1D- C^0 Linear Element)

Example T4L3-1



Mixed BC

$$c_a = -h$$

$$d_a = hT_{\infty}$$

EBC

$$k_1 = 20 \frac{\text{W}}{\text{m} \cdot ^\circ\text{C}}$$

$$k_2 = 30 \frac{\text{W}}{\text{m} \cdot ^\circ\text{C}}$$

$$k_3 = 50 \frac{\text{W}}{\text{m} \cdot ^\circ\text{C}}$$

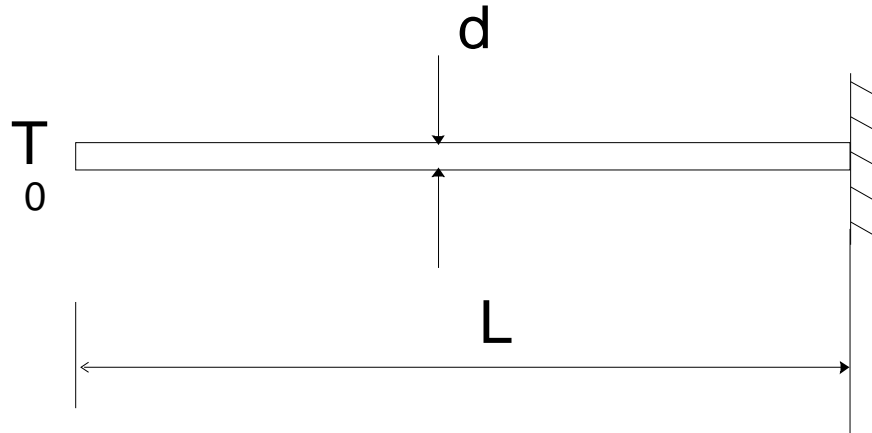
$$T_{\infty} = 800^\circ\text{C}$$

$$h = 25 \frac{\text{W}}{\text{m}^2 \cdot ^\circ\text{C}}$$

Example 4A

(1D- C^0 Linear Element)

Example T4L3-2



EBC

$$\alpha = k$$

NBC

$$\beta = \frac{hl}{A}$$

$$f = \frac{hl}{A} T_\infty$$

$$\bar{k} = 24.8 \frac{BTU}{h \cdot ft \cdot ^\circ F}$$

$$L = 0.516 ft$$

$$A = \pi \frac{d^2}{4} = 5.326(10^{-4}) ft^2$$

$$T_0 = 150^\circ F$$

$$T_\infty = 80^\circ F$$

$$h = 6 \frac{BTU}{h \cdot ft^2 \cdot ^\circ F}$$

$$l = \pi d = 0.0818 ft$$

Example T4L3-2

Mesh	Right Temp. (F)	Right Flux (Btu/h-ft ²)	
1-element	78.4	4268.3	
2-elements	88.97	1174.4	
4-elements	90.5	541.6	
64-elements	91.02	33.0	
256-elements	91.02	8.27	

Example 4B

(1D- C^0 Higher Order Elements)