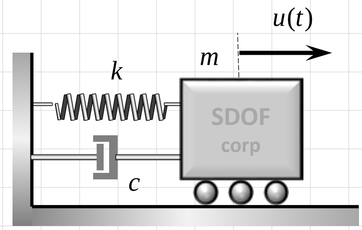
Analyze the *central difference method* for solving the damped vibration problem by finding the exact solution to the discrete equations. The equation of motion and initial conditions are

$$\ddot{u}(t) + 2\xi\omega\dot{u}(t) + \omega^2 u(t) = 0$$
$$u(0) = u_o$$
$$\dot{u}(0) = v_0$$



Damped System

and the classical solution is

$$u(t) = \mathrm{e}^{-\xi\omega t}\left(u_o \cos\omega_D t + \frac{v_o + \xi\omega u_o}{\omega_D} \sin\omega_D t \right)$$

ω	natural frequency
ξ	damping ratio
$\omega_D = \omega\sqrt{1-\xi^2}$	damped frequency

The *central difference method* satisfies the equation of motion at the discrete time points and approximates the velocity and acceleration with symmetric difference equations. Hence, the discrete equations are

$$a_n + 2\xi\omega v_n + \omega^2 u_n = 0$$
$$u_{n+1} - u_{n-1} - 2hv_n = 0$$
$$u_{n+1} - 2u_n + u_{n-1} - h^2 a_n = 0$$

$u_n \leftrightarrow u(t_n)$	displacement at time t_n
$v_n \leftrightarrow \dot{u}(t_n)$	velocity at time t_n
$a_n \leftrightarrow \ddot{u}(t_n)$	acceleration at time t_n
$h = t_{n+1} - t_n$	time step size

Find the exact solution to the difference equations. Does damping affect the stability limit found for the undamped system? What are the numerical features of this method. Write a MATLAB code to implement this numerical integrator and compare the exact numerical analysis with the results computed from the time-stepping algorithm.