Guided Study Log on Astrophysics: radiative transfer

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Abstract

This document is a record of my guided study on Astrophysics in 2022 fall semester, under the supervision of Dr. LEUNG Po Kin.

For the guided study, the detailed topic is radiative transfer. The content consists of two parts, one of which is some important chapters in *Radiative Processes in Astrophysics* [1], written by GEORGE B. RYBICKI and ALAN P. LIGHTMAN, while the other is the documentation and some related literature of a library: RADMC3D [2], which is widely used in radiative transfer.

For this document, the first section is an introduction, illustrating the relationship among all the things. The second section is for the book, five chapters of which have been selected by us to discuss. The third section is about the library RADMC3D, the 6th and 7th chapters of which contain the most physics principle of the library, so we mainly talked about these two chapters. In addition, two papers related to RADMC3D have been discussed too. The forth section is about some important abbreviations which have appeared during my study. I recorded them here just for further reference.

For every chapter in this document, I would firstly conclude the knowledge in this chapter, then show the questions arose from our guided study discussion and our answers.

Contents

1 Introduction

Our topic is **radiative transfer**, to know radiative transfer, we need to know what radiation is firstly, which is also the first question I was asked in this semester. Actually my first thought, is considering EM wave from the view of energy, because there is a chapter called “Radiation” in David Griffiths’ *Introduction to Electrodynamics*. But from Wikipedia we could know it includes: electromagnetic radiation, particle radiation, acoustic radiation, gravitational radiation, etc. But luckily, all the radiation talked about in this book refers to the EM radiation, so what David Griffiths taught me is still useful.

Since we have stressed above, EM radiation focuses on the properties of EM wave from the perspective of **energy**. As a result, the basic idea for radiative transfer is actually **law of conservation of energy**: a radiation source has generated some radiation, then the radiation passes some medium, which has processed the radiation(by absorption, emission and scattering), causing the change of energy of the radiation(Here I do not use "the change of total energy", since in some cases the total one is conserved but the distribution in different frequency, different solid angle direction would also change), thus the final received energy may differ from the initial one.

Now we could understand the core equation better, whose normal form could be written as:

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

(**Equation** *formal radiative transfer equation*)

It is also the Equation (1.23) in [1].

Here I_ν is the intensity of radiation, s is the distance the radiation has passed, α_ν is the **absorption coefficient**, while the j_ν is called **emission coefficient**. Don't worry, all the concepts would be discussed in detail later. Now we just need to conclude the core idea the **Equation** *formal radiative transfer equation* told us: the changed intensity during a small fraction of passed distance, has contributions from two aspects, one is absorption which performs subtracting role, the other is emission playing an adding role.

Now, two questions may be arisen from my readers:

1. Q: from your description of radiative transfer, it seems to have two parts, the initial radiation, and then it propagates through a medium. But the **Equation** *formal radiative transfer equation* only has the second part.

   A: that's because this one is in derivative form, if we do a integral to it, then we have the integral form, where the $I_\nu(0)$ term appears.

2. Q: About the processes take place in the medium, you said they include absorption, emission and scattering, so **where is the scattering term**?

   A: Actually the equation(3.66) in book [3] has given the more general form Radiative transfer equation:

$$\frac{dI_\nu}{ds} = -(\alpha_\nu + \alpha_\nu^s)I_\nu + j_\nu + j_\nu^s$$

(**Equation** *radiative transfer equation considering emission, absorption and elastic scattering*)

   Although it only considers a special type of scattering, we could found the character scattering plays-if we just simply new two variables: $\alpha_\nu^{total} = \alpha_\nu + \alpha_\nu^s, j_\nu^{total} = j_\nu + j_\nu^s$ and substitute them into the **Equation** *radiative transfer equation considering emission, absorption and elastic* *scattering*, then it would get the same form as **Equation** *formal radiative transfer equation*, which is thus qualified to be a "formal" one.

As we have discussed in the question 2 above, scattering could be unified into the radiative transfer equation, but how about other physics mechanisms we saw in the content of this book? They include black body radiation, Bremsstrahlung, Cyclotron, Synchrotron and Curvature, etc. What are their correspondence in the radiative transfer equation?

Actually we could see all of them the same as scattering, they are just offering their contributions through the absorption coefficient α_ν and the emission coefficient j_ν. We may decide which mechanism dominating the major part of the two coefficients based on the studied problem. For example, Synchrotron is the fundamental process in radio astronomy, while Compton scattering is the fundamental process in X-ray Astronomy and γ-ray Astronomy [3, 219].

After the discussion above, we could conclude the thread of thoughts in *Radiative Processes in Astrophysics*: centering the radiative transfer equation, there are many radiation mechanisms could make contributions to the absorption and emission coefficients, thus influencing our radiative transfer.

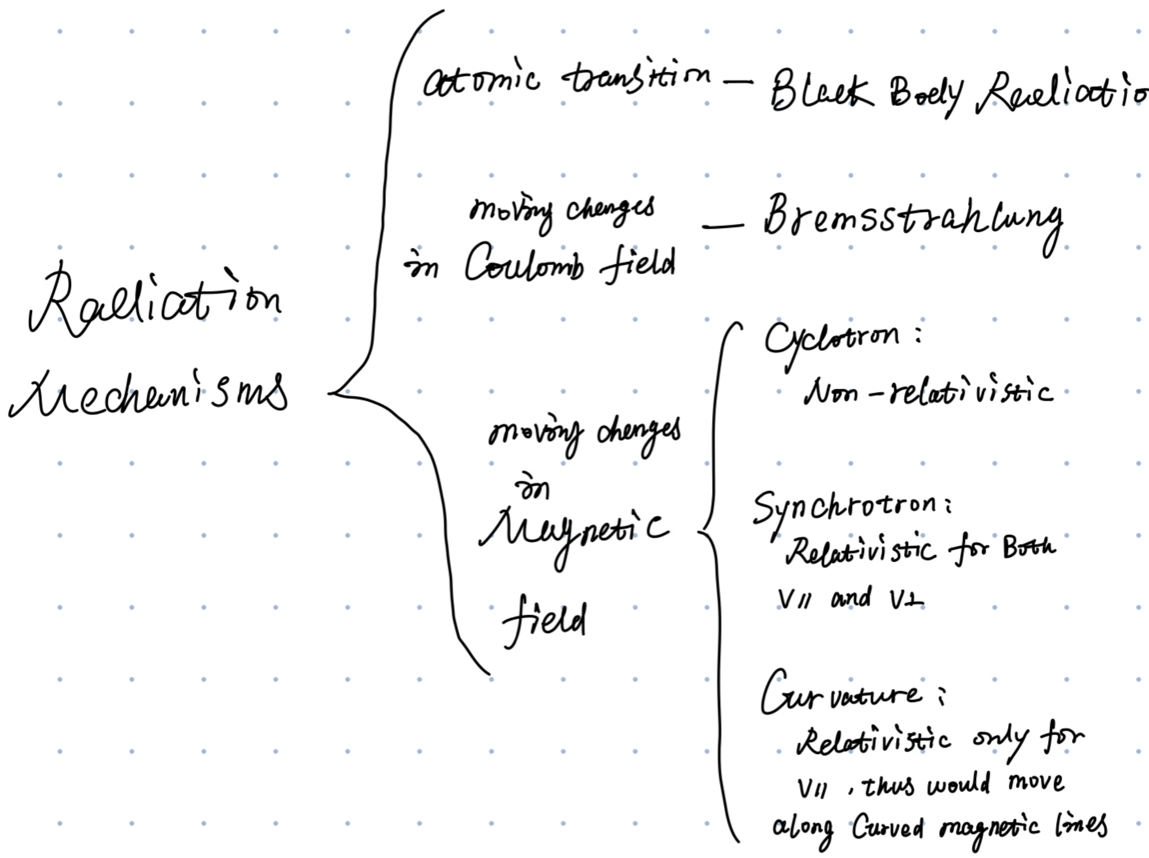


Figure 1: radiation mechanisms

In detail, Chapter 1 discussed some basic concepts, based on which we could get the radiative transfer equation; Chapter 2 is the theory of Radiation fields, we start from Maxwell Equations to get some common representations for radiation fields like radiation spectrum (this chapter does not talk about the specific situation, so the result is represented by $\vec{E}$ and $\vec{B}$ thus could be further used); Chapter 3 and 4 accordingly talked about the specific occasions under non-relativistic and relativistic, using the general result of chapter 2; chapter 5 and 6 selected two typical mechanisms: Bremsstrahlung and Synchrotron to illustrate how different mechanisms contribute to the coefficients in radiative transfer equation like I just said, using the math tools in chapter 3 and 4 accordingly.

Finally, although many things' explicit solutions should be under quantum theory, our classical theory could give a good approximation too. The following paragraph tells us when we could say the classical theory is still valid.

Criterion of whether classical theory is valid Here "classical" means using the classical orbiting theory to describe the behaviors of particles, so the uncertainty of the position of the particles needs to be smaller than the characteristic scale of the studied system r(here r is subject to the specific problem, it could be the distance between particles or the wavelength of the radiation from the particles), namely $\Delta x \ll r$

Use the uncertainty relationship $\Delta x \Delta p \simeq h$, we could have $\Delta x = \frac{h}{\Delta p} \ll r$. At the same time, $\Delta p \simeq p$, so $\lambda = \frac{h}{p} \ll r$. It means, only the de Broglie wavelength is much smaller than the characteristic scale length, then the classical theory is valid.

But de Broglie wavelength is not the directly measured quantity, so we need to transform it to radiative frequency and the energy of the particle: let us imagine a particle vibrating near its balancing position, whose momentum is p, energy is W, velocity is v, frequency is $\nu \simeq \frac{v}{r}$ then we have:

$\frac{h}{p} \ll r \Longrightarrow$

$\frac{h}{r} \ll p = \frac{W}{v} \Longrightarrow$

$h\frac{v}{r} \ll W \Longrightarrow$

$h\nu \ll W$

In language it is: the radiated photon's energy is only a small portion compared to the particle's energy, then we could use the classical theory. If the photon's energy is comparable to the particle's energy, we are supposed to use quantum theory to treat it.

2 Radiative processes in astrophysics

2.1 Chapter 1 Fundamentals of radiative transfer

2.1.1 Some basic concepts

Chapter 1 has discussed many similar concepts, so, to avoid being messy, we classify the concepts with their units.

1. $erg \cdot cm^{-3}$
   (a) total radiation density, $u = \int u_\nu d\nu = \frac{4\pi}{c} \int J_\nu d\nu$
   (b) total momentum flux, $p = \int p_\nu d\nu$
2. $erg \cdot cm^{-2} \cdot s^{-1}$
   (a) Energy flux, $F = \frac{dE}{dA \cdot dt} = \int F_\nu d\nu$
3. $erg \cdot cm^{-3} \cdot Hz^{-1}$
   (a) *radiation density for a specific frequency, $u_\nu = \int u_\nu(\Omega) d\Omega$
   (b) momentum flux along the ray at angle θ, $dp_\nu = \frac{dF_\nu}{c} \cdot cos\theta$ (divided by c to transform energy to momentum, multiply $cos\theta$ to get the component normal to dA)
   (c) *momentum flux for a specific frequency, † $p_\nu = \int dp_\nu = \frac{1}{c} \int I_\nu(\nu, \Omega) cos^2\theta d\Omega$
4. $erg \cdot cm^{-2} \cdot s^{-1} \cdot Hz^{-1}$
   (a) differential amount of flux(from the solid angle $d\Omega$), † $dF_\nu = I_\nu(\nu, \Omega) cos\theta d\Omega$ (multiply $cos\theta$ because of the reduction of effective area [1, 4])
   (b) Net flux(in the direction $\vec{n}$), †$F_\nu = \int dF_\nu = \int I_\nu(\nu, \Omega) cos\theta d\Omega$
5. $erg \cdot cm^{-3} \cdot ster^{-1} \cdot Hz^{-1}$
   (a) Radiative energy density, $u_\nu(\Omega) = \frac{dE}{dV \cdot d\Omega \cdot d\nu}$
6. $erg \cdot cm^{-2} \cdot s^{-1} \cdot ster^{-1} \cdot Hz^{-1}$
   (a) Specific intensity or brightness, $I_\nu(\nu, \Omega) = \frac{dE}{dA \cdot dt \cdot d\Omega \cdot d\nu}$

*Actually this one does not have a name in this book, so I just name it by myself

†here for the intensity, the author only used I_ν, but I think specifying the solid angle would be more clear, so I use $I_\nu(\nu, \Omega)$ instead

(b) mean intensity, $J_\nu = \frac{1}{4\pi}\int I_\nu(\nu,\Omega)d\Omega$

With so many concepts above, let me give some explanations firstly:

1. For all the concepts with a $*$, their names have not appeared in the book, so I just gave them the names

2. For all the equations with a †, the (ν,Ω) after I_ν inside them are added by me for clear purpose.

3. For the concept 4a, one question may be proposed: since this concept is from "the solid angle $d\Omega$", why there is not a "ster" in its unit?
   A: I think the author's meaning may be there is a new physical quantity equals to $\frac{dF_\nu}{d\Omega}$, which is the real owner of the description: momentum flux along the ray at angle θ. So for dF_ν, it is naturally not to have a solid angle unit with it. 3b is the same.

Then we could give a description about the relationship among them(Figure 2):

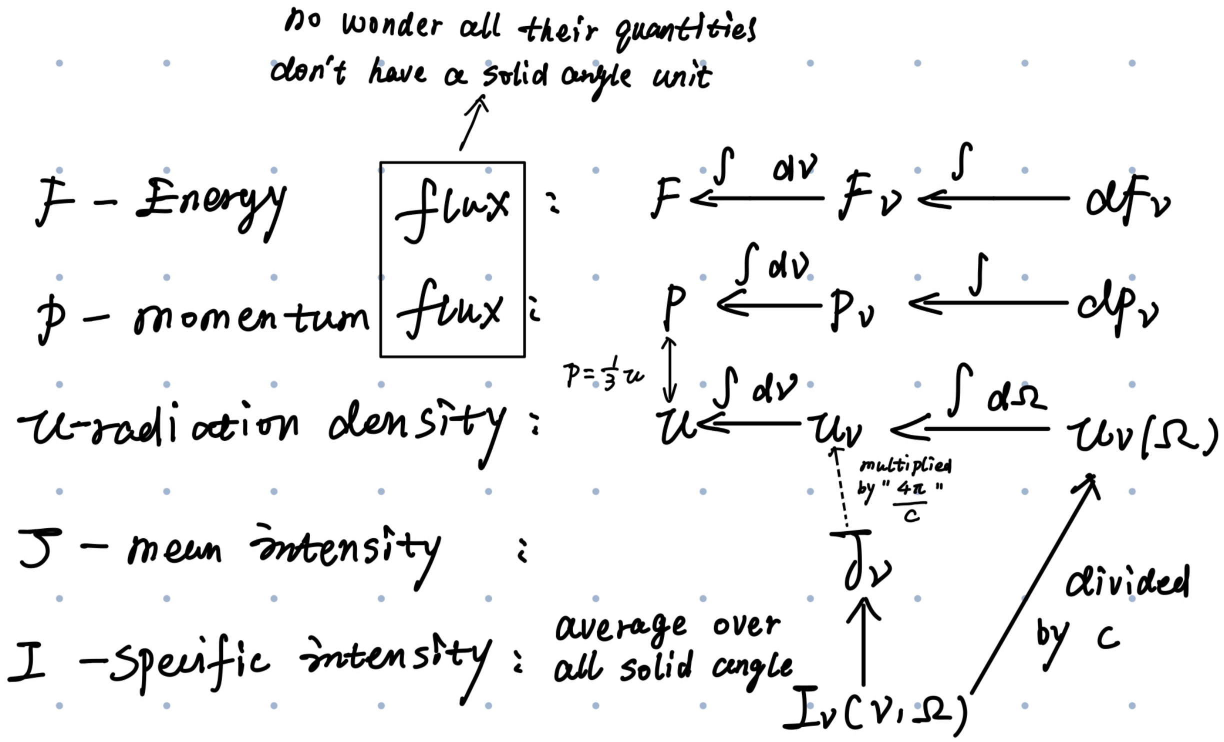


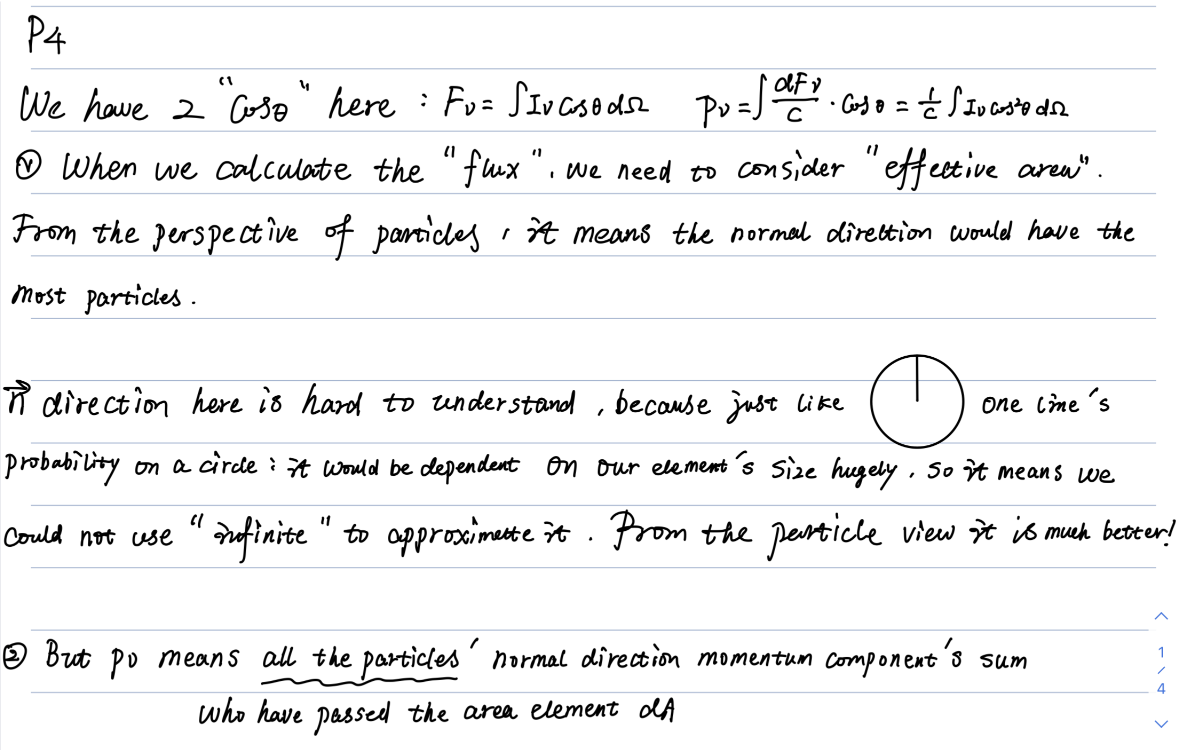
Figure 2: The relationship among those quantities

2.1.2 Some things we could conclude about these concepts

1. **The radiation pressure of an isotropic radiation field is one-third the energy density**

Actually we are talking about the momentum flux not the radiation pressure, but it has the same unit as the radiation pressure. So now energy density, radiation pressure and momentum flux would share the same unit. The detailed proof is in 3.

2. **There is one $cos\theta$ in flux formula but square in the momentum flux formula**



P4

We have 2 "$\cos\theta$" here : $F_\nu = \int I_\nu \cos\theta \, d\Omega$ $\quad p_\nu = \int \frac{dF_\nu}{c} \cdot \cos\theta = \frac{1}{c} \int I_\nu \cos^2\theta \, d\Omega$

① When we calculate the "flux", we need to consider "effective area".
From the perspective of particles, it means the normal direction would have the most particles.

$\vec{n}$ direction here is hard to understand, because just like one line's probability on a circle: it would be dependent on our element's size hugely. So it means we could not use "infinite" to approximate it. From the particle view it is much better!

② But p_ν means all the particles' normal direction momentum component's sum who have passed the area element dA

Figure 3: cos theta square's explanation

3. **The inverse square law for energy flux from an Isotropic source**
Select the example of a spherically symmetric star, there is one reference sphere S_1 whose radius is r_1, then for any sphere S with radius r we have:

$$F(r_1) \cdot 4\pi {r_1}^2 = F(r) \cdot 4\pi r^2 \tag{1}$$

So if we regard the reference sphere as a default constant, then we have:

$$F(r) = \frac{constant}{r^2} \tag{2}$$

4. **Constancy of Specific Intensity Along Rays in Free Space** In the definition of specific intensity, we considered the rays from a point or to be more precise, a small area element. But in the proof of this principle on the book, the small element area seems to have different solid angles(although they may be equal), thus is not "small element area" anymore.

Lightman P7 Constancy of specific Intensity along rays in Free Space

Full process of Proof:

$$dE_1 = \sum_{i}^{n} I_{\nu 1 i}\, dA_{1i}\, \underbrace{dt\, d\Omega_1\, d\nu_1}_{\text{the same for all}}$$

$$= dt\, d\Omega_1\, d\nu_1 \sum_{i}^{n} I_{\nu 1 i}\, dA_{1i}$$

We have $dA_{11} = dA_{12} = dA_{13} = \cdots = dA_{1n} = \frac{dA_1}{n}$

$\therefore dE_1 = dt\, d\Omega_1\, d\nu_1\, dA_1 \sum_{i}^{n} \frac{I_{\nu 1 i}}{n}$

So if n is big enough and dA_1 is small enough, $I_{\nu 1 i}$ is a constant:

$I_{\nu 11} = I_{\nu 12} = \cdots = I_{\nu 1 i} = \cdots I_{\nu 1 n} = I_{\nu 1}$

$\therefore dE_1 = dt\, d\Omega_1\, d\nu_1\, I_{\nu 1}\, dA_1$

The same reason: $dE_2 = dt\, d\Omega_2\, d\nu_2\, I_{\nu 2}\, dA_2$

And By Energy conservation Law: $dE_1 = dE_2$

$$\cancel{dt}\, d\Omega_1\, \cancel{d\nu_1}\, I_{\nu 1}\, dA_1 = \cancel{dt}\, d\Omega_2\, \cancel{d\nu_2}\, I_{\nu 2}\, dA_2$$

$$\frac{\cancel{dA_2}}{\cancel{R^2}} I_{\nu 1}\, \cancel{dA_1} = \frac{\cancel{dA_1}}{\cancel{R^2}} I_{\nu 2}\, \cancel{dA_2} \Rightarrow I_{\nu 1} = I_{\nu 2}$$

Figure 4: Constancy of specific intensity along rays in free space

5. **Is there a conflict between inverse square law and the Constancy of Specific Intensity Along Rays in Free Space?** We use Proof of the Inverse Square Law for a uniformly bright sphere to illustrate this problem.

   We could use the result to get an answer of the special case: $r = R$, then $F = \pi B$. The flux at a surface of uniform brightness B is simply πB.

P8 Proof of the Inverse Square Law for a Uniformly Bright Sphere

This problem was to show there is no conflict between the constancy of specific intensity and the inverse square law.

This is for Flux, so we need to calculate flux to verify it.

So constancy of specific intensity is our tool to prove

the distance between sphere and viewer point is free space

$\Downarrow$

For a certain ray, its start and end have the same intensity B

$$F = \int I \cos\theta \, d\Omega$$

$I \to$ a Constant B

$$= \int B \cos\theta \, d\Omega$$

$$= B \int \cos\theta \, d\Omega = B \int_0^{2\pi} d\phi \int_0^{\theta_c} \sin\theta \cos\theta \, d\theta$$

$$= \pi B \left(\frac{R}{r}\right)^2$$

One question here: what's "ϕ"?

2π in rotating direction

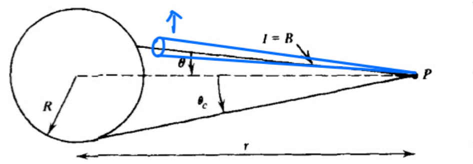


Figure 1.6 *Flux from a uniformly bright sphere.*

Wait! It seems to be the way we plotted solid angle!

How comes ϕ?

Because solid angle has: $d\Omega = \sin\theta \, d\theta \, d\phi$

Figure 5: Proof of the Inverse Square Law for a uniformly bright sphere

6. Following by the above question, here comes a new question, what is the small element of solid angle?

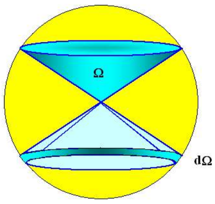


Figure 6: the small element of solid angle

I have found Figure 6 from `https://www.tf.uni-kiel.de/matwis/amat/elmat_en/kap_3/basics/b3_2_2.html`, is it a conflict with the $d\Omega$ in Figure 5?

A: No, actually one was the $d\Omega = (\Omega + d\Omega) - \Omega$, but another one is the special case of it: just set $\Omega = 0$ then $d\Omega = (0 + d\Omega) - 0 = d\Omega$

7. **Why we need to introduce concept of intensity when we have flux**

Flux : Astronomers can't agree on flux

$F_\nu = \int I_\nu \cos\theta \, d\Omega$ → But distance would influence $d\Omega = \frac{dA}{R^2}$

direction of observation

① So same star could be observed into different results of flux by different astronomers

on different planets (R) to observe a same star

② One dimmer / one brighter

Different stars may be observed in same flux by same astronomer because of the different sizes. (dA)

So we need Brightness / Specific Intensity

$$I_\nu = \frac{dE}{dA \, dt \, d\Omega \, d\nu}$$

This variable has been controlled same

the unit of dt and dν could cancel with each other, but that would be misleading.

Figure 7: The reason we introduced the concept of Intensity

2.1.3 Emission coefficient and Absorption coefficient

In this part we have derived the emission coefficient and the absorption coefficient, which are not only for these two mechanisms, but could be applied to any mechanism satisfying these two rule: the changed intensity is proportional to the intensity and the distance element, or only proportional to the distance element. So for example, we could use Einstein's coefficients to describe the black body emission. In his theory, there are two types of emission called spontaneous emission and stimulated emission, the latter of which actually should be applied the same rule as absorption instead of emission.

As I said in the intro part, we could only consider Emission and Absorption. (scattering could be included in this frame)

Emission: For all frequencies: $dE = j\, dV\, d\Omega\, dt$

Monochromatic: $dE = j_\nu\, dV\, d\Omega\, dt\, d\nu$

thus containing direction

So $P_\nu = 4\pi j_\nu$ works for isotropic emitter.

the total radiated power

And if change $dV \to dm$ we have $\varepsilon_\nu = \frac{P_\nu}{\rho}$

Use j_ν to represent $dI_\nu = \frac{dE}{dA\, d\Omega\, dt\, d\nu} = \frac{dE \cdot ds}{dV\, d\Omega\, dt\, d\nu}$

$= j_\nu \cdot ds$

Figure 8: emission coefficient

Absorption

Unlike emission, Absorption would be proportional to the Intensity at current position. We use the Cross Section model to get this conclusion.

Absorber: each element has an area $= \sigma_\nu$

volume density $= n$ (number per unit volume)

$$-dI_\nu \, dA \, d\Omega \, dt \, d\nu = I_\nu (n \sigma_\nu \, dA \, ds) \, d\Omega \, dt \, d\nu$$

definition of dI_ν from the cross section model

two ways to represent energy loss

$$\therefore \quad dI_\nu = -\underbrace{n \sigma_\nu}_{\alpha_\nu} I_\nu \, ds$$

α_ν : unit — $\frac{\text{number}}{\text{cm}}$

If want to take out the variable of density:

$$k_\nu = \frac{\alpha_\nu}{\rho} \; (\text{cm}^2/\text{g}) \qquad dI_\nu = -k_\nu \rho I_\nu \, ds$$

more precisely: number $\cdot$ cm^2/g

it means for a certain length of the same substance, if we compress it the total absorbed energy stays the same (since the total number of absorbers is not changed.

And ($\uparrow\rho \cdot ds\downarrow$) stays the same, so k_ν stays the same

k_ν describes the property of substance —— what's absorbing ability of "1g" substance.

Figure 9: absorption coefficient

2.1.4 Radiative Transfer Equation

Radiative Transfer Equation

Combining absorption and emission, we could have $\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$

Firstly we could have two special solutions:

$$\begin{cases} I_\nu(s) = I_\nu(s_0) + \int_{s_0}^{s} j_\nu(s')ds' & \text{, emission only} \\ I_\nu(s) = I_\nu(s_0)\exp\left[-\int_{s_0}^{s}\alpha_\nu(s')ds'\right] & \text{, absorption only} \end{cases}$$

One more general treatment: introduce optical depth

$$d\tau_\nu = \alpha_\nu ds \quad \text{or} \quad \tau_\nu(s) = \int_{s_0}^{s}\alpha_\nu(s')ds'$$

Then we could have: $\frac{dI_\nu}{d\tau_\nu} = -I_\nu + \underbrace{S_\nu}_{\frac{j_\nu}{\alpha_\nu}}$

We want to express everything in terms of τ_ν

So we could ~~introduce~~ $I' = I_\nu e^{\tau_\nu}$ $S' = S_\nu e^{\tau_\nu}$

then $\frac{dI'}{d\tau_\nu} = S'$

$$LHS = \frac{d(I_\nu \cdot e^{\tau_\nu})}{d\tau_\nu} = \frac{dI_\nu}{d\tau_\nu}\cdot e^{\tau_\nu} + \frac{d(e^{\tau_\nu})}{d\tau_\nu}\cdot I_\nu$$

$$= (-I_\nu + S_\nu)e^{\tau_\nu} + e^{\tau_\nu}\cdot I_\nu$$

$$= S_\nu \cdot e^{\tau_\nu} = RHS = S'$$

This form is just like Emission only Equation, which is easy to solve: $I'(\tau_\nu) = I'(0) + \int_0^{\tau_\nu} S'(\tau_\nu')d\tau_\nu'$

$\downarrow$ Substituting I_ν and S_ν

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau_\nu')} S_\nu(\tau_\nu')d\tau_\nu' \quad \text{where } \tau_\nu(s) = \int_{s_0}^{s}\alpha_\nu(s')ds'$$

Figure 10: Radiative Transfer Equation

2.1.5 Einstein coefficients, BBR and LTE

As we have derived, $h\nu \ll E$ is the condition classical theory is valid.

But many processes would not observe this condition:

Suppose there are two states: $E_m > E_n$, then $\nu = \dfrac{E_m - E_n}{h}$

Let dp_s be the probabicity from $m \to n$ per second per solid angle for

Spontaneous emission: $dp_s = a_{mn} \dfrac{d\Omega}{4\pi}$

Similarly for stimulated emission: $dp_a = b_{nm} I_\nu(\Omega) \dfrac{d\Omega}{4\pi}$

$$dp_i = b_{mn} I_\nu(\Omega) \frac{d\Omega}{4\pi}$$

Explicitly we should use QM to solve it. but we could get some relationships among them by thermodynamics: Boltzman distribution requires:

$$\frac{N_n}{N_m} = \left(\frac{g_n}{g_m}\right) \exp\left(\frac{E_m - E_n}{kT}\right) \quad ①$$

where N_m, N_n is the number of atoms on E_m and E_n

g_m, g_n is the statistical weight (degeneracy)

At the same time, if it is in thermal equilibrium, the absorption number should = emission number :

$$N_m (a_{mn} + b_{mn} I_\nu(\Omega)) = N_n b_{nm} I_\nu(\Omega) \quad ②$$

Combining ① and ②: $b_{nm} I_\nu(\Omega) = \dfrac{g_m}{g_n} (a_{mn} + b_{mn} I_\nu(\Omega)) \exp\left(-\dfrac{E_m - E_n}{kT}\right)$ ③

The above discussion was under a certain temperature "T", and $T \to \infty$ we have

$I_\nu(\Omega, T) \to \infty$ then ③ becomes: $b_{nm} = \dfrac{g_m}{g_n} b_{mn}$ ④

use ④ back into ③ $I_\nu(\Omega, T) = \left(\dfrac{a_{mn}}{b_{mn}}\right) \left(e^{\frac{h\nu}{kT}} - 1\right)^{-1}$ ⑤

use another approximation to get the relationship between a_{mn} and b_{mn} :

$KT \gg \hbar\omega$: ⑤ becomes $I_\nu(\Omega, T) = \frac{a_{mn}}{b_{mn}} \frac{KT}{h\nu}$ ⑤'

And if we use one formula for $KT \gg \hbar\omega$ approximation : Rayleigh-Jeans

$$I_\nu(\Omega, T) = 2KT\,\nu^2/c^2 \quad ⑥$$

⑤' Compares with ⑥ : $a_{mn} = \frac{2h\nu^3}{c^2} b_{mn}$ ⑦

As a result, for ④ and ⑦ we know : if any one of the three is known, then we could get the rest two.

Besides, ⑦ back into ⑤ we could have Black Body Radiation :

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/KT} - 1} \quad ⑧$$

Kirchhoff's Law of thermal radiation :

if the medium was in "LTE" (not have to be "whole" TE)

then for every local region : $S_\nu = \frac{j_\nu}{\alpha_\nu} = B_\nu(T)$

it is the property of matter. if we want to make sure $I_\nu^{out} = B_\nu(T)$

then it should be optically thick (just like above, $\tau_\nu \to \infty$ we have $I_\nu^{out} \to S_\nu = B_\nu(T)$)

So this result needs two conditions: ① LTE (or TE)

② $\tau_\nu \gg 1$ (optically thick)

2.1.6 Q and A

1. Page 5-About the derivation of the "component of momentum flux normal to dA"

   Q:I am kind of confused about: $cos\theta$ has two powers in Equation 1.4.

   As the author said before Equation (1.3b), $F_\nu(\mathbf{n})$ is the net flux in the direction $\mathbf{n}$. So when he wanted "to get the component of momentum flux normal to dA", why he still "multiply by another factor of $cos\theta$" and finally got Equation 1.4?

   $$p_\nu(dynes \cdot cm^{-2} \cdot Hz^{-1}) = \frac{1}{c}\int I_\nu cos\theta^2 d\Omega \qquad \text{(Equation 1.4)}$$

   I think only one power of $cos\theta$ is enough.

   A: It has been answered in Figure 3.

2. Page 5-About the unit of p_ν in Equation 1.4

   Q: As the author said, p_ν is just the sum of $\frac{dF_\nu}{c}$, so the two one should hold the same unit, but why not? Based on Equation (1.3a), the unit of dF_ν is $erg \cdot s^{-1} \cdot cm^{-2} \cdot Hz^{-1}$, so $\frac{dF_\nu}{c}$ holds the unit of $erg \cdot cm^{-3} \cdot Hz^{-1}$, which contradicts with the unit in Equation 1.4.

   A: Actually the two units are different: please note, it is "dynes" in the Equation 1.4. And 1 dyne equals to 1 erg/cm.

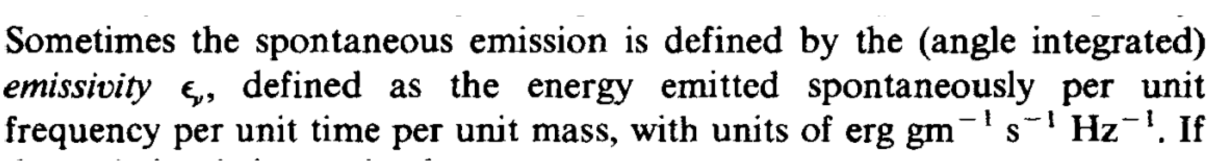
3. **Page 6-About the derivation of the radiation pressure formula.**

   Q:From equation 1.9 we know that $u = \frac{4\pi}{c}\int J_\nu d\nu$. Meanwhile, at the bottom of Page 6, the book shows that $p = \frac{2}{c}\int J_\nu d\nu \int cos\theta^2 d\Omega$. For the solid angle, we know that $d\Omega = sin\theta d\theta d\phi$, substituting which into the former equation we could get $p = \frac{2}{c}\int J_\nu d\nu \int cos\theta^2 sin\theta d\theta d\phi = -\frac{2}{c}\int J_\nu d\nu \int cos\theta^2 dcos\theta \int d\phi = -\frac{4\pi}{c}(\int J_\nu d\nu)\cdot(\frac{cos\theta^3}{3}\Big|_{\theta=0}^{\theta=\pi}) = -\frac{4\pi}{c}(\int J_\nu d\nu)\cdot(-\frac{1}{3}-\frac{1}{3}) = \frac{8\pi}{3c}\int J_\nu d\nu = \frac{2}{3}(\frac{4\pi}{c}\int J_\nu d\nu) = \frac{2}{3}u$, which contradicts with Equation 1.10 $p = \frac{1}{3}u$

   A: Since we are talking about the radiation pressure in an **enclosure containing an isotropic radiation field**, so the integral of θ should be from 0 to $\frac{\pi}{2}$.

4. Page 9-About the unit of emissivity ϵ_ν

   Q: As Figure 11 shows, there is a "gm" in the unit, is it a typo?



Sometimes the spontaneous emission is defined by the (angle integrated) *emissivity* ϵ_ν, defined as the energy emitted spontaneously per unit frequency per unit time per unit mass, with units of erg gm^{-1} s^{-1} Hz^{-1}. If

Figure 11: Page 9:the unit of emissivity

A: He wants to say "gram" whose abbreviation is "gm".

5. Page 11-About the scale comparison of the cross section and mean inter-particle distance.

   Q:The author said:"There are some conditions of validity for this microscopic picture: The most important are that (1) the linear scale of the cross section must be small in comparison to the mean interparticle distance d."

   But the cross section looks much bigger than the interparticle distance. It could not be smaller than the interparticle distance.

   A: The cross section area does not denote the cross section of the medium, but the cross section of the particle. It is also shown in Figure 1.7b.

6. Page 14-About the probability of a photon traveling an optical depth.

   Q: The author said:" the probability of a photon traveling at least an optical depth τ_ν is simply $e^{-\tau_\nu}$", but why "at least" instead of "on average"?

   A: The formula was derived when there is absorption only. So when considering emission, the photon would travel further since there is complement from emission.

7. Page 25-About the relationship of wavelength peak and frequency peak.

   Q:The author said:"One should be careful to note that the peaks of B_λ and B_ν do not occur at the same places in wavelength or frequency; that is, $\lambda_{max} \cdot \nu_{max} \neq c$, so why? A: The two Planck Law equations are:

$$B_\nu(T) = \frac{2h\nu^3/c^2}{exp(h\nu/kT) - 1} \qquad \text{(Equation 1.51)}$$

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{exp(hc/\lambda kT) - 1} \qquad \text{(Equation 1.52)}$$

   And the physics meaning of the first one is: the peak radiance during a unit frequency interval, while the second one is the peak radiance during a unit wavelength interval.

   So the relationship between them should be: $B_\nu(T) \cdot d\nu = B_\lambda(T) \cdot d\lambda$. And $\frac{d_\nu}{d_\lambda} = \frac{d(c/\lambda)}{d\lambda} = \frac{-c}{\lambda^2}$, thus finally the equation becomes $B_\nu(T) = B_\lambda(T) \cdot \lambda^2/(-c)$, where the peaks of the two functions would not appear simultaneously.

2.2 Chapter 2 Basic theory of radiation fields

1. Page 52-About the Maxwell Equations the book quoted.

   Q: Why are the constants in the Maxwell Equations different from those in my Electrodynamics book?

   A: It is in the Gaussian Units instead of the International System of Units.

2. Page 62-About the description of the relationship between Electric field and the Power spectrum

   Q: For the Figures 2.1 2.2 2.3, the author said:"Second, the existence of a sinusoidal time dependence within the pulse shape causes the spectrum to be concentrated near $\omega \sim \omega_0$" But I think this sentence only worked for 2.1 and 2.2. For Figure 2.3, the Electric field is not even a standard sinusoidal shape, how could the Power spectrum still follow the description?

   A: What the author said is just sinusoidal function with $\omega = \omega_0$ is the main part of the electric field, not the only part. Actually, though for the Figure 2.2, the electric field there is not a "real" sinusoidal function, since it only lasts a time "T", which means the Electric field has many additional sinusoidal parts to cancel the shape to zero in the rest "non-T" part. That's also the reason why the power spectrum is not a delta function, but a "wide delta function".

3. Q: How to understand unpolarized wave? For example, why would the intensity of an unpolarized wave passing a certain polarizing filter be exactly half of the total intensity?

   A: The definition of unpolarized wave(vibrating in multiple directions) is not accurate, the real situation is:"At any instant of time at one location there is a definite direction to the electric and magnetic fields, however it implies that the polarization changes so quickly in time that it will not be measured or relevant to the outcome of an experiment."

   That is, even in a very small time interval, the direction of the wave would vary randomly many times. So if the amplitude of the unpolarized wave is A, then the amplitude of the wave passing the certain filter is $cos\theta$(θ is the angle between the wave's current plane and the filter). And the intensity is proportional to the square of amplitude, so $I_{observed} =< cos^2\theta > I = \frac{1}{2}I$.

2.3 Chapter 3 Radiation from moving charges

1. Q: What is the relationship between those "physics formulas" in the section 1-3? There are too many math derivations but I only want to know the conclusion–which I should use in different situations.

   A: From ElectroDynamics, we have gained another symbol system–**the vector potential and the scalar potential**($\vec{A}$ and V) from the two field vectors($\vec{E}$ and $\vec{B}$). Of course, due to the decrease in freedom, we need to add equations to eliminate the uncertainty(for certain $\vec{E}$ and $\vec{B}$, there are many pairs of $\vec{A}$ and V). There are usually two candidates, **Lorenz gauge condition** and **Coulomb gauge condition**.

   For a EM wave, there would always be a time interval between it was produced and we received it. So we need to introduce the concept of **retarded potential** when we studied EM wave's production from a source. By the way, usually Lorenz gauge condition is used in the calculation(at least in David Griffiths' book).

From the formula of $\vec{A}$ and V, here "one source is known" refers to its charge density(ρ) and current density($\vec{J}$) are known. And the simplest case is that the source is a single moving charge. The retarded potential for the single moving charge is called **Liénard–Wiechert potential**. When we do derivative of the Liénard–Wiechert potential($\vec{A}$ and V), we could get $\vec{E}$ and $\vec{B}$ for the single moving charge.

When we know $\vec{E}$ and $\vec{B}$, we can calculate the power of radiation emitted by a single moving particle into all angles, the non-relativistic approximation of which is called **Larmor's Formula**.

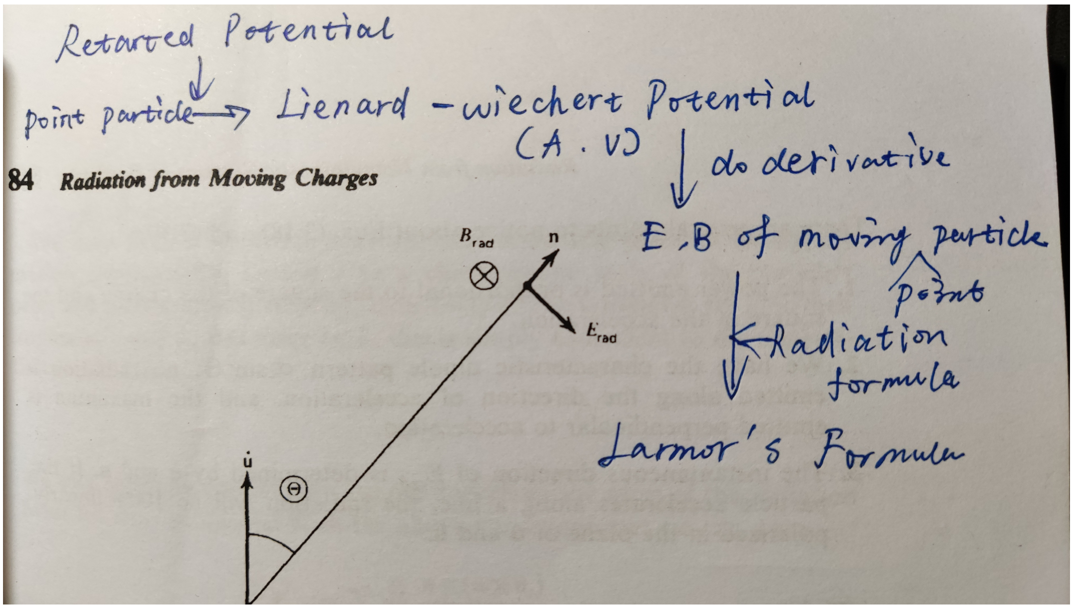


Figure 12: Relationships among all the physical quantities in Ch3

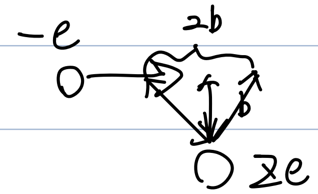
2. Q: A question about the Thomson Scattering: How to understand that there is a cross section in Thomson Scattering? Since we know Thomson Scattering is a non-quantum thing, why could it have different "possibility" in different scattering solid angle?

   A: Because our derivations of Thomson Scattering is based on the radiation formula of dipole, which is subject to solid angle. And the derivation of dipole radiation is non-quantum, thus not conflicting with the non-quantum property of Thomson Scattering.

2.4 Chapter 5 Bremsstrahlung

2.4.1 Derivation of Bremsstrahlung

Bremsstrahlung



Since it is non-relativistic we could use Larmor's formula:

$$P = \frac{2}{3}\frac{e^2 a^2}{c^3}$$

So we need to get "a"

$$\ddot{x} \sim \frac{Ze^2}{b^2 m_e}$$

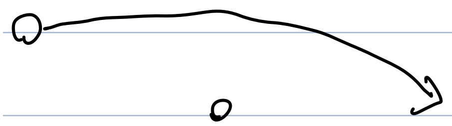
then $P \sim \frac{2}{3}\frac{Z^2 e^6}{b^4 m_e^2 c^3}$

$$E = P\Delta t$$

So we need to get "Δt"

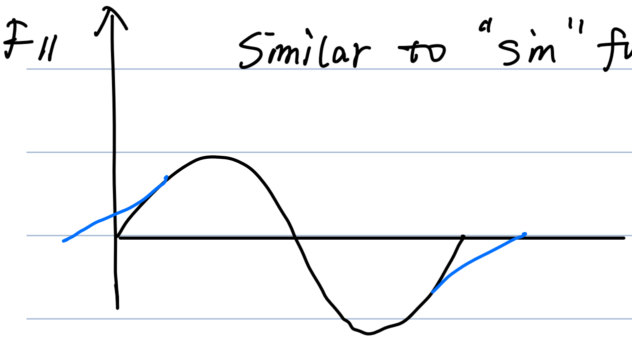
$$\Delta t \sim \frac{2b}{v}$$

then $E \sim \frac{4}{3}\frac{Z^2 e^6}{b^4 m_e^2 c^3} \cdot \frac{b}{v}$



We want spectrum, so frequency is more important than "b", which should be replaced. $b = ?(\omega)$

Let us decompose the force into path direction and perpendicular direction, then:

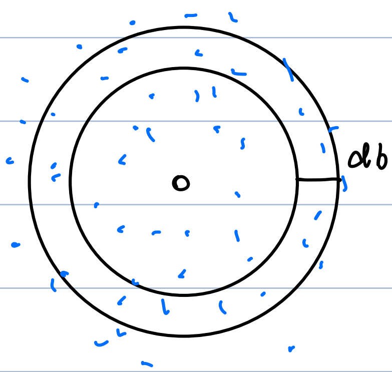


Similar to "sin" function

$$\frac{1}{2}T \sim \frac{2b}{v} \Rightarrow \nu \sim \frac{v}{4b}$$

$$d\nu = \frac{v}{4b^2}\, db$$

Now consider a bunch of electrons



electron moving fast would contribute more!

$$dP \sim E \cdot 2\pi b \cdot db \cdot n_e \cdot v$$

electron density

$$\sim \frac{4}{3}\frac{Z^2 e^6}{b^4 m_e^2 c^3} \cdot \frac{b}{v} \cdot 2\pi b \cdot \frac{4b^2}{v} \cdot d\nu \cdot n_e \cdot v$$

"v" is considered to be "fixed" here

Why we select a "ring" instead of a "shell"?

Because every single electron's path is 2D instead of 3D.

Let us divide the whole space into "N" rings

$$\sum_{i=1}^{N} E \cdot 2\pi b \cdot db \cdot n_e' \cdot V = E \cdot 2\pi b \cdot db \cdot n_e \cdot V$$

so there is no problem about ring model, only the n_e here is equal to $\sum_{i=1}^{N} n_e'$

Do derivative of V to P $\quad \frac{dP}{dV} \sim -\frac{32}{3}\pi \frac{z^2 e^6 n_e}{m_e^2 c^3 v}$

Then the next thing is to consider "v", since electrons may observe different velocity distribution. Maxwellian velocity distribution is good enough in a plasma.

$$f(v) = \left(\frac{m_e}{2\pi kT}\right)^{\frac{3}{2}} 4\pi v^2 e^{-\frac{m_e v^2}{2kT}}$$

And since "free-free", we need to define a lower limit velocity to prevent the electron to be captured by the nucleus : $h\nu \sim \frac{1}{2} m v_{min}^2$

then $$\left\langle \frac{dP}{dV} \right\rangle = \int_{v_{min}}^{\infty} \frac{dP}{dV} 4\pi \left(\frac{m_e}{2\pi kT}\right)^{\frac{3}{2}} e^{-\frac{m_e v^2}{2kT}} v^2 dv$$

$$= \frac{64\sqrt{\pi}}{3\sqrt{2}} \frac{z^2 e^6 n_e}{m_e^{\frac{3}{2}} c^3 (kT)^{\frac{1}{2}}} e^{-\frac{h\nu}{kT}}$$

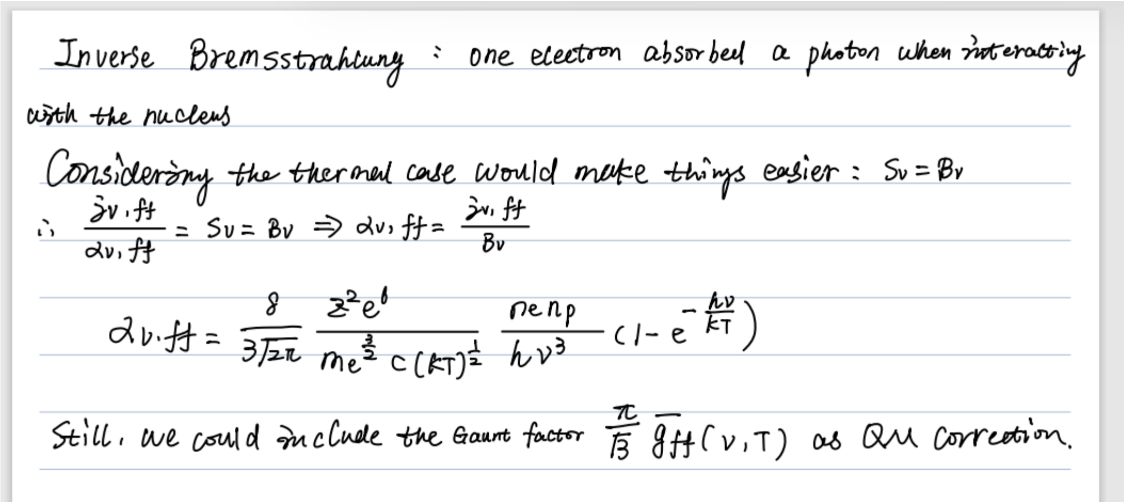
$$j_{\nu,ff} = \frac{16}{3\sqrt{2}\pi} \frac{z^2 e^6}{m_e^{\frac{3}{2}} c^3 (kT)^{\frac{1}{2}}} n_e n_p e^{-\frac{h\nu}{kT}}$$

$\left(j_\nu = \frac{1}{4\pi} P_\nu \ (1.16)\right)$ there may be multiple nuclei

power per volume per frequency

Above is the thermal Bremsstrahlung result. to be more precise, a quantum mechanical correction factor could be used : $\frac{\pi}{\sqrt{3}} g_{ff}(\nu, T)$, Gaunt factor

2.4.2 Inverse Bremsstrahlung



Inverse Bremsstrahlung : one electron absorbed a photon when interacting with the nucleus

Considering the thermal case would make things easier : $S_\nu = B_\nu$

$\therefore \frac{j_{\nu,ff}}{\alpha_{\nu,ff}} = S_\nu = B_\nu \Rightarrow \alpha_{\nu,ff} = \frac{j_{\nu,ff}}{B_\nu}$

$$\alpha_{\nu,ff} = \frac{8}{3\sqrt{2\pi}} \frac{Z^2 e^6}{m_e^{\frac{3}{2}} c (kT)^{\frac{1}{2}}} \frac{n_e n_p}{h\nu^3} (1 - e^{-\frac{h\nu}{kT}})$$

Still, we could include the Gaunt factor $\frac{\pi}{\sqrt{3}} \bar{g}_{ff}(\nu, T)$ as QM correction.

Figure 13: inverse bremsstrahlung

2.4.3 Q and A

1. Page 155-About the reason why the bremsstrahlung of two same particles is zero

   Q: Because the two particles have the same mass and charge, we could get the two things out. Then in this case, the dipole becomes $d = \sum_{i=1}^{2} q\vec{r_i} = \sum_i (\frac{q}{m}) m\vec{r_i} = \frac{q}{m} \sum m\vec{r_i}$. This form is equal to $(\frac{q}{m})\vec{R} \sum_i m$, where the $\vec{R}$ is vector of the center of mass of this system.

   Everything seems good now, but for radiation we need to look at the second order derivative of the dipole, namely $\ddot{\vec{d}}$, which is proportional to $\ddot{\vec{R}}$. And because there is not external force on the two-particle system, the center of mass's acceleration is zero, thus $\ddot{\vec{d}} = 0$, and finally $P = \frac{2\ddot{d}^2}{3c^3} = 0$.

2.5 Chapter 6 Synchrotron radiation

2.5.1 Total emit power

Synchrotron

Total emit power P

$$\frac{d}{dt}(\gamma m v) = F = \frac{q\vec{v}\times\vec{B}}{c} \Rightarrow m\gamma\frac{d\vec{v}}{dt} = \frac{q}{c}\vec{v}\times\vec{B}$$

$$a_{//} = 0 \quad a_{\perp} = \frac{q}{\gamma m c}\vec{v}_{\perp}\times\vec{B}$$

Get a constant : $\frac{qvB}{c} = \gamma m\frac{v^2}{R} \Rightarrow qB = c\gamma m\omega_B \Rightarrow \omega_B = \frac{qB}{\gamma m c}$

Then $a_{\perp}$ could be represented into $a_{\perp} = \omega_B v_{\perp}$

So from the total emission formula of relativistic particles :

$P = \frac{2q^2}{3c^3}\gamma^4(a_{\perp}^2 + \gamma^2 a_{//}^2)$ we have $P = \frac{2q^2}{3c^3}\gamma^4\omega_B^2 v_{\perp}^2$

$$= \frac{2q^2}{3c^3}\gamma^4\frac{q^2B^2}{\gamma^2 m^2 c^2}v_{\perp}^2 \quad \text{use } \beta_{\perp} = \frac{v_{\perp}}{c}$$

$$= \frac{2q^2}{3c^3}\gamma^4\frac{q^2B^2}{\gamma^2 m^2 c^2}\beta_{\perp}^2\cdot c^2 = \frac{2}{3}\frac{q^4B^2\gamma^2\beta_{\perp}^2}{c^3 m^2}$$

$$= \frac{2}{3}\Big(\underbrace{\frac{q^4}{c^4 m^2}}_{r_0^2}\Big)\, c\,\beta_{\perp}^2\gamma^2 B^2$$

$$= \frac{2}{3}r_0^2 c\beta_{\perp}^2\gamma^2 B^2$$

if the isotropic distribution is $\langle\beta_{\perp}^2\rangle = \frac{\beta^2}{4\pi}\int \sin^2\alpha\, d\Omega = \frac{2}{3}\beta^2$

Then $P = (\frac{2}{3})^2 r_0^2 c\beta^2\gamma^2 B^2$

Figure 14: Synchrotron total emit power

2.5.2 Spectrum $\mathbf{P}(\omega)$

Spectrum $P(\omega)$:

1. A qualitative analysis

From the right figure,

we could know the radiation

we received must be

periodically impulses.

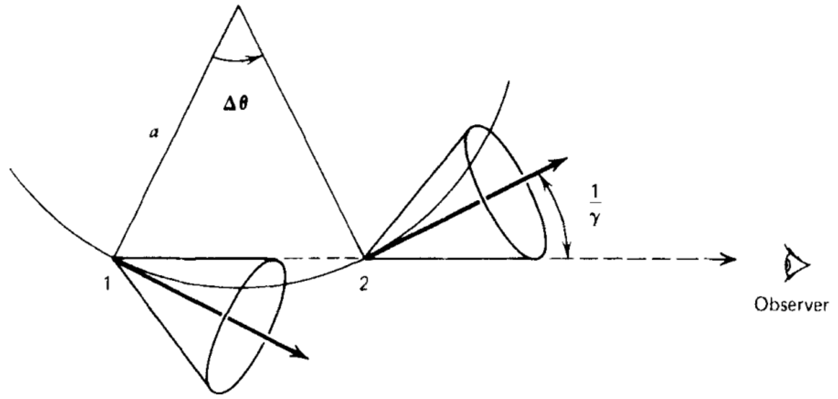
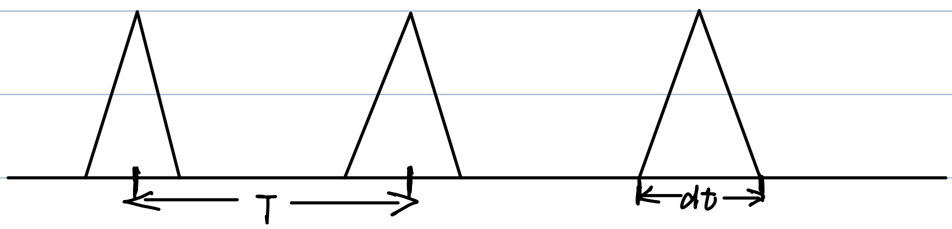


Figure 6.2 Emission cones at various points of an accelerated particle's trajectory.



T corresponds to the rotation period $T = \frac{1}{\nu_0} = \frac{2\pi}{\omega_0} = 2\pi \cdot \frac{\gamma mc}{eB} = 2\pi \cdot \gamma \cdot \frac{mc}{eB}$

$\frac{mc}{eB} = \frac{1}{\omega_L}$. ω_L is the rotation period of non-relativistic particle

$\therefore T = \frac{2\pi\gamma}{\omega_L}$, $\omega_B = \frac{\omega_L}{\gamma}$

dt is the time interval of an impulse

Firstly we could calculate the time interval electron moving from point 1 to 2 :

we call it $dt' = \frac{2\Delta\theta}{\omega_B} = \frac{2}{\gamma\omega_B}$

Secondly, dt is not equal to dt' because the distance between 1 and viewer is different from the distance between 2 and viewer.

$$dt \neq dt'$$

$$dt = \frac{(c-v)\,dt'}{c}$$, where $v\,dt'$ is the correction

$$\therefore dt = \left(1 - \frac{v}{c}\right) dt' = \frac{c\left(1-\frac{v}{c}\right)\left(1+\frac{v}{c}\right)}{1+\frac{v}{c}}\, dt' \approx \frac{1}{2\gamma^2}\, dt'$$

$$= \frac{1}{\gamma^3 \omega_B} = \frac{1}{\gamma^2 \omega_L}$$

Based on Fourier Theory, we could know this impulse could be decomposed of a series of waves: $(\omega_B, 2\omega_B, 3\omega_B, \cdots\cdots)$

$\omega_B = \frac{1}{\gamma}\omega_L$ is the base angular frequency $\Leftrightarrow$ "T"

$\omega_m = \frac{1}{dt} = \gamma^2\omega_L = \gamma^3\omega_B$ is the max-amplitude angular frequency $\Leftrightarrow$ "dt"

So the spectrum of relativistic electron seems to be a series of discrete spectrum, but because the radiation with frequencies far from ω_m have very small amplitude, and $\omega_m \gg \omega_B$, so the spectrum would look like continuous spectrum.

But the above discussion is only for $v_{\parallel} = 0$

if there is an angle α between $\vec{v}$ and $\vec{B}$, then:

the base frequency $= \sin\alpha\, \omega_B$

the peak frequency $= \gamma^3 \omega_B \cdot \sin\alpha$

in fact, we usually use $\omega_c = \frac{3}{2}\omega_m = \frac{3}{2}\gamma^3\omega_B \cdot \sin\alpha$ to represent the critical angular frequency, the frequencies above which could be cut off! (very small)

2. Get the quantitative formula

There is more analytical method, but we could make good use of Tools we have to get a simpler derivation:

① the angular distribution of Emitted and Received Power

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3}\frac{(\gamma^2 a_{\parallel}^2 + a_{\perp}^2)}{(1-\beta\mu)^4}\sin^2 \Theta' \quad (4.99)$$

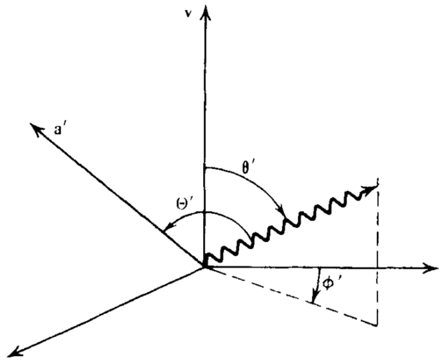


Figure 4.10 *Geometry for dipole emission from a particle instantaneously at rest.*

and its relativistic limit:

$$\frac{dP_{\parallel}}{d\Omega} \approx \frac{16 q^2 a_{\parallel}^2}{\pi c^3}\gamma^{10}\frac{\gamma^2\theta^2}{(1+\gamma^2\theta^2)^6} \quad (4.104)$$

$$\frac{dP_{\perp}}{d\Omega} \approx \frac{4 q^2 a_{\perp}^2}{\pi c^3}\gamma^{8}\frac{1-2\gamma^2\theta^2\cos 2\phi + \gamma^4\theta^4}{(1+\gamma^2\theta^2)^6} \quad (4.105)$$

(4.104) and (4.105) gave us an information: when θ appears, "γ" would also appear with the same order, so we could regard "$\gamma\theta$" as a whole

② the ~~total~~ emitted power formula P

because there is a relationship: $P = \int_0^\infty P(\omega)\, d\omega$

which could be used to do Normalization

—————————————————— Start

As we have said in the qualitative analysis, we could use an observer system to record the impulse: $E(t)$

But what we care is $\gamma\theta$: θ could be connected to "t"

let s be the distance of journey, a is the radius

then $\theta = \frac{s}{a}$, and $t = \frac{s}{v}(1-\frac{v}{c}) = \frac{s}{v}\frac{1}{2\gamma^2}$

So $S = 2\gamma^2 v t$, ∴ $\theta = \frac{S}{a} = \frac{2\gamma^2 v t}{a} = 2\gamma^2 t \cdot (\omega_B \cdot \sin\alpha)$

∴ $\gamma\theta = 2\gamma^3 t(\omega_B \cdot \sin\alpha) = 2\gamma^3 \omega_B \sin\alpha \cdot t = \frac{4}{3}\cdot \underbrace{\frac{3}{2}\gamma^3 \omega_B \sin\alpha}\cdot t$

$= \frac{4}{3}\omega_c t$

So $\gamma\theta \propto \omega_c t \Rightarrow E(t) \propto g(\omega_c t)$

BTW actually at first I wanna use $\theta = \omega_B \cdot \sin\alpha \cdot t$

Why not? Because we select the observer's system, we need to consider the "$2\gamma^2$" brought by relativistic.

We want spectrum. so do Fourier Transform:

$\hat{E}(\omega) \propto \int_{-\infty}^{\infty} g(\omega_c t) e^{i\omega t} dt \Rightarrow \hat{E}(\omega) \propto \int_{-\infty}^{\infty} g(\xi) e^{i\xi\omega/\omega_c} d\xi$

$\xi \overset{\Downarrow}{=} \omega_c t$ So $\hat{E}(\omega)$ is a function of $(\omega_c t)$

And we know $\frac{dW}{dA\, d\omega} = c|\hat{E}(\omega)|^2 \Rightarrow \frac{dW}{d\omega} \propto |\hat{E}(\omega)|^2$

So for $P(\omega)$, we could at least use a function of $(\frac{\omega}{\omega_c})$ to represent it:

$P(\omega) = C_1 F(\frac{\omega}{\omega_c})$

Now we could use total power function to normalize it:

$P(\omega) = \frac{\sqrt{3}}{2\pi}\frac{q^3 B\sin\alpha}{mc^2} F(\frac{\omega}{\omega_c})$

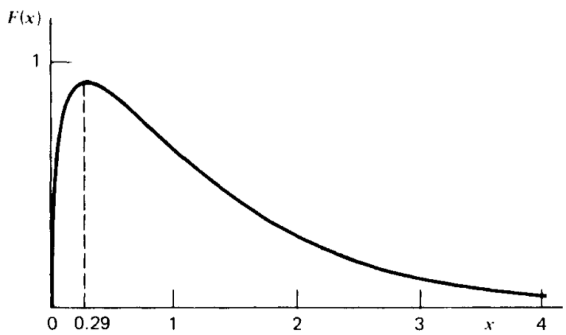


Figure 6.6 Function describing the total power spectrum of synchrotron emission. Here $x=\omega/\omega_c$. (Taken from Ginzburg, V. and Syrovatskii, S. 1965, Ann. Rev. Astron. Astrophys., 3, 297.)

2.5.3 Q and A

1. P171-Q: Why does the critical frequency in (6.11a) have "3" as the power?

   A: This question has been answered in 2.5.2.

2. P171-Q: What's the physical meaning of critical frequency in synchrotron radiation?

   A: This question has been answered in 2.5.2.

3. P174-About the total synchrotron radiation spectrum of power-law distribution electrons

   Q: How to understand the fact in a intuitive way–When p=1, the total Synchrotron Radiation emitted by the electrons obeying the distribution in (6.20) is equivalent in different frequency.

   A: Please note: although the LHS of (6.18) is written as $P(\omega)$, when we consider "energy" we should notice there is a energy factor in (6.18), which is hidden in the critical frequency(6.17c). Therefore, for our problem we could write it as $P_{single}(\omega,\gamma)$. Then $P_{total}(\omega) = \int P_{single}(\omega,\gamma)N(\gamma)d\gamma$. Thus we can see the total emission at a specific frequency interval could be determined by two factors. Since the distribution part is normalized to 1, we could imagine the graph of $P_{single}(\omega,\gamma)$ firstly and regard the distribution function as a "weight" for different γ interval. So the result could be regarded a kind of "weighted average".

   It could be shown in a matrix form, like what Figure 15 did.

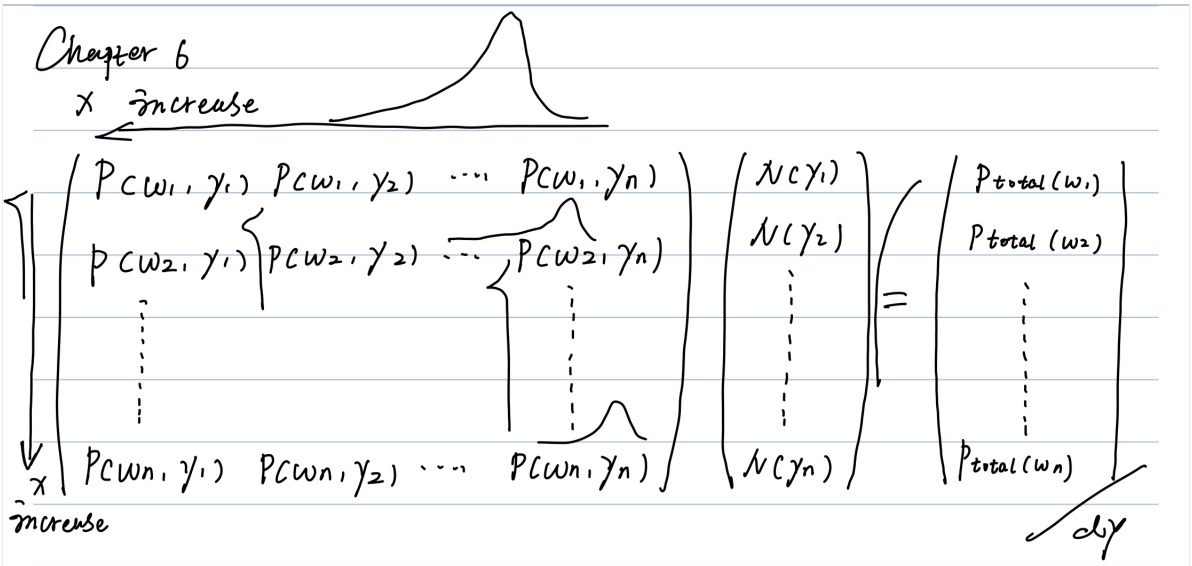


Figure 15: My drawing for P_{total} in a matrix form

Let me give some explanations of this graph:In this graph, $\omega and \gamma$ with smaller indexes have the smaller values.

We could get the size relationship in each row or column from the graph of the function

F(x)(Since other parts in (6.33) hold the same value in our problem, thus the relationship between final values is the same as that between F(x)).

For each row, γ becomes bigger from left to right, so the ω_c becomes bigger from left to right and thus the x becomes bigger from right to left. So we could do an horizontal flip of Figure 6.6 to express each row; what's more, for different rows, the x would become bigger from up to down, so the peak would arrive earlier and earlier, like what I drew.

For each column, ω becomes bigger from up to down, so the ω_c becomes bigger from up to down and thus the x becomes bigger from up to down. So we could do an horizontal flip and then a 90 degrees anticlockwise rotation of Figure 6.6 to express each row; what's more, for different columns, the x would become smaller from left to right, so the peak would arrive later and later, like what I drew.

When p in $N(\gamma) = C\gamma^{-p}$ becomes larger than one(compared to the p=1 case), then the distribution function would focus more on the smaller γ range, which means the first several columns would have the most weights. Thus the P_{total} matrix would act the same property like the first several columns, namely, the peak would appear in smaller ω. Then the result would show more like a Monotonically decreasing function, like (6.22a) suggests: $P_{tot}(\omega) \propto \omega^{-(p-1)/2} = \omega^{negative}$.

When p in $N(\gamma) = C\gamma^{-p}$ becomes less than one(compared to the p=1 case), then the distribution function would focus more(it is compared to the former case, not a arbitrary thing, since as long as p is a positive number thus -p is a negative number, then the smaller frequency range would get a bigger weight) on the bigger γ range, which means the last several columns would also have some non-negligible weights. Thus the P_{total} matrix would act more property like the last several columns, namely, the peak would appear in bigger ω. Then the result would show more like a Monotonically increasing function, like (6.22a) suggests: $P_{tot}(\omega) \propto \omega^{-(p-1)/2} = \omega^{positive}$.

3 RADMC3D

3.1 chapter 6 DUST CONTINUUM RADIATIVE TRANSFER

1. Page 27,28-About the simulation mechanism

   Q: The author said: "Once it escapes, a new photon package is launched, until also it escapes. After all photon packages have been launched and escaped, the dust temperature that remains is the final answer of the dust temperature."

So is it kind of strange only having one photon package at the same time? How to prove it is equivalent to the real situation?

A: Note: we want to prove the final temperature is equivalent to the "equilibrium dust temperature". So here the condition we consider should be divided into two parts: about equilibrium, and other things which are not about equilibrium.

For the first, the most important property for equilibrium is, "each dust grain acquires as much energy as it radiates away", that is, no addition or subtraction in energy; for the second, we also know "the heating/cooling time scales for dust grains are typically very short compared to any time-dependent dynamics of the system", where heating/cooling just corresponds to the "equilibrium" thing-so we could know the other physics mechanism would not have much influence to our estimations.

4 Some abbreviation

1. SED: spectral energy distribution

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