



Zoom-In Cosmological Simulation for a Blue-Tilted Primordial Power Spectrum

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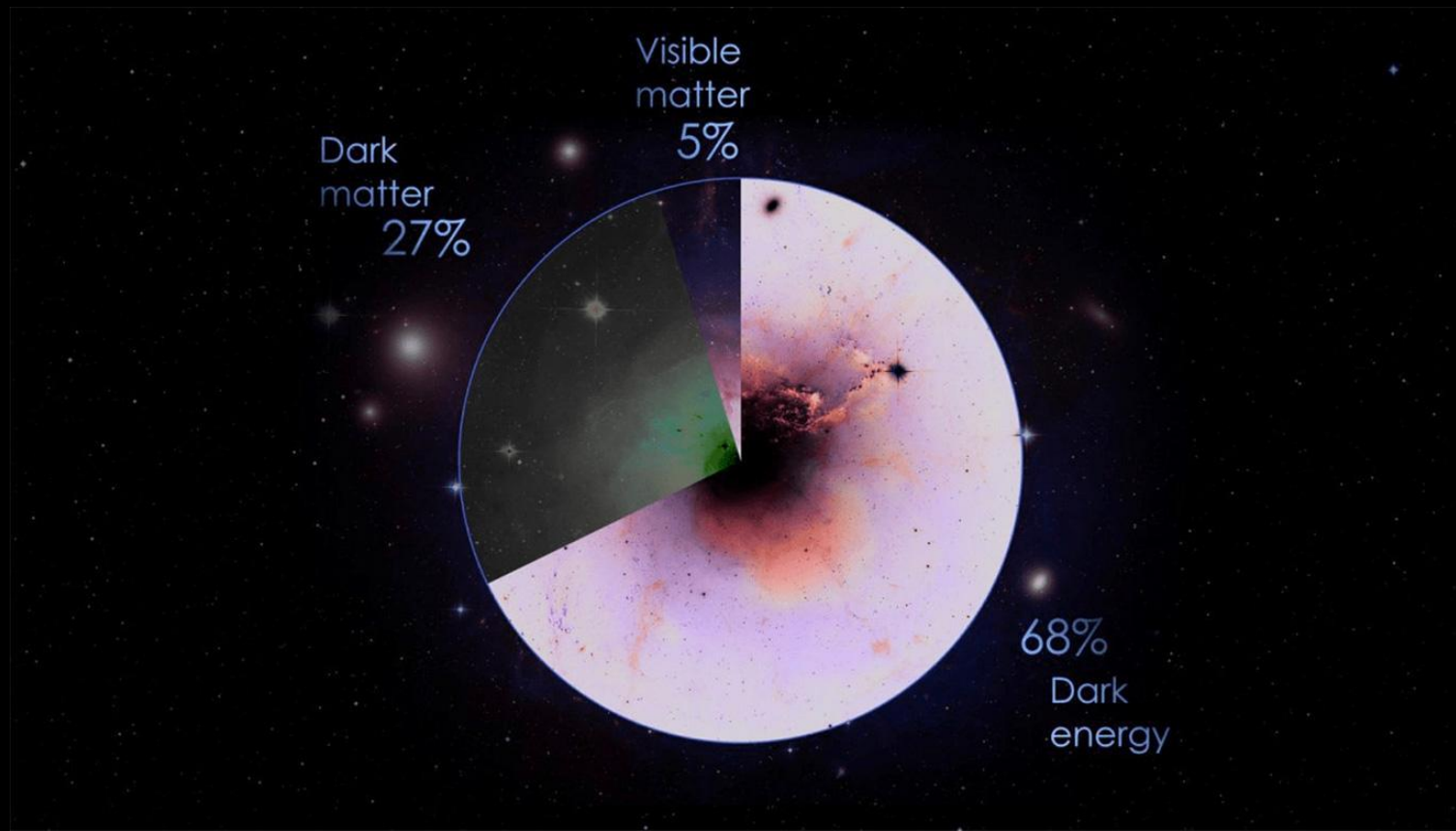
The latest JWST results have shown the early formation of massive galaxies. This could be explained by the blue-tilted primordial power spectrum, which has a small-scale enhancement compared to that generated by the traditional single-field slow-roll inflation model. Along with the strong lensing studies and nearby galaxy observations, we are motivated to conduct cosmological zoom-in dark matter-only simulation of the MW host size halo with blue tilted primordial power spectra.

We find that the blue-tilted subhalo mass function is enhanced by more than a factor of two for subhalo masses $M_{\text{sub}} \lesssim 10^{10} M_{\text{sun}}$ and maximum circular velocities $V_{\text{max}} \lesssim 30$ km/s. The blue-tilted subhalos are a factor of three more centrally concentrated, while the blue-tilted scaled cumulative substructure fraction can be an order of magnitude higher at $\sim 10\%$ virial radius. The blue-tilted subhalos also have higher central density, since for the same V_{max} , the blue tilted subhalos have smaller R_{max} . We have also verified these findings with higher-resolution simulations.



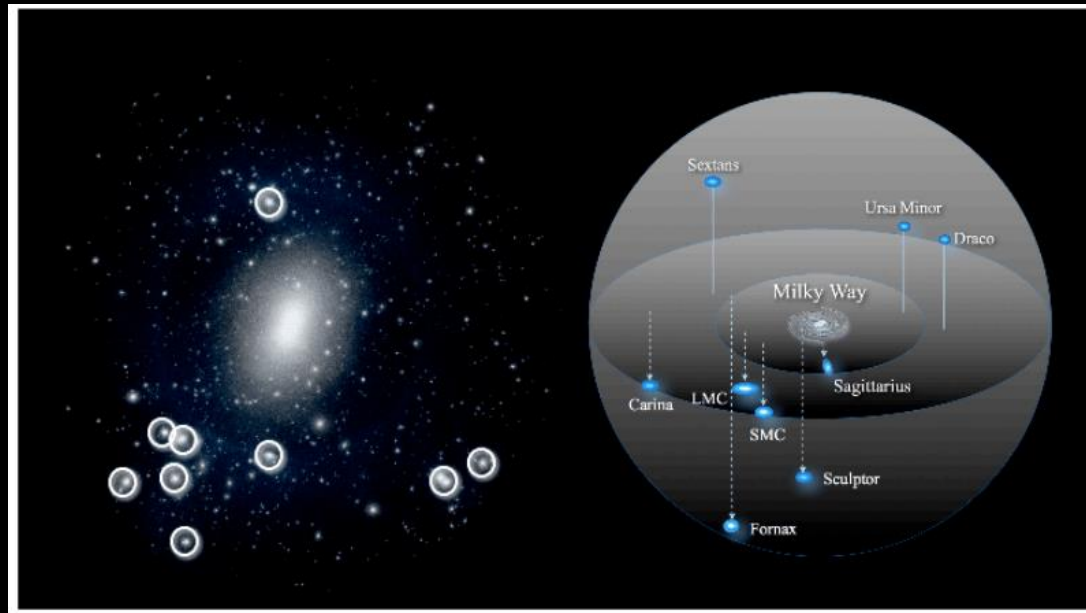
Background about dark matter Zoom-In simulation

- The standard model of cosmology believes dark matter composed of the major part of matter in universe



Background about dark matter Zoom-In simulation

- Dark matter halo is one form of dark matter existing in the universe, containing clumps of gravitationally bound dark matter. It could host galaxies which are luminous and observable
- Dark matter subhalo is “halo in halo”. Just like:
 - Halo (could host) --> galaxy
 - Subhalo (could host) --> satellite galaxy
- So though “dark”, we could still infer dark matter via visible matter!



Background about dark matter Zoom-In simulation

- Different early universe model/dark matter model/dark energy model, could all give different result on observations today! (Look at figure's upper side)

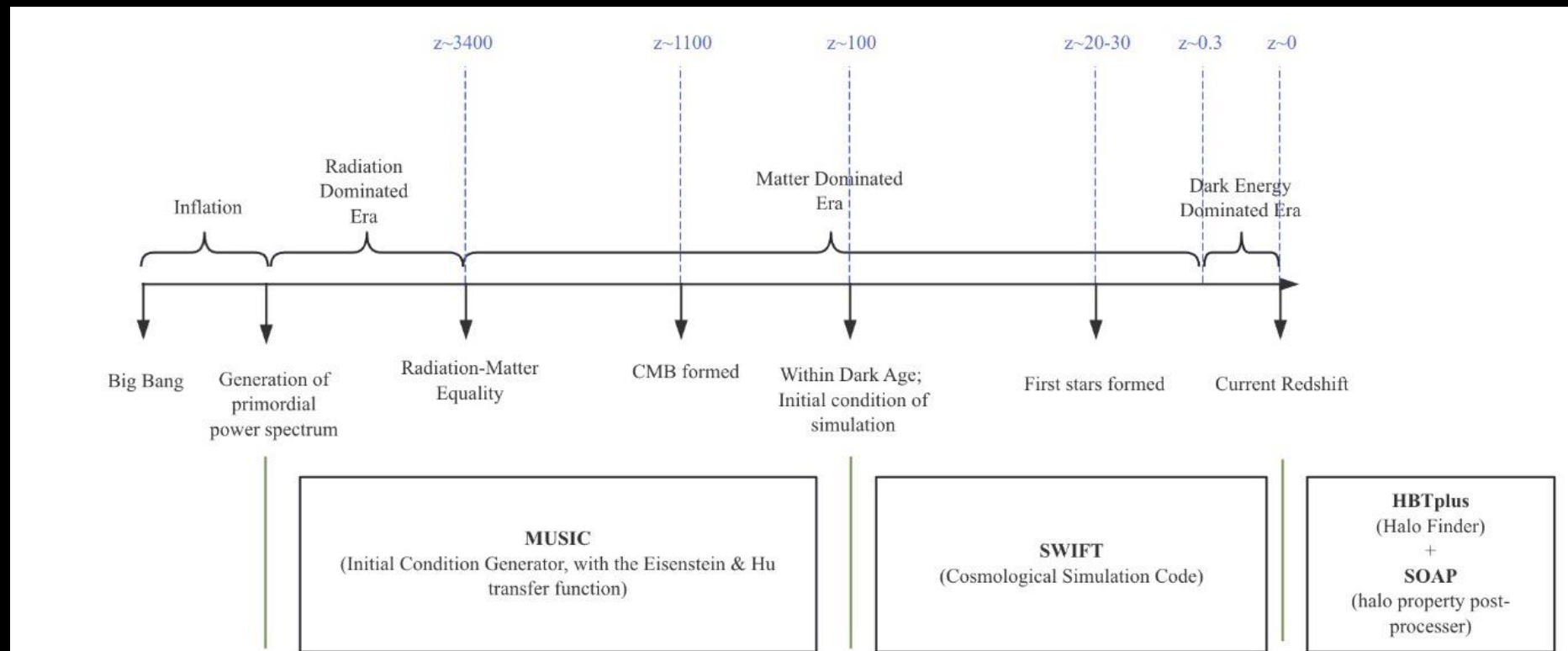


FIG. 1. The conceptual flow of the cosmological stages and our simulation steps. The *upper panel* shows the universe's chronology, along with redshifts of some critical era; the *lower panel* shows the numerical tools we used at different cosmological stages.

Motivation

- Standard cosmology model(single field slow roll inflation + LCDM) has achieved great success during the past several decades, on large scale of universe
- However, multiple observations are in favor of a small scale enhanced cosmological model:
 - JWST has observed early formation of massive galaxies
 - Strong Lensing result requires a larger fraction mass of substructure
 - A too-many-satellite-galaxies problem appeared in nearby galaxy observation

Method: Change the early universe model

- To be more specific, we adopt a **small-scale enhanced primordial power spectrum**

the growth factor. In the traditional single-field slow-roll inflation, the PPS follows the PL model:

$$P_i(k) \propto k^{n_s}, \quad (2)$$

with the spectral index $n_s \sim 0.96$ (see [section III B 2](#)).

Ref. [23] gave the following formalism for the BT models:

$$P_i(k) \propto k^{n_s} (k \leq k_p), \quad (3)$$

$$\propto k_p^{n_s - m_s} \cdot k^{m_s} (k > k_p), \quad (4)$$

which is a broken power law modification as [Equation 2](#). We introduce k_p as the pivot scale and m_s as the spectral index on the small scale.

We could also write it more intuitively:

$$P_i(k) \propto k^{n_s} (k \leq k_p), \quad (5)$$

$$\propto k^{n_s} \cdot \left(\frac{k}{k_p}\right)^{m_s - n_s} (k > k_p), \quad (6)$$

Models	Related parameters
PL	Power Law Primordial Power Spectrum $n_s = 0.961$
BT_deep	$k_p = 3.51 \text{ Mpc}^{-1}$ $m_s = 1.5$
BT_soft	$k_p = 0.702 \text{ Mpc}^{-1}$ $m_s = 1.5$

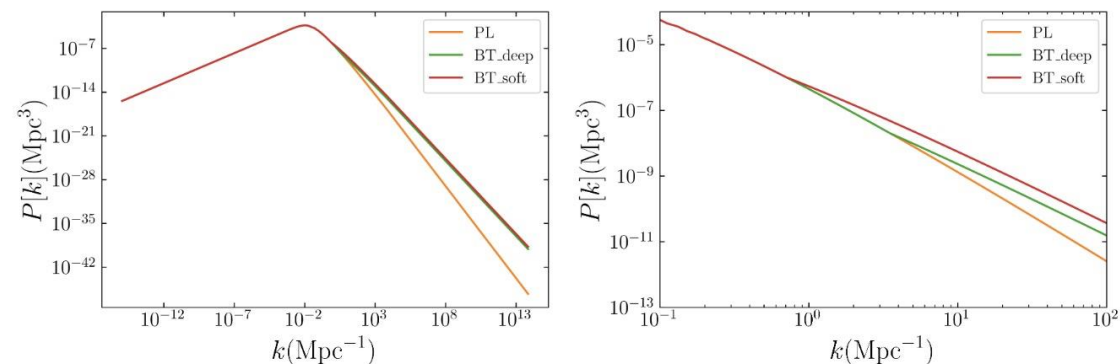


FIG. 2. The power spectra for matter density perturbation at $z=1089$ with PL (orange), BT_deep (green), and BT_soft (red). The left panel is for the whole wave vector k range, and the right panel focuses on model differences. The models' parameters are listed in [Table I](#).

Method: Change the early universe model

- Our prediction based on pivot scale calculation:
 - Both deep and soft model could enhance the substructure in MW host size halo
 - But only soft model could enhance the MW host size main halo itself!

$$\begin{aligned} M_l &= \frac{4\pi}{3} r_l^3 \rho_m = \frac{\Omega_m H_0^2}{2G} r_l^3 \\ &= 1.71 \times 10^{11} \left(\frac{\Omega_m}{0.3} \right) \left(\frac{H_0}{70} \right)^2 \left(\frac{r_l}{1 \text{ Mpc}} \right)^3 M_{\text{sun}}. \end{aligned} \quad (7)$$

We could further give the pivot mass M_p under which the BT model could enhance the structure formation:

$$M_p = 5.29 \times 10^{12} \left(\frac{\Omega_m}{0.3} \right) \left(\frac{H_0}{70} \right)^2 \left(\frac{k_p}{1 \text{ Mpc}^{-1}} \right)^{-3} M_{\text{sun}}. \quad (8)$$

For the BT_{deep} and BT_{soft} models, M_p would be $\sim 1.1 \times 10^{11} M_{\text{sun}}$ and $\sim 1.4 \times 10^{13} M_{\text{sun}}$ respectively, using the cosmological parameters in our paper (see [section III B 2](#)). The host halo of an MW-size galaxy is believed to be around $M_{\text{halo}} \sim 10^{12} M_{\text{sun}}$. Therefore, while

Method: Change the early universe model

- After changing the primordial power spectrum, then use cosmological simulation to evolve to current redshift! (Look at figure's lower side)

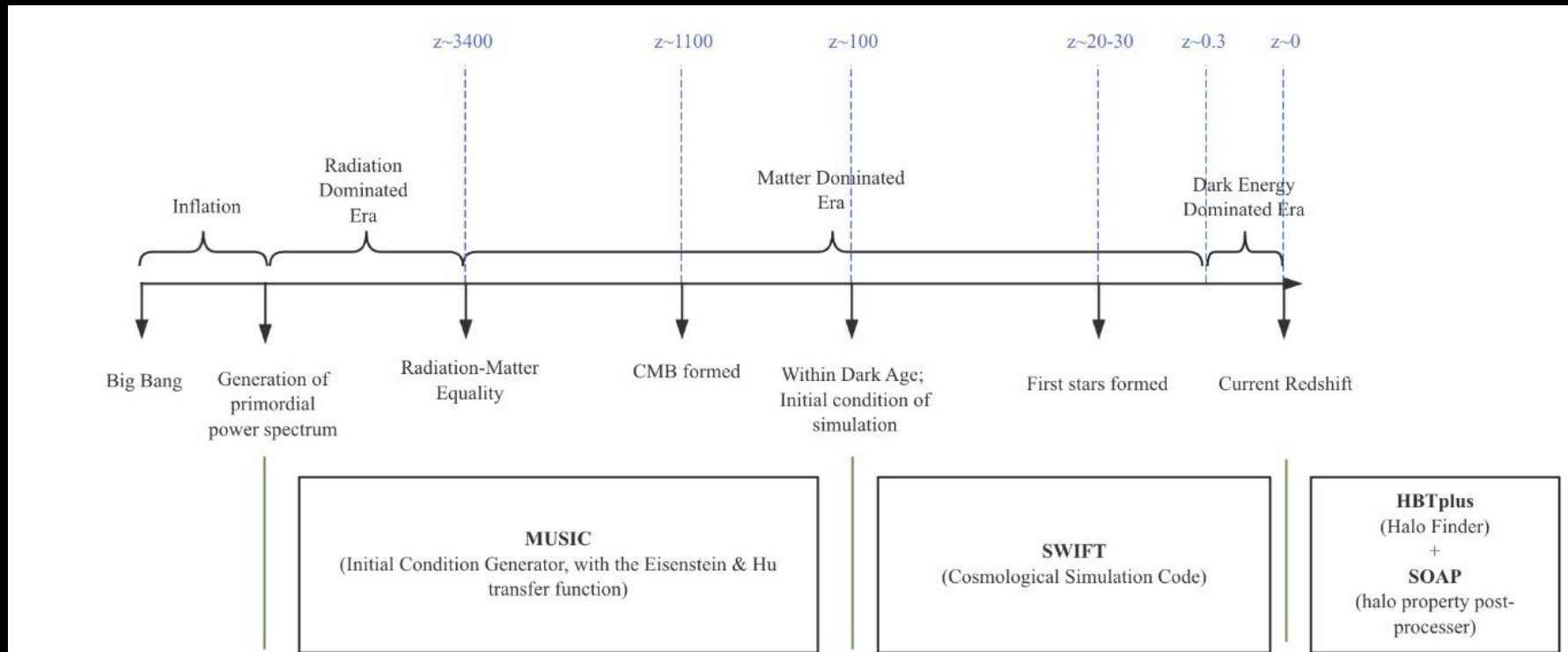
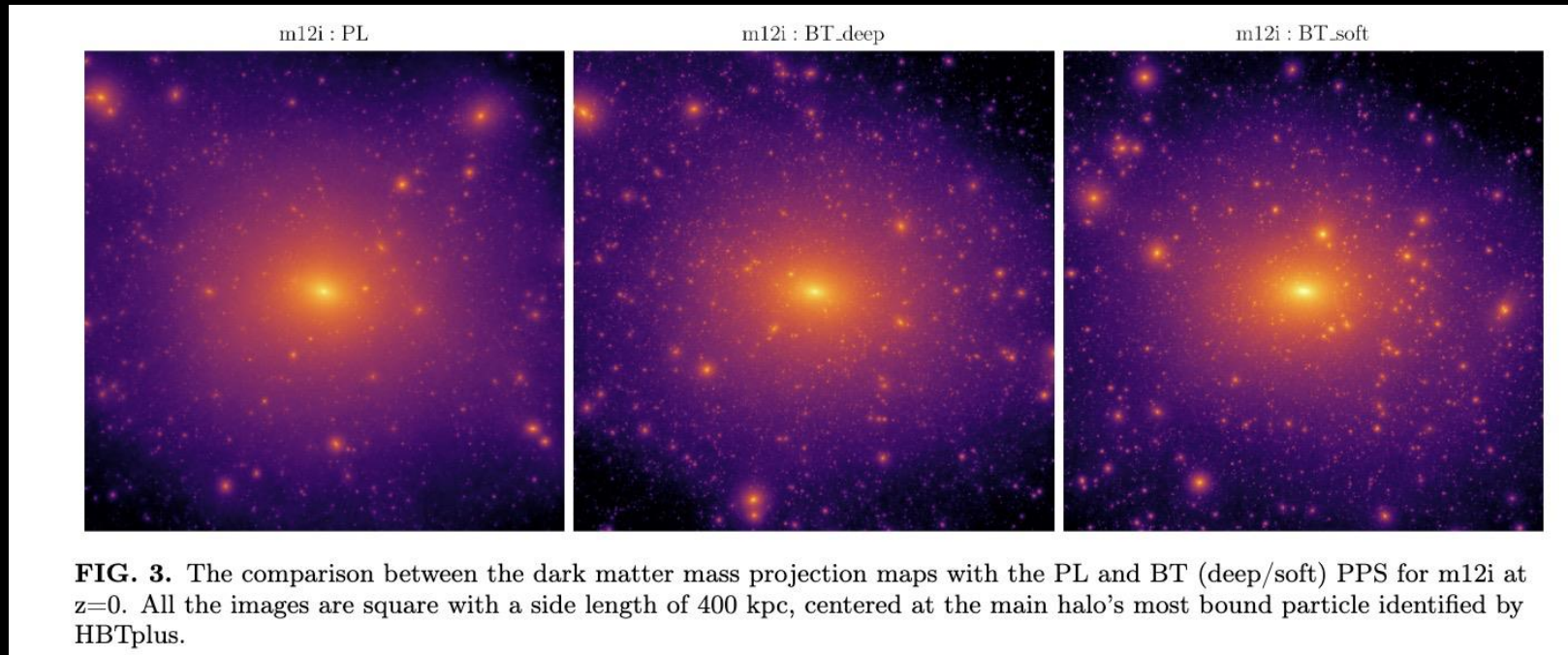


FIG. 1. The conceptual flow of the cosmological stages and our simulation steps. The *upper panel* shows the universe's chronology, along with redshifts of some critical era; the *lower panel* shows the numerical tools we used at different cosmological stages.

Result: from projection maps

- dark matter projection map
 - Indeed deep and soft could have more substructures
 - It is an intuitive but not accurate enough way



Result: for main halo

- The main halo radial density profile:
 - While deep model behaves alike, soft model could make main halo more concentrated

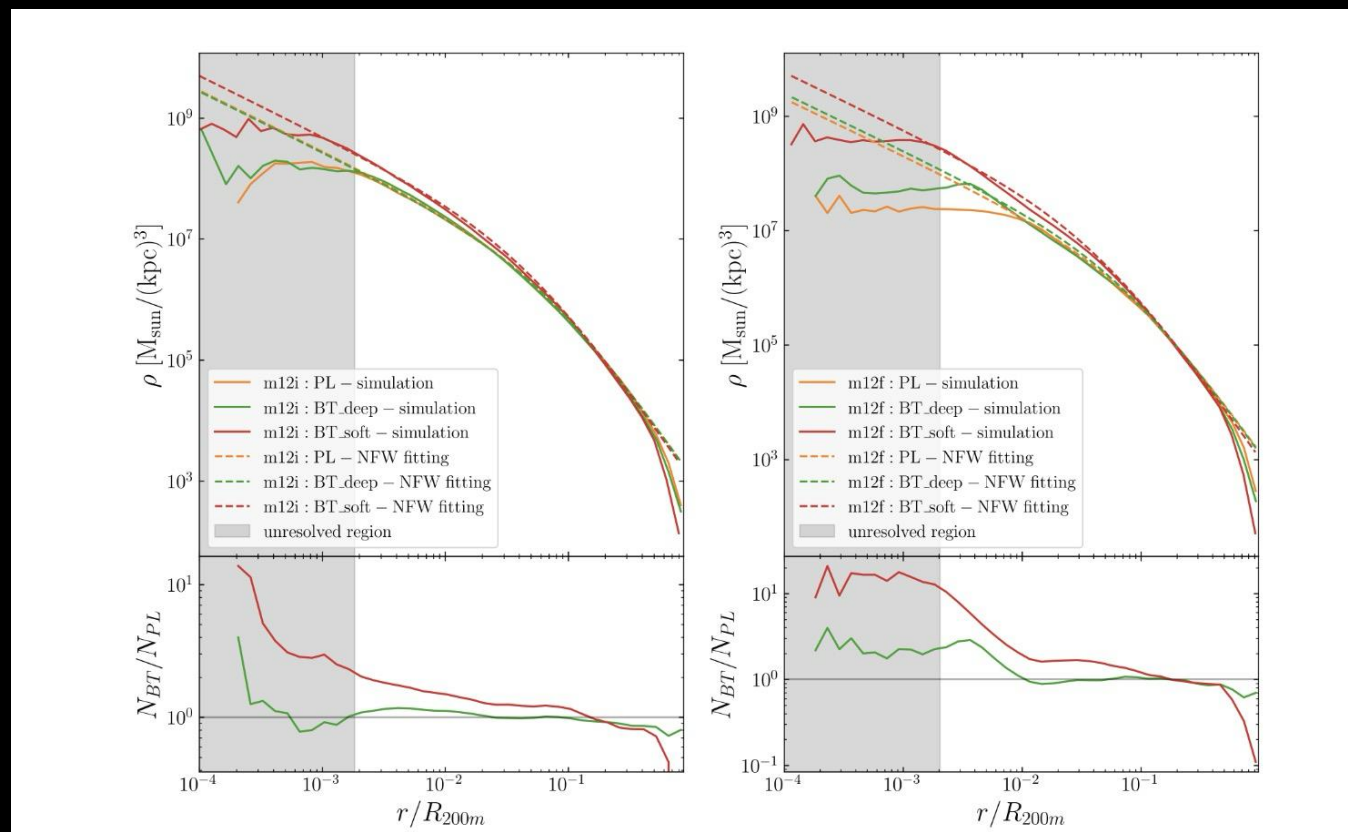


FIG. 4. The density profiles of the main halos for m12i (left) and m12f (right) respectively. For both figures: The *upper panel* shows the density profile of the main halo with the PL (orange), BT_deep (green), and BT_soft (red) models; the NFW fitting line is also shown for each model. The *bottom panel* shows the ratios of the BT density to the PL density. The *shaded area* shows the unresolved region given by Equation 9.

Result: for subhalos

- More subhalos in terms of
 - subhalo mass
 - subhalo V_{\max}
- We could further give a fitting formula for the enhancement ratio:

$$f(x) = \frac{1}{2} \cdot (m - 1) \cdot \left(\frac{-x}{\sqrt{1+x^2}} + 1 \right) + 1, \quad (12)$$

where
for mass

$$x = 4 \cdot \log_{10} \frac{\mu}{\mu_c}. \quad (13)$$

for velocity

$$x = 4 \cdot \log_{10} \frac{\nu}{\nu_c}. \quad (14)$$

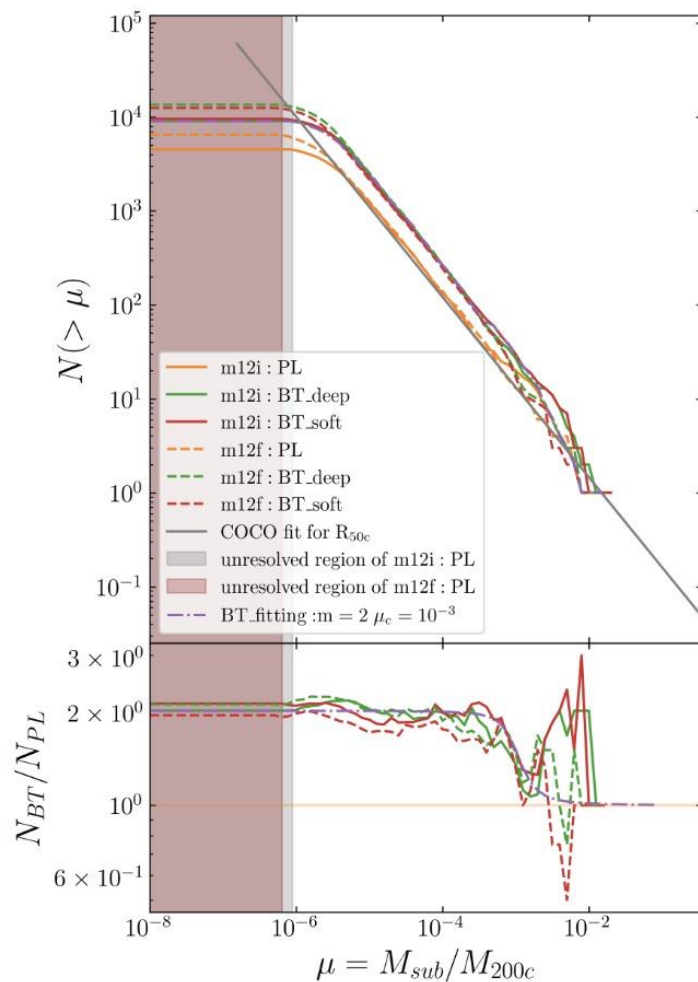


FIG. 5. The cumulative scaled subhalo mass functions for m12i (colored solid) and m12f (colored dashed) respectively. The *upper panel* shows the subhalo mass function for all the subhalos within R_{50c} from the main halo center in the PL (orange), BT_deep (green), and BT_soft (red) simulations, with the subhalo mass M_{sub} scaled by M_{200c} of the main halo. The *bottom panel* shows the ratios of the BT counting number to the PL counting number. The *shaded areas* show the subhalo mass cutoff due to the halo finder's setting for minimum halo size, where we set it to 20 particles.

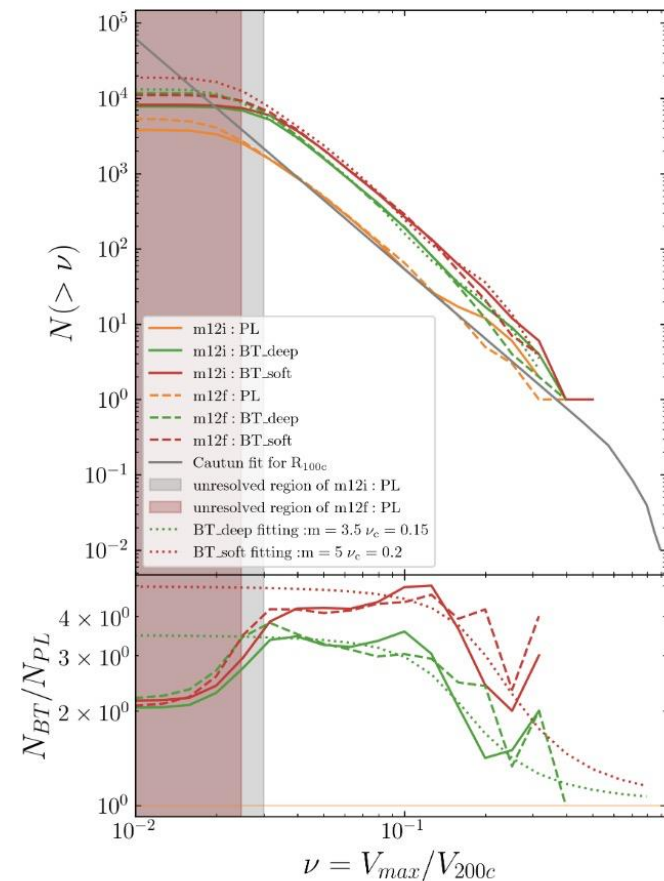


FIG. 6. The cumulative scaled subhalo velocity functions for m12i (colored solid) and m12f (colored dashed) respectively. The *upper panel* shows the subhalo velocity function for all the subhalos within R_{100c} from the main halo center in PL (orange), BT_deep (green), and BT_soft (red) simulations, with V_{\max} of the subhalo scaled by V_{200c} of the main halo. The *bottom panel* shows the ratios of the BT counting number to the PL counting number. The *shaded areas* show the unresolved V_{\max} range, the limit of which is the median V_{\max} value among all the subhaloes with 100 particles in the PL simulations. 100 is an empirical lower limit of particle number for dark matter halo to be resolved [79].

Result: for subhalos

- BT subhalo number density is higher within the inner region than PL

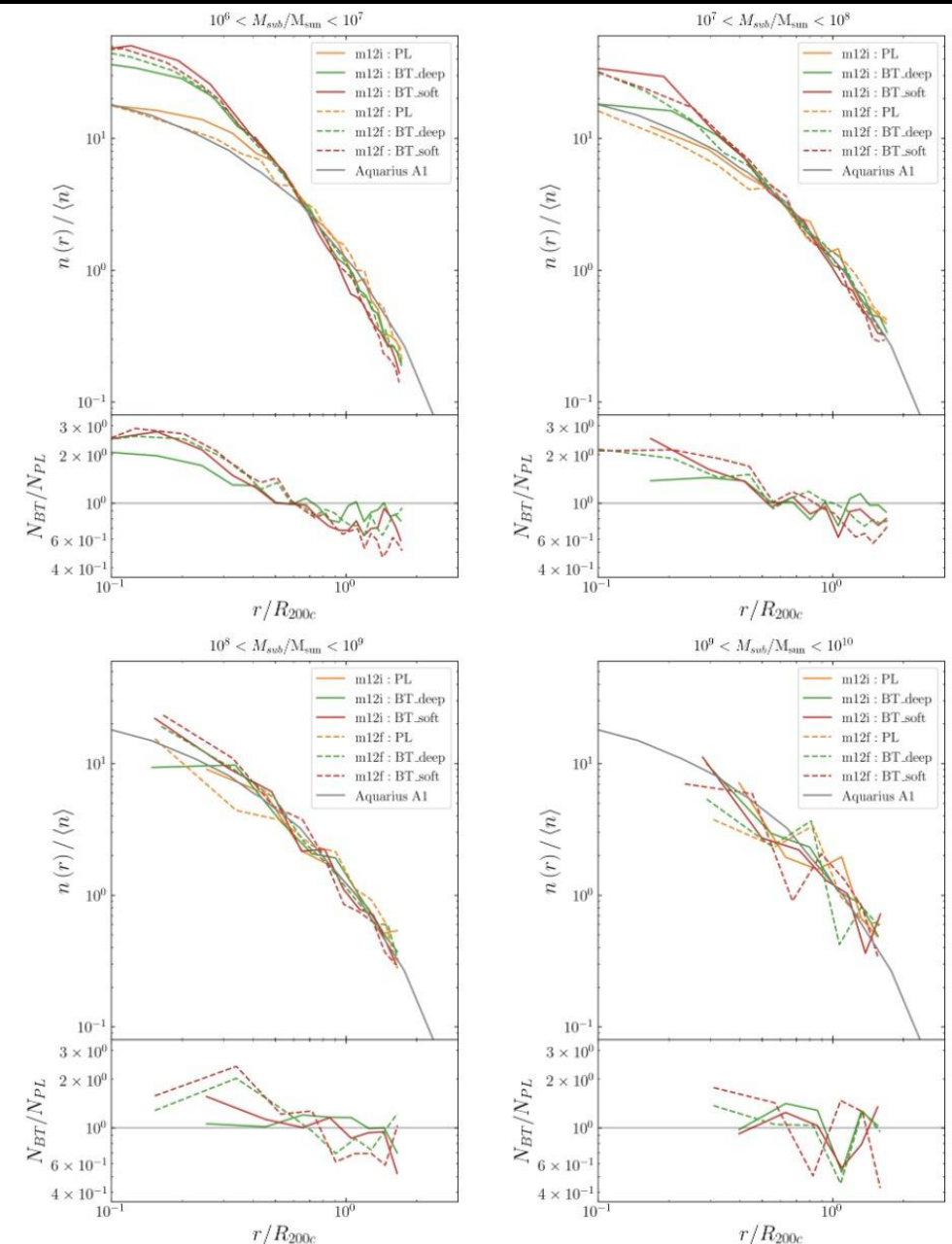


FIG. 7. The subhalo radial number density profiles at four different mass bins for m12i (colored solid) and m12f (colored dashed). In each subfigure (also in each mass bin): The *upper panel* shows the radial number density $n(r)$ normalized by $\langle n \rangle$ in Equation 15, for the PL (orange), the BT_deep (green), and the BT_soft (red) models. The grey solid line shows the equivalent substructure profile from Figure 12 of Ref. [38]. The *bottom panel* shows the ratios of the BT normalized number density to the PL normalized number density.

Result: for subhalos

- The cumulative substructure mass fraction is higher in BT than in PL.
- which could explain the violation of asymptotic flux ratio relation in strong lensing

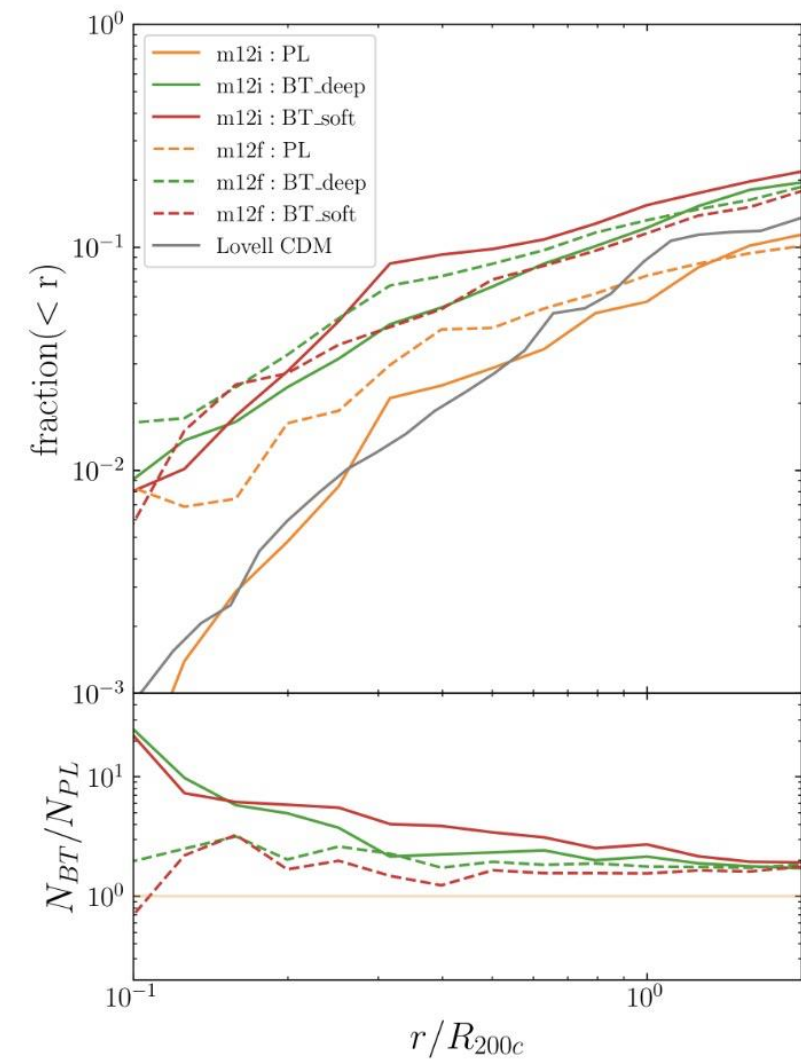


FIG. 8. The cumulative mass fraction in substructures as a function of the radius for PL (orange), BT_deep (green), and BT_soft (red). The *upper panel* shows the scaled cumulative substructure fraction function, with r (the subhalo distance from the main-halo center) scaled by R_{200c} of the main halo. The grey solid line shows the equivalent substructure profile from Ref. [81]. The *bottom panel* shows the ratios of the BT substructure fractions to the PL substructure fractions.

Result: for subhalos

- For subhalo V_{max} - R_{max} relations:
 - For a certain V_{max} , BT could have a smaller R_{max}
 - which means the BT subhalos are more concentrated

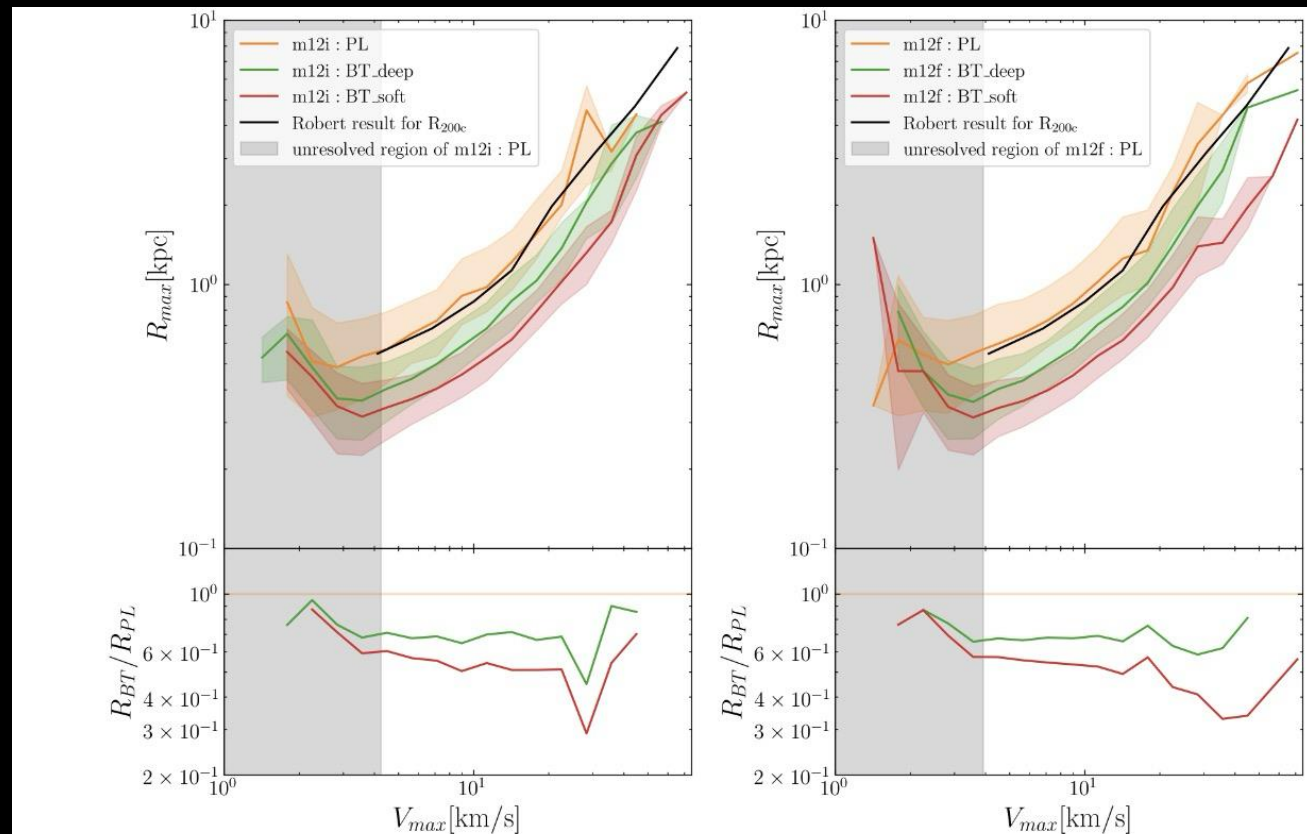


FIG. 9. The subhalo V_{max} - R_{max} relations for m12i (left figure) and m12f (right figure) respectively. For both figures: The *upper panel* shows the subhalo R_{max} - V_{max} relations for all the subhalos within R_{200c} from the main halo center with the PL (orange), BT_deep (green), and BT_soft (red) models. The colored solid lines depict the median relation found by binning the subhalos according to their V_{max} values. The colored regions surrounding the colored lines illustrate the 16th-84th percentiles around the medians. The black solid line is the result of Ref. [82] for the PL LCDM model, taking account of all the subhalos within R_{200c} of the main halo. The *bottom panel* shows the ratios of R_{max} between the BT and PL models. The *shaded area* is the same as that in Figure 6.

Conclusion

In this study, we conducted cosmological zoom-in simulations of MW host-size halos with the blue-tilted primordial power spectrum. We considered two blue-tilted models from Ref. [32]: BT_soft with the enhancement at $\gtrsim 1 \text{ Mpc}^{-1}$ and BT_deep with the enhancement at $\gtrsim 4 \text{ Mpc}^{-1}$. They are all within the accepted parameter space according to the JWST observation [32] and the dwarf galaxy central density observation [46].

Our main results are summarized in the following:

1. We found that BT_soft could also enhance the mass, radius, V_{max} , concentration (Table II), and the radial density profile (section IV B 1) of the main halo. However, the effects of the BT_deep model on the main halo are much smaller, since it is blue-tilted at a smaller scale in the primordial power spectrum, which corresponds to a smaller mass scale Equation 8.
2. BT enhance the subhalo mass function by ~ 2 and the subhalo velocity function by $\gtrsim 3$ for a wide range of halo masses ($\sim 10^6 \text{ M}_{\text{sun}} - 10^9 \text{ M}_{\text{sun}}$) and maximum circular velocities ($\sim 5 \text{ km/s} - 50 \text{ km/s}$) (section IV B 2 and section IV B 3). The enhancement ratios, defined as the numbers of subhalos in BT over that in PL, follow inverse S shape functions (Equation 12) in both mass and velocity functions.

In both the BT_deep and BT_soft simulations, the enhancements in the subhalo mass function are similar. However, the BT_soft simulation has a stronger enhancement in the subhalo velocity function than the BT_deep simulation.

3. The BT PPS increases the number density of subhalos in the inner region of the main host halo by more than a factor of two, compared to PL (section IV B 4). In the BT simulations, the number density of subhalos in the lower mass bins is more centrally concentrated.
4. We find that BT could boost the mass fraction of the substructures in the inner region ($< 0.1 R_{200c}$) by an order of magnitude (section IV B 5). Even around R_{200c} , the BT models could have a two-fold increase in the substructure mass fraction, compared to PL.
5. At a fixed Vmax, BT reduce the mean R_{max} values of the subhalos by 30%–50% (section IV B 6). This