ACTIVE SUSPENSION OF QUARTER CAR MODEL USING LQR

ACT PROJECT FINAL REPORT

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ABSTRACT

The main aim of Suspension System is to provide the passenger comfort by minimizing the vertical acceleration of the body and to isolate the body irrespective of road profile. The suspension system has to balance the trade-off between ride comfort and handling performance. A linear quarter-car model is used for the analysis and simulation. The performance of the LQR controller is compared with a system without LQR and PID type control.

INTRODUCTION

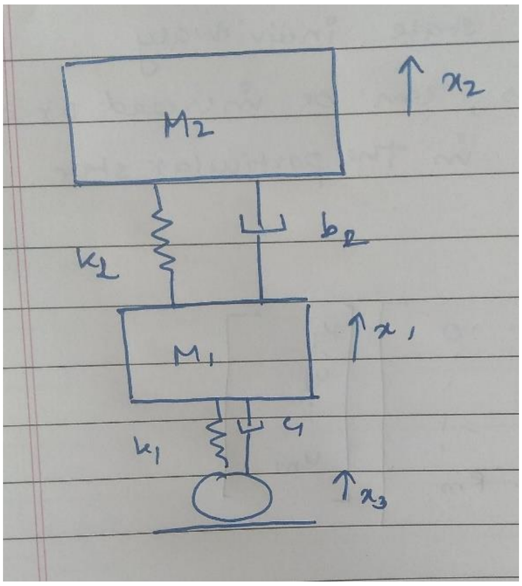
The automobile is a combination of a variety of complex systems. One such system is the suspension system. The suspension system has been widely applied to vehicles, from horse-drawn carriages with flexible leaf springs to modern automobiles with complex control algorithms. Passive suspension systems are a trade-off between ride comfort and performance. A car with a nice cushy ride usually wallows through the corners, whereas a car with high performance suspension, like F1 cars, will hang on tight through the corners but will make the passengers feel every little dip and bump in the road. The intent of the active suspension system is to replace the classical passive elements by a controlled system, an active suspension system, which can supply unlimited force to the system. The active suspension system dynamically responds to the changing road surface due to its ability to supply energy which is used to achieve the relative motion between the body and wheel. This project presents a relatively simple active suspension control strategy – a Linear Quadratic Regulator (LQR). The LQR controller is used with a passive suspension system to improve the vehicle ride comfort. The model is subjected to disturbances like step, sine, white noise, bump, etc. The passive suspension system is used as a reference system. The performance of the LQR active suspension system is compared with the passive suspension system.

UNDERSTANDING THE QUARTER CAR MODEL:

- It is a mode to study vehicle suspension system

  Assumptions taken in the model :
- Tyre is modelled as a linear spring without damping
- Damping of spring and damper are linear
- Tyre is always in contact with the road
- Effect of friction is neglected
- Modelling of the suspension is done on a horizontal plane, longitudinal deflections of the suspension are taken to be negligible compared to the vertical deflection

The quarter car model used in the project is given below :



In the model mass m2 is sprung mass or body mass of the vehicle , the portion of the vehicle which gets isolated from the shock ,eg : fuel tank , transmission etc

Mass m1 is the unsprung mass , the parts of the vehicle which feel the disturbance , tyre wheel , break etc

The suspension weight is distributed evenly between the sprung and unsprung mass and mostly the division is done experimentally.

1. The system shown above has 2 degree of freedom

STATE SPACE MODEL:

Solving the above system we get the following equations , where x1 and x2 are displacement of the block and x3 is displacement due to road input.

$$m_1\ddot{x}_1 + (c_1+c_2)\dot{x}_1 + (k_1+k_2)x_1 - c_2\dot{x}_2 - k_2x_2 - c_1\dot{x}_3 - k_1x_3 = 0$$

$$m_2\ddot{x}_2 + c_2\dot{x}_2 + k_2x_2 - c_2\dot{x}_1 - k_2x_1 = 0$$

STATE VARIABLES :

4 state variables are decided : position and velocity of the two blocks

$$X_1(t) = x_2(t)$$
$$X_2(t) = x_1(t) \qquad \dot{X}_3(t) = \ddot{x}_1(t) = \ddot{X}_2$$
$$X_3(t) = \dot{x}_1(t) \qquad \dot{X}_4(t) = \ddot{x}_2(t) = \ddot{X}_1$$
$$X_4(t) = \dot{x}_2(t)$$

STATE SPACE OBTAINED :

$$\therefore A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k_2/m_2 & k_2/m_2 & -c_2/m_2 & c_2/m_2 \\ k_2/m_1 & \dfrac{-(k_1+k_2)}{m_1} & c_2/m_1 & -(c_1+c_2)/m_1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \cancel{0}-c_2/m_2 \\ k_1/m_1 \end{bmatrix} ; \; C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \; D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

PARAMETERS:

K2	Spring constant between car and suspension block	16,000 N/m
K1	Spring constant between suspension block and ground	1,90,000N/m
M2	Mass of car	300kg
M1	Mass of suspension block	60kg
B2	Damping factor of damper	1000N-s/m
B1	Damping factor of damper	1000N-s/m
X2	Displacement of car	
X1	Displacement of suspension block	
X3	Input disturbance at road	INPUT U

UNDERSTANDING LQR:

In designing control systems, one is often interested in choosing the control vector u(t) such that a given performance index (cost function) is minimized.

- J_{LQR} (cost function) = $\int_0^\infty (\underbrace{x^T Q x} + u^T R u)\, dx$

quadratic states taken, as they not only penalize large errors more but also makes the cost function a quadratic function, which are convexed so they have a set minimum.

$$J = \int_0^\infty (\underbrace{x^T Q x}_{\text{matrix to penalize bad performance}} + \underbrace{u^T R u}_{\text{penalize actuator effor.}})\, dt$$

where the matrix Q is a positive-definite (or positive-semi definite) Hermitian or real symmetric matrix, R is a positive definite Hermitian or real symmetric matrix. The first term on the right-hand side of the equation accounts for the error between the initial and final state, and the second term accounts for the expenditure of the energy of the control signal. The matrices Q and R determine the relative importance of the error and expenditure of the performance index. The control vector u(t) is considered to be unconstrained.

$$\begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix} \begin{bmatrix} Q_1 & 0 & \cdots & \\ 0 & Q_2 & 0 & \cdots \\ \vdots & 0 & Q_3 & \\ \vdots & & & \ddots \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$x^T Q x > 0$$

We can target individual state , q respected to individual states can increase or decreased to make the system more or less aggressive when it comes to state error.

R matrix helps us restrict actuator effort low r value for a state means the actuator can produce large effort with less cost whereas a high R value indicates high cost for actuation .

We have to select Q and R matrix values appropriately the best estimate.

SOLVING LQR

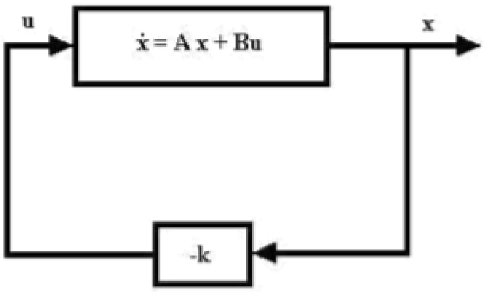
according to law $\bar{u}(t) = -k\bar{x}(t)$

Where $k = R^{-1}B^{T}S$

S = solution of the algebric Ricati eqn of the following

$$A^{T}S + SA - SBR^{-1}B^{T}S + Q = 0$$

Where u is the control vector .



DESIGN :

As Q can be any positive semi-definite matrix and R can be any positive definite matrix, we start our testing with the following values:

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } R = [1]$$

By testing different values of ‘Q and ‘R’ by simple hit and trial method, we arrived at a satisfactory value of:

Q = 1000*[1 0 0 0; 0 1 0 0; 0 0 1 0; 0 0 0 1]

R = [0.001]

MATLAB CODE AND IMPLEMENTATION :

[X,K,L] = icare(A,B,Q,R,S,E,G) can be used to solve the riccati pdf from which we will aquire multiple solutions from which we need to check which solutions give us stable systems (negative poles)

Instead we can directly use the lqr in built command in matlab which gives the gain as output directly , also we can acquire the riccati soln and eigen vectors from this command .

MATLAB CODE:

```
clc
clear all;
close all
mw=60;
mb=300;
kt=190000;
ks=16000;
bs=1000;
bw = 1000;
A=[0 0 1 0;0 0 0 1;(-ks/mb) (ks/mb) (-bs/mb)
(bs/mb);(ks/mw) (-(ks+kt)/mw) (bs/mw) -(bw+bs)/mw]
B=[0;0;1/mb;-1/mw]
C=[1 0 0 0; 0 1 0 0; 0 0 1 0;
0 0 0 1]
D=[0;0;0;0]
Q=1000*eye(4);
R=0.0001;
k=lqr(A,B,Q,R)
t = 0:0.01:10;
sys_withLQR = ss(A-B*k, 0.1*B, C, D);
sys_woLQR = ss(A, 0.1*B, C, D);
figure(1)
step(sys_woLQR,t)
hold on
figure(2)
step(sys_withLQR,t)
hold on
figure(3)
step(sys_woLQR,sys_withLQR,t)
```

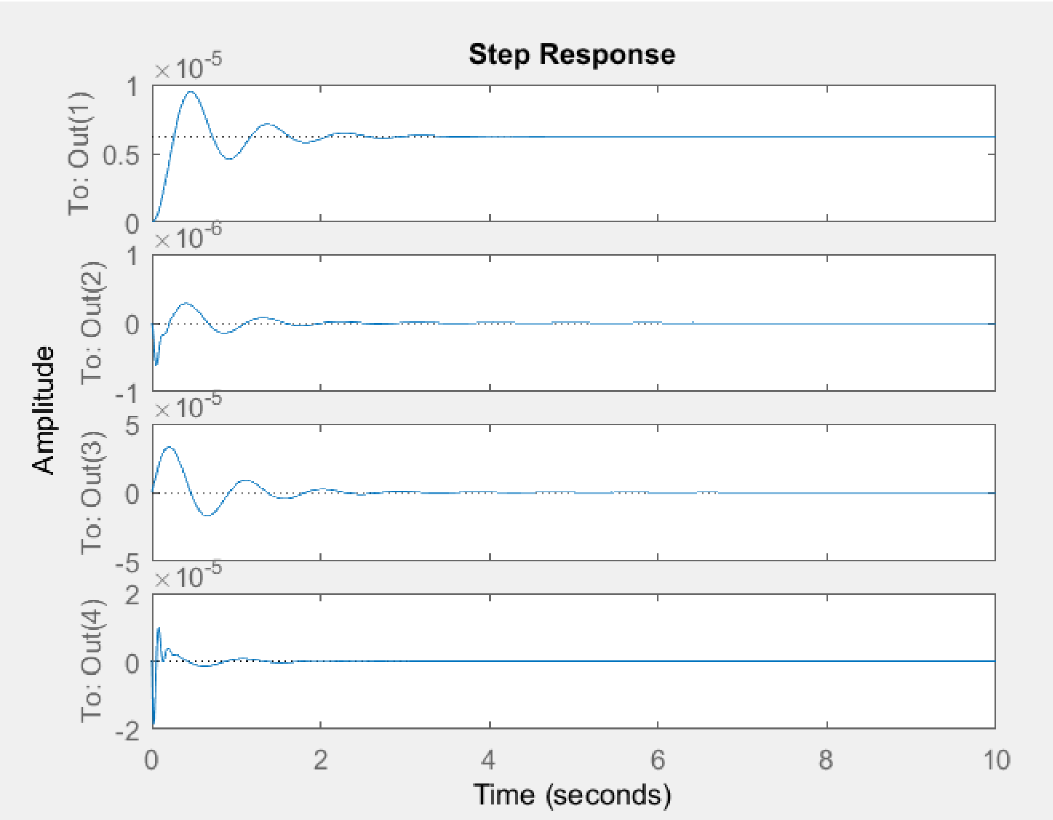
RESULTS

```
k =

   1.0e+04 *

    0.0310   -2.3239    0.2521   -0.1660
```

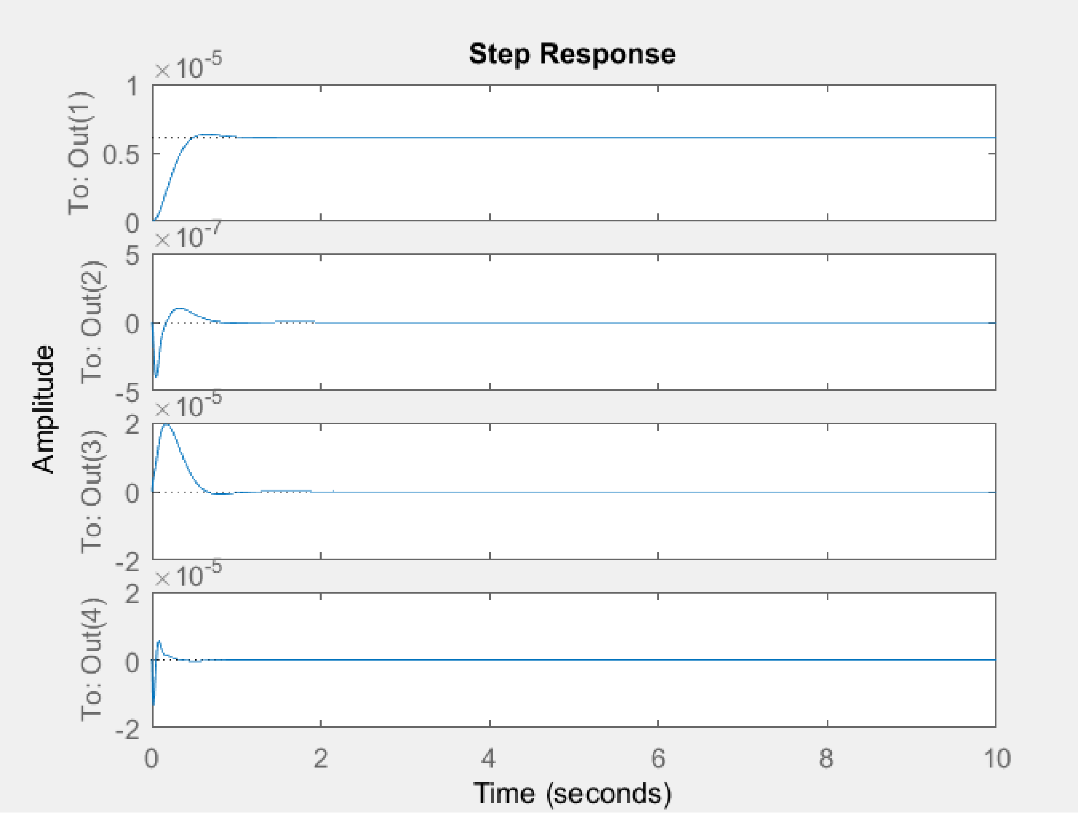
INPUT: STEP SIGNAL

OUTPUT:

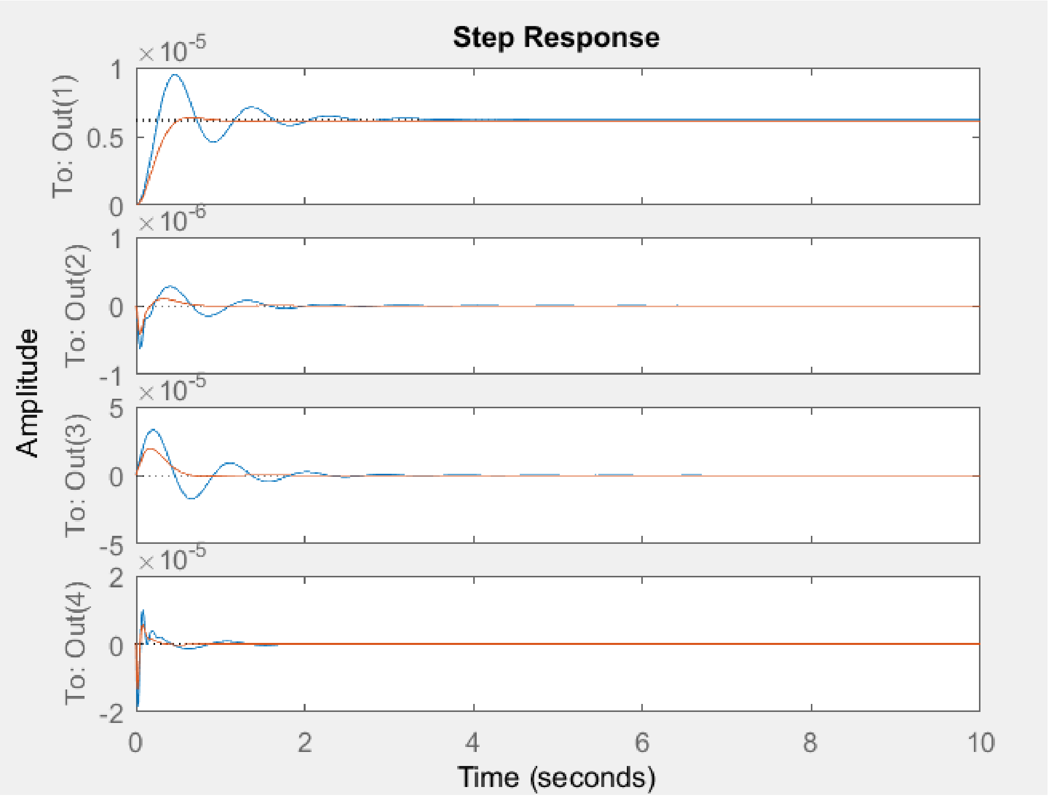
1. WITHOUT LQR:



2. WITH LQR:

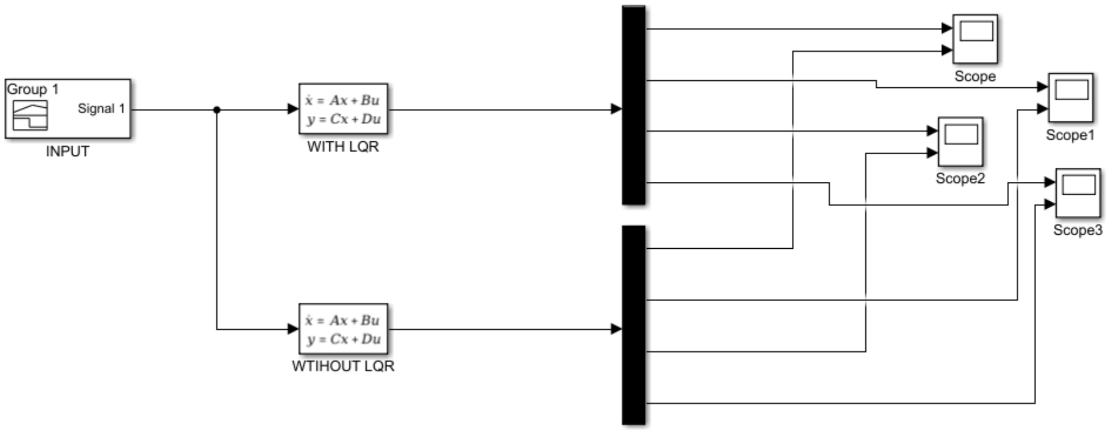


3. COMPARISION:

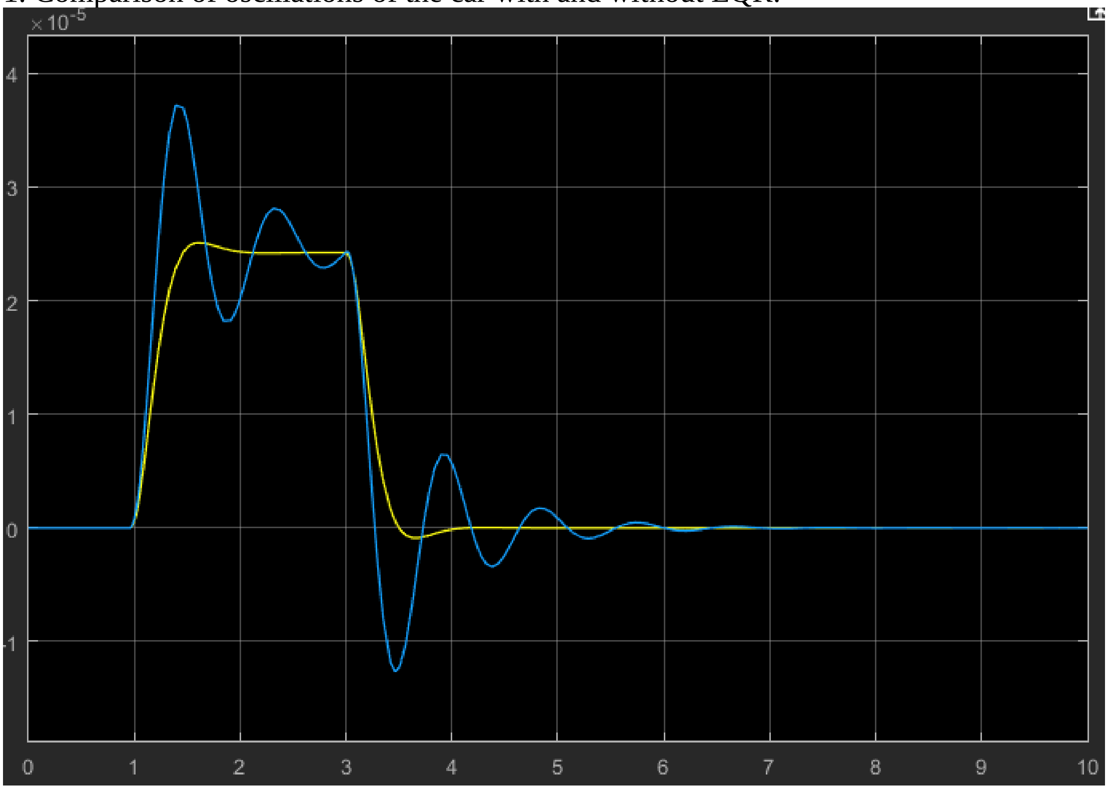


NOTE:
1st graph from the top is a displacement vs time graph of the car.
2nd graph represents the displacement vs time curve of the suspension block.
3rd graph represents the velocity vs time curve of the car.
4th graph represents the velocity vs time curve of the suspension block.

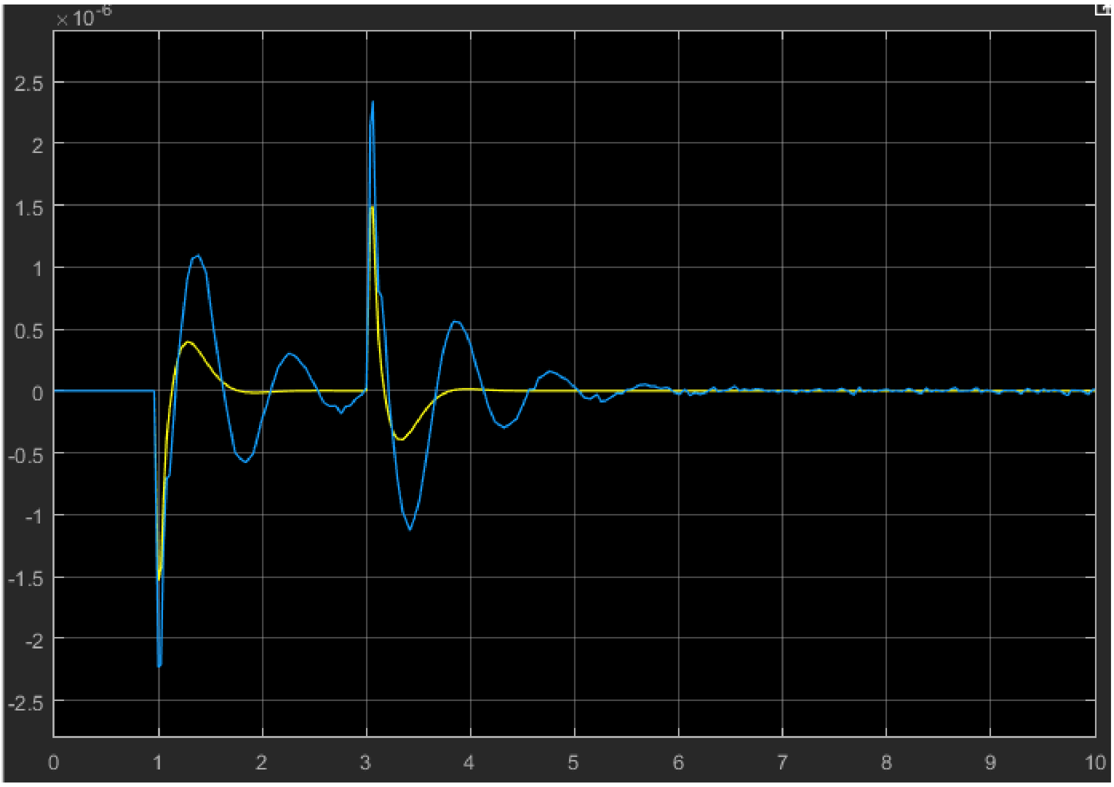
SIMULINK :



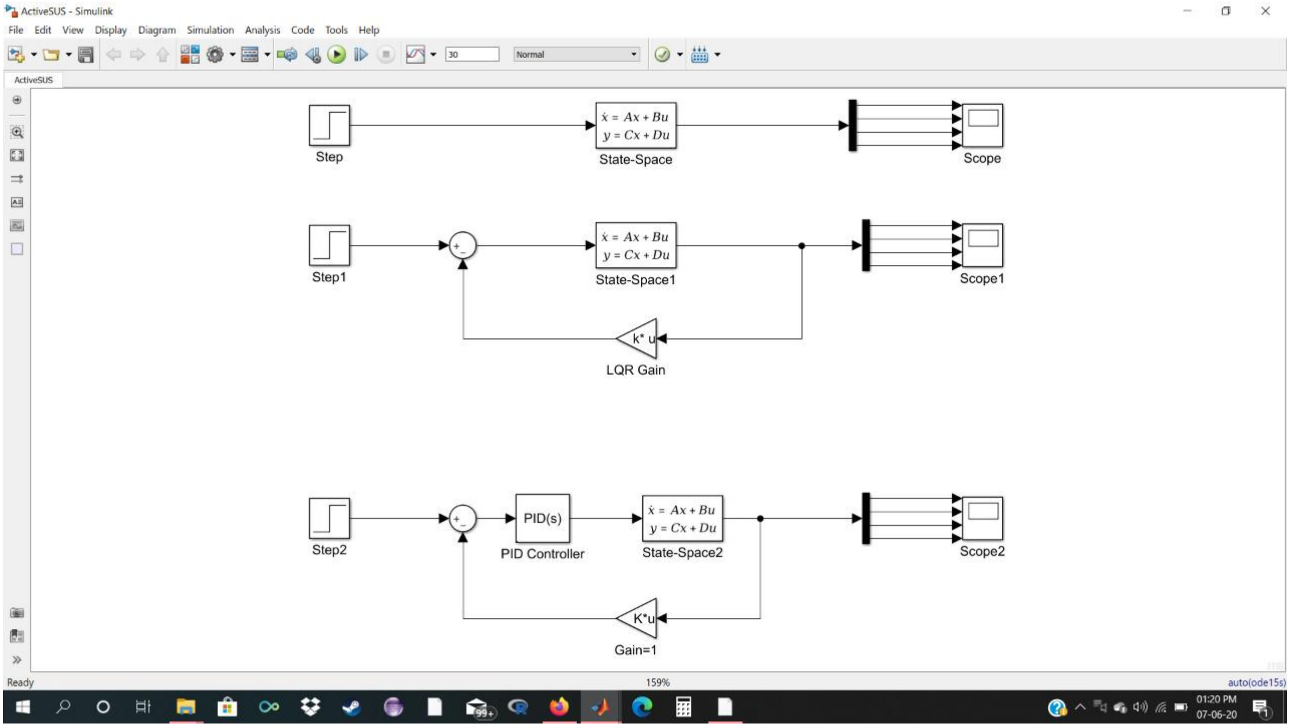
OUTPUT:
1. Comparison of oscillations of the car with and without LQR:



2. Comparison of oscillations of the suspension block with and without LQR:



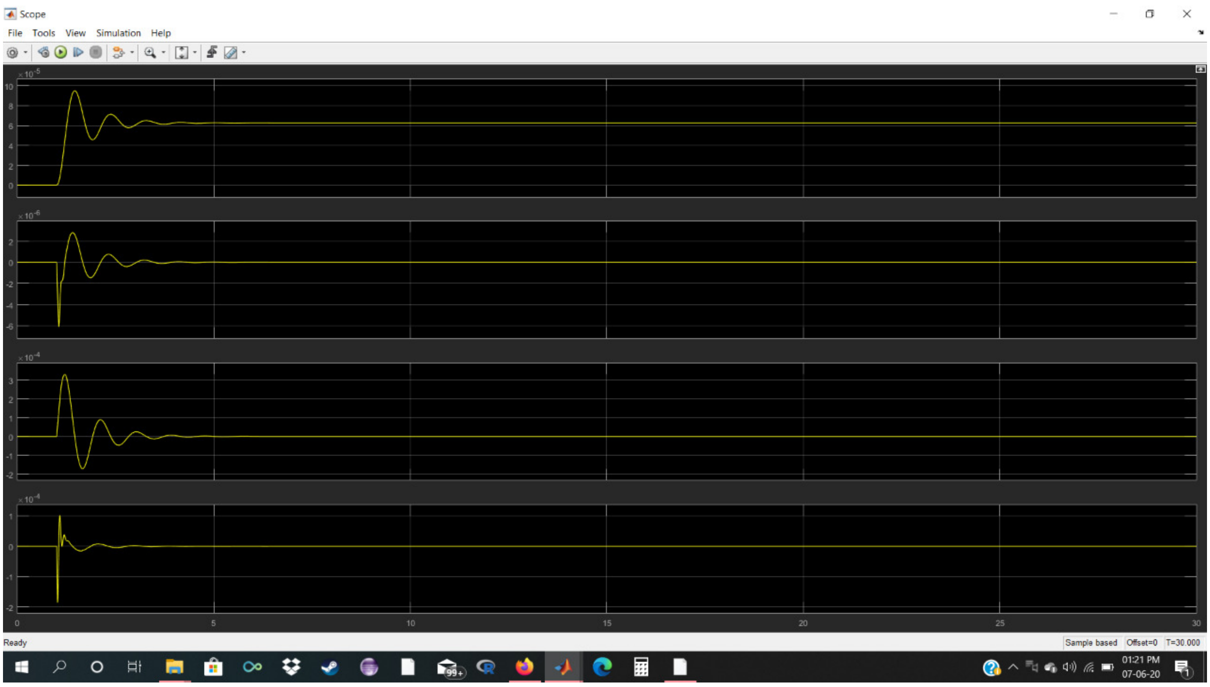
SIMULINK COMPARISON OF SYSTEM BETWEEN OPEN LOOP RESPONSE ,PID ,LQR



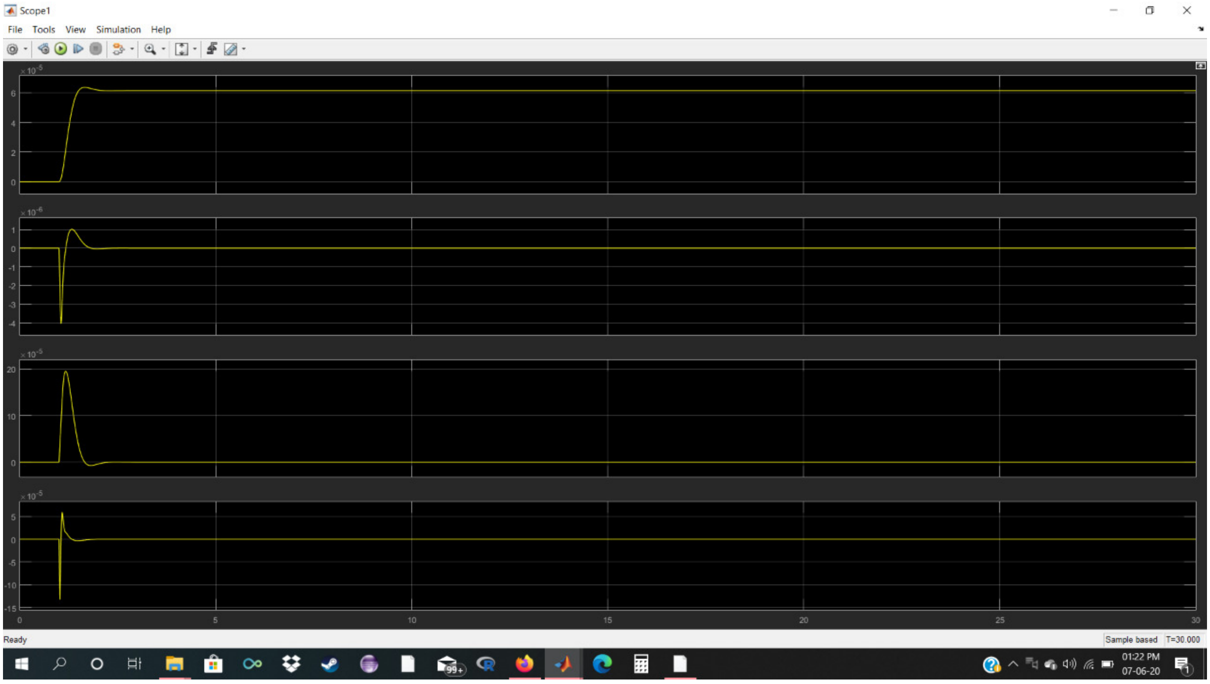
Pid is tuned using in built tuning options.

4 outputs are shown each for response of each state in the system.

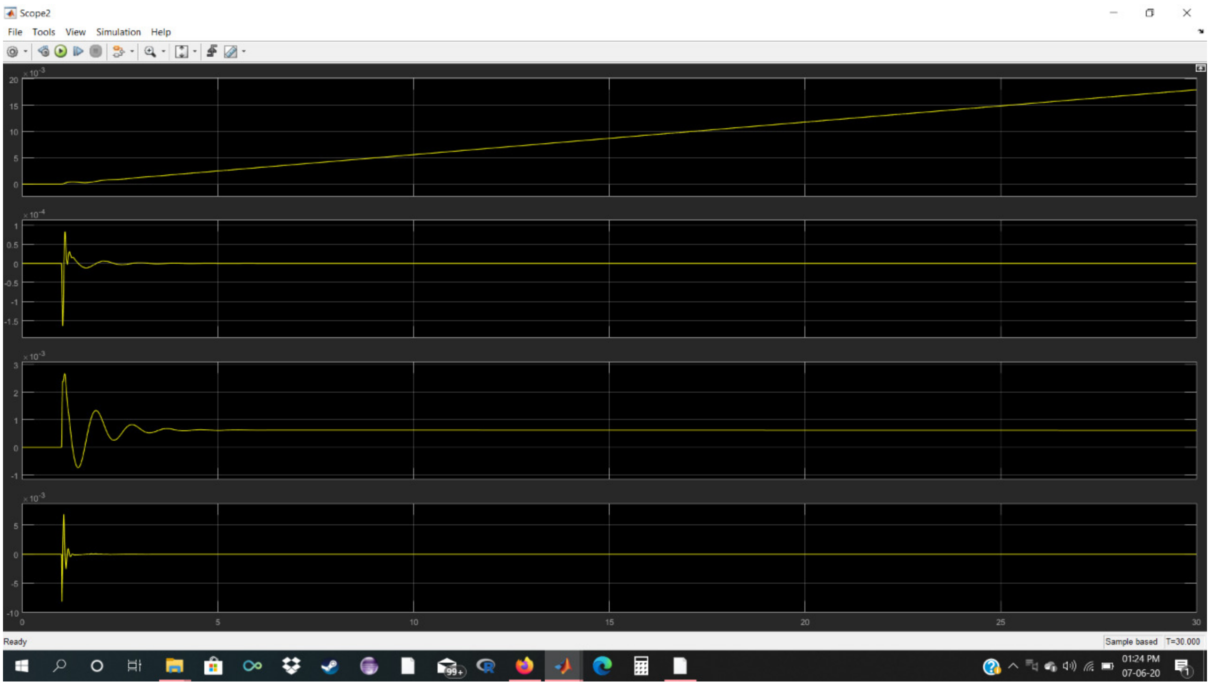
1. Open loop output :



2.LQR Response



3.PID Response



RESULTS AND CONCLUSION:

The main aim of our project was to demonstrate the active control strategy (here LQR controller) against a passive suspension system. We developed a LQR controller on a quarter-car system to enhance the ride comfort of the passengers. A passive suspension system without any controller and an active suspension system with our LQR controller were modelled and simulated using a Matlab/Simulink environment. The simulation results show that the designed active suspension system can improve the ride quality by minimizing the displacements and velocity more efficiently than the passive suspension model.

More over comparison is done between PID and LQR and it is noticed that even if we use the tuning tool present in matlab the PID control has a optimal gain in which one of the states becomes unstable , hence lqr is preferred and gives much more stable and a balanced ride

REFERENCES

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