Probabilistic Discrepancies between Kalshi and Traditional Options

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1 Introduction

This project looks at the lead-lag relationship between event based prediction markets (Kalshi) and standard financial derivatives. Treating Kalshi contracts as binary options makes it possible to extract an implied probability signal to compare it to the risk-neutral density from the volatility surface of the S&P 500.

2 Mathematical Methodology

2.1 Probability Extraction from Prediction Markets

Prediction markets like Kalshi trade binary contracts which pay \$1 if an event occurs, otherwise \$0. Given the market price $P_K \in [0,1]$ and assuming risk neutrality under the measure $\mathbb{P}$, the price of a contract for an event E at time t with expiration T is:

$$P_K(t) = e^{-r(T-t)}\mathbb{E}^{\mathbb{P}}[\mathbb{I}_E \mid \mathcal{F}_t] \tag{1}$$

where $\mathbb{I}_E$ is the indicator function for the event. For index-level targets (e.g., CPI above x), Kalshi provides a cumulative distribution. We recover the probability density function (PDF) by differencing adjacent contract thresholds:

$$f_K(x) \approx \frac{P_K(X > x) - P_K(X > x + \Delta x)}{\Delta x} \tag{2}$$

2.2 Risk-Neutral Density from Options Markets

To derive a comparable signal from the traditional options market, we utilize the Black-Scholes framework. The probability that an option expires in-the-money (the risk-neutral probability) is given by the $N(d_2)$ component:

$$P_{opt} = N(d_2) = N\left(\frac{\ln(S/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}}\right) \tag{3}$$

where σ is the implied volatility (IV) extracted from the volatility surface. In our empirical analysis, we use the VIX or VIX1D as a proxy for the short-term aggregate IV to measure institutional fear or expectation of a regime shift.

2.3 The Discrepancy Coefficient (λ)

We define the *Arb Gap* or Discrepancy Coefficient, λ, as the absolute difference between the Kalshi-implied probability and the options-implied probability:

$$\lambda_t = |P_{Kalshi,t} - P_{Options,t}| \tag{4}$$

A high λ indicates a large gap between retail sentiment and institutional hedging behavior.

2.4 Lead-Lag Analysis and Granger Causality

To test whether Kalshi is a leading indicator of market volatility we use a Vector Autoregression (VAR) model and perform a Granger causality test. We assess whether or not past values of the Kalshi probability signal (X) help predict future values of the VIX (Y):

$$Y_t = \alpha + \sum_{i=1}^{p} \beta_i Y_{t-i} + \sum_{j=1}^{p} \gamma_j X_{t-j} + \epsilon_t \tag{5}$$

The null hypothesis $H_0 : \gamma_1 = \gamma_2 = \cdots = \gamma_p = 0$ is tested to evaluate whether the prediction market provides unique information prior to the options market "correcting" its volatility pricing.