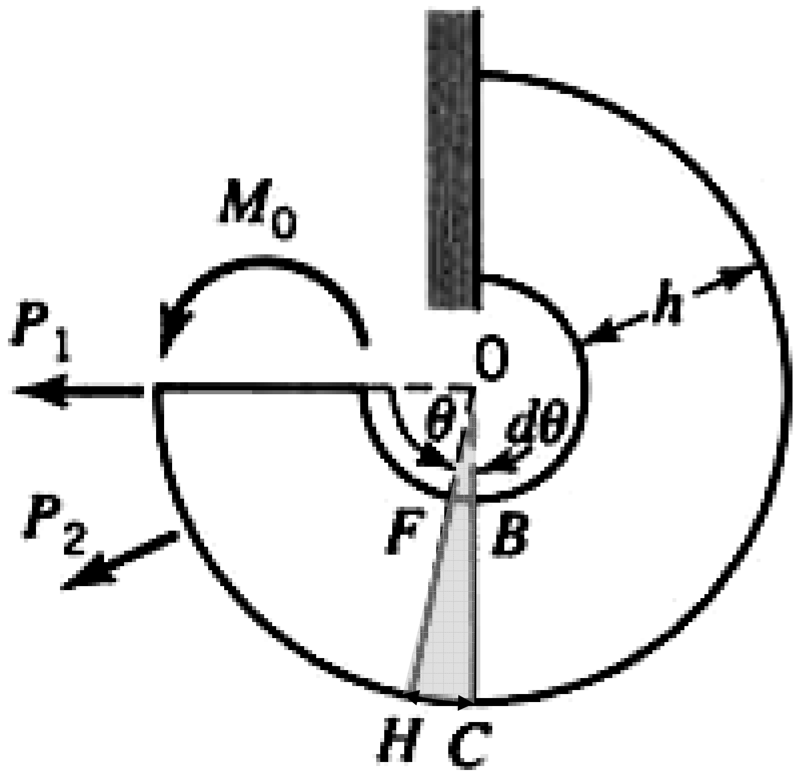
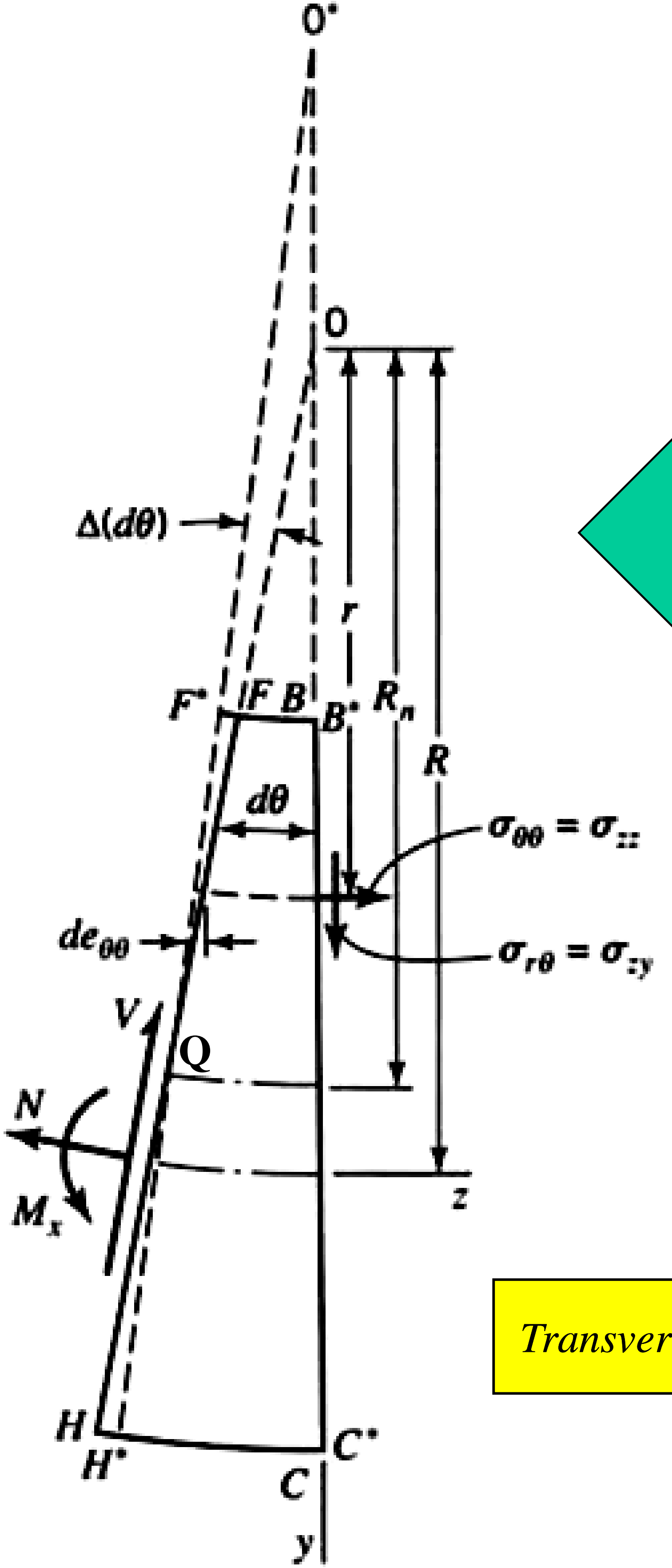
Curved Beams

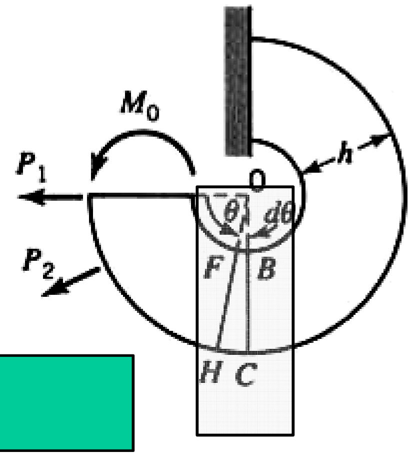


O : Center of curvature of undeformed beam
h : Thickness of beam
θ : Angle at which stress is to be found
dθ : Angular extent of section

Magnified view of the shaded area in next slide

Curved Beams

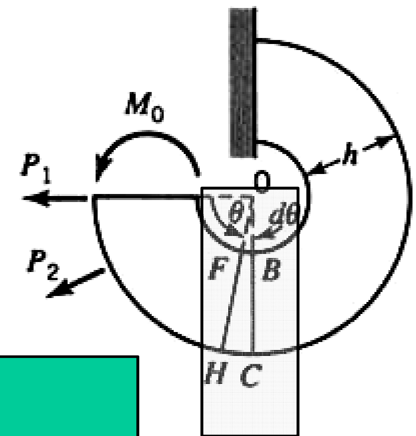


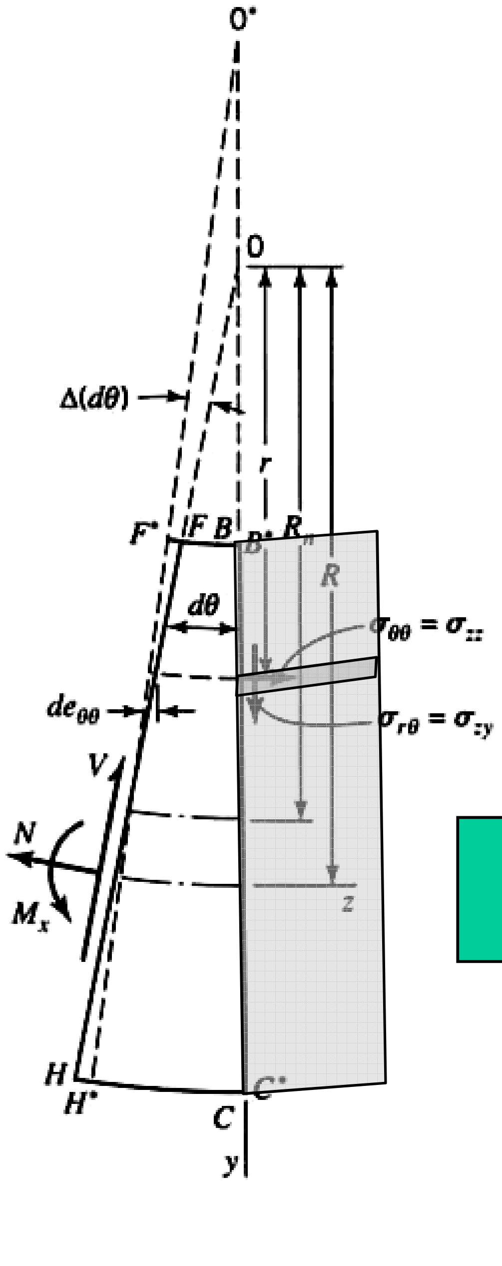


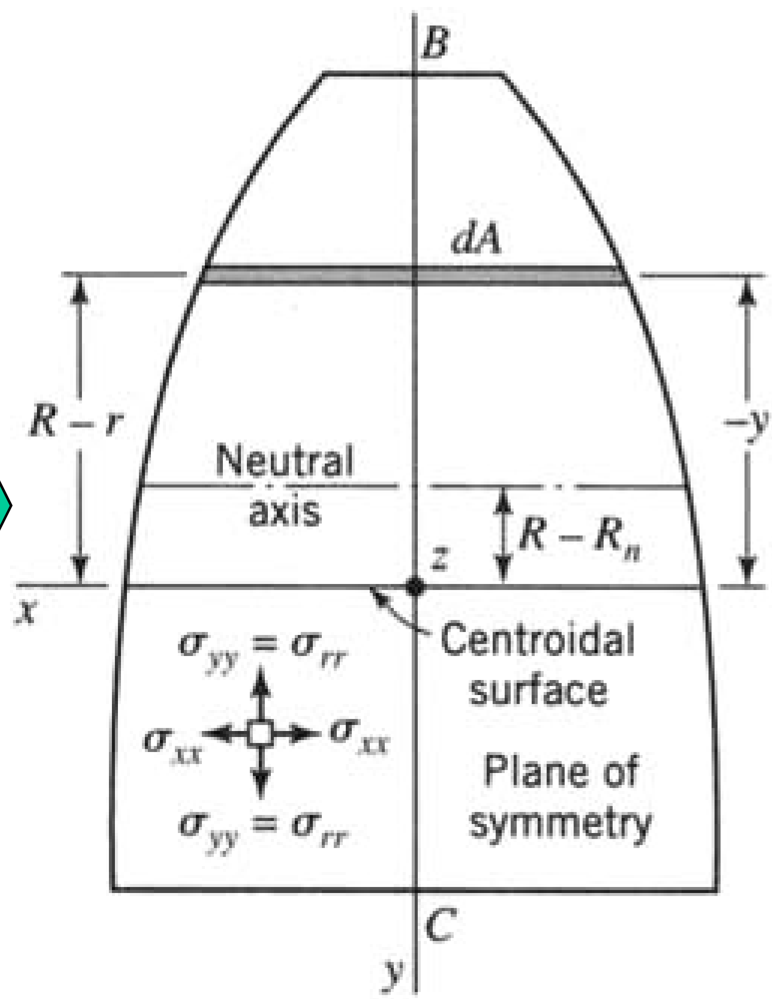
O' : Center of curvature of deformed beam
R : Distance of center of curvature from centroid of cross section
R_n : Distance of center of curvature from neutral axis of cross section (Not yet known)
r : radial distance at which stress is to be found
Δdθ : Angular rotation of deformed section about the point (Q) of intersection of the neutral axis with the section.

Transverse section in next slide

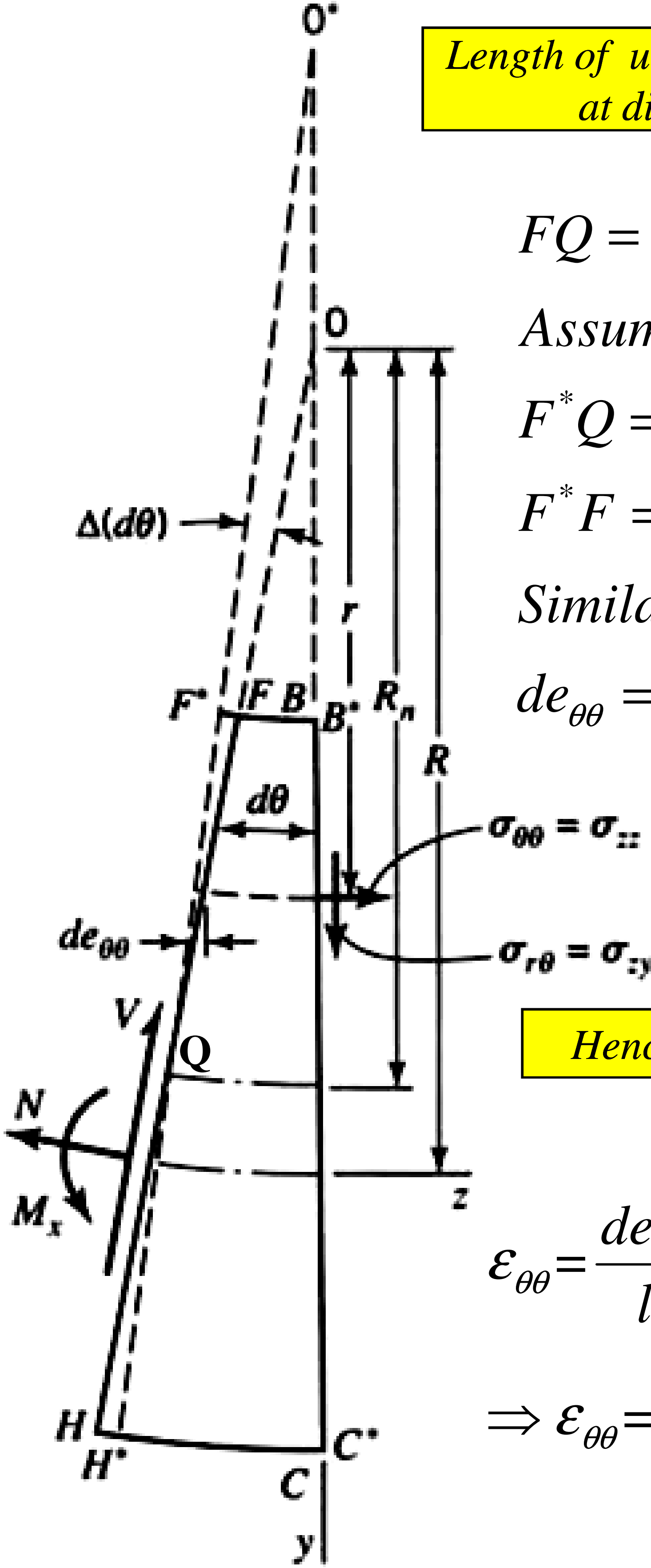
Curved Beams







Curved Beams



Length of undeformed fiber at distance r

$$l = r d\theta$$

$$FQ = R_n - r_i$$

Assuming OO^* *is very small*

$$F^*Q = R_n - r_i$$

$$F^*F = \left(R_n - r_i\right)\Delta d\theta$$

Similarly at distance r

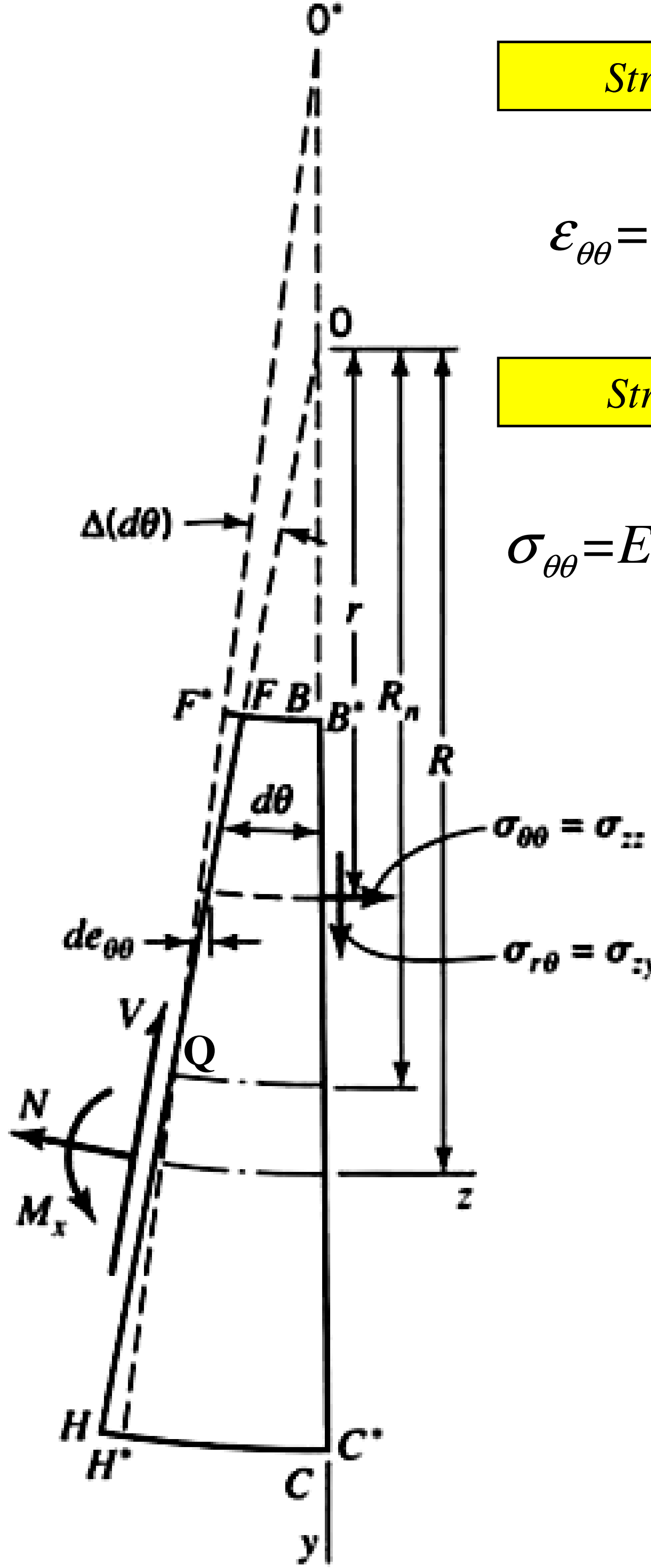
$$de_{\theta\theta} = \left(R_n - r\right)\Delta d\theta$$

Hence strain at distance r

$$\varepsilon_{\theta\theta} = \frac{de_{\theta\theta}}{l} = \frac{\left(R_n - r\right)\Delta d\theta}{r d\theta}$$

$$\Rightarrow \varepsilon_{\theta\theta} = \left(\frac{R_n}{r} - 1\right)\omega, \quad \omega = \frac{\Delta d\theta}{d\theta}$$

Curved Beams



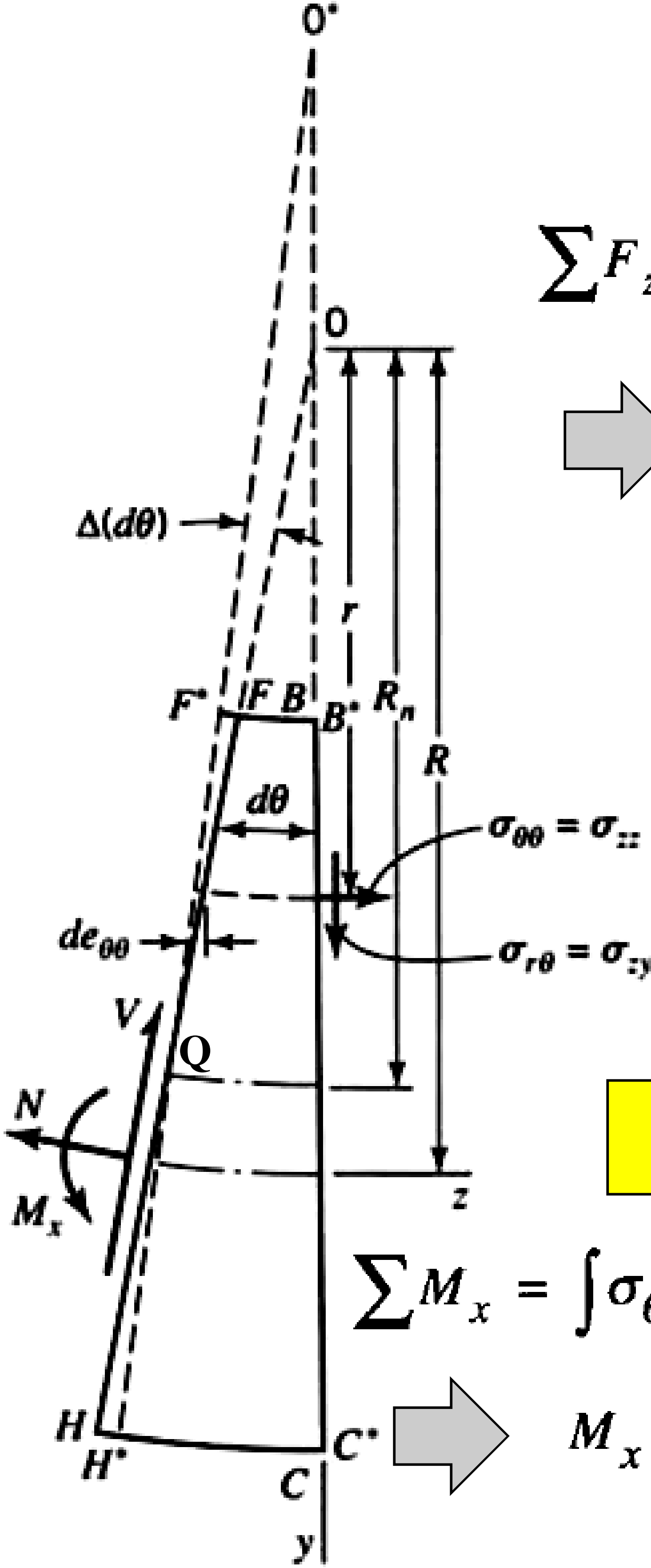
Strain at distance r

$$\varepsilon_{\theta\theta}=\left(\frac{R_n}{r}-1\right)\omega$$

Stress at distance r

$$\sigma_{\theta\theta}=E\varepsilon_{\theta\theta}=E\left(\frac{R_n}{r}-1\right)\omega$$

Curved Beams



Equilibrium of forces normal to the section

$$\sum F_z = \int \sigma_{\theta\theta}\, dA - N = 0$$

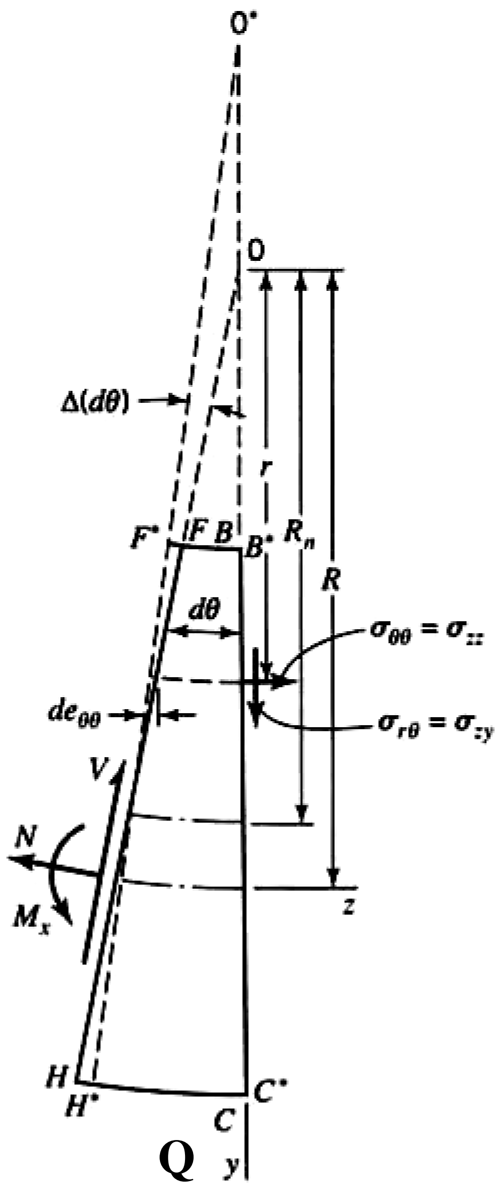
$$N = \int \sigma_{\theta\theta}\, dA$$

Equilibrium of moments about centroidal axis

$$\sum M_x = \int \sigma_{\theta\theta}(R - r)\, dA - M_x = 0$$

$$M_x = \int \sigma_{\theta\theta}(R - r)\, dA$$

Curved Beams



$$\sigma_{\theta\theta}=E\left(\frac{R_n}{r}-1\right)\omega$$

$$N=\int\sigma_{\theta\theta}dA$$

$$\Rightarrow N=\int E\left(\frac{R_n}{r}-1\right)\omega dA$$

$$\Rightarrow N=ER_n\omega\int\frac{dA}{r}-E\omega A$$

$$\Rightarrow N=E\omega\left(R_nA_m-A\right)$$

$$A_m=\int\frac{dA}{r}$$

Curved Beams

$$\sigma_{\theta\theta}=E\left(\frac{R_n}{r}-1\right)\omega$$

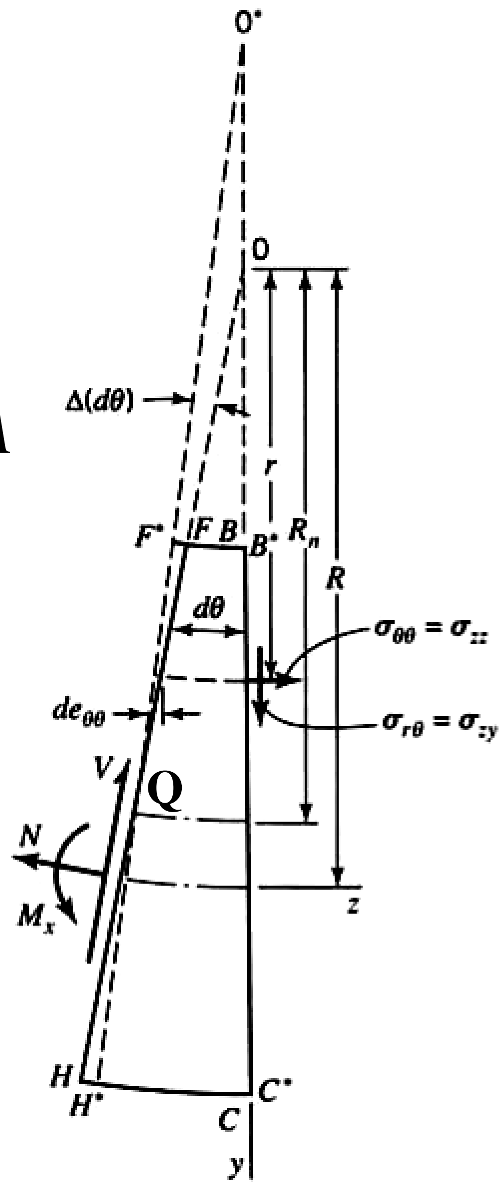
$$M_x=\int\sigma_{\theta\theta}\left(R-r\right)dA$$

$$\Rightarrow M_x=\int E\left(\frac{R_n}{r}-1\right)\omega\left(R-r\right)dA$$

$$\therefore M_x$$

$$=E\omega\int\left[\frac{R_nR}{r}-\left(R+R_n\right)+r\right]dA$$

By definition $\int rdA=RA$

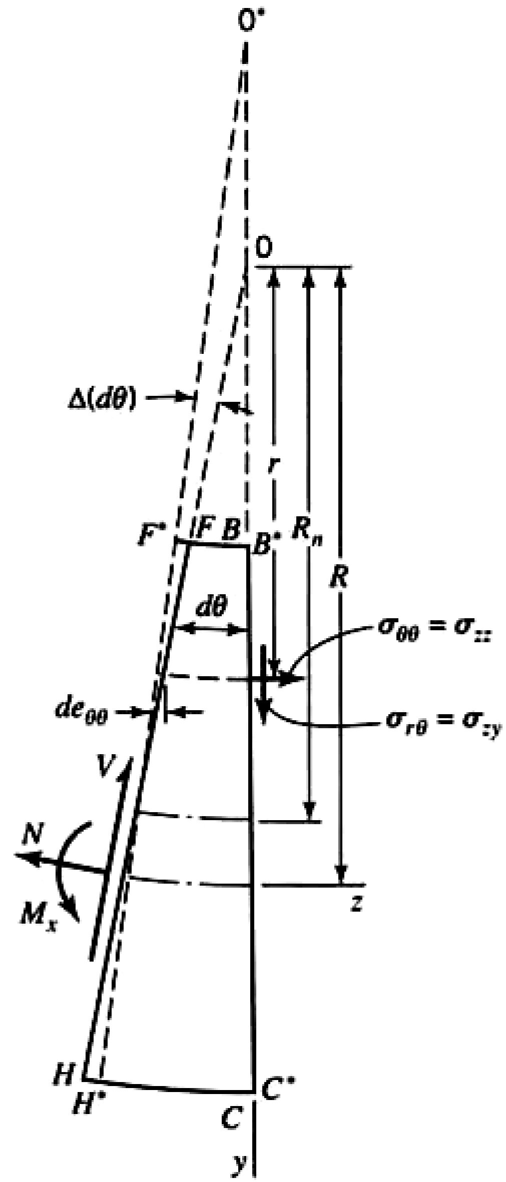


$$\therefore M_x=E\omega R_nR\int\frac{dA}{r}-\left(R+R_n\right)E\omega A+E\omega\int rdA$$

$$\Rightarrow M_x=E\omega R_nRA_m-\left(R+R_n\right)E\omega A+E\omega RA$$

$$\Rightarrow M_x=E\omega R_n\left(RA_m-A\right)$$

Curved Beams



$$M_x = E\omega R_n\left(RA_m - A\right)$$

$$\Rightarrow E\omega R_n = \frac{M_x}{RA_m - A}$$

$$\therefore N = E\omega R_n A_m - E\omega A$$

$$\Rightarrow N = \left(\frac{M_x}{RA_m - A}\right)A_m - E\omega A$$

$$\Rightarrow E\omega A = \left(\frac{M_x}{RA_m - A}\right)A_m - N$$

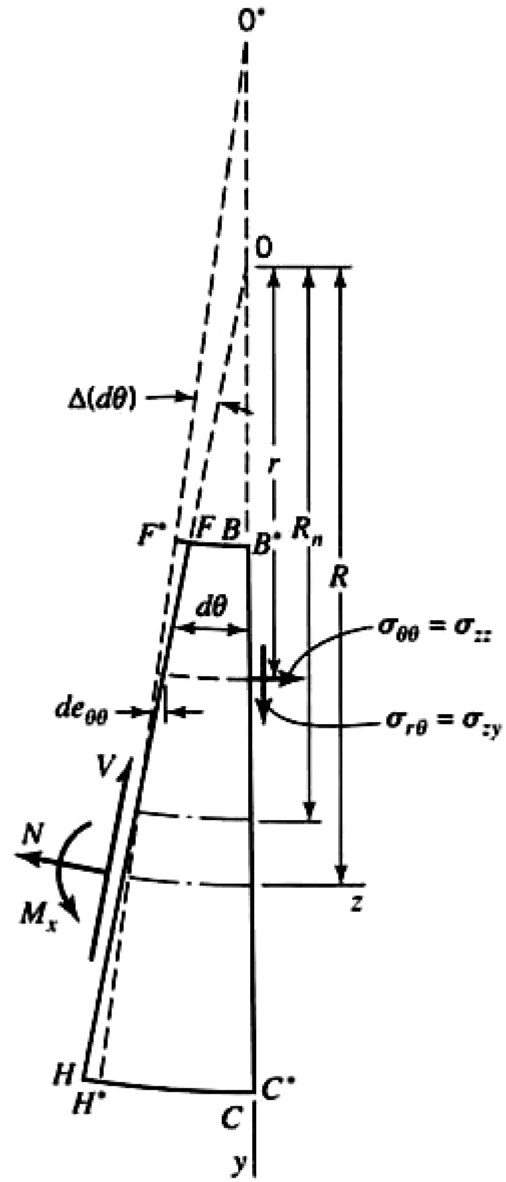
$$\Rightarrow E\omega = \frac{A_m}{A\left(RA_m - A\right)}M_x - \frac{N}{A}$$

$$\because \sigma_{\theta\theta} = E\omega R_n\frac{1}{r} - E\omega$$

$$\therefore \sigma_{\theta\theta} = \frac{M_x}{r\left(RA_m - A\right)} - \left[\frac{A_m M_x}{A\left(RA_m - A\right)} - \frac{N}{A}\right]$$

$$\Rightarrow \sigma_{\theta\theta} = \frac{N}{A} + \frac{M_x\left(A - rA_m\right)}{Ar\left(RA_m - A\right)}$$

Curved Beams



$$\sigma_{\theta\theta} = \frac{N}{A} + \frac{M_x\left(A - rA_m\right)}{Ar\left(RA_m - A\right)}$$

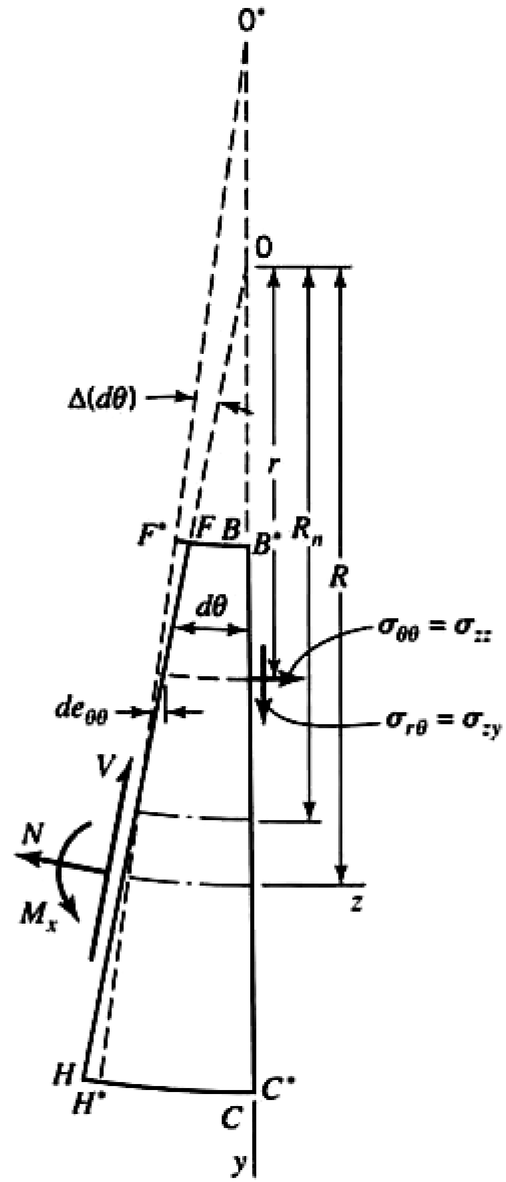
$$\sigma_{\theta\theta} = 0 \Rightarrow r = R_n$$

$$\therefore \frac{N}{A} + \frac{M_x\left(A - R_n A_m\right)}{AR_n\left(RA_m - A\right)}$$

$$\Rightarrow NAR_n\left(RA_m - A\right) + A^2 M_x - R_n A_m A M_x = 0$$

$$\Rightarrow R_n = \frac{AM_x}{A_m M_x + N\left(A - RA_m\right)}$$

Curved Beams



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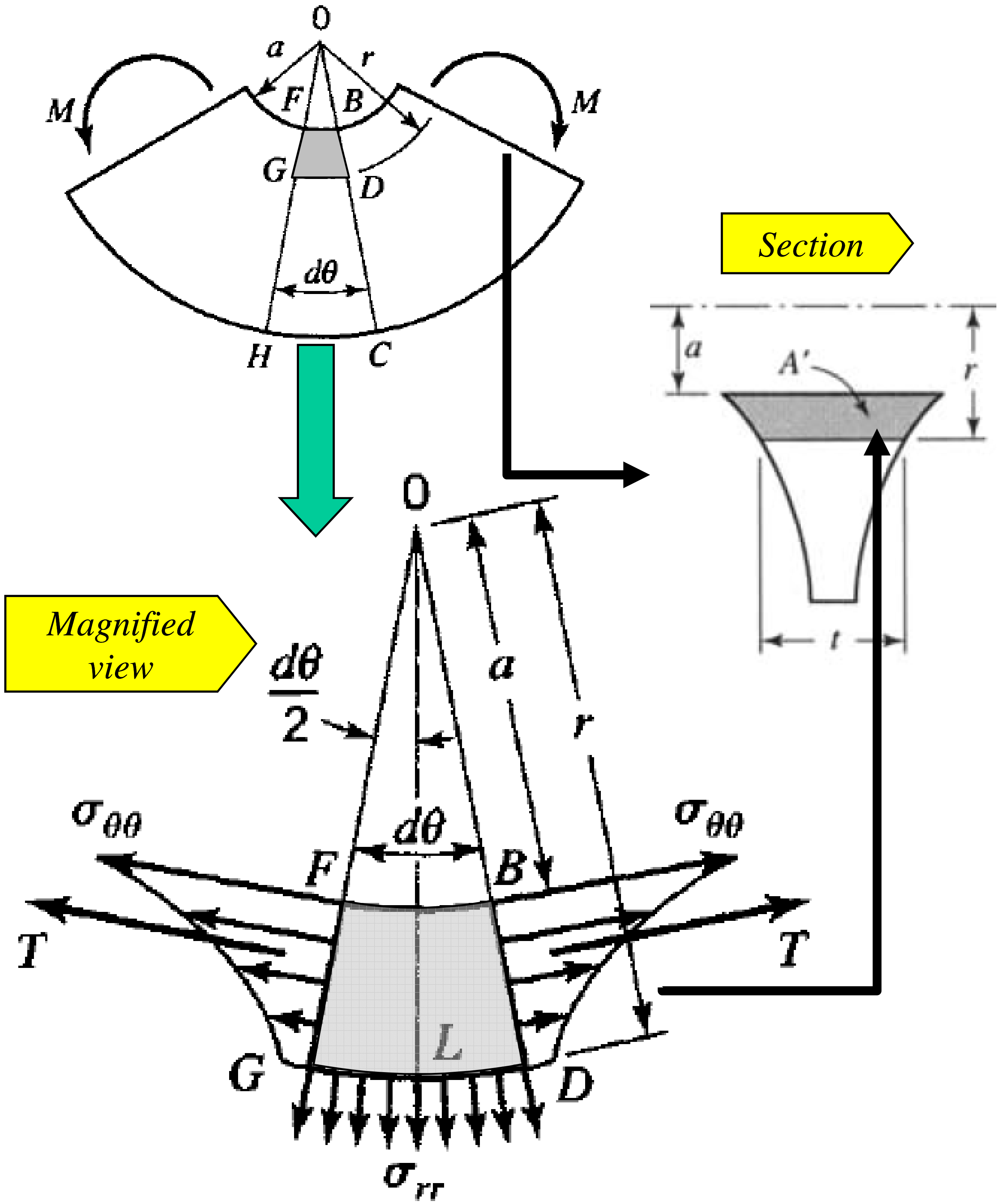


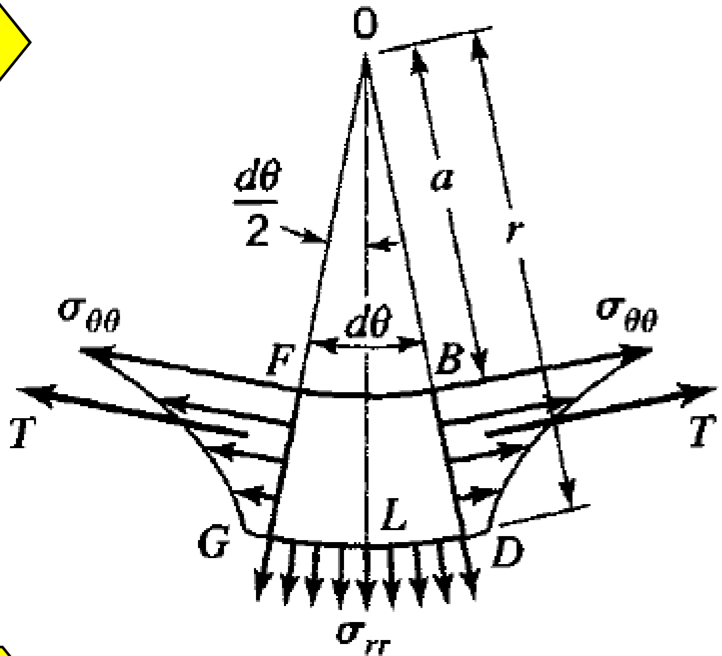
Curved Beams

Circumferential force

$$T = \int_a^r \sigma_{\theta\theta} dA$$

Radial force

$$F_r = \sigma_{rr}\left(trd\theta\right)$$



Equilibrium of forces

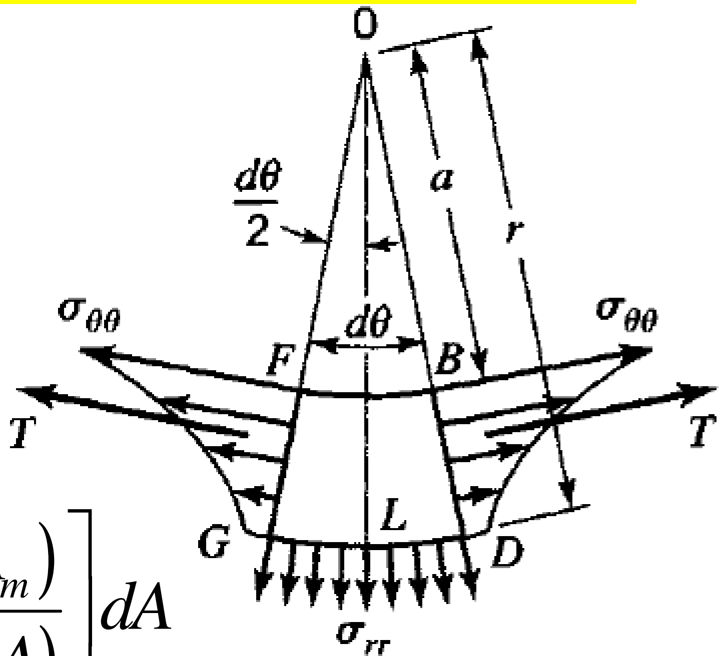
$$2T\ sin\left(\frac{d\theta}{2}\right) = F_r \Rightarrow 2T\ sin\left(\frac{d\theta}{2}\right) = \sigma_{rr}\left(trd\theta\right)$$

$$\because d\theta \text{ is very small}$$

$$\therefore 2T\left(\frac{d\theta}{2}\right) = \sigma_{rr}\left(trd\theta\right) \Rightarrow Td\theta = \sigma_{rr} trd\theta$$

$$\Rightarrow \sigma_{rr} = \frac{T}{tr}$$

Curved Beams



$$\sigma_{\theta\theta} = \frac{N}{A} + \frac{M_x\left(A - rA_m\right)}{Ar\left(RA_m - A\right)}$$

$$T = \int_a^r \sigma_{\theta\theta} dA$$

$$\Rightarrow T = \int_a^r \left[\frac{N}{A} + \frac{M_x\left(A - rA_m\right)}{Ar\left(RA_m - A\right)}\right] dA$$

$$\Rightarrow T = \frac{N}{A}\int_a^r dA + \frac{M_x}{\left(RA_m - A\right)}\int_a^r \frac{\left(A - rA_m\right)}{Ar} dA$$

$$\Rightarrow T = \frac{N}{A}\int_a^r dA + \frac{M_x}{\left(RA_m - A\right)}\left[\int_a^r \frac{dA}{r} - \frac{A_m}{A}\int_a^r dA\right]$$

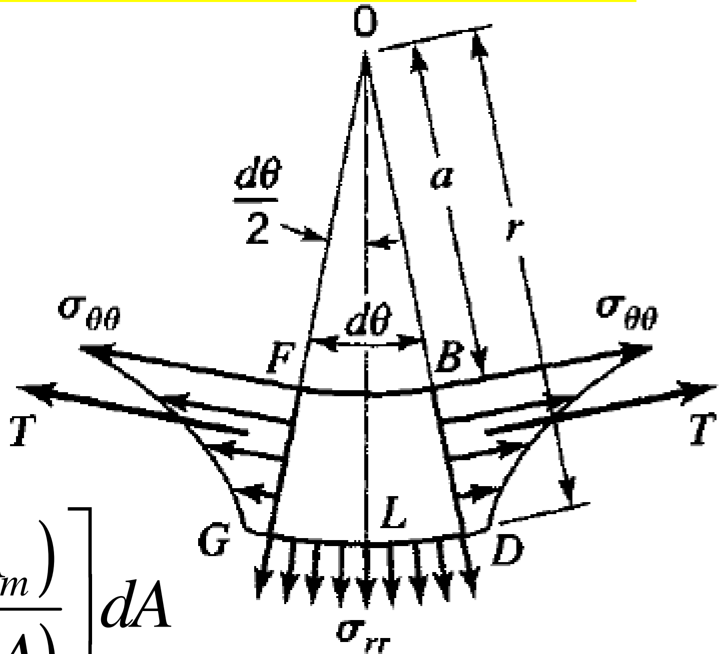
Define

$$A_m' = \int_a^r \frac{dA}{r}, \qquad A' = \int_a^r dA$$

$$\therefore T = N\left(\frac{A'}{A}\right) + \frac{M_x}{\left(R - \frac{A}{A_m}\right)}\left(\frac{A_m'}{A_m} - \frac{A'}{A}\right)$$

Curved Beams

$$\sigma_{\theta\theta} = \frac{N}{A} + \frac{M_x\left(A - rA_m\right)}{Ar\left(RA_m - A\right)}$$



$$T = \int_a^r \sigma_{\theta\theta} dA$$

$$\Rightarrow T = \int_a^r \left[\frac{N}{A} + \frac{M_x\left(A - rA_m\right)}{Ar\left(RA_m - A\right)}\right] dA$$

$$\Rightarrow T = \frac{N}{A}\int_a^r dA + \frac{M_x}{\left(RA_m - A\right)}\int_a^r \frac{\left(A - rA_m\right)}{Ar} dA$$

$$\Rightarrow T = \frac{N}{A}\int_a^r dA + \frac{M_x}{\left(RA_m - A\right)}\left[\int_a^r \frac{dA}{r} - \frac{A_m}{A}\int_a^r dA\right]$$

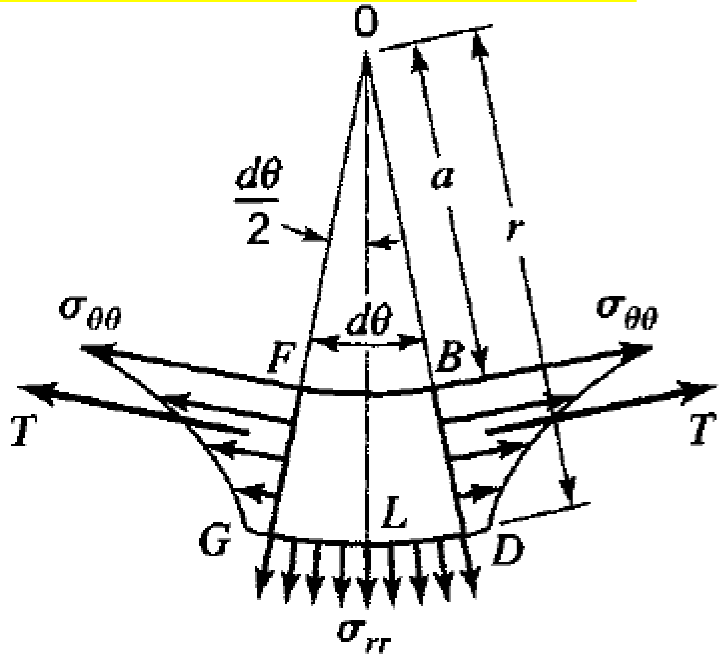
Define

$$A_m' = \int_a^r \frac{dA}{r}, \qquad A' = \int_a^r dA$$

$$\therefore T = N\left(\frac{A'}{A}\right) + \frac{M_x}{\left(R - \dfrac{A}{A_m}\right)}\left(\frac{A_m'}{A_m} - \frac{A'}{A}\right)$$

Curved Beams

$$A_m' = \int_a^r \frac{dA}{r}, \qquad A' = \int_a^r dA$$



$$T = N\left(\frac{A'}{A}\right) + \frac{M_x}{\left(R - \frac{A}{A_m}\right)}\left(\frac{A_m'}{A_m} - \frac{A'}{A}\right)$$

$$\therefore \sigma_{rr} = \frac{T}{tr} = \frac{N}{tr}\left(\frac{A'}{A}\right) + \frac{M_x}{tr\left(R - \frac{A}{A_m}\right)}\left(\frac{A_m'}{A_m} - \frac{A'}{A}\right)$$