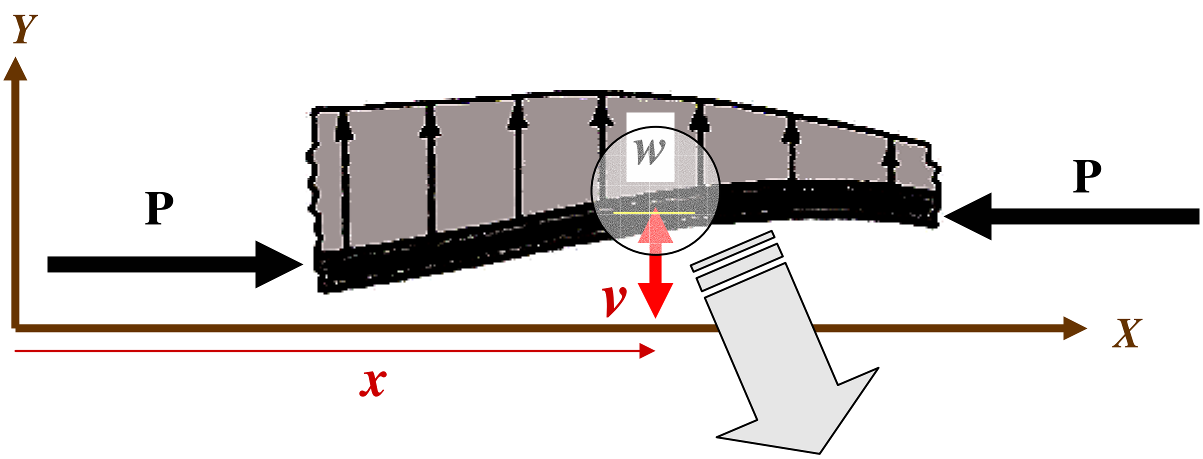
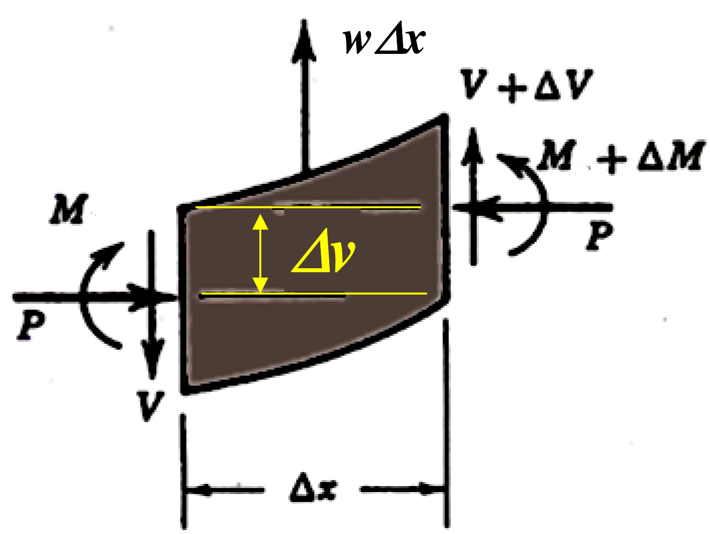
Equation of beam column

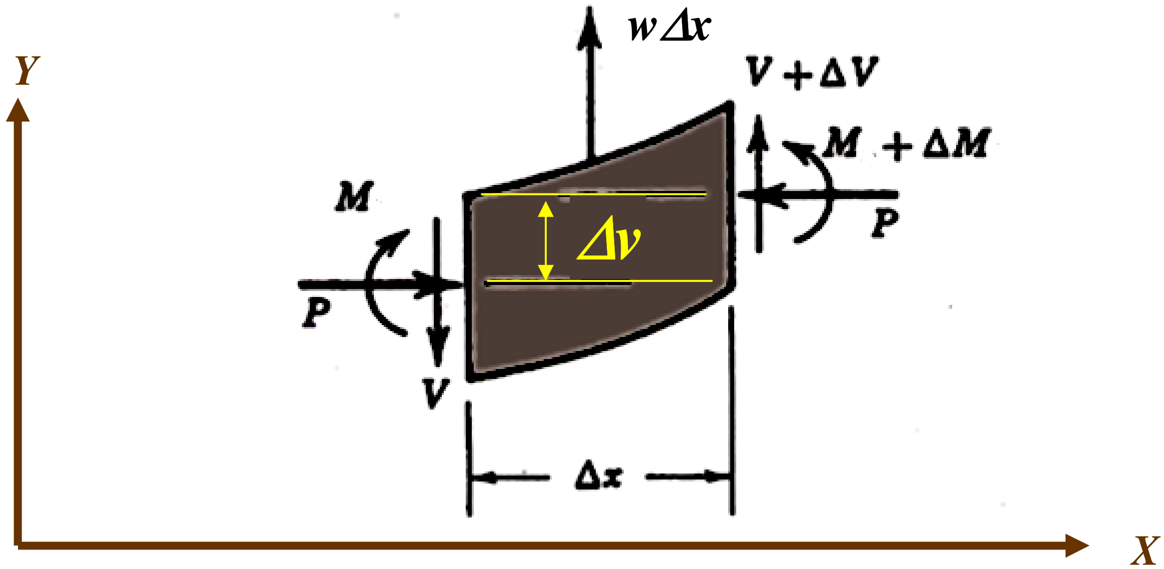
A segment of a beam column is considered. A coordinate system is set up as shown. Unstretched length, of the beam column is taken as L. A general transverse loading is considered along with axial concentrated loads. The undeformed beam is initially coincident with the x axis. The vertical deflection is v. Shearing deformation is neglected.



A small section



Equation of beam column



Force Equilibrium in Y

$$\sum F_y = 0$$

$$\Rightarrow \left(V + \Delta V\right) - V + w\Delta x = 0$$

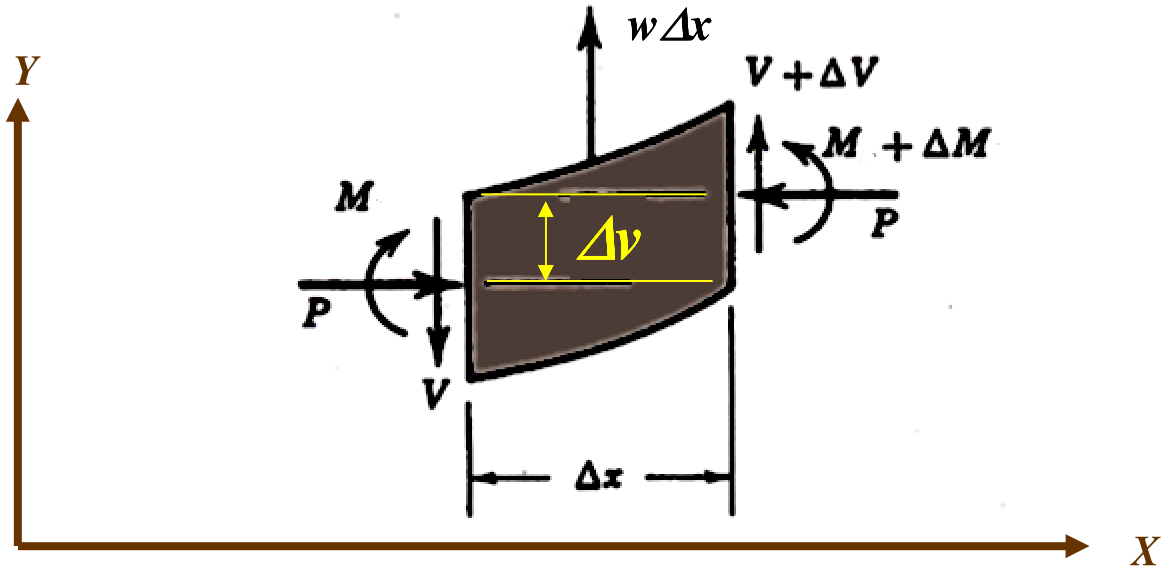
$$\Rightarrow \Delta V = -w\Delta x$$

$$\Rightarrow \frac{\Delta V}{\Delta x} = -w$$

In the limit

$$\boxed{\frac{dV}{dx} + w = 0}$$

Equation of beam column



Moment Equilibrium about center of segment

$$\left(M+\Delta M\right)-M+V\frac{\Delta x}{2}+\left(V+\Delta V\right)\frac{\Delta x}{2}+P\Delta v=0$$

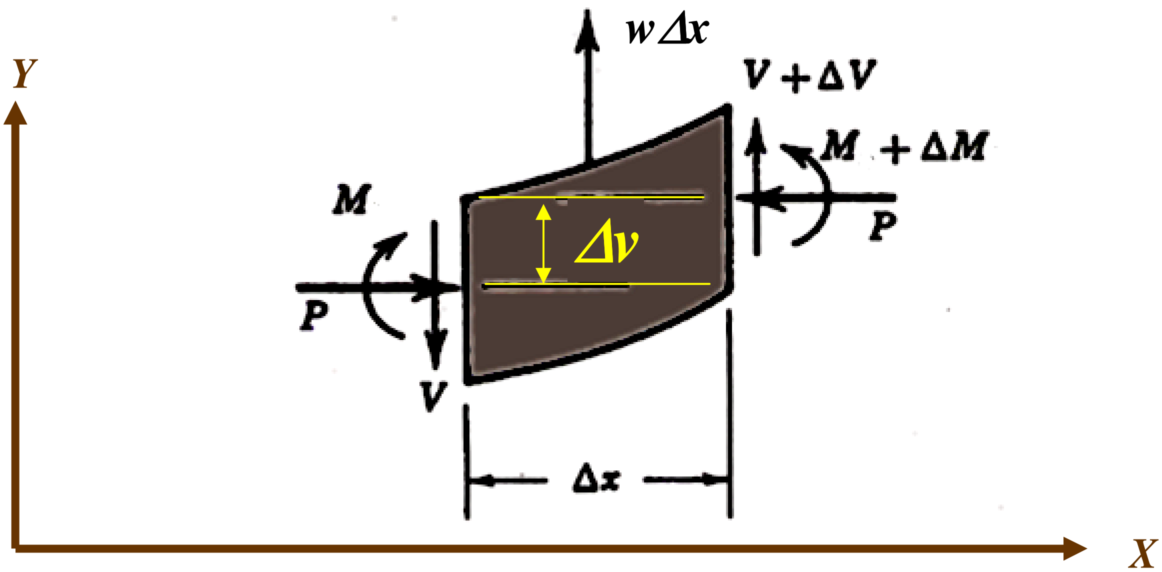
$$\Rightarrow \Delta M+2V\frac{\Delta x}{2}+\Delta V\frac{\Delta x}{2}+P\Delta v=0$$

$$\Rightarrow \frac{\Delta M}{\Delta x}+V+\frac{\Delta V}{2}+P\frac{\Delta v}{\Delta x}=0$$

In the limit

$$\boxed{\frac{dM}{dx}+V+P\frac{dv}{dx}=0}$$

Equation of beam column



Assuming only bending moment is responsible for deformation we assume that like in case of a pure beam

$$EI\frac{d^2v}{dx^2} = M$$

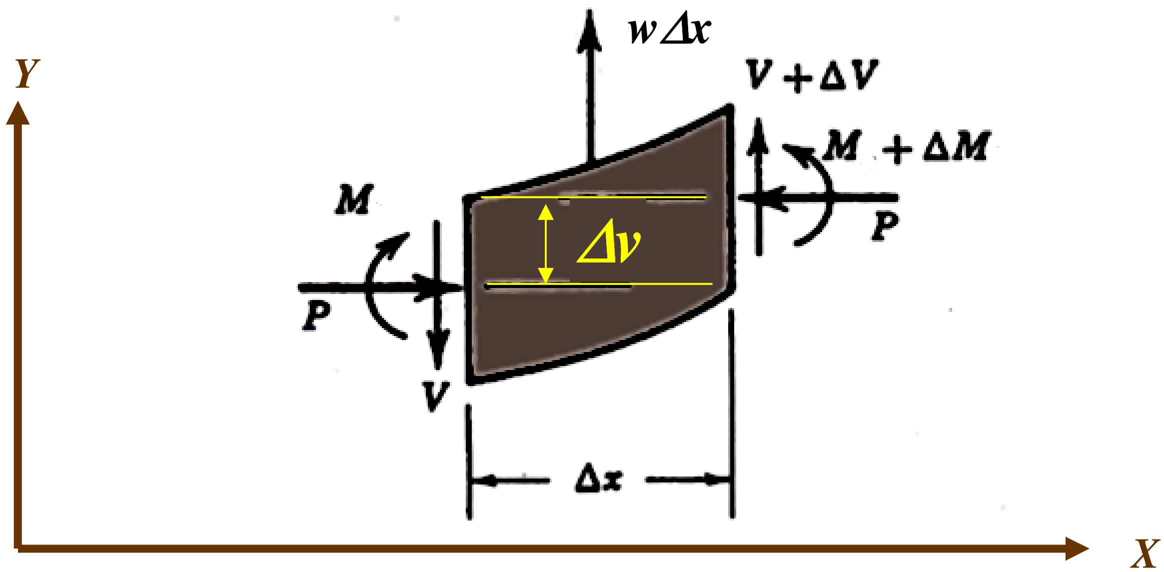
Hence

$$\frac{dM}{dx} + V + P\frac{dv}{dx} = 0 \Rightarrow \frac{d^2M}{dx^2} + \frac{dV}{dx} + \frac{d}{dx}\left(P\frac{dv}{dx}\right) = 0$$

$$\frac{dV}{dx} + w = 0 \Rightarrow \frac{d^2M}{dx^2} - w + \frac{d}{dx}\left(P\frac{dv}{dx}\right) = 0$$

$$EI\frac{d^2v}{dx^2} = M \Rightarrow \frac{d^2}{dx^2}\left(EI\frac{d^2v}{dx^2}\right) + \frac{d}{dx}\left(P\frac{dv}{dx}\right) = w$$

Equation of beam column



General Equation of a Beam Column

$$\frac{d^2}{dx^2}\left(EI\frac{d^2v}{dx^2}\right)+\frac{d}{dx}\left(P\frac{dv}{dx}\right)=w$$

Where EI and P are constants

$$EI\frac{d^4v}{dx^4}+P\frac{d^2v}{dx^2}=w$$

Boundary conditions are similar to pure beam problem