Tidal dissipation in planet-hosting stars: damping of spin–orbit misalignment and survival of hot Jupiters

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ABSTRACT

Observations of hot Jupiters around solar-type stars with very short orbital periods (~ 1 d) suggest that tidal dissipation in such stars is not too efficient so that these planets can survive against rapid orbital decay. This is consistent with recent theoretical works, which indicate that the tidal quality factor, $Q_\star$, of planet-hosting stars can indeed be much larger than the values inferred from the circularization of stellar binaries. On the other hand, recent measurements of Rossiter–McLaughlin effects in transiting hot Jupiter systems not only reveal that many such systems have misaligned stellar spin with respect to the orbital angular momentum axis, but also show that systems with cooler host stars tend to have aligned spin and orbital axes. Winn et al. suggested that this obliquity–temperature correlation may be explained by efficient damping of stellar obliquity due to tidal dissipation in the convection zone of the star. This explanation, however, is in apparent contradiction with the survival of these short-period hot Jupiters. We show that in the solar-type parent stars of close-in exoplanetary systems, the effective tidal $Q_\star$ governing the damping of stellar obliquity can be much smaller than that governing orbital decay. This is because, for misaligned systems, the tidal potential contains a Fourier component with frequency equal to the stellar spin frequency (in the rotating frame of the star) and rotating opposite to the stellar spin. This component can excite inertial waves in the convective envelope of the star, and the dissipation of inertial waves then leads to a spin–orbit alignment torque and a spin-down torque, but not orbital decay. By contrast, for aligned systems, such inertial wave excitation is forbidden since the tidal forcing frequency is much larger than the stellar spin frequency. We derive a general effective tidal evolution theory for misaligned binaries, taking account of different tidal responses and dissipation rates for different tidal forcing components.

Key words: hydrodynamics – waves – binaries: close – planetary systems – stars: rotation.

1 INTRODUCTION

1.1 Tidal dissipation in planet-hosting stars: a conundrum

Recent measurements of stellar obliquity in transiting exoplanetary systems using the Rossiter–McLaughlin effect have shown that a significant fraction of hot Jupiter systems have misaligned stellar spin with respect to the planetary angular momentum axis (e.g. Hébrard et al. 2008; Johnson et al. 2009; Narita et al. 2009; Winn et al. 2009, 2010, 2011; Pont et al. 2010; Triaud et al. 2010; Moutou et al. 2011; Simpson et al. 2011). This suggests that a large population of hot Jupiters are formed through dynamical planet–planet scatterings (e.g. Rasio & Ford 1996; Weidenschilling & Marzari 1996; Zhou, Lin & Sun 2007; Chatterjee et al. 2008; Juric &

Tremaine 2008) and, more importantly, through secular interactions between multiple planets or Kozai effect induced by a distant companion (e.g. Wu & Murray 2003; Fabrycky & Tremaine 2007; Nagasawa, Ida & Bessho 2008; Katz, Dong & Malhotra 2011; Naoz et al. 2011a,b; Wu & Lithwick 2011), although other effects involving star–disc interactions (Foucart & Lai 2011; Lai, Foucart & Lin 2011) and the assembly of protoplanetary discs (Bate, Lodato & Pringle 2010; Thies et al. 2011) may also play a role in producing spin–orbit misalignment.

Recent observations have also revealed an intriguing correlation between stellar obliquity Θ and effective temperature $T_{\rm eff}$: Winn et al. (2010) found that the misaligned systems tend to have hotter parent stars ($T_{\rm eff} \gtrsim 6250$ K, corresponding to stellar mass $M_\star \gtrsim 1.3\,{\rm M_\odot}$), while the systems containing cooler stars have small obliquities. This trend was also found from a recent analysis of the stellar rotation velocities (Schlaufman 2010). Although this Θ–$T_{\rm eff}$ correlation may indicate different planet formation mechanisms for

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stars of different masses, Winn et al. (2010) pointed out that $T_{eff} = 6250$ K corresponds to the temperature below which stars contain a large subsurface convective envelope, and suggested that tidal dissipation in these stars leads to the damping of their obliquities. Indeed, the facts that effective temperature is more closely related to obliquity than stellar mass and that a few low-mass and long-period planets are exceptions to the Θ–T_{eff} correlation (Winn et al. 2010, 2011) support the idea of tidal damping of spin–orbit misalignment in solar-type stars. Most recently, Triaud (2011) found a correlation between the obliquity and stellar age, suggesting a tidal alignment time-scale of about 2.5 Gyr.

It has long been recognized that tidal dissipation in planet-hosting stars could play an important role in the evolution and survival of hot Jupiter systems (e.g. Lin, Bodenheimer & Richardson 1996; Rasio et al. 1996; Marcy et al. 1997; Sasselov 2003; Dobbs-Dixon, Lin & Mardling 2004; Barker & Ogilvie 2009; Jackson, Barnes & Greenberg 2009; Levrard, Winisdoerffer & Chabrier 2009; Leconte et al. 2010; Matsumura, Peale & Rasio 2010). The strength of tidal dissipation is usually parametrized by a dimensionless quality factor $Q_\star$. When the orbital mean motion Ω is larger than the stellar spin frequency Ω_s, the orbital decay time-scale (for circular orbits) is

$$t_a = \left| \frac{a}{\dot{a}} \right| = \frac{2Q'_\star}{9} \left(\frac{M_\star}{M_p} \right) \left(\frac{a}{R_\star} \right)^5 \frac{1}{\Omega}$$

$$\simeq 1.28 \left(\frac{Q'_\star}{10^7} \right) \left(\frac{M_\star}{10^3 M_p} \right) \left(\frac{\bar{\rho}_\star}{\bar{\rho}_\odot} \right)^{5/3} \left(\frac{P}{1\,\text{d}} \right)^{13/3} \text{Gyr}, \quad (1)$$

where $Q'_\star = 3Q_\star/(2k_2)$ is the reduced tidal quality factor, k_2 is the Love number, and $\bar{\rho}_\star$ is the mean density of the star and $\bar{\rho}_\odot$ is the solar value. The inspiral time for a planet into its host star is $(2/13)t_a$. The observations of hot Jupiters with the shortest orbital periods (such as WASP-18b, 0.94 d; WASP-19b, 0.79 d; and WASP-43b, 0.81 d; Hellier et al. 2009, 2011; Hebb et al. 2010) suggest $Q'_\star \gtrsim 10^8$ (see Brown et al. 2011). Such a large $Q'_\star$ value is consistent with recent theoretical works (see Section 1.2) on the physics of tidal dissipation in planet-hosting solar-type stars (Ogilvie & Lin 2007; Barker & Ogilvie 2010, 2011; Penev & Sasselov 2011). On the other hand, in the often-used tidal evolution equations (see Section 2.5), the damping time for stellar obliquity Θ is

$$t_\Theta = \left| \frac{\sin \Theta}{\dot{\Theta}} \right| \simeq \left(\frac{2S}{L} \right) t_a$$

$$\simeq 1.13\, t_a \left(\frac{\kappa}{0.1} \right) \left(\frac{M_\star}{10^3 M_p} \right) \left(\frac{\bar{\rho}_\odot}{\bar{\rho}_\star} \right)^{2/3} \left(\frac{10\,\text{d}}{P_s} \right) \left(\frac{1\,\text{d}}{P} \right)^{1/3}, \quad (2)$$

where S and L are the stellar spin and orbital angular momenta, respectively, P_s is the spin period and the moment of inertia of star is $\kappa M R^2$. Thus, for typical parameters of hot Jupiter systems, $t_\Theta \sim t_a$, and a tidal quality factor $Q'_\star \gtrsim 10^8$ would not cause significant damping of Θ. To put it in another way, a reduction in the stellar obliquity is accompanied by a similar amount of orbital decay, $\Delta a/a \simeq (2S/L)\Delta\Theta/\sin\Theta$. This poses a severe problem to the (otherwise appealing) tidal damping interpretation of the stellar obliquity–effective temperature correlation (Winn et al. 2010). One way out of this problem is to assume that the star's convective envelope is weakly coupled to its radiative core, thus reducing the obliquity damping time for the stellar envelope (Winn et al. 2010). This assumption, however, is difficult to substantiate, as fluid instabilities may develop in the presence of large differential rotation (especially when the directions of rotation vary across the star) to quickly couple the rotations of the core and the envelope.

To recapitulate, there is a conundrum concerning the efficiency of tidal dissipation in planet-hosting solar-type stars. On the one hand,

the survival of hot Jupiters with shortest orbital periods and recent theoretical works both indicate that stellar tidal dissipation induces only modest or negligible orbital decay. On the other hand, the observed stellar obliquity–effective temperature correlation suggests that tidal dissipation is important in damping stellar obliquity.

In this paper, we show that tidal damping of spin–orbit misalignment can be much more efficient than tidal damping of the orbit. In another word, the effective tidal quality factor for the former process can be much smaller than the latter. This provides a natural resolution to the conundrum discussed above.

1.2 Basic idea

Many previous works on tidal evolution in hot Jupiter systems (e.g. Rasio et al. 1996; Sasselov 2003; Dobbs-Dixon et al. 2004; Barker & Ogilvie 2009; Jackson et al. 2009; Levrard et al. 2009; Hansen 2010; Matsumura et al. 2010) were based on the weak friction theory of equilibrium tides. This theory considers large-scale quadrupole distortion of the star, and parametrizes tidal dissipation by a dimensionless quality factor $Q_\star$ or, more generally, by a constant tidal lag time Δt_L. The theory was first formulated by Darwin (1880), and extensively applied to Solar system bodies (e.g. Goldreich & Soter 1966) and stellar binaries (see Zahn 2008, for a review). These applications have proved very useful since they provide empirical estimates or constraints on the values of $Q_\star$ for various systems. The most general (arbitrary orbital eccentricity and spin–orbit inclination angle) and correct equations for tidal evolution based on this theory were derived by Alexander (1973), and were also elaborated by others (e.g. Hut 1981; Eggleton, Kiseleva & Hut 1998; Correia & Laskar 2010).

Although it is well recognized that the equilibrium tide theory is a parametrized theory, with all the physics of tidal dissipation hidden in a single parameter $Q_\star$ or Δt_L (the Love number of the body can be absorbed into the definition of these parameters), it is not widely appreciated that the effective tidal $Q_\star$ for different processes (e.g. spin–orbit alignment and orbital decay) can be different. In another word, the widely used tidal evolution equations based on equilibrium tide theory can be incorrect even at the parametrized level. Indeed, the conundrum discussed in Section 1.1 arises because equation (2) assumes that the tidal $Q_\star$ for stellar obliquity damping is similar to the $Q_\star$ for orbital decay. In fact, this is incorrect, as we explain below.

There are three channels of tidal dissipations in solar-type stars.

(i) *Equilibrium tides*. The large-scale quasi-static tidal bulge can be damped by turbulent viscosity in the star's convective envelope (Zahn 1977, 1989). The major uncertainty involves how the effective viscosity derived from the mixing-length theory, $\sim v_t l_t/3$ (where v_t and l_t are the velocity and size of convective eddies, respectively), is reduced when the tidal forcing period P_{tide} is shorter than the convective turnover time $\tau_t = l_t/v_t$ (see Goodman & Oh 1997). Recent simulations (Penev, Barranco & Sasselov 2009, 2011) suggest that the reduction factor is about $P_{tide}/(2\pi\tau_t)$ (for a limited range of P_{tide}), and the corresponding tidal $Q_\star$ well exceeds 10^8 (Penev & Sasselov 2011). An even larger $Q_\star$ will result if the reduction factor $(P_{tide}/2\pi\tau_t)^2$ is used (see Ogilvie & Lin 2007).

(ii) *Excitation and damping of internal gravity waves* (Goodman & Dickson 1998; Ogilvie & Lin 2007; Barker & Ogilvie 2010, 2011). These waves (also called Hough waves when modified by rotation) are launched at the bottom of the star's convective envelope and propagate towards the stellar centre. If they attain sufficient amplitudes at the centre, wave breaking will occur; this will

produce significant tidal dissipation, corresponding to $Q_\star \sim$ a few $\times 10^5 (P/1\,\text{d})^{8/3}$ (assuming the orbital period P is much shorter than the spin period; see Barker & Ogilvie 2010). If the waves are reflected coherently at the stellar centre (e.g. by a small convective core) before non-linear breaking (Terquem et al. 1998), only weak dissipation will result ($Q_\star \gtrsim 10^8$). The latest calculations by Barker & Ogilvie (2010, 2011) suggest that while the non-linear wave breaking certainly occurs for binary stars, it is probably unimportant for exoplanetary systems – this would explain the survival of short-period hot Jupiters against orbital decay (see Weinberg et al. 2011).

(iii) *Excitation and damping of inertial waves.* Recent theoretical works on dynamical tides in rotating planets (Ogilvie & Lin 2004, 2007; Ogilvie 2005, 2009; Papaloizou & Ivanov 2005; Wu 2005a,b; Goodman & Lackner 2009) and stars (Papaloizou & Savonije 1997; Savonije & Papaloizou 1997; Savonije & Witte 2002; Ogilvie & Lin 2007) have emphasized the importance of inertial waves driven by Coriolis force. When the tidal forcing frequency (in the rotating frame) $\tilde{\omega}$ is less than twice the spin frequency (Ω_s), short-wavelength inertial waves can be excited. In particular, when these waves are confined to a spherical shell (as in the convection zone outside the solid core of a giant planet or in the convective envelope of a solar-type star), tidal disturbances are concentrated in very narrow regions (called 'wave attractors') where dissipation takes place (Ogilvie & Lin 2004; Goodman & Lackner 2009; Ogilvie 2009). It appears that this mechanism can explain the tidal Q ($\sim 10^6$) of giant planets and, when combined with internal gravity wave damping [see (ii) above], can also explain the dissipation required for the circularization of stellar binaries. However, for solar-type stars in hot Jupiter systems, the inertial wave dissipation mechanism is not expected to operate, since the tidal frequency $\tilde{\omega} = 2(\Omega - \Omega_s)$ (assuming circular orbit and aligned stellar spin) is larger than $2\Omega_s$ for typical parameters (e.g. $P_s \sim 10\,\text{d}$ and $P \sim 1\,\text{d}$).

The main point of our paper concerns inertial wave dissipation in the parent stars of hot Jupiter systems when the stellar spin S is misaligned with the orbital angular momentum L. For a circular binary, in the inertial coordinate system with the z-axis along L, the tidal potential has two components (to the quadrupole order), with frequencies $\omega_{m'} = m'\Omega$ (where $m' = 0, 2$). As seen in the rotating frame of the star, the tidal frequencies become

$$\tilde{\omega}_{mm'} = m'\Omega - m\Omega_s, \tag{3}$$

with $m = 0, \pm 1, \pm 2$. For an aligned system, only the $m = m' = 2$ component of the tidal potential is non-zero and involved in tidal dissipation. (The $m = m' = 0$ component is also non-zero, but it does not transfer energy and angular momentum since it is completely static.) For misaligned systems, however, all seven tidal components [all combinations of (m, m') except $m = m' = 0$; note that for $m' = 0$, the $m = -1(-2)$ component is physically identical to the $m = 1(2)$ component] contribute to the transfers of tidal energy and/or angular momentum. In general, each of the seven distinct components will generate tidal disturbance with its own quality factor $Q_{mm'}$. Of particular interest is the $(m, m') = (\pm 1, 0)$ components. They have $\tilde{\omega}_{mm'} = \mp\Omega_s$ and the angular pattern frequency is $\tilde{\omega}/m = -\Omega_s$ (the negative sign means it is retrograde with respect to the spin). They can generate inertial waves in the convection zone, and therefore can make a significant or dominant contribution to the tidal alignment of spin–orbit inclination. These components, however, do not contribute to orbital decay since they are static in the inertial frame and do not transfer energy (see Section 3).

In summary, while for aligned hot Jupiter systems, the effective stellar tidal $Q_\star$ governing orbital decay (and contributing to orbital circularization[1]) may be quite large ($\gtrsim 10^8$) because channel (i) and channel (ii) (i.e. equilibrium tides and gravity waves) are ineffective and channel (iii) (inertial waves) is forbidden, for misaligned systems, inertial wave excitation becomes possible, which may provide an efficient damping mechanism for the spin–orbit misalignment.

The remainder of this paper is organized as follows. In Section 2, we develop a general effective theory for tidal evolution of misaligned binaries. It is 'general' because the theory takes account of the different responses (including both equilibrium and dynamical tides) of the star to different frequency components of the tidal potential. It is 'effective' because the responses and dissipations of different components are treated in a parametrized way. In Section 3, we present the tidal evolution equations due to inertial wave dissipation for misaligned hot Jupiter systems. We conclude in Section 4.

2 TIDAL EVOLUTION OF MISALIGNED BINARIES: A GENERAL EFFECTIVE THEORY

2.1 Tidal potential

We consider a star of mass M, radius R and spin Ω_s (along the direction $\hat{S}$) orbited by a companion (planet) of mass M'. The orbital semimajor axis is a and the orbital angular frequency is Ω. We allow for general spin–orbit inclination angle Θ (the angle between the spin angular momentum $S = S\hat{S}$ and the orbital angular momentum $L = L\hat{L}$), but consider circular orbit for simplicity. In the spherical coordinate system centred on M with the z-axis along $\hat{L}$, the tidal potential produced by M' can be expanded in terms of spherical harmonics:

$$U(\mathbf{r}, t) = -GM' \sum_{m'} \frac{W_{2m'} r^2}{a^3} e^{-im'\Omega t} Y_{2m'}(\theta_L, \phi_L), \tag{4}$$

where $m' = 0, \pm 2$, with $W_{20} = -(\pi/5)^{1/2}$ and $W_{2\pm 2} = (3\pi/10)^{1/2}$. To study the dynamical response of stellar fluid to the tidal forcing, we need to express $U(\mathbf{r}, t)$ in terms of $Y_{lm}(\theta, \phi)$, the spherical harmonic function defined in the inertial frame centred on M with the z-axis along $\hat{S}$. This is achieved by the relation

$$Y_{2m'}(\theta_L, \phi_L) = \sum_m \mathcal{D}_{mm'}(\Theta) Y_{2m}(\theta, \phi), \tag{5}$$

where $\mathcal{D}_{mm'}(\Theta)$ is the Wigner $\mathcal{D}$-matrix of $l = 2$ (e.g. Wybourne 1974), and we have chosen the y-axis along the direction $\hat{S} \times \hat{L}$. The relevant $\mathcal{D}_{mm'}$ values are

$$\mathcal{D}_{2\pm 2} = \frac{1}{4}(1 \pm \cos\Theta)^2, \tag{6}$$

$$\mathcal{D}_{2\pm 1} = -\frac{1}{2}\sin\Theta(1 \pm \cos\Theta), \tag{7}$$

$$\mathcal{D}_{20} = \frac{\sqrt{6}}{4}\sin^2\Theta, \tag{8}$$

$$\mathcal{D}_{10} = -\frac{\sqrt{6}}{2}\sin\Theta\cos\Theta, \tag{9}$$

$$\mathcal{D}_{00} = \frac{1}{2}(3\cos^2\Theta - 1), \tag{10}$$

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[1] Tidal dissipation in the planet also contributes to circularization.

and

$$\mathcal{D}_{m'm} = (-1)^{m-m'}\mathcal{D}_{mm'} = \mathcal{D}_{-m,-m'}. \tag{11}$$

The tidal potential then becomes

$$U(\boldsymbol{r}, t) = -\sum_{mm'} U_{mm'}\, r^2 Y_{2m}(\theta, \phi)\, e^{-im'\Omega t}, \tag{12}$$

where

$$U_{mm'} \equiv \frac{GM'}{a^3}\mathcal{U}_{mm'} \equiv \frac{GM'}{a^3} W_{2m'}\mathcal{D}_{mm'}(\Theta). \tag{13}$$

Note that when expressed in terms of $\phi_{\rm r}$ (the azimuthal angle in the rotating frame of the star), each term in equation (12) has the dependence $e^{im\phi_{\rm r}+im\Omega_{\rm s}t-im'\Omega t}$. Thus, the tidal potential is composed of various (mm') components, each with forcing frequency $m'\Omega$ in the inertial frame. In the frame corotating with the star, the forcing frequency of the (mm') component is $\tilde{\omega}_{mm'} = m'\Omega - m\Omega_{\rm s}$ (equation 3), with the pattern rotation frequency

$$\frac{\tilde{\omega}_{mm'}}{m} = \left(\frac{m'}{m}\right)\Omega - \Omega_{\rm s}. \tag{14}$$

(Obviously, the pattern frequency has no meaning for the axisymmetric $m = 0$ components.) Note that, physically, there are seven distinct components: $(m, m') = (0, 2), (\pm 1, 2), (\pm 2, 2), (1, 0), (2, 0)$ (see also Barker & Ogilvie 2009). The $(0, 0)$ component is static and does not contribute to tidal dissipation. The $(-m, -m')$ component is physically identical to the (m, m') component.

2.2 Ansatz for tidal response

Each (mm') component of the tidal potential drives fluid perturbation inside the star, which can be specified by the Lagrangian displacement $\boldsymbol{\xi}_{mm'}(\boldsymbol{r}, t)$ and Eulerian density perturbation $\delta\rho_{mm'}(\boldsymbol{r}, t)$. In the absence of dissipation, these perturbations are proportional to $(U_{mm'}/\omega_0^2)e^{im\phi-im'\Omega t}$ (where $\omega_0 \equiv \sqrt{GM/R^3}$ is the dynamical frequency of the star), exactly in phase with the tidal potential. When fluid dissipation is present, there will be phase shift between the fluid perturbation and the tidal potential. This phase shift, in general, depends on m, the forcing frequency (in the rotating frame) $\tilde{\omega}_{mm'}$, as well as the intrinsic property (including the rotation rate) of the star. We write this phase shift as

$$\Delta_{mm'} = \tilde{\omega}_{mm'}t_{mm'}, \tag{15}$$

where $t_{mm'}$ is the 'lag time' associated with the (mm') perturbation. Thus, without loss of generality, we can write (mm') component of the fluid displacement as

$$\boldsymbol{\xi}_{mm'}(\boldsymbol{r}, t) = \frac{U_{mm'}}{\omega_0^2}\bar{\boldsymbol{\xi}}_{mm'}(\boldsymbol{r})\, \exp(-im'\Omega t + i\Delta_{mm'}). \tag{16}$$

The corresponding density perturbation is

$$\delta\rho_{mm'}(\boldsymbol{r}, t) = \frac{U_{mm'}}{\omega_0^2}\delta\bar{\rho}_{mm'}(\boldsymbol{r})\, \exp(-im'\Omega t + i\Delta_{mm'}), \tag{17}$$

with

$$\delta\bar{\rho}_{mm'} = -\nabla \cdot (\rho\bar{\boldsymbol{\xi}}_{mm'}). \tag{18}$$

Note that $\bar{\boldsymbol{\xi}}_{mm'}(\boldsymbol{r})$ and $\delta\bar{\rho}_{mm'}(\boldsymbol{r})$ are proportional to $e^{im\phi}$; except for this factor, they are real functions. Also note that

$$\frac{U_{mm'}}{\omega_0^2} = \frac{M'}{M}\left(\frac{R}{a}\right)^3 \mathcal{U}_{mm'}, \tag{19}$$

with $\mathcal{U}_{mm'} \equiv W_{2m'}\mathcal{D}_{mm'}(\Theta)$. Thus, $\bar{\boldsymbol{\xi}}_{mm'}(\boldsymbol{r})$ and $\delta\bar{\rho}_{mm'}(\boldsymbol{r})$ specify the amplitudes (and shapes) of the fluid perturbations after factoring out the 'equilibrium' dimensionless tidal distortion, $(M'/M)(R/a)^3$.

Note that although our ansatz of the tidal responses is motivated by the weak friction theory of equilibrium tides, it is actually quite general, provided that one keeps in mind that the lag time $t_{mm'}$ depends on m, $\tilde{\omega}_{mm'}$ and the intrinsic property of the star.

2.3 Tidal torque and energy transfer rate

The tidal torque on the star is

$$\boldsymbol{T} = \int {\rm d}^3x\, \delta\rho(\boldsymbol{r}, t)\, \boldsymbol{r} \times \left[-\nabla U^*(\boldsymbol{r}, t)\right], \tag{20}$$

and the tidal energy transfer rate (from the orbit to the star) is

$$\dot{E} = \int {\rm d}^3x\, \rho(\boldsymbol{r})\frac{\partial\boldsymbol{\xi}(\boldsymbol{r}, t)}{\partial t} \cdot \left[-\nabla U^*(\boldsymbol{r}, t)\right]. \tag{21}$$

Using equations (12) and (17), the z-component (along the spin axis) of the tidal torque reduces to

$$T_z = T_0 \sum_{mm'} \mathcal{U}_{mm'}^2\, m\, \kappa_{mm'}\, \sin\Delta_{mm'}, \tag{22}$$

where

$$T_0 \equiv G\left(\frac{M'}{a^3}\right)^2 R^5, \tag{23}$$

and $\kappa_{mm'}$ is the 'Love coefficient':

$$\kappa_{mm'} = \frac{1}{MR^2}\int {\rm d}^3x\, \delta\bar{\rho}_{mm'}(\boldsymbol{r})r^2 Y_{2m}^*(\theta, \phi). \tag{24}$$

Similarly, the energy transfer rate is given by

$$\dot{E} = T_0\Omega \sum_{mm'} \mathcal{U}_{mm'}^2\, m'\kappa_{mm'}\, \sin\Delta_{mm'}. \tag{25}$$

Note that the (mm') component of energy transfer and tidal torque satisfies

$$(\dot{E})_{mm'} = \frac{m'\Omega}{m}(T_z)_{mm'}. \tag{26}$$

This is expected since the pattern rotation rate (in the inertial frame) of the tidal force is $m'\Omega/m$.

Equations (22) and (25) can be worked out explicitly. Assuming $|\Delta_{mm'}| \ll 1$, we have

$$\begin{aligned}
T_z = \frac{(W_{22})^2}{2}T_0\Big[&(1 + c_\theta)^4(\Omega - \Omega_{\rm s})\tau_{22} \\
&+ s_\theta^2(1 + c_\theta)^2(2\Omega - \Omega_{\rm s})\tau_{12} \\
&- s_\theta^2(1 - c_\theta)^2(2\Omega + \Omega_{\rm s})\tau_{-12} \\
&- (1 - c_\theta)^4(\Omega + \Omega_{\rm s})\tau_{-22}\Big] \\
&- 3(W_{20})^2T_0\,\Omega_{\rm s}\left(s_\theta^4\,\tau_{20} + s_\theta^2\,c_\theta^2\,\tau_{10}\right)
\end{aligned} \tag{27}$$

and

$$\begin{aligned}
\dot{E} = \frac{(W_{22})^2}{2}T_0\Omega\Big[&(1 + c_\theta)^4(\Omega - \Omega_{\rm s})\tau_{22} \\
&+ 2s_\theta^2(1 + c_\theta)^2(2\Omega - \Omega_{\rm s})\tau_{12} + 6\,s_\theta^4\,\Omega\,\tau_{02} \\
&+ 2s_\theta^2(1 - c_\theta)^2(2\Omega + \Omega_{\rm s})\tau_{-12} \\
&+ (1 - c_\theta)^4(\Omega + \Omega_{\rm s})\tau_{-22}\Big],
\end{aligned} \tag{28}$$

where we have defined

$$\tau_{mm'} = t_{mm'}\kappa_{mm'}, \tag{29}$$

and $c_\theta \equiv \cos\Theta$, $s_\theta \equiv \sin\Theta$, and we have used the identity $\tau_{-m,-m'} = \tau_{m,m'}$.

The perpendicular component of the tidal torque $\boldsymbol{T}$ is less straightforward to evaluate. The y-component does not depend on tidal dissipation (for $|\Delta_{mm'}| \ll 1$) and contributes to the spin precession. Since T_y does change the spin–orbit elements, we will not consider it further. The x-component of $\boldsymbol{T}$ is

$$T_x = T_0 \sum_{m\bar{m}m'} \mathcal{U}_{mm'} \mathcal{U}_{\bar{m}m'} \Delta_{mm'} \kappa_{m\bar{m}m'}, \tag{30}$$

where

$$\kappa_{m\bar{m}m'} \equiv \frac{1}{iMR^2} \int d^3x \, \delta\bar{\rho}_{mm'}(\boldsymbol{r}) \, r^2 \left(\frac{\partial Y^*_{2\bar{m}}}{\partial\theta} \sin\phi - \frac{i\bar{m}}{\tan\theta} Y^*_{2\bar{m}} \cos\phi \right). \tag{31}$$

Since $\delta\bar{\rho}_{mm'}(\boldsymbol{r}) \propto e^{im\phi}$, $\kappa_{m\bar{m}m'}$ is non-zero only for $\bar{m} = m \pm 1$. If we define

$$\kappa^{\pm}_{mm'} \equiv \kappa_{m,\bar{m}=m\pm1,m'}, \tag{32}$$

then equation (30) can be written as

$$T_x = T_0 \sum_{mm'} \mathcal{U}_{mm'} \Delta_{mm'} \left(\mathcal{U}_{m+1,m'} \kappa^+_{mm'} + \mathcal{U}_{m-1,m'} \kappa^-_{mm'} \right). \tag{33}$$

A direct calculation shows that the coefficients $\kappa^{\pm}_{mm'}$ are related to the Love coefficients (equation 24) by

$$\kappa^-_{2m'} = \kappa_{2m'}, \quad \kappa^+_{-2m'} = \kappa_{-2m'},$$
$$\kappa^+_{1m'} = \kappa_{1m'}, \quad \kappa^-_{1m'} = \sqrt{3/2}\,\kappa_{1m'},$$
$$\kappa^{\pm}_{0m'} = \sqrt{3/2}\,\kappa_{0m'},$$
$$\kappa^+_{-1m'} = \sqrt{3/2}\,\kappa_{-1m'}, \quad \kappa^-_{-1m'} = \kappa_{-1m'}. \tag{34}$$

Thus, T_x can be worked out explicitly:

$$\begin{aligned}
T_x = \frac{(W_{22})^2}{2} T_0 \Big[&s_\theta(1+c_\theta)^3(\Omega - \Omega_s)\tau_{22} \\
&+ s_\theta(1+c_\theta)^2(2-c_\theta)(2\Omega - \Omega_s)\tau_{12} \\
&+ 6\,s_\theta^3\,\Omega\,\tau_{02} \\
&+ s_\theta(1-c_\theta)^2(2+c_\theta)(2\Omega + \Omega_s)\tau_{-12} \\
&+ s_\theta(1-c_\theta)^3(\Omega + \Omega_s)\tau_{-22} \Big] \\
&+ 3(W_{20})^2 T_0 \Omega_s \left(s_\theta^3 c_\theta\,\tau_{20} + s_\theta\,c_\theta^3\,\tau_{10} \right).
\end{aligned} \tag{35}$$

2.4 Tidal evolution equations

Given the tidal torque T_z and energy transfer rate $\dot{E}$, the tidal evolution equations for the stellar spin Ω_s and the orbital semimajor axis a are

$$\dot{\Omega}_s = \frac{T_z}{I}, \qquad \frac{\dot{a}}{a} = -\frac{2a\dot{E}}{GMM'}, \tag{36}$$

where I is the moment of the inertia of the star. The spin–orbit misalignment angle Θ is given by $\cos\Theta = \boldsymbol{S}\cdot\boldsymbol{L}/(SL)$, where $S = I\Omega_s$ and $L = \mu a^2\Omega$. Using $\dot{\boldsymbol{S}} = -\dot{\boldsymbol{L}} = \boldsymbol{T}$, we find

$$\dot{\Theta} = -\frac{N_x}{S} - \frac{N_x}{L}\cos\Theta + \frac{N_z}{L}\sin\Theta. \tag{37}$$

Note that the rate of change for the magnitude of the orbital angular momentum satisfies

$$\dot{L} = -T_z\cos\Theta - T_x\sin\Theta = -\frac{\dot{E}}{\Omega} = \frac{\dot{E}_{\text{orb}}}{\Omega}. \tag{38}$$

This can be checked directly using equations (27), (28) and (35). Thus, a circular orbit will remain circular, as it should be.

These evolution equations for $\dot{\Omega}_s$, $\dot{a}$ and $\dot{\Theta}$ [with T_z, $\dot{E}$ and T_x given by equations (27), (28) and (35), respectively] are the most general tidal equations for circular binaries. They depend on seven independent 'reduced' tidal lag times $\tau_{mm'} = t_{mm'}\kappa_{mm'}$, corresponding to the seven independent tidal forcing components. In general, each $\tau_{mm'}$ depends on m, $\tilde{\omega}_{mm'}$ and the intrinsic property (including Ω_s) of the star.

2.5 Special case: weak friction theory of equilibrium tide

When $\tau_{mm'} = \tau$ are the same for all seven tidal forcing components, we find

$$\dot{E} = \frac{12\pi}{5} T_0 \Omega \left(\Omega - \Omega_s\cos\Theta\right)\tau, \tag{39}$$

$$T_z = \frac{6\pi}{5} T_0 \left[2\Omega\cos\Theta - \left(1 + \cos^2\Theta\right)\Omega_s \right]\tau, \tag{40}$$

$$T_x = \frac{6\pi}{5} T_0 \sin\Theta(2\Omega - \Omega_s\cos\Theta)\tau. \tag{41}$$

These give

$$\frac{\dot{a}}{a} = -\frac{1}{t_a}\left(1 - \frac{\Omega_s}{\Omega}\cos\Theta\right), \tag{42}$$

$$\frac{\dot{\Omega}_s}{\Omega_s} = \frac{1}{t_a}\left(\frac{L}{2S}\right)\left[\cos\Theta - \left(\frac{\Omega_s}{2\Omega}\right)(1 + \cos^2\Theta)\right], \tag{43}$$

$$\dot{\Theta} = -\frac{1}{t_a}\left(\frac{L}{2S}\right)\sin\Theta\left[1 - \left(\frac{\Omega_s}{2\Omega}\right)\left(\cos\Theta - \frac{S}{L}\right)\right], \tag{44}$$

where

$$\frac{1}{t_a} \equiv \frac{3k_2}{Q}\left(\frac{M'}{M}\right)\left(\frac{R}{a}\right)^5\Omega, \tag{45}$$

and we have defined

$$k_2\Delta t_L \equiv \frac{4\pi}{5}\tau, \quad Q \equiv (2\Omega\Delta t_L)^{-1}. \tag{46}$$

Here k_2 and Δt_L have the usual meanings as in the equilibrium tide theory: k_2 is the Love number and Δt_L is the tidal lag time [which is related to the viscous time t_{vis} by $\Delta t_L = 1/(\omega_0^2 t_{\text{vis}})$]. These equations agree with those given in Alexander (1973) and others (Hut 1980; Eggleton et al. 1998).

3 TIDAL TORQUE DUE TO INERTIAL WAVE DISSIPATION

The convective envelope of a solar-type star supports inertial waves, which are driven entirely by Coriolis force. The frequency $\tilde{\omega}$ (in the rotating frame) of an inertial wave is related to its (local) wavenumber vector $\boldsymbol{k}$ by the dispersion relation (e.g. Greenspan 1968)

$$\tilde{\omega}^2 = (2\boldsymbol{\Omega}_s\cdot\boldsymbol{k}/|\boldsymbol{k}|)^2. \tag{47}$$

Thus, inertial waves exist only when $|\tilde{\omega}| < 2\Omega_s$.

In hot Jupiter systems with $\Omega \gg \Omega_s$, the only tidal forcing component that is capable of exciting inertial waves in the star is $(m, m') = (1, 0)$, with the forcing frequency (in the rotating frame) $\tilde{\omega} = -\Omega_s$. [The $(-1, 0)$ component has $\tilde{\omega} = \Omega_s$ and is physically identical.] The tidal torque associated with this component can be read off directly from equations (27) and (35):

$$(T_z)_{10} = -\frac{3\pi}{5} T_0 \Omega_s \tau_{10}\,(\sin\Theta\cos\Theta)^2, \tag{48}$$

$$(T_x)_{10} = \frac{3\pi}{5} T_0 \Omega_s \tau_{10} \, \sin\Theta \cos^3\Theta, \tag{49}$$

with $T_0 = G(M'/a^3)^2 R^5$. Similar to equation (46), we define the relevant tidal Love number k_{10}, lag time Δt_{10} and quality factor Q_{10}:

$$k_{10}\Delta t_{10} \equiv \frac{4\pi}{5}\tau_{10}, \quad Q_{10} \equiv (\Omega_s \Delta t_{10})^{-1}. \tag{50}$$

Using equations (36) and (37), we find

$$\left(\frac{\dot{\Omega}_s}{\Omega_s}\right)_{10} = -\frac{1}{t_{s10}} (\sin\Theta \cos\Theta)^2, \tag{51}$$

$$(\dot{\Theta})_{10} = -\frac{1}{t_{s10}} \sin\Theta \cos^2\Theta \left(\cos\Theta + \frac{S}{L}\right), \tag{52}$$

where

$$\frac{1}{t_{s10}} = \frac{3\pi\tau_{10}T_0}{5I}$$

$$= \frac{3k_{10}}{4Q_{10}} \left(\frac{M'}{M}\right) \left(\frac{R}{a}\right)^5 \left(\frac{L}{S}\right) \Omega. \tag{53}$$

For $M = M_\star$ and $M' = M_p$, the corresponding time-scale is

$$t_{s10} = 4.3 \left(\frac{\kappa}{0.1}\right) \left(\frac{Q_{10}/k_{10}}{10^7}\right) \left(\frac{M_\star}{10^3 M_p}\right)^2 \left(\frac{\bar{\rho}_\star}{\bar{\rho}_\odot}\right)$$

$$\times \left(\frac{10\,\mathrm{d}}{P_s}\right) \left(\frac{P}{1\,\mathrm{d}}\right)^4 \mathrm{Gyr} \tag{54}$$

(cf. equation 2).

Since the (1, 0) component of the tidal potential is static in the inertial frame, the associated energy transfer rate $\dot{E}$ associated with this component is zero, giving

$$(\dot{a})_{10} = 0. \tag{55}$$

This does not imply zero tidal dissipation. In fact, the tidal energy dissipation rate equals the energy transfer rate in the rotating frame:

$$\dot{E}_r = -\Omega_s T_z > 0. \tag{56}$$

This is exactly balanced by $(\mathrm{d}/\mathrm{d}t)(S^2/2I) = \Omega_s\dot{S} = \Omega_s T_z < 0$, so that $\dot{E} = \dot{E}_r + \Omega_s T_z = 0$. Also note that the (1, 0) tidal force does not change the magnitude of the orbital angular momentum, $\dot{L} = -T_z \cos\Theta - T_x \sin\Theta = 0$. So a circular orbit will remain circular.

Two interesting features of equation (52) are worth noting. First, $\dot{\Theta} = 0$ when $\Theta = 90°$. Thus, if the (1, 0) tidal component is the only tidal force operating in the system, there could be many systems with the spin–orbit misalignment angle stalled around $90°$. Secondly, $\dot{\Theta} > 0$ when $\cos\Theta < -S/L$. This implies that a retrograde system ($\Theta > \pi/2$) may evolve towards anti-alignment. Of course, other tidal components can also contribute to $\dot{\Theta}$ (Section 2) and will weaken these features. However, with enough statistics of misaligned systems, it may be possible to test these or constrain k_{10}/Q_{10}.

A detailed calculation of Q_{10}/k_{10} is beyond the scope of this paper. Previous works of tidal dissipation in rotating solar-type stars have focused on the $m = 2$ tide for aligned binaries (Papaloizou & Savonije 1997; Savonije & Papaloizou 1997; Savonije & Witte 2002; Ogilvie & Lin 2007) and demonstrated the importance of inertial waves. In the numerical study by Ogilvie & Lin (2007), inertial waves affect tidal dissipation in two ways: (i) short-wavelength inertial waves in the convection zone are damped by turbulent viscosity and (ii) inertial waves influence the excitation of

gravity (Hough) waves in the radiative zone. Regarding (ii), they assumed that the inward-propagating Hough waves are damped near the stellar centre; this is appropriate for binary stars, but may lead to an overestimate of stellar tidal dissipation in hot Jupiter systems (Barker & Ogilvie 2010, 2011; see Section 1.2). Ogilvie & Lin (2007) found that when inertial waves are excited ($|\tilde{\omega}| < 2\Omega_s$), the tidal dissipation rate depends on $\tilde{\omega}$ in an erratic manner. On average, the energy dissipation rate in the convection zone is significantly increased (by one to three orders of magnitude, depending on the spin period) compared to equilibrium tides. For example, figs 3–6 of Ogilvie & Lin (2007) show that for a solar-type star at $\tilde{\omega} = -\Omega_s$, the tidal quality factor associated with inertial wave dissipation is $Q'_\star \simeq 6 \times 10^7$ for $P_s = 10\,\mathrm{d}$ and $Q'_\star \simeq 5 \times 10^6$ for $P_s = 3\,\mathrm{d}$.

Barker & Ogilvie (2009) reported the result of a calculation of the $l = 2, m = 1$ tidal dissipation for a specific F-type stellar model (appropriate for the hot Jupiter system XO-3) using the numerical method of Ogilvie & Lin (2007). Intriguingly, they found that at $\tilde{\omega}/\Omega_s = -1$, tidal dissipation is significantly enhanced, with the effective tidal $Q \sim 10^6$ (see their fig. 7). They suggested that this prominent feature arises from resonant excitation of the $l = m = 1$ Rossby mode. More systematic studies on the $(m, m') = (1, 0)$ tide for a range of stellar models (with different sizes of the convective envelope) would be useful.

4 CONCLUSION

The main point of this paper is that in close-in exoplanetary systems, when the stellar spin axis is misaligned with the orbital angular momentum axis, a new tidal dissipation channel opens up. This channel involves the excitation of inertial waves in the stellar convection zone, and is forbidden for aligned systems. Thus, tidal damping of spin–orbit misalignment can be more efficient than orbital decay. This may explain the stellar obliquity–effective temperature correlation observed by Winn et al. (2010) and the obliquity–age correlation noted by Triaud (2011), while still being consistent with the survival of hot Jupiters with very short orbital period (see Section 1.1).

On a more general level, this paper highlights the importance of treating tidal dissipation as being dependent on the tidal forcing frequency, the strength of tidal potential and the tidal processes involved (e.g. orbital decay versus spin–orbit alignment), in contrast to the equilibrium tide equations widely used in many applications and empirical works (see references in Section 1.2). Indeed, various studies of the physical mechanisms of tidal dissipation (e.g. Ogilvie & Lin 2004, 2007; Goodman & Lackner 2009; Barker & Ogilvie 2010, 2011; see also Zahn 2008, for a review of earlier works) have already made this point clear. While it is recognized that the weak friction theory of equilibrium tides (Darwin 1880; Goldreich & Soter 1966; Alexander 1973; Hut 1981; Eggleton et al. 1998) is a parametrized theory (with the tidal Q or lag time being the single parameter), we have shown in this paper that even at the parametrized level, its equations are sometimes inadequate or misleading, since different tidal processes (e.g. orbital decay versus spin–orbit alignment) may involve very different tidal dissipation mechanisms.

Given the complicated nature of tidal dissipation, a parametrized 'effective' theory of tidal evolution remains useful. We have derived such an effective theory in this paper (see Section 2). For misaligned circular binaries, there are seven independent tidal quality factors $Q_{mm'}$ or lag times $\tau_{mm'}$ in the theory, related to the tidal responses for different Fourier components of the tidal potential. Obviously, such a system of equations with seven parameters (Section 2.4) is

not convenient to use in real applications. For hot Jupiter systems, we suggest that a reduced system of equations involving two tidal Q parameters may be adopted. They are

$$\frac{\dot{a}}{a} = \left(\frac{\dot{a}}{a}\right)_{eq}, \tag{57}$$

$$\frac{\dot{\Omega}_s}{\Omega_s} = \left(\frac{\dot{\Omega}_s}{\Omega_s}\right)_{eq} + \left(\frac{\dot{\Omega}_s}{\Omega_s}\right)_{10} - \left(\frac{\dot{\Omega}_s}{\Omega_s}\right)_{10,eq}, \tag{58}$$

$$\dot{\Theta} = (\dot{\Theta})_{eq} + (\dot{\Theta})_{10} - (\dot{\Theta})_{10,eq}, \tag{59}$$

where $(\dot{a}/a)_{eq}$, $(\dot{\Omega}_s/\Omega_s)_{eq}$ and $(\dot{\Theta})_{eq}$ are given by equations (42)–(44) and characterized by the parameter k_2/Q, while $(\dot{\Omega}_s/\Omega_s)_{10}$ and $(\dot{\Theta})_{10}$ are given by equations (51) and (52) and characterized by the parameter k_{10}/Q_{10}. The expressions for $(\dot{\Omega}_s/\Omega_s)_{10,eq}$ and $(\dot{\Theta})_{10,eq}$ are the same as equations (51) and (52), except that the parameter k_{10}/Q_{10} should be replaced by k_2/Q. These equations (extended to eccentric orbits), combined with similar equations for planetary tides, can be used to assess and constrain the effects of tidal evolution in close-in exoplanetary systems.

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