

$x y z$	$x + y + z$	$(x + y + z)'$	x'	y'	z'	$x' y' z'$	$x y z$	(xyz)	$(xyz)'$	x'	y'	z'	$x' + y' + z'$
0 0 0	0	1	1	1	1	1	0 0 0	0	1	1	1	1	1
0 0 1	1	0	1	1	0	0	0 0 1	0	1	1	1	0	1
0 1 0	1	0	1	0	1	0	0 1 0	0	1	1	0	1	1
0 1 1	1	0	1	0	0	0	0 1 1	0	1	1	0	0	1
1 0 0	1	0	0	1	1	0	1 0 0	0	1	0	1	1	1
1 0 1	1	0	0	1	0	0	1 0 1	0	1	0	1	0	1
1 1 0	1	0	0	0	1	0	1 1 0	0	1	0	0	1	1
1 1 1	1	0	0	0	0	0	1 1 1	1	0	0	0	0	0

(b)

$x y z$	$x + yz$	$(x + y)$	$(x + z)$	$(x + y)(x + z)$
0 0 0	0	0	0	0
0 0 1	0	0	1	0
0 1 0	0	1	0	0
0 1 1	1	1	1	1
1 0 0	1	1	1	1
1 0 1	1	1	1	1
1 1 0	1	1	1	1
1 1 1	1	1	1	1

(c)

$x y z$	$x(y + z)$	xy	xz	$xy + xz$
0 0 0	0	0	0	0
0 0 1	0	0	0	0
0 1 0	0	0	0	0
0 1 1	0	0	0	0
1 0 0	0	0	0	0
1 0 1	1	0	1	1
1 1 0	1	1	0	1
1 1 1	1	1	1	1

(d)

$x y z$	x	$y + z$	$x + (y + z)$	$(x + y)$	$(x + y) + z$
0 0 0	0	0	0	0	0
0 0 1	0	1	1	0	1
0 1 0	0	1	1	1	1
0 1 1	0	1	1	1	1
1 0 0	1	0	1	1	1
1 0 1	1	1	1	1	1
1 1 0	1	1	1	1	1
1 1 1	1	1	1	1	1

(e)

$x y z$	yz	$x(yz)$	xy	$(xy)z$
0 0 0	0	0	0	0
0 0 1	0	0	0	0
0 1 0	0	0	0	0
0 1 1	1	0	0	0
1 0 0	0	0	0	0
1 0 1	0	0	0	0
1 1 0	0	0	1	0
1 1 1	1	1	1	1

2.4

(a) Reduce the Boolean expression $A'C' + ABC + AC'$ to three literals.

Rewrite the expression and apply the postulate $x + x' = 1$.

$$\begin{aligned}A'C' + ABC + AC' &= C'(A + A') + ABC \\&= C'(1) + ABC \\&= C' + ABC\end{aligned}$$

Use the postulate $x + yz = (x + y)(x + z)$

$$\begin{aligned}C' + (AB)C &= (C' + AB)(C' + C) \\&= (C' + AB)(1) \\&= C' + AB\end{aligned}$$

Hence, we get

$$\boxed{A'C' + ABC + AC' = C' + AB}$$

(b) Simplify $(x'y' + z)' + z + xy + wz$ to three literals.

By De Morgan's law, we have $(AB)' = A' + B'$ and $(A + B)' = A'B'$.

$$\begin{aligned}(x'y' + z)' + z + xy + wz &= [(x'y')' z'] + z + xy + wz \\ &= [(x + y)z'] + z + xy + wz\end{aligned}$$

Use the postulate $x + yz = (x + y)(x + z)$.

$$\begin{aligned}[(x + y)z'] + z + xy + wz &= \{(x + y)z' + z\} + xy + wz \\ &= \{(z + z')(z + x + y)\} + xy + wz\end{aligned}$$

Simplify.

$$\begin{aligned}[(z + z')(z + x + y)] + xy + wz &= z + x + y + xy + wz \\ &= z(1 + w) + x(1 + y) + y \\ &= z + x + y\end{aligned}$$

Thus, we get

$$(x'y' + z)' + z + xy + wz = x + y + z$$

(c) Reduce the Boolean expression $A'B(D' + C'D) + B(A + A'CD)$ to one literal.

Clear parentheses and rearrange the terms as

$$\begin{aligned}A'B(D' + C'D) + B(A + A'CD) &= A'BD' + A'BC'D + AB + A'BCD \\&= B(A'D' + A'C'D + A + A'CD) \\&= B(A'D' + A'D(C' + C) + A)\end{aligned}$$

Use the postulate $x + x' = 1$.

$$\begin{aligned}B(A'D' + A'D(C' + C) + A) &= B(A'D' + A'D(1) + A) \\&= B(A'(D' + D) + A) \\&= B(A' + A) \\&= B\end{aligned}$$

Thus, we get

$$\boxed{A'B(D' + C'D) + B(A + A'CD) = B}$$

(d) Reduce the expression $(A' + C)(A' + C')(A + B + C'D)$ to four literals.

Apply the postulates $x + x' = 1$, $x \cdot x' = 0$, and $x \cdot x = x$.

$$\begin{aligned}(A' + C)(A' + C')(A + B + C'D) &= (A'A' + A'C' + CA' + CC')(A + B + C'D) \\&= (A' + A'(C' + C) + 0)(A + B + C'D) \\&= (A' + A'(1))(A + B + C'D) \\&= (A' + A')(A + B + C'D) \\&= A'(A + B + C'D)\end{aligned}$$

Simplify.

$$\begin{aligned}A'(A + B + C'D) &= A'A + A'B + A'C'D \\&= 0 + A'(B + C'D) \\&= A'(B + C'D)\end{aligned}$$

Thus, we get

$$\boxed{(A' + C)(A' + C')(A + B + C'D) = A'(B + C'D)}$$

(e) Simplify the Boolean expression $ABCD + A'BD + ABC'D$ to two literals.

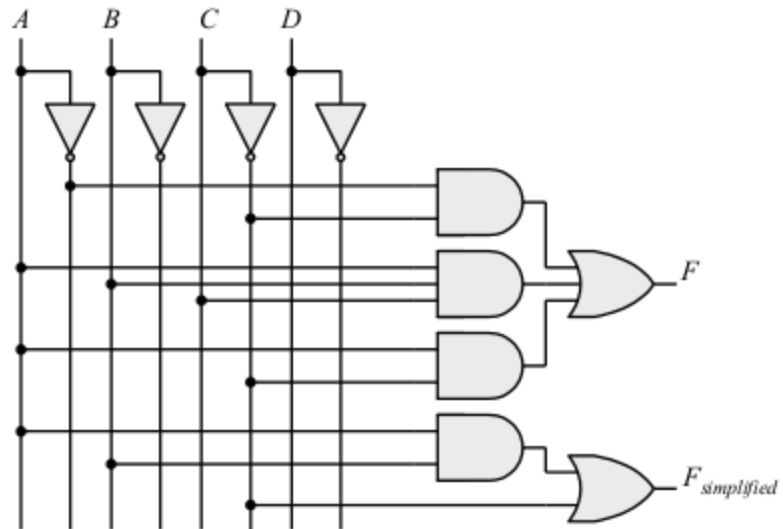
Use the postulate $x + x' = 1$.

$$\begin{aligned}ABCD + A'BD + ABC'D &= ABD(C + C') + A'BD \\&= ABD(1) + A'BD \\&= BD(A + A') \\&= BD\end{aligned}$$

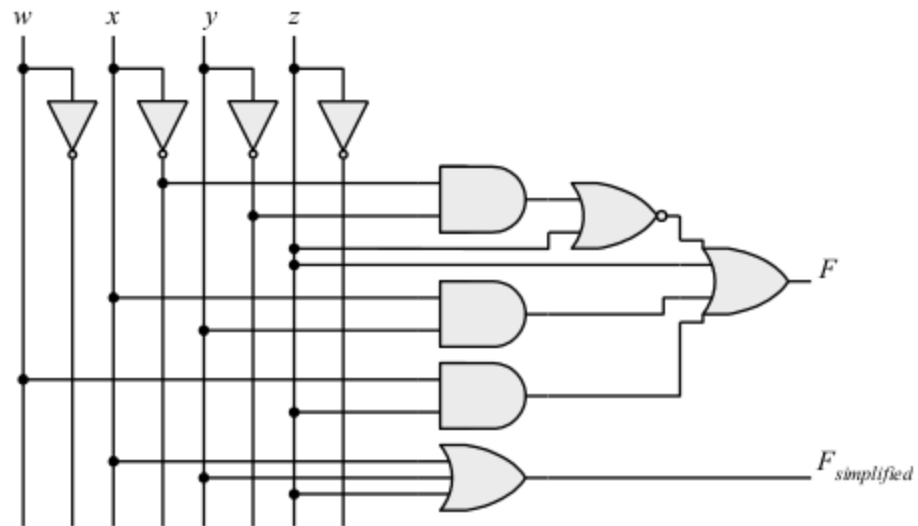
Thus, we get

$$\boxed{ABCD + A'BD + ABC'D = BD}$$

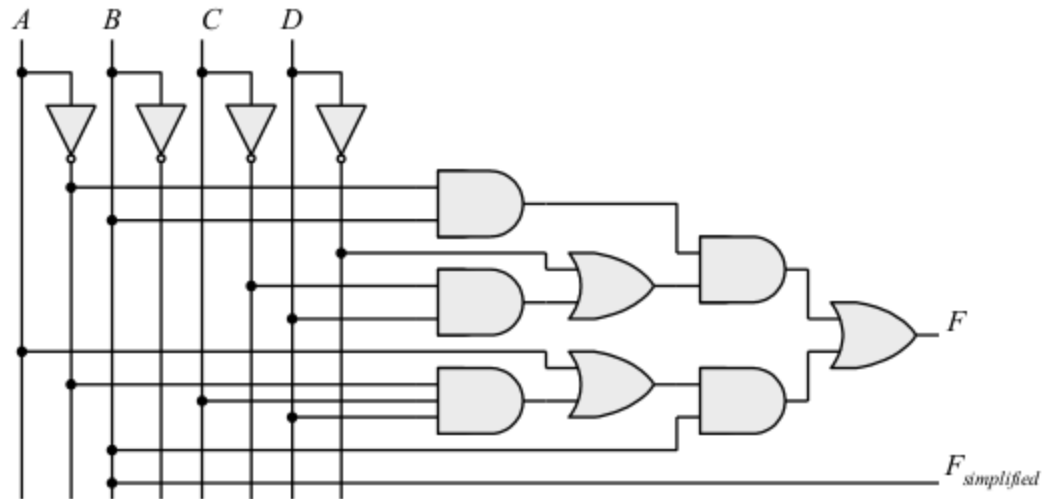
2.7 (a)



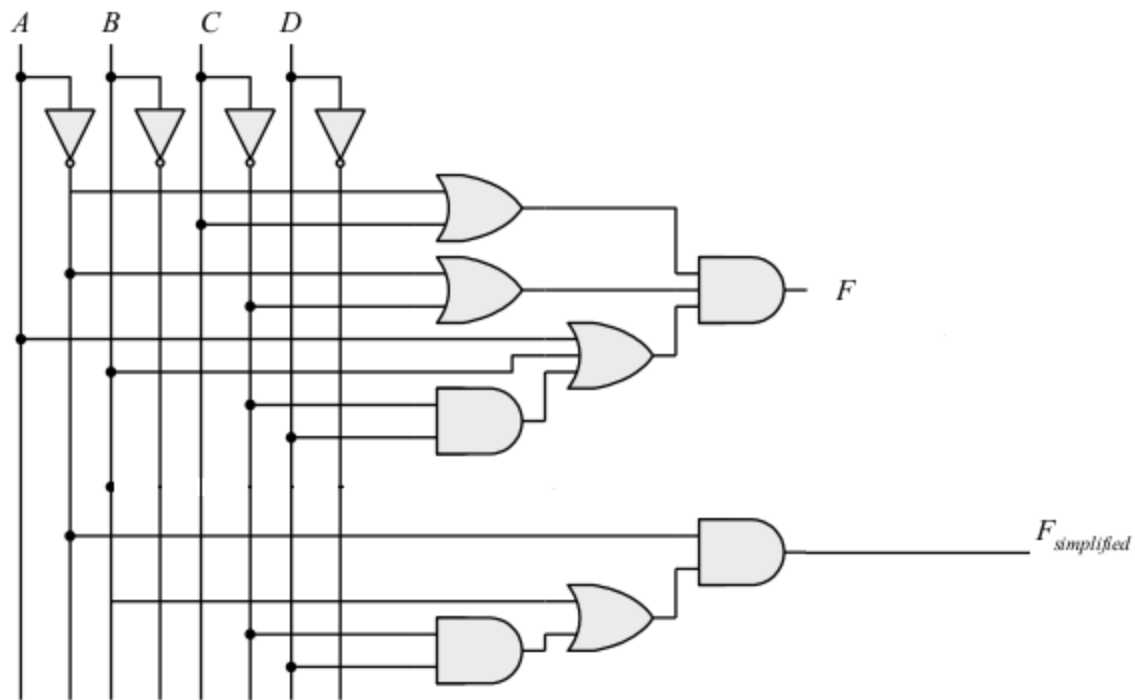
2.7 (b)



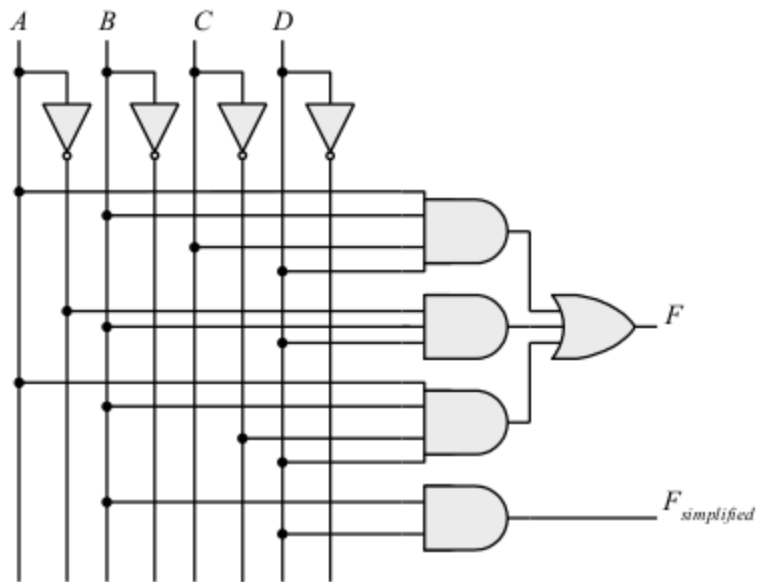
2.7 (c)



2.7 (d)



2.7 (e)



2.9

(a)

Consider the expression: $xy' + x'y$

Complement of the given expression is,

$$F' = (xy' + x'y)' \dots\dots (1)$$

Apply DeMorgan's theorem in equation (1), we get,

$$\begin{aligned} F' &= (xy')' \cdot (x'y)' \\ &= (x' + y) \cdot (x + y') \\ &= x'x + x'y' + yx + yy' \\ &= xy + x'y' \end{aligned}$$

Note: $xx' = 0$

Therefore the complement of $xy' + x'y$ is $\boxed{xy + x'y'}$.

(b)

Consider the expression: $(a + c)(a + b')(a' + b + c')$

Complement of the given expression is,

$$F' = [(a + c)(a + b')(a' + b + c')] \dots\dots (2)$$

Apply DeMorgan's theorem in equation (2), we get,

$$\begin{aligned} F' &= (a + c)' + (a + b')' + (a' + b + c')' \\ &= a'c' + a'b + ab'c \end{aligned}$$

Therefore the complement of $(a + c)(a + b')(a' + b + c')$ is $\boxed{a'c' + a'b + ab'c}$.

(c)

Consider the expression: $z + z'(v'w + xy)$

Complement of the given expression is,

$$F' = [z + z'(v'w + xy)]' \dots\dots (3)$$

Apply DeMorgan's theorem in equation (3), we get,

$$\begin{aligned} F' &= (z') \cdot [z'(v'w + xy)]' \\ &= (z') \cdot [z + (v'w + xy)'] \\ &= (z') \cdot [z + (v'w)' \cdot (xy)'] \\ &= (z') \cdot [z + (v + w') \cdot (x' + y')] \end{aligned}$$

Further simplifying, we get,

$$\begin{aligned} F' &= z'z + z'(v + w')(x' + y') \\ &= z'(v + w')(x' + y') \end{aligned}$$

Therefore the complement of $z + z'(v'w + xy)$ is $\boxed{z'(v + w')(x' + y')}$.

2.11 **(a)** $F(x, y, z) = \Sigma(1, 4, 5, 6, 7)$

(b) $F(x, y, z) = \Sigma(0, 2, 3, 7)$

$$F = xy + xy' + y'z$$

x y z	F
0 0 0	0
0 0 1	1
0 1 0	0
0 1 1	0
1 0 0	1
1 0 1	1
1 1 0	1
1 1 1	1

$$F = x'z' + yz$$

x y z	F
0 0 0	1
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	0
1 1 0	0
1 1 1	1

2.13

(a)

Consider the expression: $Y = [(u + x') (y' + z)]$

The logic diagram to implement the Boolean expression is shown in Figure 1.

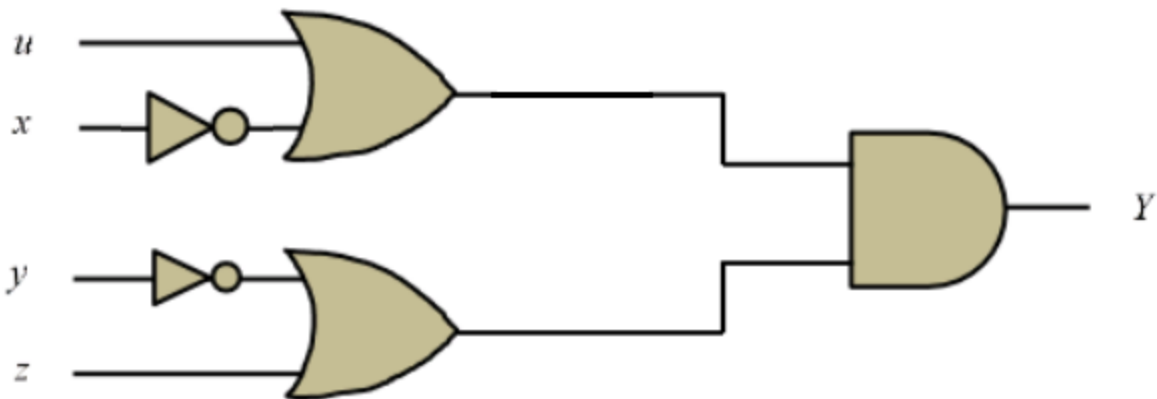


Figure 1 Logic diagram to implement $Y = [(u + x') (y' + z)]$

Therefore, the required logic diagrams to implement the Boolean expression is as shown in [Figure 1](#).

(b)

Consider the expression: $Y = (u \oplus y)' + x$

The logic diagram to implement the Boolean expression is shown in Figure 2.

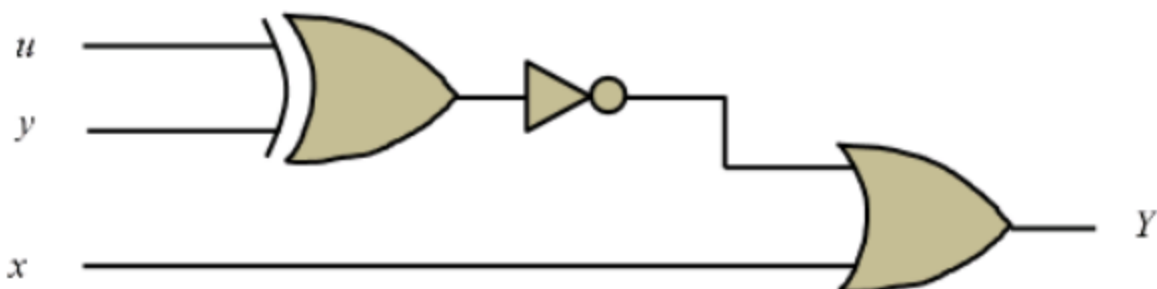


Figure 2 Logic diagram to implement $Y = (u \oplus y)' + x$

Therefore, the required logic diagrams to implement the Boolean expression is as shown in [Figure 2](#).

(c)

Consider the expression: $Y = (u' + x')(y + z')$

The logic diagram to implement the Boolean expression is shown in Figure 3.

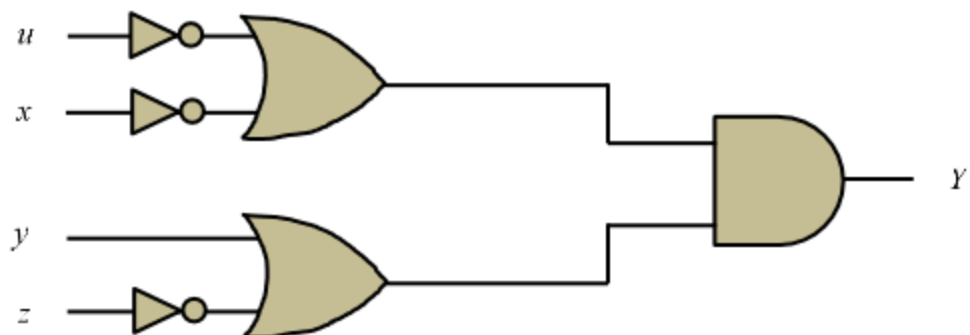


Figure 3 Logic diagram to implement $Y = (u' + x')(y + z')$

Therefore, the required logic diagrams to implement the Boolean expression is as shown in Figure 3 .

(d)

Consider the expression: $Y = u(x \oplus z) + y'$

The logic diagram to implement the Boolean expression is shown in Figure 4.

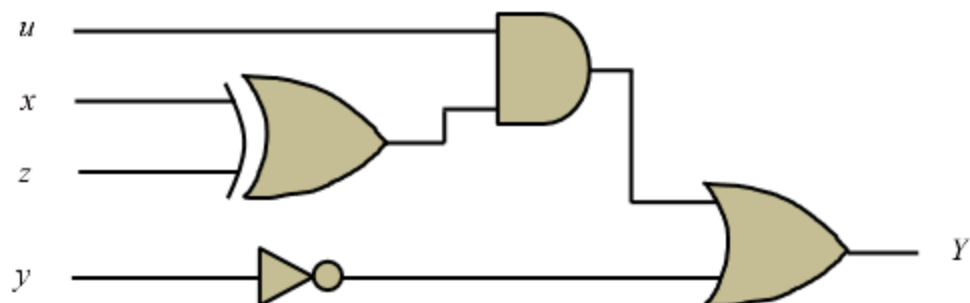


Figure 4 Logic diagram to implement $Y = (u \oplus y)' + x$

Therefore, the required logic diagrams to implement the Boolean expression is as shown in Figure 4 .

(e)

Consider the expression: $Y = u + yz + uxy$

The logic diagram to implement the Boolean expression is shown in Figure 5.

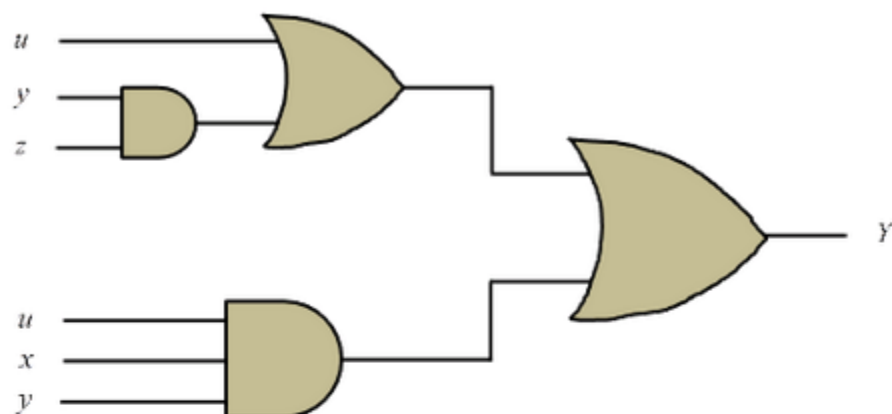


Figure 5 Logic diagram to implement $Y = u + yz + uxy$

Therefore, the required logic diagrams to implement the Boolean expression is as shown in Figure 5.

(f)

Consider the expression: $Y = u + x + x'(u + y')$

The logic diagram to implement the Boolean expression is shown in Figure 6.

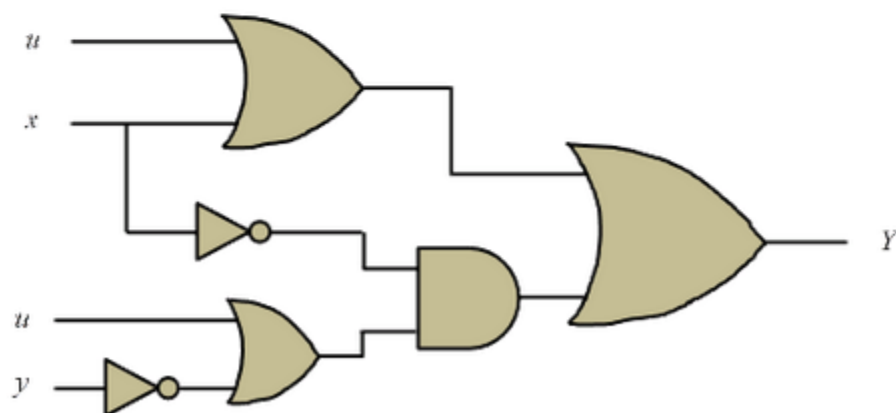


Figure 6 Logic diagram to implement $Y = u + x + x'(u + y')$

Therefore, the required logic diagrams to implement the Boolean expression is as shown in Figure 6.

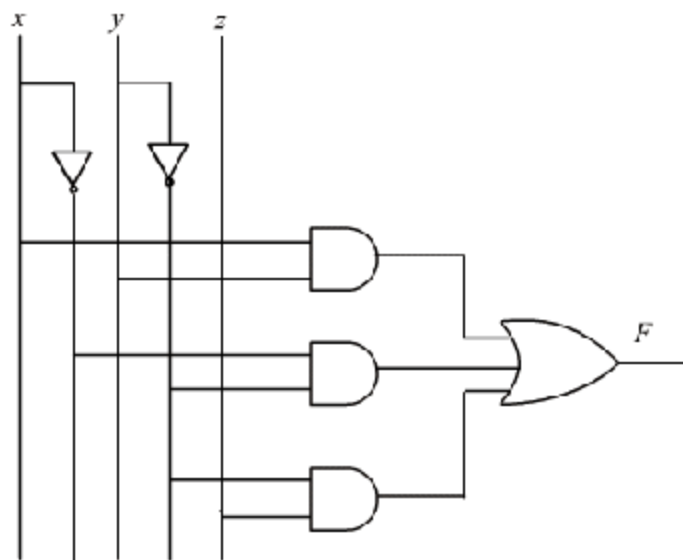
2.14

Given Boolean function

$$F = xy + x'y' + y'z.$$

(a)

Implementation of the Boolean expression using AND, OR and inverter gates.



(b)

Implementation of the Boolean expression using OR and inverter gates:

In order to implement using OR and inverter gates we have to complement the expression

$$F = xy + x'y' + y'z.$$

Now, find the complement of the function

$$F' = (xy + x'y' + y'z)'$$

$$= (xy)' (x'y')' (y'z)'$$

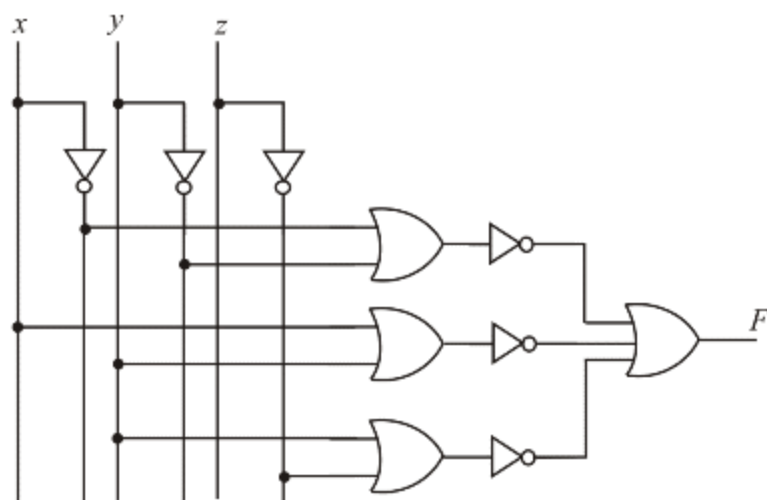
$$F' = (x' + y')(x + y)(y + z')$$

Again complementing the above expression we get

$$(F')' = \left((x' + y')(x + y)(y + z') \right)'$$

$$F = (x' + y')' + (x + y)' + (y + z')'$$

Now sketch the logic diagram for the expression obtained.



(c)

Implementation of the Boolean expression using AND and inverter gates:

In order to implement using AND and inverter gates we have to simplify the expression

$$F = xy + x'y' + y'z.$$

Now, find the complement of the function

$$F' = (xy + x'y' + y'z)'$$

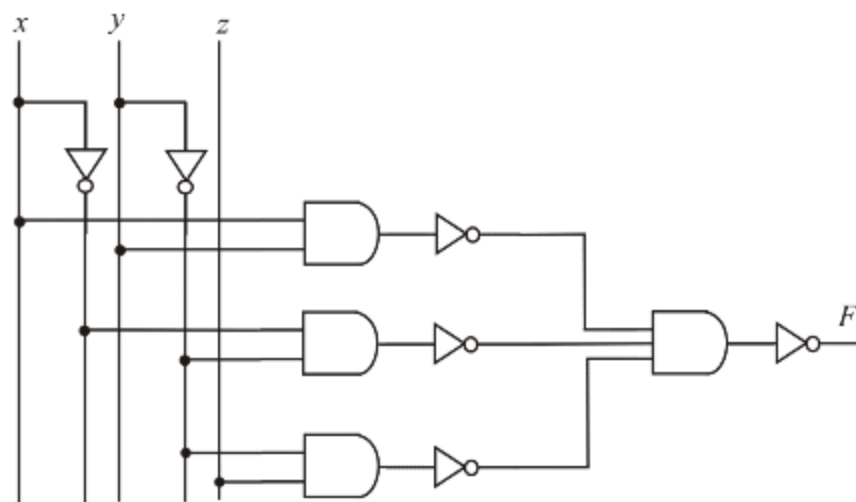
$$F' = (xy)' (x'y')' (y'z)'$$

Again complementing the above expression we get

$$(F')' = ((xy)' (x'y')' (y'z)')'$$

$$F = ((xy)' (x'y')' (y'z)')'$$

Now sketch the logic diagram for the expression obtained.



(d)

Implementation of the Boolean expression using NAND and inverter gates

In order to implement using OR and inverter gates we have to simplify the expression

Now, find the complement of the function

$$F' = (xy + x'y' + y'z)'$$

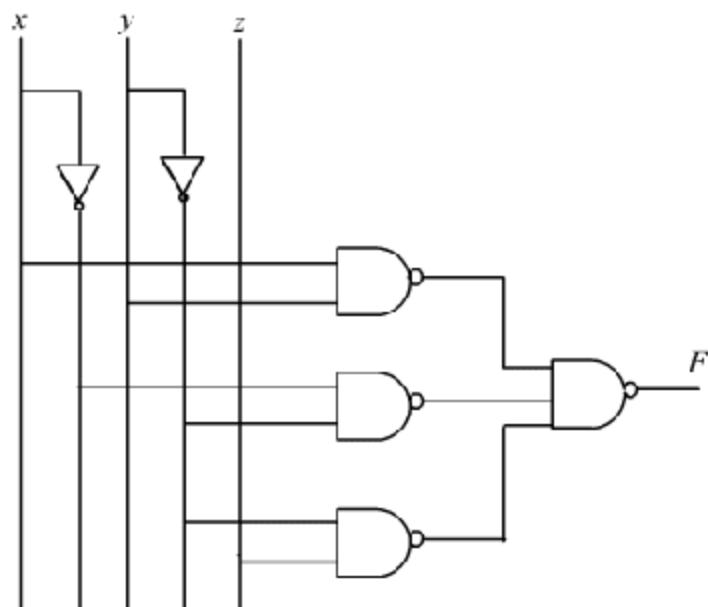
$$F' = (xy)' (x'y')' (y'z)'$$

Again complementing the above expression we get

$$(F')' = ((xy)' (x'y')' (y'z)')'$$

$$F = ((xy)' (x'y')' (y'z)')$$

Now sketch the logic diagram for the expression obtained.



(e)

Implementation of the Boolean expression using NOR and inverter gates

In order to implement using NOR and inverter gates we have to simplify the expression

$$F = xy + x'y' + y'z.$$

Now, find the complement of the function

$$\begin{aligned} F' &= (xy + x'y' + y'z)' \\ &= (xy)' (x'y')' (y'z)' \end{aligned}$$

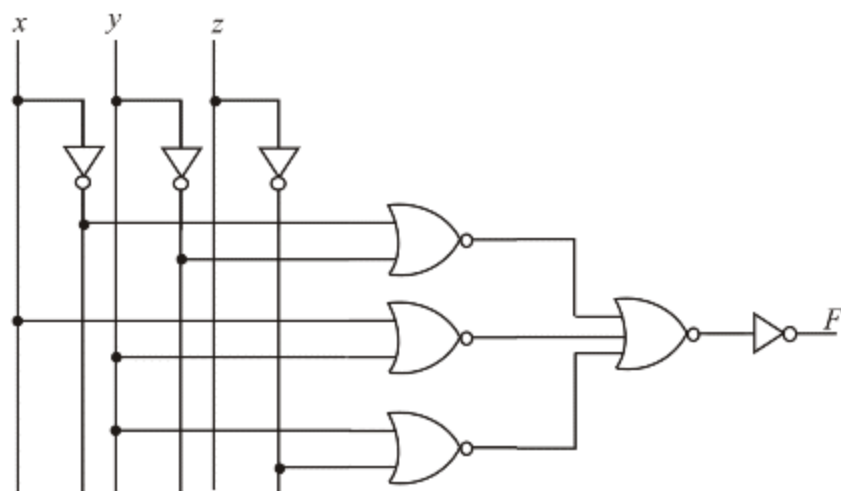
$$F' = (x' + y')(x + y)(y + z')$$

Again complementing the above expression we get

$$(F')' = \left((x' + y')(x + y)(y + z') \right)'$$

$$F = (x' + y')' + (x + y)' + (y + z')'$$

Now sketch the logic diagram for the expression obtained.



2.18

Given function

$$F = xy'z + x'y'z + w'xy + wx'y + wxy$$

(a)

In order to obtain the truth table we have to find the minterms.

The given function has one variable missing in each term, therefore

$$xy'z = xy'z(w + w')$$

$$= wxy'z + w'xy'z$$

$$x'y'z = x'y'z(w + w')$$

$$= wx'y'z + w'x'y'z$$

$$w'xy = w'xy(z + z')$$

$$= w'xyz + w'xyz'$$

$$wx'y = wx'y(z + z')$$

$$= wx'yz + wx'yz'$$

$$wxy = wxy(z + z')$$

$$= wxyz + wxyz'$$

Combining all terms, we have

$$F = \left(wxy'z + w'xy'z + wx'y'z + w'x'y'z + w'xyz + w'xyz' + wx'yz + wx'yz' + wxyz + wxyz' \right)$$

Therefore

$$F = m_1 + m_5 + m_6 + m_7 + m_9 + m_{10} + m_{11} + m_{13} + m_{14} + m_{15}$$

From the obtained simplified function we can find the sum-of-minterms as

$$F = \Sigma(1, 5, 6, 7, 9, 10, 11, 13, 14, 15).$$

Now the truth table of F is

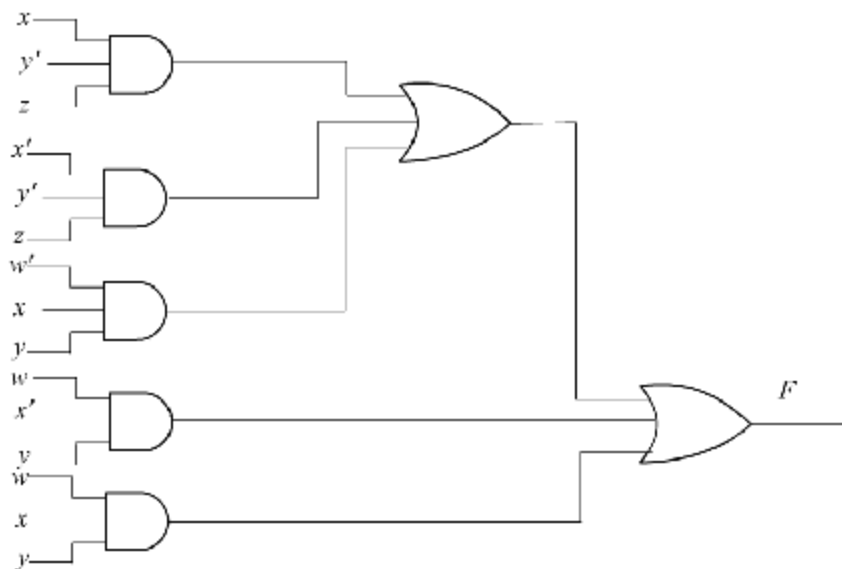
w	x	y	z	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

(b)

Given Boolean expression

$$F = xy'z + x'y'z + w'xy + wx'y + wxy$$

Logic diagram for the given expression is:



(c)

Simplify the function into minimum number of literals.

$$\begin{aligned}F &= xy'z + x'y'z + w'xy + wx'y + wxy \\&= y'z(x + x') + xy(w + w') + wx'y \quad \left(\text{Since } x + x' = 1 \right) \\&= y'z + xy + wx'y \\&= y'z + y(x + wx') \\&= y'z + y(x + w)(x + x') \quad \left(\text{Since } (A + BC) = (A + B)(A + C) \right) \\&= y'z + y(x + w) \\F &= y'z + xy + wxy\end{aligned}$$

Therefore

$$\boxed{F = y'z + y(w + x)}.$$

(d)

The truth table for the expression obtained in (c).

In order to obtain the truth table we have to find the minterms.

The given function has one variable missing in each term, therefore

$$\begin{aligned}y'z &= y'z(w'x' + w'x + wx' + wx) \\&= w'x'y'z + w'xy'z + wx'y'z + wxy'z \\yw &= yw(x'z' + x'z + xz' + xz) \\&= wx'y'z' + wx'yz' + wxyz' + wxyz \\xy &= xy(w'z' + w'z + wz' + wz) \\&= w'xyz' + w'xyz + wxyz' + wxyz\end{aligned}$$

Now combine all the terms, we get

$$\begin{aligned}
 F &= \left(w'x'y'z + w'xy'z + wx'y'z + wxy'z + wx'yz' + wx'yz + wxyz' + wxyz + \right. \\
 &\quad \left. w'xyz' + w'xyz + wxyz' + wxyz \right) \\
 &= \left(w'x'y'z + w'xy'z + wx'y'z + wxy'z + wx'yz' + wx'yz + wxyz' + wxyz + \right. \\
 &\quad \left. w'xyz' + w'xyz \right)
 \end{aligned}$$

Therefore

$$F = m_1 + m_5 + m_6 + m_7 + m_9 + m_{10} + m_{11} + m_{13} + m_{14} + m_{15}$$

From the obtained simplified function we can find the sum-of-minterms as

$$F = \Sigma(1, 5, 6, 7, 9, 10, 11, 13, 14, 15).$$

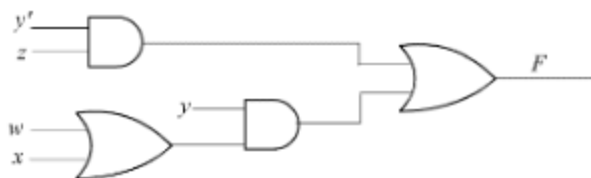
Truth table of simplified function F is

w	x	y	z	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

The truth table obtained for the simplified expression in (c) is same as that of the truth table obtained for the expression (a)

(e)

The logic diagram for the simplified expression is shown.



Step 10 of 10

In the logic diagram drawn at **(b)** the number of gates used is high when compared to the diagram shown in **(e)**.

The total number of gates used in **(b)** is five AND gate and 2 OR gates, whereas in **(e)** the number of AND gates has been reduced to 2 and the use of OR gates is same that is 2.