Mobile Robot Localization Using Extended Kalman Filter

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***Abstract*— Localizing the mobile robot in an indoor environment is one of the problems encountered repeatedly. Achieving the target precisely in any environment is not an easy task since there are noises and obstacles in the surrounding environment. Therefore, filtering the signals to reduce noises is essential for more accurate and precise motion. In this paper, we selected the extended Kalman filter, which is used for non-linear models' signals to predict the coordinates of a wheeled mobile robot. We tested the efficiency of this filter under three noise cases: no noise, Gaussian noise and non-Gaussian noise using MATLAB software.**

Keywords-component; Mobile robot- localization- extended Kalman filter-indoor environment

I. INTRODUCTION [1]

The localization of mobile robots is a cumbersome process. It is even more difficult in closed environments like the indoor environment due to the presence of obstacles. In situations like this, determining the flow line of the robot must be more accurate and precise.

The localization of the mobile robot is a key factor in any mobile robot navigation; it requires some information for localization, representation, and the local environment. However, the problem of navigation includes three main factors: self-localization, map building, and path planning.

Self-localization requires the system state estimation in real-time, but this is not directly measurable since it might be corrupted by different noise. Therefore, this data must be filtered since it is collected by the robot to continuously update its position and decide the correct way of motion at any time until accomplishing it's given task.

II. EXTENDED KALMAN FILTER

"The extended Kalman filter (EKF) is the nonlinear version of the Kalman filter, exploiting the assumption that all transformations are quasi-linear, the EKF simply linearizes all nonlinear transformations and substitutes Jacobian matrices for the linear transformations in the KF equations "[1]. It is used to fuse the inner position estimation and outer measurement, and to locate the mobile robot with the wheeled wheels. The extended Kalman filter has been a popular choice to deal with mobile robot problems with sequential localization and mapping. The efficiency of EKF relies on the theory of covariance matrices for system and measurement noise. Inaccurate knowledge of these statistical analyses can cause a major reduction in performance [2]. EKF gives a solution to the estimation problem for a nonlinear model. The technique is based on a Gaussian probability density of the robot's position. It may be used with many sensors to get a precise estimation of the pose when the environment is affected by randomly surrounding noises [3]. The extended Kalman filter was developed in 1970 as the initial approach to estimate the state for a nonlinear system [4]. It is an approximation method for nonlinear state estimation based on the Taylor series expansion to overcome the nonlinearity system and measurement models. EKF approximation of the system function linearizes the nonlinear functions around the estimated trajectory [5]. It is well known that the EKF is prone to divergence, mainly for the bad initial estimate and high noise [6]. The work of [7] propose an Extended Kalman filter as a new method for localization by combining it with an infrared system to reduce further the estimation errors. The authors in [5] use the extended Kalman filter to solve the localization problem when the measurement data are available at a specific time. In G. Atali et all [8], they used extended Kalman filter to combine the data which was gathering from many sensors for a long time to estimate the position with more accuracy. Also, the Kalman filter is used in the least complex system with a two-wheeled mobile robot for indoor environment navigation to minimize the errors accrue when collecting the data of estimate the position [9].The discrete-time model of the WMR can be written as:

$$X_{k+1} = f(X_k, U_k) + w_{k,} \tag{1}$$

$$Z_k = h(X_k) + v_k \tag{2}$$

where $X = [x \quad y \quad \theta]^T$ is the state vector to be estimated, $U = [\mu \quad \omega]^T$ is the control vector, Z is the measured output,

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and (w,v) is the system input and v_k is a measurement disturbances. The extended Kalman filter is based on the linearization of the nonlinear equations (1,2). Further, it is assumed that the initial state and noises are Gaussian and uncorrelated to each other. Figure (1) shows the steps of EKF. The prediction and update of the EKF are given as follows [6]:

$$\hat{\hat{X}}_{k+1} = f(\hat{X}_k, U_k) \quad (3)$$

$$\hat{P}_{k+1} = A_k P_k A_k^T + Q \quad (4)$$

$$K_k = \hat{P}_k H_k^T \left(H_k \hat{P}_k H_k^T + R\right)^{-1} \quad (5)$$

$$\hat{X}_{k+1} = \hat{\hat{X}}_{k+1} + K_k \left(Z_{k+1} - h(\hat{\hat{X}}_{k+1})\right) \quad (6)$$

$$P_{k+1} = (1 - K_k H_k)\hat{P}_{k+1} \quad (7)$$

Where the dynamic and the output matrices are

$$A_k = \frac{\partial f(X_k, U_k)}{\partial X_k} \quad \text{and} \quad H_k = \frac{\partial h(X_k)}{\partial X_k}$$

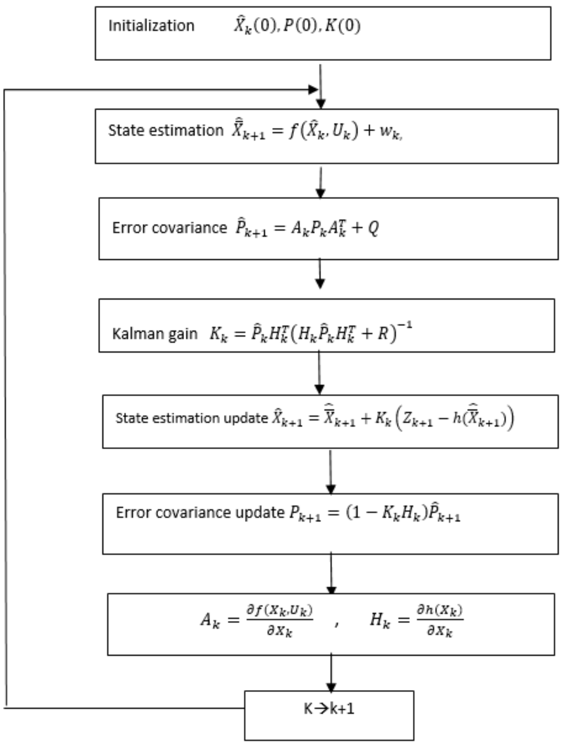

Figure 1 Extended Kalman filter flowchart.

III. SYSTEM MODELING

For the system addressed in this study, the kinematic nonlinear model of the wheeled mobile robot is used. The equations of this kinematic model are given as [12]:

$$\begin{cases} \dot{x} = \mu\cos(\theta) \\ \dot{y} = \mu\sin(\theta) \\ \dot{\theta} = \omega \end{cases} \quad (8)$$

where (x,y) is the position and θ is the orientation, μ is the linear speed, and ω is the steering angle.

Using the Taylor approximation of the previous model, the position and orientation at any future time $t = k+1$ can be calculated from the previous values at $t=k$ [12] as follows:

$$\begin{cases} x(k+1) = x(k) + \mu(k)T_s \cos(\theta(k)) \\ y(k+1) = y(k) + \mu(k)T_s \sin(\theta(k)) \\ \theta(k+1) = \theta(k) + \omega(k)T_s \end{cases} \quad (9)$$

where $T_s = t_{k+1} - t_k$ is the sampling time.

From these equations, we can get the Jacobian matrices:

$$A_k = \frac{\partial f(X_k, U_k)}{\partial X_k} = \begin{vmatrix} 1 & 0 & -\mu(k)T_s \sin(\theta(k)) \\ 0 & 1 & \mu(k)T_s \cos(\theta(k)) \\ 0 & 0 & 1 \end{vmatrix} \quad (10)$$

$$H_k = \frac{\partial h(X_k)}{\partial X_k} = [1\ 1\ 1] \quad (11)$$

A. *Testing the EKF with different noises*

There are many factors that affect the accuracy of localization estimation. The primary factor is the unwanted data in the environment, which is called noise. It is not easy to get high accuracy localization in a noisy environment; therefore, data needs a filtering process to measure it correctly. In the following, we discuss the sensitivity of extended Kalman filter under three different noises conditions.

1) Zero noise

In this case, we tested our model when the input to the kinematic model has no noise at all. This case serves as a validation for our model to make sure that the algorithm is working properly.

2) Gaussian distributed noise

The Gaussian distribution (also called normal distribution) is widely used in natural and social sciences. Its importance is partly due to the central limited theorem [10]. It is often used to represent real-valued random variables with unknown distributions. Generally, the numerical mean of a large number of independent and identically distributed random variables, each with a finite expected value and variance, is distributed roughly in Gaussian [11]. The probability density of Gaussian distribution is given by:

$$F(x|\mu,\sigma) = \left(\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\pi)^2}{2\sigma^2}}\right) \quad (12)$$

where μ is the mean and σ is the standard deviation.

3) Non-Gaussian distributed noise

Gaussian models expose challenges in fitting data that often have distinct spiky and compulsive features that differ from Gaussian distributions, which is known as non-Gaussian distributions [13]. Large measured data show that the impulse noise process has probability density distribution similar to the Gaussian process: symmetrical, smooth, and ring-shaped, but its tail is stronger than the Gaussian distribution [13]. In our study, besides Gaussian noise, we tested cases where random non-Gaussian noise affects the data.

B. Simulation scenarios and input data

To measure the sensitivity of the extended Kalman filter to noise and in order to calculate the processing time in different cases for the robot that moves in an indoor environment as a large area like company ,or small area like an office, we performed a simulation using the three different scenarios as follows:

1- The optimal case without any noise.
2- Gaussian distributed noise with zero means.
3- Non-Gaussian distributed noise.

The codes were implemented using MATLAB software including the above equations and the inputs were selected to be a Cartesian coordinates system of the wheeled mobile robot locations at different time steps. The x, y, and θ coordinates are initially assigned and then the Extended Kalman filter processing is done. The following initial conditions were assumed: $X(0) = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$, and initial estimate $X\_(0) = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$, The error covariance for position: $P(0) = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$, the time step is T_s=0.1 *s*, the linear speed is *v*=5 m/s, the Jacobean of the sensor transfer functions H = [1 1 1], the steering angle is ω=1 rad/s, the covariance coefficient for the position error, $\sigma_v = 0.05$ and the covariance coefficient for the frequency error, $\sigma_w = 0.05$ and the number of steps equals10.

C. Simulation results and discussion

In this section, the results of the different simulations are discussed.

1) Testing scenario

In this scenario, the covariance coefficients for position and frequency errors were set to zero, so that the input data has no noise.

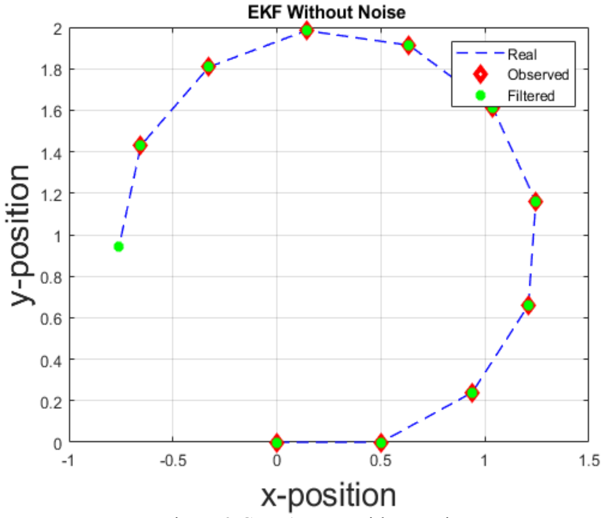

Figure 2 Case 1, EKF with no noise.

As shown in figure 2, the extended Kalman filter on the perfect environment without any noise affection predicts the positions perfectly. The observed and filtered data are in complete agreement with the real positions. This step was important to make sure that the algorithm is working properly.

2) First scenario of noise.

In the first case, we added Gaussian noise using (randn) built-in MATLAB function. To check that the added noise follows the Gaussian distribution, we run the code for 1000 steps and built a histogram using the error (the difference between the real and observed values) for both x and y coordinates. The results are shown in figure3. The error mean values in both cases was 0.0012, which is very close to zero and the histogram clearly shows the shape of the normal distribution.

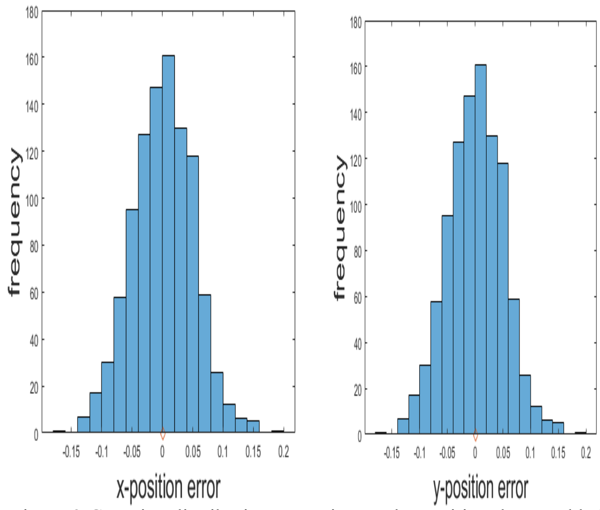

Figure 3 Gaussian distribution errors in x and y position due to added noise.

Under these noises, the extended Kalman filter performance is shown in figure (4).

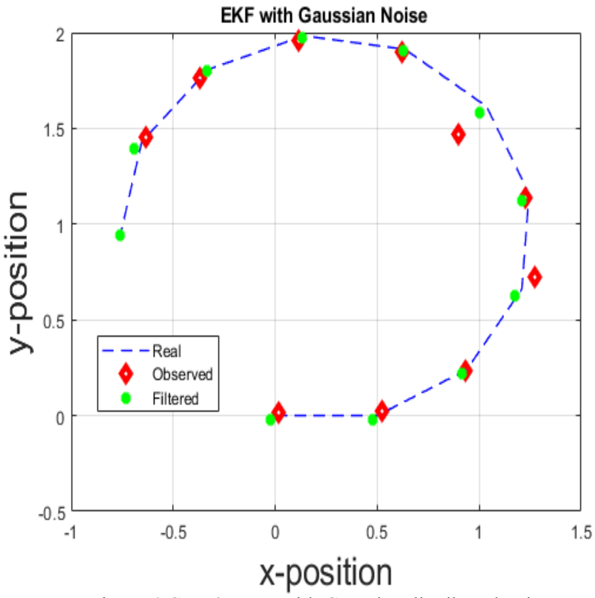

Figure 4 Case 2, EKF with Gaussian distributed noise.

we can see that the EKF is performing well in this case, even for observed points with significant error.

3) Second scenario of noise.

In the second case, we added noise with the non-Gaussian distribution. The errors due to this noise are shown in figure (5).

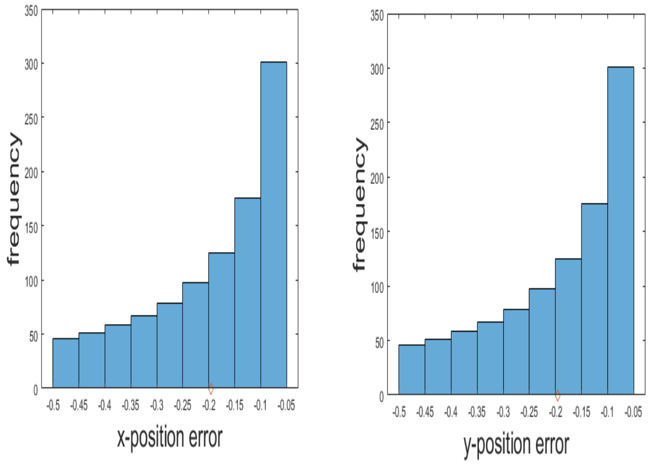

Figure5 Non-Gaussian distribution errors in x and y position due to added noise.

We added the non-Gaussian noise by adding logarithmically spaced noise to the input data. The performance of the EKF under this noise is shown in figure (6). We can see that the EKF is also performing very well under this non-Gaussian distributed noise. Although the red points are greatly deviating from the real trajectory, the predicted position using EKF is in perfect match with the real position.

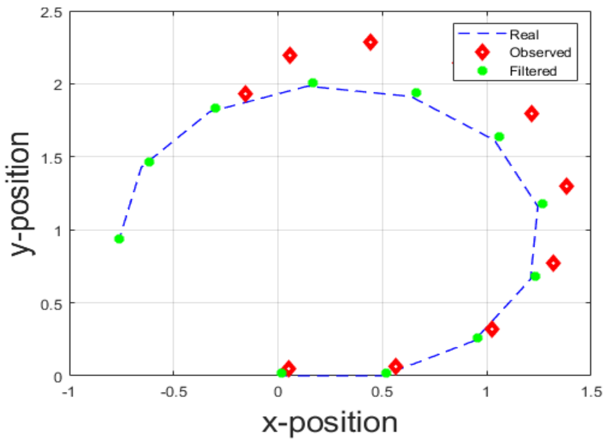

Figure 6 Case3, EKF with non-Gaussian distributed noise.

We compared the performance of the EKF in the three cases using the mean of the error, the standard deviation and root mean square error. The results are shown in Table 1.

Table 1 Comparing the EKF performance in the three noise cases

Cases	Processing time(sec)	X			Y		
		μ	*σ*	*RMSE*	*μ*	*σ*	*RMSE*
Case1	0.854	0	0	0	0	0	0
Case2	0.039	9.34e-04	6.02e-04	0.036	9.48e-04	7.87e-04	0.0040
Case3	0.163	0.0628	0.1017	0.3830	- 0.0136	0.1221	0.3860

As shown in table (1), the EKF gives no errors in case of no noise as expected. The errors are much less in the case of Gaussian distributed noise (case 2). These results are also presented using figures (7) and (8) below.

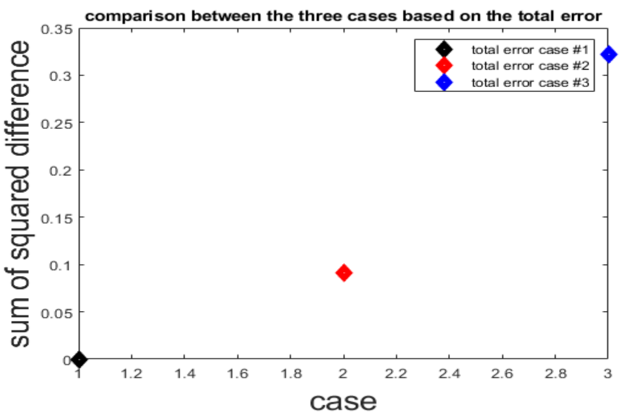

Figure 7 The error of EKF in the three cases using the sum of squared differences between predicted and real values (distance).

In figure (7) we used the sum of the squared difference between the predicted value using EKF and the real value (i.e. the sum of squared distance between the points), it is clear that the error is less in the case of Gaussian noise. In figure (8), the error is calculated by the squared difference between the predicted and real value at each step. We can see that EKF performance is better in the case of Gaussian noise.

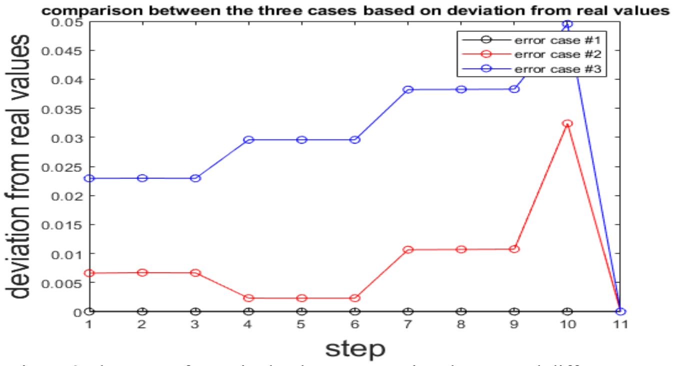

Figure 8 The error of EKF in the three cases using the squared difference at each step.

The accuracy and the efficiency of the estimator can be seen easily in the figures above, the no noise model shows a great estimation as the estimator line lies directly over the actual trajectory in the figure and the errors are zero. The Gaussian distributed noise case shows a very close estimation. The non-Gaussian distributed noise shows relatively worse estimation as shown by the errors.

IV. CONCLUSION

This paper investigates the use of the extended Kalman filter for the non-linear process under two different noise cases to measure the accuracy of the estimation position for the robot movement in an indoor environment. The kinetics model of the wheeled mobile robot was used. Noises were added to the observation data and process model in two different ways: Gaussian noise, and non-Gaussian noise. The case without noise has been also, tested. The findings show that the EKF is a powerful estimation tool for noisy signals in all cases. However, the results confirmed that this filter works better in the case of the Gaussian noise compared to the non-Gaussian noise and that other filters may need to be considered such as the particle filter.

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