A Comparison Between the Classic Extended Kalman Filter and a Range-Bearing-Only Approach for Robot Localization

by

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Project Report

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Autonomous Robotics

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1. Introduction

Localization allows robots to estimate their pose within an environment. This helps them to navigate, plan, and interact with the world [1]. Probabilistic algorithms, such as the Extended Kalman Filter (EKF), have been widely used for this task because they can fuse noisy measurements from different sources [2]. In a typical EKF-based system, the prediction step uses proprioceptive data (e.g., wheel encoders) to estimate motion, and the correction step updates the estimate using exteroceptive sensors like cameras or laser range-finders [3]. While powerful, this architecture has known drawbacks: accumulated error from drift in the prediction phase, overconfidence in estimates due to linearization, and challenges such as the kidnapped robot problem, where misplacement of the robot in the prediction step invalidates pose estimates [4].

This project explores an alternative approach that removes the dependency on proprioceptive data during prediction. Instead, range-bearing cameras mounted on known landmarks help estimate the robot's position externally, whereas the robot uses its camera to estimate its orientation. This simplifies and decouples the localization process from the robot's kinematics model—whether wheeled, legged, or otherwise. Additionally, it offers a more consistent estimation framework by relying solely on externally observable quantities. A comparison between Range-Bearing-Only (RBO) estimator to the classical EKF under identical simulated conditions is made. Additionally, their statistical consistency and robustness using the ANEES metric is evaluated.

2. Scope

This project's objective is to evaluate and compare two localization approaches: the classical Extended Kalman Filter (EKF), which relies on proprioceptive prediction and exteroceptive correction, and a Range-Bearing-Only (RBO) method that eliminates dependence on proprioceptive inputs. The comparison is performed entirely in simulation under controlled conditions with identical noise and sensor parameters. Initially, the project aimed to incorporate landmark observations as exteroceptive control inputs in the robot's motion model during the EKF prediction step, effectively replacing encoder-based prediction with external sensing. However, this approach was later abstracted into a simpler two-stage estimation pipeline to isolate better and evaluate the consistency of RBO versus EKF.

Several simplifying assumptions were made to constrain the scope of the project. First, the robot's motion dynamics are abstracted by simple wheel displacements. Second, all landmarks are static with known positions. It is assumed that the landmarks and the robot operate on the same horizontal plane (i.e., at the same height) and that mutual visibility exists throughout the simulation, that is, landmark cameras and the camera mounted on the robot are always within the field of view of each other. The simulation does not account for occlusion, sensor dropout, or landmark misidentification. Furthermore, no real-world hardware integration is included; all results are based on synthetic data generated within a noise-controlled environment.

3. Methodology

3.1. System Overview

This project presents a simulation-based comparison between the classical Extended Kalman Filter (EKF) and a Range-Bearing-Only (RBO) approach for mobile robot localization.

3.2. Simulation Tools

The simulations and analysis for this project were implemented entirely in Python using the Google Colab environment. Core libraries used include `NumPy` and `pandas` for numerical operations and data management, `matplotlib` and `seaborn` for visualization, and `scipy` for statistical analysis. The simulator is also available in Github [1].

3.3. EKF Implementation

The EKF used in this simulation is modeled identically to a previously developed version by the author [5], presented in Table 3.1 on pages 29-30 of the thesis titled *Multirobot Localization Using Heuristically Tuned Extended Kalman Filter*[2].

[1] `https://github.com/ruslanmasinjila/comp6912_final_project.git`

[2] `https://ruor.uottawa.ca/items/f33b7c30-ca34-4856-874a-9bf3f93fb868`

3.4. RBO Implementation

The Range-Bearing-Only (RBO) estimator eliminates the use of robot-mounted proprioceptive sensors by relying exclusively on exteroceptive range-bearing observations. The robot's position is estimated through observations from landmark-mounted cameras that observe the robot directly. These measurements are fused using a weighted least-squares approach. The robot's orientation is then estimated using its own camera by comparing the observed bearings of known landmarks with their expected global directions.

The estimation procedure is as follows:

1. For each landmark, simulate noisy range and bearing (ρ_i, ϕ_i) to the robot.

2. Convert each measurement into a global coordinate estimate:

$$x_i = l_x + \rho_i \cos(\phi_i), \quad y_i = l_y + \rho_i \sin(\phi_i)$$

3. Estimate each covariance Σ_i using Jacobian-based error propagation.

4. Fuse all landmark estimates using weighted least squares:

$$\Sigma^{-1} = \sum_i \Sigma_i^{-1}, \quad \hat{p} = \Sigma \left(\sum_i \Sigma_i^{-1} [x_i, y_i]^T \right)$$

5. Estimate the robot's orientation by computing $\psi_i = \theta_i - \phi_i$ for each landmark, where θ_i is the global angle to the landmark.

6. Compute the orientation estimate $\hat{\theta}$ using a weighted circular mean:

$$\hat{\theta} = \text{atan2}\left(\sum w_i \sin(\psi_i), \sum w_i \cos(\psi_i) \right)$$

3.5. Shared Parameters

Both the EKF and RBO pipelines were implemented using a shared configuration file, defined as a Python dictionary `config`, which centralizes all experiment parameters. This includes sensor noise levels, wheel base and diameter, landmark positions, number of simulation steps, and random seed values for motion and measurement noise generation. The use of a shared configuration ensures that both pipelines operate under identical conditions, enabling a fair and meaningful comparison.

3.6. Logging & Evaluation

At each simulation step, relevant data were stored in `pandas` DataFrames, including the ground truth pose, estimated pose, and the full 3×3 state covariance matrix. For the EKF pipeline, the covariance matrix was directly obtained from the Kalman filter. For the RBO pipeline, position covariance was computed through Jacobian-based propagation from range-bearing measurements, and orientation variance was estimated using inverse-variance weighting from multiple landmark observations.

To evaluate estimator consistency, the Average Normalized Estimation Error Squared (ANEES) was computed using the full robot state, including position and orientation. This required proper alignment of the error vector with its corresponding covariance matrix and normalization of angular residuals. ANEES values were compared against the expected chi-squared bounds to assess whether each estimator produced statistically consistent outputs under the assumed noise models.

4. Results

To evaluate the consistency and accuracy of the two localization pipelines, the simulation was repeated 20 times across four experimental categories. Each category was defined by varying the number of time steps and the random seed values used for motion and sensor noise generation. Specifically, Category 1 uses 100 simulation steps, while Category 2 increases this to 500. Within each category, five runs were conducted by varying either the motion seed or the sensor seed from 1 to 5, while holding the other fixed at 1024. This configuration yields four sets of results, summarized in Tables 4.1 to 4.4. Each table reports the Root Mean Square Error (RMSE), the Average Normalized Estimation Error Squared (ANEES), and a consistency label indicating whether the estimator's ANEES falls within the expected chi-squared bounds. Overall, the Range-Bearing-Only (RBO) estimator consistently exhibited higher RMSE values than the Extended Kalman Filter (EKF). For simulations with 100 time steps, the RBO method produced consistent estimates across all trials, whereas the EKF occasionally yielded some pessimistic estimates—meaning its estimated uncertainty (covariance) exceeded the actual estimation error. In contrast, for simulations with 500 time steps, both estimators became inconsistent in all trials. Interestingly, the EKF remained mostly pessimistic, except in one case (Table 4.4), while the RBO estimator became optimistic, indicating that its predicted uncertainty underestimated the actual error. In practical terms, this means the EKF tended to be more accurate than it estimated, while the RBO was less accurate than it believed.

MOTION SEED	EXTENDED KALMAN FILTER (EKF)			RANGE-BEARING-ONLY (ROB)		
	RMSE	ANEES	CONSISTENCY	RMSE	ANEES	CONSISTENCY
1	0.037	2.3305	PESSIMISTIC	0.0697	3.1931	CONSISTENT
2	0.0351	2.1923	PESSIMISTIC	0.0703	3.1822	CONSISTENT
3	0.0368	2.3719	PESSIMISTIC	0.071	3.1713	CONSISTENT
4	0.0362	2.6915	CONSISTENT	0.0681	3.118	CONSISTENT
5	0.0363	2.5734	CONSISTENT	0.0714	3.2428	CONSISTENT

TABLE 4.1: Results for: | Time Steps = 100 | Sensor Seed = 1024 | Chi-Squared Bounds = [2.5391, 3.4987] |

SENSOR SEED	EXTENDED KALMAN FILTER (EKF)			RANGE-BEARING-ONLY (ROB)		
	RMSE	ANEES	CONSISTENCY	RMSE	ANEES	CONSISTENCY
1	0.0407	2.8599	CONSISTENT	0.0835	3.3681	CONSISTENT
2	0.0386	2.5961	CONSISTENT	0.0835	3.3681	CONSISTENT
3	0.0447	3.372	CONSISTENT	0.0835	3.3681	CONSISTENT
4	0.0518	4.242	PESSIMISTIC	0.0835	3.3681	CONSISTENT
5	0.0388	2.4867	PESSIMISTIC	0.0835	3.3681	CONSISTENT

TABLE 4.2: Results for: | Time Steps = 100 | Motion Seed = 1024 | Chi-Squared Bounds = [2.5391, 3.4987] |

MOTION SEED	EXTENDED KALMAN FILTER (EKF)			RANGE-BEARING-ONLY (ROB)		
	RMSE	ANEES	CONSISTENCY	RMSE	ANEES	CONSISTENCY
1	0.0381	2.6202	PESSIMISTIC	0.0793	3.4819	OPTIMISTIC
2	0.0401	2.3956	PESSIMISTIC	0.1059	3.4199	OPTIMISTIC
3	0.0381	2.5229	PESSIMISTIC	0.079	3.3392	OPTIMISTIC
4	0.0373	2.4925	PESSIMISTIC	0.0834	3.5139	OPTIMISTIC
5	0.0371	2.7	PESSIMISTIC	0.0738	69.1034	OPTIMISTIC

TABLE 4.3: Results for: | Time Steps = 500 | Sensor Seed = 1024 | Chi-Squared Bounds = [2.7891, 3.2185] |

SENSOR SEED	EXTENDED KALMAN FILTER (EKF)			RANGE-BEARING-ONLY (ROB)		
	RMSE	ANEES	CONSISTENCY	RMSE	ANEES	CONSISTENCY
1	0.0426	2.3913	PESSIMISTIC	0.1459	4.0381	OPTIMISTIC
2	0.0432	2.5449	PESSIMISTIC	0.1459	4.0381	OPTIMISTIC
3	0.0459	2.6351	PESSIMISTIC	0.1459	4.0381	OPTIMISTIC
4	0.0492	2.96	CONSISTENT	0.1459	4.0381	OPTIMISTIC
5	0.0459	2.4724	PESSIMISTIC	0.1459	4.0381	OPTIMISTIC

TABLE 4.4: Results for: | Time Steps = 500 | Motion Seed = 1024 | Chi-Squared Bounds = [2.7891, 3.2185] |

5. Conclusion

This project compared the classical Extended Kalman Filter (EKF) and a Range-Bearing-Only (RBO) approach for robot localization through a series of controlled simulations. Using identical sensor configurations and random seeds, the evaluation focused on accuracy and statistical consistency.

Results showed that the EKF generally achieved lower RMSE, but the RBO method demonstrated more consistent performance in shorter runs (100 steps). In longer simulations (500 steps), both methods became inconsistent: EKF tended to overestimate uncertainty (pessimistic), while RBO underestimated it (optimistic). These trends reflect the RBO's simplified structure, which avoids reliance on a motion model.

A key advantage of RBO is its independence from the robot's kinematics, making it adaptable to systems with complex or unknown motion models. Future work could explore its use in multi-robot settings or environments with dynamic landmarks.

Bibliography

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[5] Ruslan Masinjila. Multirobot localization using heuristically tuned extended kalman filter. Master's thesis, University of Ottawa, 2016. See Table 3.1, page 29-30.