Exercise 1. Diffusion limited aggregation

Goal: In this exercise we learn how to perform a DLA simulation i.e. how to describe the process of aggregating particles (forming a cluster) in cases where the single particle diffusion is the dominating mechanism.

As ususal we restrict ourselves to the two-dimensional case. Consider a system consisting of a seed point (or seed surface) sitting within some boundaries e.g. a box, a sphere, etc. In every step a particle is released from a random position on the boundaries of the system and performs a random walk until it reaches the perimeter of the structure where the particle is attached to the structure. For the first particle this means that the random walk is performed until it reaches the seed. For the next particle(s), however, the random walk is stopped when either the seed is reached or a previous particle that is already sticking to the seed. This way the particles start to form a cluster around the seed. In principle we can either simulate this in continuous space or on a lattice.

Task: Generate a DLA-cluster on a two-dimensional lattice (square lattice or triangular lattice). Color sites according to their aggregation time. Which sites of the perimeter are occupied more frequently?

Hint: It is usually reasonable to imagine two different boundaries. One which is used as a starting point for the particles (inner boundary) and one which is used as 'maximal distance' where the random walk is stopped (outer boundary) (because the particle is too far away from the cluster) and a new walker starts at the start boundary.

Hint: For efficiency, the start radius and the maximal radius can be adjusted relative to the radius of the cluster at every time step.

Optional Task 1: Implement a 'noise reduction' method, i.e. a perimeter site has to be visited m times before it is occupied. What is the qualitative difference when varying m?

Optional Task 2: Determine the fractal dimension with the methods you already know (box counting method or sandbox method).

Optional Task 3: Generate a DLA cluster in continuous space.