Exercise 1. Forest fires

Goal: In this exercise we study a simple model describing forest fires.

Consider a 2D square lattice of size $L \times L$. Every site is occupied by a tree with probability p. By arbitrarily setting a fire in the forest we are going to investigate how the fire spreads. The system is modelled as follows:

- Consider a 2D array of integers. '0': site is not occupied, '1': site is occupied by a tree, '2': site is occupied by a burning tree, '3': site is occupied by a burnt tree.
- Time step: Consider a burning tree positioned at site (i, j) at time t_k. At time t_{k+1} the tree at site (i, j) is burnt $(2 \to 3)$ and the neighbouring trees $(i+1, j)$, $(i, j+1)$, $(i-1, j)$, $(i, j-1)$ are set on fire $(1 \to 2)$.

Task 1: Write a function that takes a 2D array containing 0s (empty), 1s (occupied), 2s (burning) and 3s (burnt) and creates an output image in `ppm` format that is a graphical representation of the lattice. Use the colors white, green, red and black for the array values 0, 1, 2 and 3, respectively.

Task 2: Write a function that determines whether the sample contains a spanning cluster (i.e. it percolates). This is done as follows:

- Create an empty lattice of size $L \times L$ (all values set to 0).
- For every lattice site draw a random number $r \in [0, 1)$. If $r \leq p$ set the value of the lattice site to 1.
- In the first time step burn all the trees in the first row.
- In the next time step burn all the trees neighbouring the burning trees and burn out the previously burning trees.
- Repeat the same until the fire stops.

Task 3: Use the program of task 2 to obtain statistics for the probability of finding a spanning cluster, the shortest path length and the life time of the fire. This is done as follows:

- For a given N and p make several measurements (e.g. 1000) using different seeds for the random number generator, and compute the fraction of samples with percolative cluster, the average shortest path length (when percolation occurs), and the life time of the fire.
- Repeat the same for different values of p. Plot the results against p.
- Change N and compare the results. (For each N you should obtain a separate curve.)
- Find the value of the threshold probability p_c for which the system changes its behaviour.

Hint: The shortest path length is defined as the smallest number of time steps that the fire needs to reach the other side (in the case the sample has spanning clusters). The life time of the fire is defined as the total number of time steps needed before the fire stops.