**Exercise 1. Fractal dimension of the percolating cluster**

*Goal: Here, we compute the fractal dimension of a percolating cluster at (/close to) $p_c$.*

Set up the system at (/close to) $p_c$. To find the percolating cluster you may use the burning method:

- The procedure is exactly the same as discussed in exercise sheet 02, except that only one occupied site is set to fire (instead of the whole first row).
- If the fire reaches the other side of the system, the system percolates and all the burned sites belong to the percolating cluster. (Note: To find all sites of this cluster, you will have to burn further until the fire dies out.)
- If the fire dies out without reaching the other side, the chosen starting site did not belong to the percolating cluster. Choose another site from the first row. If the fire does not reach the other side starting from any (filled) site in the first row, throw away the sample, make a new one, and try again.

To compute the fractal dimension of the obtained spanning cluster, assign a '1' to all sites that belong to the spanning cluster and a '0' to all remaining sites.

**Task 1:** Compute the fractal dimension of the percolating cluster using the box counting method.

Box counting method:

- Measure the number $N$ of occupied cells on different grids. (A cell is considered to be occupied if it contains at least one site of the percolating cluster.)
- The finest grid, here called the "fine grid" is the original lattice (lower cut-off). Coarser grids consist of larger cells which contain several cells of the finer one. A cell of a coarser grid is occupied if it contains at least one occupied cell of the finer grid.
- Plot $N$ (log) as a function of the inverse size of the cell (log), the slope of which is the fractal dimension.

**Task 2:** Compute the fractal dimension of the percolating cluster using the sandbox method.

Sandbox method:

- Compute the mass $N(R)$ (number of sites) of that part of the percolating cluster which lies within a square box of size $R$ that is located at the center of the system, where $R \leq L$ ($L$: system size of a square lattice). (Note: For the sandbox method, the center of the boxes must be an occupied site.)
- Vary $R$ from small values $R \ll L$ (e.g. $R = 3$) to the maximal value $R = L$.
- Plot $N$ (log) as a function of $R$ (log) and measure the fractal dimension (the slope).