1.5.5 Repeat the above excercise for the sides AB and AC.

Solution: We know the value of $\mathbf{I}$ is

$$\mathbf{I} = \frac{1}{\sqrt{37}+4+\sqrt{61}} \begin{pmatrix} \sqrt{61}-16-3\sqrt{37} \\ -\sqrt{61}+24-5\sqrt{37} \end{pmatrix} \tag{1}$$

from the problem 1.5.2 . The equation of AB is:

$$\begin{pmatrix} 7 & 5 \end{pmatrix} \mathbf{x} - 2 = 0 \tag{2}$$

Let r_1 be the distance between $\mathbf{I}$ and AB, then

$$r_1 = \frac{\left| \begin{pmatrix} 7 & 5 \end{pmatrix} \mathbf{I} - 2 \right|}{\left\| \begin{pmatrix} 7 \\ 5 \end{pmatrix} \right\|} \tag{3}$$

$$= \frac{\left| \frac{1}{\sqrt{37}+4+\sqrt{61}} \begin{pmatrix} 7 & 5 \end{pmatrix} \begin{pmatrix} \sqrt{61}-16-3\sqrt{37} \\ -\sqrt{61}+24-5\sqrt{37} \end{pmatrix} - 2 \right|}{\sqrt{7^2+5^2}} \tag{4}$$

$$= \frac{\frac{2\sqrt{61}-46\sqrt{37}+8}{\sqrt{37}+4+\sqrt{61}} - 2}{\sqrt{74}} \tag{5}$$

$$= \frac{48\sqrt{37}}{\sqrt{74}(\sqrt{37}+4+\sqrt{61})} \tag{6}$$

$$= \frac{48}{\sqrt{2}(\sqrt{37}+4+\sqrt{61})} \tag{7}$$

$$= \frac{24\sqrt{2}}{\sqrt{37}+4+\sqrt{61}} \tag{8}$$

$$= 1.8969 \tag{9}$$

Similarly, the equation of AC is

$$\begin{pmatrix} 4 & -4 \end{pmatrix} \mathbf{x} - 8 = 0 \tag{10}$$

Let r_2 be the distance between $\mathbf{I}$ and AC, then

$$r_2 = \frac{\left| \begin{pmatrix} 4 & -4 \end{pmatrix} \mathbf{I} - 8 \right|}{\left\| \begin{pmatrix} 4 \\ -4 \end{pmatrix} \right\|} \tag{11}$$

$$= \frac{\left| \frac{1}{\sqrt{37}+4+\sqrt{61}} \begin{pmatrix} 4 & -4 \end{pmatrix} \begin{pmatrix} \sqrt{61} - 16 - 3\sqrt{37} \\ -\sqrt{61} + 24 - 5\sqrt{37} \end{pmatrix} - 8 \right|}{\sqrt{4^2 + (-4)^2}} \tag{12}$$

$$= \frac{\left| \frac{8\sqrt{61}+8\sqrt{37}-160}{\sqrt{37}+4+\sqrt{61}} - 8 \right|}{4\sqrt{2}} \tag{13}$$

$$= \frac{192}{4\sqrt{2}(\sqrt{37}+4+\sqrt{61})} \tag{14}$$

$$= \frac{48}{\sqrt{2}(\sqrt{37}+4+\sqrt{61})} \tag{15}$$

$$= \frac{24\sqrt{2}}{\sqrt{37}+4+\sqrt{61}} \tag{16}$$

$$= 1.8969 \tag{17}$$