1.4.1. The equation of the perpendicular bisector of BC is

$$\left(\mathbf{B} - \mathbf{C}\right)^{\top} \left(\mathbf{x} - \frac{\mathbf{B} + \mathbf{C}}{2}\right) = 0 \tag{1}$$

Substitute numerical values and find the equations of the perpendicular bisectors of $AB,\,BC$ and CA.

Solution: It is given that

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{2}$$

1. The equation of perpendicular bisector of BC is given as

$$\left(\mathbf{B} - \mathbf{C}\right)^{\top} \left(\mathbf{x} - \frac{\mathbf{B} + \mathbf{C}}{2}\right) = 0 \tag{3}$$

$$\left(\begin{pmatrix} -4 \\ 6 \end{pmatrix} - \begin{pmatrix} -3 \\ -5 \end{pmatrix} \right)^{\mathsf{T}} \left(\mathbf{x} - \frac{1}{2} \left(\begin{pmatrix} -4 \\ 6 \end{pmatrix} + \begin{pmatrix} -3 \\ -5 \end{pmatrix} \right) \right) = 0$$
(4)

$$\begin{pmatrix} -1\\11 \end{pmatrix}^{\mathsf{T}} \left(\mathbf{x} - \frac{1}{2} \begin{pmatrix} -7\\1 \end{pmatrix} \right) = 0 \tag{5}$$

$$\begin{pmatrix} -1 & 11 \end{pmatrix} \begin{pmatrix} \mathbf{x} - \frac{1}{2} \begin{pmatrix} -7 \\ 1 \end{pmatrix} \end{pmatrix} = 0 \tag{6}$$

$$\begin{pmatrix} -1 & 11 \end{pmatrix} \mathbf{x} = \frac{1}{2} \begin{pmatrix} -1 & 11 \end{pmatrix} \begin{pmatrix} -7 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 11 \end{pmatrix} \mathbf{x} = \frac{7+11}{2} \tag{8}$$

$$\begin{pmatrix} -1 & 11 \end{pmatrix} \mathbf{x} = 9 \tag{9}$$

2. Similarly, the equation of the perpendicular bisector of AB is given as

$$\left(\mathbf{B} - \mathbf{A}\right)^{\top} \left(\mathbf{x} - \frac{\mathbf{B} + \mathbf{A}}{2}\right) = 0 \tag{10}$$

$$\left(\begin{pmatrix} -4 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right)^{\mathsf{T}} \left(\mathbf{x} - \frac{1}{2} \left(\begin{pmatrix} -4 \\ 6 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right) \right) = 0$$
(11)

$$\begin{pmatrix} -5 \\ 7 \end{pmatrix}^{\mathsf{T}} \left(\mathbf{x} - \frac{1}{2} \begin{pmatrix} -3 \\ 5 \end{pmatrix} \right) = 0$$
 (12)

$$\begin{pmatrix} -5 & 7 \end{pmatrix} \begin{pmatrix} \mathbf{x} - \frac{1}{2} \begin{pmatrix} -3\\5 \end{pmatrix} \end{pmatrix} = 0 \tag{13}$$

$$\begin{pmatrix} -5 & 7 \end{pmatrix} \mathbf{x} = \frac{1}{2} \begin{pmatrix} -5 & 7 \end{pmatrix} \begin{pmatrix} -3 \\ 5 \end{pmatrix}$$
(14)

$$(-5 \quad 7) \mathbf{x} = \frac{15+35}{2}$$
 (15)

$$\begin{pmatrix} -5 & 7 \end{pmatrix} \mathbf{x} = 25 \tag{16}$$

3. Similarly, the equation of the perpendicular bisector of AC is given as

$$\left(\mathbf{A} - \mathbf{C}\right)^{\top} \left(\mathbf{x} - \frac{\mathbf{A} + \mathbf{C}}{2}\right) = 0 \tag{17}$$

$$\left(\begin{pmatrix} 1 \\ -1 \end{pmatrix} - \begin{pmatrix} -3 \\ -5 \end{pmatrix} \right)^{\mathsf{T}} \left(\mathbf{x} - \frac{1}{2} \left(\begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} -3 \\ -5 \end{pmatrix} \right) \right) = 0$$
(18)

$$\begin{pmatrix} 4 \\ 4 \end{pmatrix}^{\top} \left(\mathbf{x} - \frac{1}{2} \begin{pmatrix} -2 \\ -6 \end{pmatrix} \right) = 0 \tag{19}$$

$$\begin{pmatrix} 4 & 4 \end{pmatrix} \begin{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} \end{pmatrix} = 0 \tag{20}$$

$$\begin{pmatrix} 4 & 4 \end{pmatrix} \mathbf{x} = -\begin{pmatrix} 4 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad (21)$$

$$(4 4) \mathbf{x} = -4 - 12 (22)$$

$$\begin{pmatrix} 4 & 4 \end{pmatrix} \mathbf{x} = -16 \tag{23}$$

$$4(1 \quad 1)\mathbf{x} = -16 \tag{24}$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = -4 \tag{25}$$

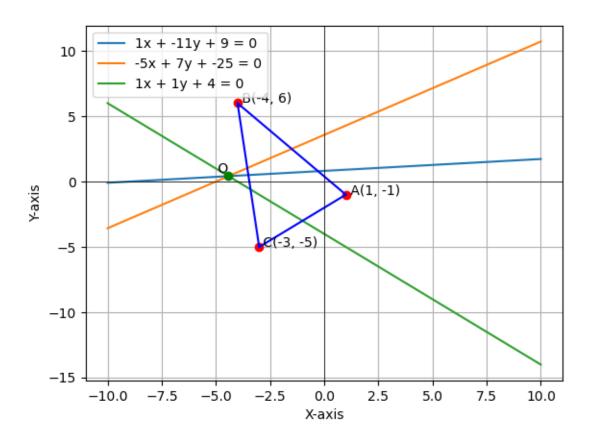


Figure 1: Perpendicular bisectors of AB, BC and CA