20. Two dice are thrown simultaneously. What is the probability that the sum of the numbers appearing on the dice is (i) 7? (ii) a prime number? (iii)

Solution: Let X and Y represent two dice. We know,

$$\Pr(X+Y=n) = \begin{cases} 0 & n \le 1\\ \frac{n-1}{36} & 2 \le x \le 7\\ \frac{13-n}{36} & 7 < n \le 12\\ 0 & n > 12 \end{cases}$$

Then,

(i) Let E be the event that the sum is 7.

$$Pr(E) = Pr(X + Y = 7) \tag{1}$$

$$=\frac{7-1}{36}\tag{2}$$

$$=\frac{1}{6}\tag{3}$$

(ii) Let F be the event that the sum is a prime number.

$$\Pr(F) = \Pr(X + Y = 2) + \Pr(X + Y = 3) + \Pr(X + Y = 5) + \Pr(X + Y = 7) + \Pr(X + Y = 11)$$

(4)

$$= \frac{2-1}{36} + \frac{3-1}{36} + \frac{5-1}{36} + \frac{7-1}{36} + \frac{13-11}{36}$$
 (5)

$$=\frac{1}{36} + \frac{2}{36} + \frac{4}{36} + \frac{6}{36} + \frac{2}{36} \tag{6}$$

$$=\frac{1+2+4+6+2}{36}\tag{7}$$

$$= \frac{2-1}{36} + \frac{3-1}{36} + \frac{5-1}{36} + \frac{7-1}{36} + \frac{13-11}{36}$$

$$= \frac{1}{36} + \frac{2}{36} + \frac{4}{36} + \frac{6}{36} + \frac{2}{36}$$

$$= \frac{1+2+4+6+2}{36}$$

$$= \frac{15}{36}$$
(8)

$$=\frac{5}{12}\tag{9}$$

(iii) Let G be the event that the sum is 1. Then,

$$\Pr(G) = \Pr(X + Y = 1) \tag{10}$$

$$=0 (11)$$