

20. Two dice are thrown simultaneously. What is the probability that the sum of the numbers appearing on the dice is (i) 7? (ii) a prime number? (iii) 1?

Solution: Let X and Y represent two dice. We know,

$$\Pr(X + Y = n) = \begin{cases} 0 & n \leq 1 \\ \frac{n-1}{36} & 2 \leq n \leq 7 \\ \frac{13-n}{36} & 7 < n \leq 12 \\ 0 & n > 12 \end{cases} \quad (1)$$

Then,

(i) Let E be the event that the sum is 7.

$$\Pr(E) = \Pr(X + Y = 7) \quad (2)$$

$$= \frac{7-1}{36} \quad (3)$$

$$= \frac{1}{6} \quad (4)$$

(ii) Let F be the event that the sum is a prime number.

$$\Pr(F) = \Pr(X + Y = 2) + \Pr(X + Y = 3) + \Pr(X + Y = 5) + \Pr(X + Y = 7) + \Pr(X + Y = 11) \quad (5)$$

$$= \frac{2-1}{36} + \frac{3-1}{36} + \frac{5-1}{36} + \frac{7-1}{36} + \frac{13-11}{36} \quad (6)$$

$$= \frac{1}{36} + \frac{2}{36} + \frac{4}{36} + \frac{6}{36} + \frac{2}{36} \quad (7)$$

$$= \frac{1+2+4+6+2}{36} \quad (8)$$

$$= \frac{15}{36} \quad (9)$$

$$= \frac{5}{12} \quad (10)$$

(iii) Let G be the event that the sum is 1. Then,

$$\Pr(G) = \Pr(X + Y = 1) \quad (11)$$

$$= 0 \quad (12)$$