Consider a triangle with vertices

$$\mathbf{A} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{1}$$

1 Vectors

parameters	values	description
$\mathbf{m_1}$	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	AB
$\mathbf{m_2}$	$\begin{pmatrix} 10 \\ -4 \end{pmatrix}$	BC
$\mathbf{m_3}$	$\begin{pmatrix} -11 \\ 3 \end{pmatrix}$	CA
$\|\|A-B\|\|$	1.41	length of AB
$\|\|B-C\|\|$	10.77	length of BC
$\|\|C-A\|\|$	11.40	length of CA
rank$\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix}$	3	non collinear
$\mathbf{n_1}$	$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	AB
c_1	-3	
$\mathbf{n_2}$	$\begin{pmatrix} -4 \\ -10 \end{pmatrix}$	BC
c_2	40	
$\mathbf{n_3}$	$\begin{pmatrix} 3 \\ 11 \end{pmatrix}$	CA
c_3	-51	
Area	7	Area of Triangle
$\angle A$	60.2°	Angles
$\angle B$	113.2°	
$\angle C$	6.5°	

TABLE 1: Vectors.

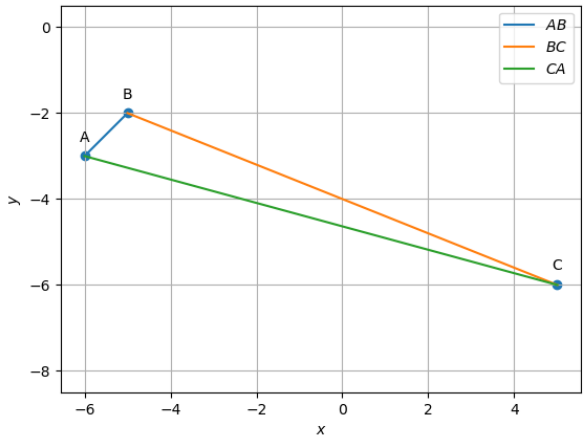

Fig. 1: triangle plotted using python

2 Median

parameters	values	description
$\mathbf{D}$	$\begin{pmatrix} 0 \\ -4 \end{pmatrix}$	BC midpoint
$\mathbf{E}$	$\begin{pmatrix} -0.5 \\ -4.5 \end{pmatrix}$	CA midpoint
$\mathbf{F}$	$\begin{pmatrix} -5.5 \\ -2.5 \end{pmatrix}$	AB midpoint
$\mathbf{m_4}$	$\begin{pmatrix} 6 \\ -1 \end{pmatrix}$	AD
$\mathbf{n_4}$	$\begin{pmatrix} -1 \\ -6 \end{pmatrix}$	
c_4	24	
$\mathbf{m_5}$	$\begin{pmatrix} 4.5 \\ -2.5 \end{pmatrix}$	BE
$\mathbf{n_5}$	$\begin{pmatrix} -2.5 \\ 4.5 \end{pmatrix}$	
c_5	21.5	
$\mathbf{m_6}$	$\begin{pmatrix} -10.5 \\ 3.5 \end{pmatrix}$	CF
$\mathbf{n_6}$	$\begin{pmatrix} 3.5 \\ 10.5 \end{pmatrix}$	
c_6	-45.5	
$\mathbf{G}$	$\begin{pmatrix} -2 \\ -3.67 \end{pmatrix}$	Centroid
$\frac{BG}{GE}$	2	Division ratio by $\mathbf{G}$
$\frac{CG}{GF}$		
$\frac{AG}{GD}$		
rank$\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{D} & \mathbf{G} \end{pmatrix}$	2	collinear
rank$\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{B} & \mathbf{E} & \mathbf{G} \end{pmatrix}$		
rank$\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{C} & \mathbf{F} & \mathbf{G} \end{pmatrix}$		

TABLE 2: Median.

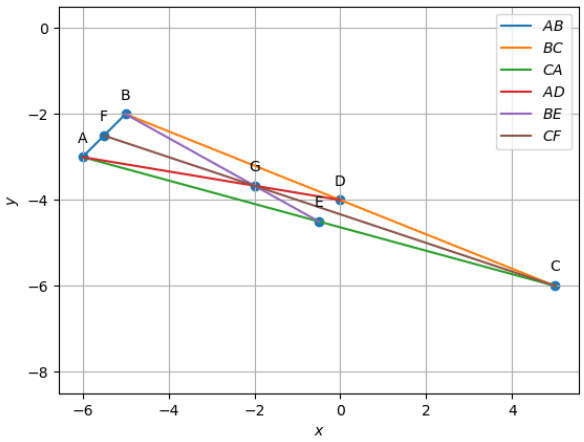

Fig. 2: medians plotted using python

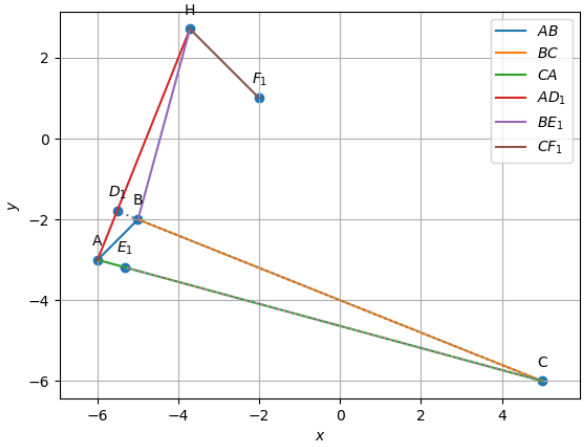

Fig. 3: altitudes plotted using python

3 Altitude

parameters	values	description
$\mathbf{D_1}$	$\begin{pmatrix}-5.52\\-1.79\end{pmatrix}$	Foot of altitude from **A**
$\mathbf{E_1}$	$\begin{pmatrix}-5.32\\-3.18\end{pmatrix}$	Foot of altitude from **B**
$\mathbf{F_1}$	$\begin{pmatrix}-2\\1\end{pmatrix}$	Foot of altitude from **C**
$\mathbf{m_7}$	$\begin{pmatrix}0.48\\1.21\end{pmatrix}$	AD_1
$\mathbf{n_7}$	$\begin{pmatrix}1.21\\-0.48\end{pmatrix}$	
c_7	-5.79	
$\mathbf{m_8}$	$\begin{pmatrix}-0.32\\-1.18\end{pmatrix}$	BE_1
$\mathbf{n_8}$	$\begin{pmatrix}-1.18\\0.32\end{pmatrix}$	
c_8	5.27	
$\mathbf{m_9}$	$\begin{pmatrix}-7\\7\end{pmatrix}$	CF_1
$\mathbf{n_9}$	$\begin{pmatrix}7\\7\end{pmatrix}$	
c_9	-7	
$\mathbf{H}$	$\begin{pmatrix}-3.71\\2.71\end{pmatrix}$	Orthocentre

TABLE 3: Altitude.

4 Perpendicular Bisector

parameters	values	description
$\mathbf{m_{10}}$	$\begin{pmatrix}-4\\-10\end{pmatrix}$	AD_1
$\mathbf{n_{10}}$	$\begin{pmatrix}10\\-4\end{pmatrix}$	
c_{10}	16	
$\mathbf{m_{11}}$	$\begin{pmatrix}-3\\-11\end{pmatrix}$	BE_1
$\mathbf{n_{11}}$	$\begin{pmatrix}11\\-3\end{pmatrix}$	
c_{11}	8	
$\mathbf{m_{12}}$	$\begin{pmatrix}-1\\1\end{pmatrix}$	CF_1
$\mathbf{n_{12}}$	$\begin{pmatrix}-1\\-1\end{pmatrix}$	
c_{12}	8	
$\mathbf{O}$	$\begin{pmatrix}-1.14\\-6.86\end{pmatrix}$	Circumcentre
$\|\mathbf{O}-\mathbf{A}\|$	6.2	$OA = OB = OC = R$
$\|\mathbf{O}-\mathbf{B}\|$		
$\|\mathbf{O}-\mathbf{C}\|$		
R		
$\angle BOC$	120.5°	$\angle BOC = 2\angle BAC$
$\angle BAC$	60.25°	
$\angle AOC$	226.4°	$\angle AOC = 2\angle ABC$
$\angle ABC$	113.2°	
$\angle AOB$	13.1°	$\angle AOB = 2\angle BCA$
$\angle BCA$	6.55°	

TABLE 4: Perpendicular Bisector.

5 Angle Bisector

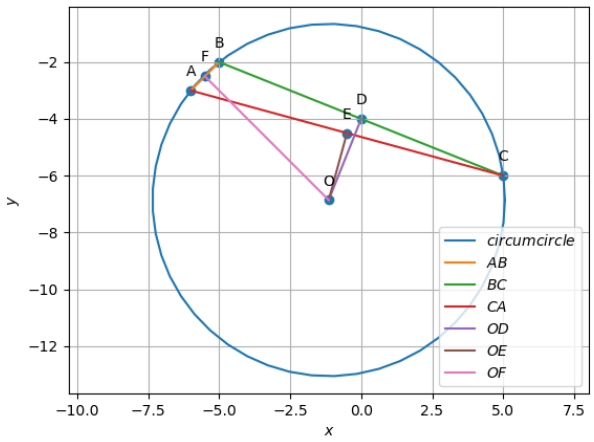

Fig. 4: perpendicular bisectors plotted using python

parameters	values	description
$\mathbf{m_{13}}$	$\begin{pmatrix}-1.67\\-0.44\end{pmatrix}$	AI
$\mathbf{n_{13}}$	$\begin{pmatrix}-0.44\\1.67\end{pmatrix}$	
c_{13}	-2.35	
$\mathbf{m_{14}}$	$\begin{pmatrix}0.22\\-1.08\end{pmatrix}$	BI
$\mathbf{n_{14}}$	$\begin{pmatrix}1.08\\0.22\end{pmatrix}$	
c_{14}	-5.84	
$\mathbf{m_{15}}$	$\begin{pmatrix}1.89\\-0.63\end{pmatrix}$	CI
$\mathbf{n_{15}}$	$\begin{pmatrix}0.63\\1.89\end{pmatrix}$	
c_{15}	-8.19	
$\mathbf{I}$	$\begin{pmatrix}-4.86\\-2.7\end{pmatrix}$	Incentre
$\mathbf{D_3}$	$\begin{pmatrix}-4.64\\-2.14\end{pmatrix}$	Point of contact with BC
$\mathbf{E_3}$	$\begin{pmatrix}-5.01\\-3.27\end{pmatrix}$	Point of contact with AC
$\mathbf{F_3}$	$\begin{pmatrix}-5.28\\-2.27\end{pmatrix}$	Point of contact with AB
$\|\mathbf{I}-\mathbf{D_3}\|$	0.59	$ID_3 = IE_3 = IF_3 = r$
$\|\mathbf{I}-\mathbf{E_3}\|$		
$\|\mathbf{I}-\mathbf{F_3}\|$		
r		
$\angle BAI$	30.12°	$\angle BAI = \angle CAI$
$\angle CAI$		
$\angle ABI$	56.6°	$\angle ABI = \angle CBI$
$\angle CBI$		
$\angle ACI$	3.27°	$\angle ACI = \angle BCI$
$\angle BCI$		

TABLE 5: Angle Bisectors.

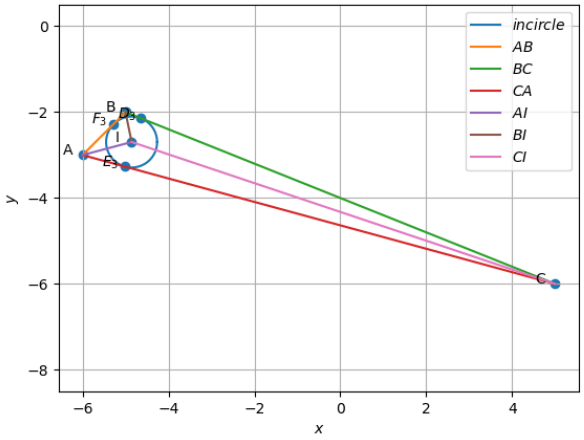

Fig. 5: Angle bisectors plotted using python