1.5.4 Find the distance from $\mathbf{I}$ to BC.

Solution : We know the value of $\mathbf{I}$ is

$$\mathbf{I} = \frac{1}{\sqrt{37}+4+\sqrt{61}} \begin{pmatrix} \sqrt{61}-16-3\sqrt{37} \\ -\sqrt{61}+24-5\sqrt{37} \end{pmatrix} \tag{1}$$

from the problem 1.5.2 . The equation of BC is:

$$\begin{pmatrix} 11 & 1 \end{pmatrix} \mathbf{x} + 38 = 0 \tag{2}$$

Let r be the distance between $\mathbf{I}$ and BC, then

$$r = \frac{\left| \begin{pmatrix} 11 & 1 \end{pmatrix} \mathbf{I} + 38 \right|}{\left\| \begin{pmatrix} 11 \\ 1 \end{pmatrix} \right\|} \tag{3}$$

$$r = \frac{\left| \frac{1}{\sqrt{37}+4+\sqrt{61}} \begin{pmatrix} 11 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{61}-16-3\sqrt{37} \\ -\sqrt{61}+24-5\sqrt{37} \end{pmatrix} + 38 \right|}{\sqrt{11^2+1^2}} \tag{4}$$

$$= \frac{\frac{10\sqrt{61}-38\sqrt{37}-152}{\sqrt{37}+4+\sqrt{61}} + 38}{\sqrt{122}} \tag{5}$$

$$= \frac{48\sqrt{61}}{\sqrt{122}(\sqrt{37}+4+\sqrt{61})} \tag{6}$$

$$= \frac{48}{\sqrt{2}(\sqrt{37}+4+\sqrt{61})} \tag{7}$$

$$= \frac{24\sqrt{2}}{\sqrt{37}+4+\sqrt{61}} \tag{8}$$

$$= 1.8969 \tag{9}$$