

1.5.4 Find the distance from  $\mathbf{I}$  to  $BC$ .

**Solution:** : We know the value of  $\mathbf{I}$  is

$$\mathbf{I} = \frac{1}{\sqrt{37} + 4 + \sqrt{61}} \begin{pmatrix} \sqrt{61} - 16 - 3\sqrt{37} \\ -\sqrt{61} + 24 - 5\sqrt{37} \end{pmatrix} \quad (1)$$

from the problem 1.5.2 . The equation of  $BC$  is:

$$\begin{pmatrix} 11 & 1 \end{pmatrix} \mathbf{x} + 38 = 0 \quad (2)$$

Let  $r$  be the distance between  $\mathbf{I}$  and  $BC$ , then

$$r = \frac{\left| \begin{pmatrix} 11 & 1 \end{pmatrix} \mathbf{I} + 38 \right|}{\left\| \begin{pmatrix} 11 \\ 1 \end{pmatrix} \right\|} \quad (3)$$

$$= \frac{\left| \frac{1}{\sqrt{37}+4+\sqrt{61}} \begin{pmatrix} 11 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{61} - 16 - 3\sqrt{37} \\ -\sqrt{61} + 24 - 5\sqrt{37} \end{pmatrix} + 38 \right|}{\sqrt{11^2 + 1^2}} \quad (4)$$

$$= \frac{\frac{10\sqrt{61}-38\sqrt{37}-152}{\sqrt{37}+4+\sqrt{61}} + 38}{\sqrt{122}} \quad (5)$$

$$= \frac{48\sqrt{61}}{\sqrt{122}(\sqrt{37} + 4 + \sqrt{61})} \quad (6)$$

$$= \frac{48}{\sqrt{2}(\sqrt{37} + 4 + \sqrt{61})} \quad (7)$$

$$= \frac{24\sqrt{2}}{\sqrt{37} + 4 + \sqrt{61}} \quad (8)$$

$$= 1.8969 \quad (9)$$