1.5.4 Find the distance from $\mathbf{I}$ to BC.

Solution : We know the value of $\mathbf{I}$ is

$$\mathbf{I} = \frac{1}{\sqrt{37}+4+\sqrt{61}} \begin{pmatrix} \sqrt{61}-16-3\sqrt{37} \\ -\sqrt{61}+24-5\sqrt{37} \end{pmatrix} \tag{1}$$

from the problem 1.5.2 . The equation of BC from Problem 1.5.1 is:

$$\mathbf{n_2}^\top \mathbf{x} + 50 = 0 \tag{2}$$

where,

$$\mathbf{n_2} = \begin{pmatrix} 11 \\ -1 \end{pmatrix} \implies \mathbf{n_2}^\top = \begin{pmatrix} 11 & -1 \end{pmatrix} \tag{3}$$

Also,

$$\|\mathbf{n_2}\| = \sqrt{11^2 + (-1)^2} = \sqrt{121+1} = \sqrt{122} \tag{4}$$

Let r be the distance between $\mathbf{I}$ and BC, then

$$r = \frac{\left|\mathbf{n_2}^\top \mathbf{I} + 50\right|}{\|\mathbf{n_2}\|} \tag{5}$$

$$r = \frac{\left| \frac{1}{\sqrt{37}+4+\sqrt{61}} \begin{pmatrix} 11 & -1 \end{pmatrix} \begin{pmatrix} \sqrt{61}-16-3\sqrt{37} \\ -\sqrt{61}+24-5\sqrt{37} \end{pmatrix} + 50 \right|}{\sqrt{122}} \tag{6}$$

$$= \frac{\frac{12\sqrt{61}-28\sqrt{37}-200}{\sqrt{37}+4+\sqrt{61}} + 50}{\sqrt{122}} \tag{7}$$

$$= \frac{62\sqrt{61}+22\sqrt{37}}{\sqrt{122}(\sqrt{37}+4+\sqrt{61})} \tag{8}$$

$$= 3.1272 \tag{9}$$