# Final Exam

## Cover Sheet

GROUND RULES:

- Complete the exam within a total time of **6 hours** (honor system)
- You may spread out the 6 hours however you like
- Journal article reading time does not count against your 6 hours
- Open note, open book, open internet
- No collaboration allowed

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## I. In Space Propulsion Mission Applications (10 points)

Match the propulsion technologies below to the mission application for which they are best-suited. Provide a short (2-3 sentence) justification for each of your responses. (Hint: no calculations are required for this question)

- Electrospray thruster
- Hall thruster
- Nuclear thermal rocket (fission)
- Liquid $H_2/O_2$ bi-propellant rocket
- Z-pinch fusion rocket

Mission application:

a) Mercury orbiter with large scientific payload requiring 4 km/s of propulsive delta-v and able to accommodate a six-year orbit transfer phase.

b) Crewed Mars mission with minimum round-trip duration less than 2 years.

c) 2,000 kg interplanetary probe to 125 AU within 10 years.

d) Precise control of the relative orbital positions of multiple satellite interferometers across distances of ~$10^6$ km.

e) Orbit transfer between low-Earth orbit and low-Lunar orbit for a crewed Moon mission.

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## II. Sheath Voltage and Hall Thruster Performance (40 points)

Suppose we are able to arbitrarily vary the wall sheath voltage of a Hall thruster relative to the plasma potential. Assume the Hall thruster has the following properties:

| | |
|---|---|
| Current efficiency: | $\eta_b = 0.75$ |
| Voltage efficiency: | $\eta_V = 0.9$ |
| Discharge current: | $I_d = 10$ A |
| Discharge voltage: | $V_d = 300$ V |
| Outer wall radius: | $r_{out} = 7$ cm |
| Inner wall radius: | $r_{in} = 3$ cm |
| Plasma length: | $L = 4$ cm |
| Wall material: | $BNSiO_2$ |

Perform the following using four different values of $\phi_s - \phi_p = [-50, -100, -150, -200]$ Volts:

(i) Use the power balance equation for the discharge chamber plasma to calculate $T_e$ (in units of eV) for each value of the sheath potential. Present your answer as a plot of $T_e$ vs. $|\phi_s - \phi_p|$.

(ii) Calculate the power lost to the wall by electrons relative to the discharge power for each value of the sheath potential. Present your answer as a plot of $P_{w,e}/P_d$ vs. $|\phi_s - \phi_p|$.

(iii) Calculate the power lost to the wall by ions relative to the discharge power for each value of the sheath potential. Present your answer as a plot of $P_{w,i}/P_d$ vs. $|\phi_s - \phi_p|$.

(iv) Calculate the total power lost to the wall relative to the discharge power for each value of the sheath potential. Present your answer as a plot of $P_w/P_d$ vs. $|\phi_s - \phi_p|$.

(v) Calculate the energy at which ions strike the wall, $\varepsilon_i$ (in units of eV), for each value of the sheath potential. Present your answer as a plot of $\varepsilon_i$ vs. $|\phi_s - \phi_p|$.

(vi) Using the plot given in Homework #3, find the sputtering yield for each value of the sheath potential. Present your answer as a plot of $Y$ vs. $|\phi_s - \phi_p|$. (Hint: use the 45 deg. impact curve above)

(vii) Assuming failure occurs after 1 cm of erosion, calculate the lifetime of the thruster Iin units of hours) for each value of the sheath potential. Present your answer as a plot of $t_{life}$ vs. $|\phi_s - \phi_p|$.

(viii) What does the above analysis tell us about the influence of the sheath potential on Hall thruster performance?

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## III. Nuclear Thermal Rocket (20 points)

In this problem we will examine the influence of pressure on frozen-flow losses and the performance of a hydrogen propellant nuclear thermal rocket. Assume the reactor is limited to a temperature $T_r = 3{,}000$ K.

(i) Using Fig. 6-2 in the text, calculate the species coefficients $\alpha_2$ and $\alpha_1$ for molecular and atomic hydrogen, respectively. Perform this calculation for the following two reactor pressures: $P_r = [0.01, 1.0]$ atm.

(ii) For the two reactor pressures, calculate the maximum ($\xi = 0$) and minimum ($\xi = 1$ ) exhaust velocity, where $\xi$ is frozen-flow dissociation fraction.

(iii) Calculate the frozen-flow efficiency, $\eta_f$, for each reactor pressure.

(iv) The value of $\xi$ is typically a complex function of the temperature and pressure of the flow in addition to the geometry of the nozzle. Assume our nozzle provides the following values:

$$\xi = 0.95 \quad \text{for} \quad P_r = 0.01 \text{ atm}$$

$$\xi = 0.50 \quad \text{for} \quad P_r = 1.0 \text{ atm}$$

Calculate the exhaust velocity of the rocket for each pressure.

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## IV. Magnetically Shielded Hall Thrusters (30 points)

In this problem you will review the first experimental comparison between magnetically shielded and unshielded Hall thruster configurations. You will need to read the following journal article:

> Hofer, R., et al. "Magnetic shielding of a laboratory Hall thruster II. Experiments." Journal of Applied Physics, 115, 043304, 2014.

After reading the article, answer the following questions:

(i) How is the magnetic field geometry different in the magnetically shielded (MS) configuration compared to the unshielded (US) configuration?

(ii) How does the structure of the plasma change between the MS and US configurations? Answer this question using both probe data and the visual appearance of the plasma.

(iii) What impact does magnetic shielding have on the propulsion performance (specific impulse, thrust efficiency, etc.)?

(iv) How do the authors experimentally determine the wall erosion rate?

(v) What impact does magnetic shielding have on the lifetime of the thruster?

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