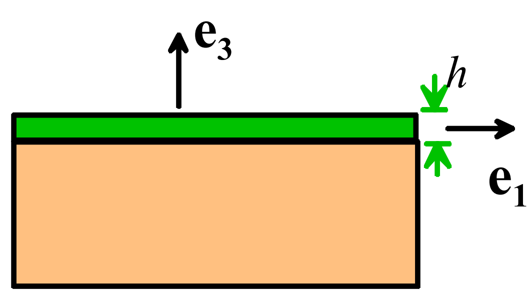
Due: 8:30 a.m. December 2, Tuesday, 2021 (No late homework accepted)

1. [10 points] (*Viscoelastic material – Maxwell model*) Plot the stress of this viscoelastic material as a function of time t (0 to 600 s), given $\varepsilon_0 = 0.1$, $k = 0.1$ GPa, and $\eta = 20$ GPa-s.

2. [10 points] (*Viscoelastic material – Voigt-Kelvin model*) Given a viscoelastic material, we impose ***constant stress*** σ_0 at $t = 0$. Derive the constitutive equation using the ***Voigt-Kelvin*** model (i.e., express strain ε as a function of stiffness k, damping factor η, time t, and given stress σ_0).

3. [10 points] (*Viscoelastic material – Voigt-Kelvin model*) Plot the strain of this viscoelastic material as a function of time t (0 to 600 s), given $\sigma_0 = 1$ MPa, $k = 0.1$ GPa, and $\eta = 20$ GPa-s. Which behavior does this model represent, retarded elastic behavior or steady-state creep behavior? (The response might not be exactly same as what we have covered in the class. But you can qualitatively judge which behavior this model represents).

4. [40 points] (*Constitutive relationship for viscoelastic materials*) A floor is covered with a pad with thickness h of viscoelastic material, as shown in the figure. The pad is perfectly bonded to the floor, so that $\varepsilon_{11} = \varepsilon_{22} = 0$. The pad can be idealized as a viscoelastic solid with time independent bulk modulus K, and has a shear modulus that can be approximated by $G(t) = G_\infty + G_1 e^{-t/t_1}$. The surface of the pad is subjected to a history of displacement $\mathbf{u} = u(t)\mathbf{e}_3$. Assume out-of-plane strain ε_{33} is uniform throughout the thickness of the pad.



4.1 Calculate the history of stresses (all six components) induced in the pad by $u(t) = 0$ for $t < 0$ and $u(t) = u_0$ for $t > 0$

4.2 Calculate the history of stresses (all six components) induced in the pad by $u(t) = 0 \quad t < 0 \quad u(t) = u_0 \sin \omega t \quad t > 0$

(Note $\int e^{ax}\cos(bx)dx = \frac{b}{a^2+b^2}e^{ax}\sin(bx) + \frac{a}{a^2+b^2}e^{ax}\cos(bx)$)

4.3 Plot the stress history in 1.2. using appropriate parameters of your choice, such that the curves show both transient and steady-state responses.

4.4 Assume that the pad is subjected to a displacement $u(t) = u_0 \sin \omega t$ for long enough for the cycles of stress and strain to settle to steady state. Calculate the total energy dissipated per unit area of the pad during a cycle of loading.

5. [15 points] (*Tresca yield criterion*) Derive a mathematical expression for a 3D Tresca yield criterion, given the uniaxial yield stress σ_0 = 200 MPa. Plot the envelop in 3D space using Matlab.

6. [15 points] (*Von Mises yield criterion*) Plot the envelop of the Von-Mises yield criterion in 3D space using Matlab, given the uniaxial yield stress σ_0 = 200 MPa.