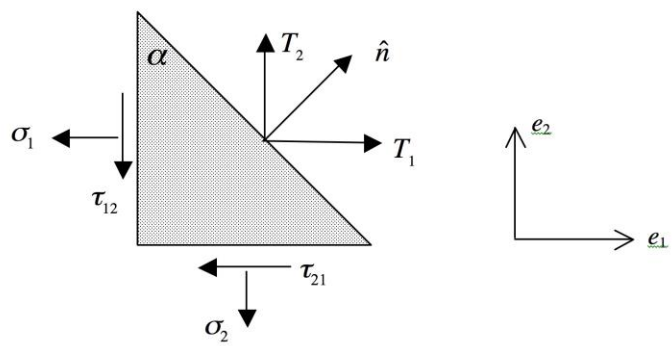
AA 530: SOLID MECHANICS Out: Oct. 14, 2021
HW #2 Due: Oct. 26, 2021

**Due: 8:30 a.m. October 26, Tuesday, 2021 (No late homework accepted)**

1. [50 points] (*Equilibrium & Cauchy Stress*) The figure below shows an infinitesimal triangular component taken from a 2D solid in equilibrium. The slanted surface has an angle $\alpha$ with respect to the vertical line.




1.1 Derive Cauchy's formula by considering equilibrium of forces (i.e., express $T_1$ and $T_2$ in terms of given stresses and $\alpha$).

1.2. Calculate normal and shear tractions (i.e., stresses) applied to the slanted surface.

1.3. In which $\alpha$, do we obtain the maximum normal stress? Given $\sigma_1$ = 30 MPa, $\sigma_2$ = 10 MPa, and $\tau_{12} = \tau_{21}$ = –10 MPa, what is this $\alpha$ value and the corresponding maximum stress ($0 \leq \alpha < 180°$)?

1.4. In which $\alpha$, do we obtain the maximum shear stress? Given $\sigma_1$ = 30 MPa, $\sigma_2$ = 10 MPa, and $\tau_{12} = \tau_{21}$ = –10 MPa, what is this $\alpha$ value and the corresponding maximum stress ($0 \leq \alpha < 180°$)?

1.5. What is the relationship between the two $\alpha$'s obtained in 1.3. and 1.4?

1.6. Given $\sigma_1$ = 30 MPa, $\sigma_2$ = 10 MPa, and $\tau_{12} = \tau_{21}$ = –10 MPa, plot the trajectory of normal (*x*-axis) and shear (*y*-axis) stresses in an x-y Cartesian coordinate under the variations of $\alpha$ from 0 to 180 degrees (Use Matlab).

1.7. Show that the normal and shear stresses derived in 1.2. are following a circular trajectory under the variation of $\alpha$ (i.e., mathematically derive Mohr's circle relationship). What are the principal stresses and maximum shear stress?

2. [50 points] (*Cauchy stress*) The stress tensor at a point is given by:

$$\sigma = \begin{bmatrix} 6 & -2 & 0 \\ -2 & 3 & 4 \\ 0 & 4 & 3 \end{bmatrix} \text{ (unit: Pa)}$$

2.1. Find the stress component perpendicular and parallel to the plane with the unit normal vector:

$$\hat{n} = (1, \quad 1, \quad 1)/\sqrt{3}$$

2.2. Determine the principal stresses and the corresponding directions (you can use Matlab).

2.3. Find the maximum shear stress (hint: use relationship between principal normal stresses and maximum shear stresses, e.g., the information in Problem 1.7).

2.4. Find hydrostatic and von-Mises stresses.