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Physica B 338 (2003) 201–205
PHYSICA B
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Locally resonant sonic materials

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Abstract

We have fabricated a new type of composite which displays localized sonic resonances at $\sim$350–2000 Hz with a microstructure size in the millimeter to centimeter range. Around the resonance frequencies the composite behaves as a material with effective negative elastic constants and as a total wave reflector—a 2 cm slab of this material is shown to break the conventional mass-law of sound transmission by order(s) of magnitude. When the microstructure is periodic, our composites exhibit large elastic wave band gaps at the sonic frequency range, with a lattice constant order(s) of magnitude smaller than the corresponding sonic wavelength in air. Good agreement is obtained between theory and experiment.

PACS: 43.40.+s; 62.30.+d; 81.40.Ji; 43.50.+y

Keywords: Localized resonance; Negative elastic constants; Elastic wave band gaps

1. Introduction

In recent years, the study of classical wave characteristics in periodic structures, the so-called spectral band-gap crystals, is an active area of research due to their novel photonic and acoustic characteristics [1–15]. Photonic band-gap crystals, for example, forbid the propagation of electromagnetic waves in certain frequency regimes and hold promise for a host of potential applications such as extremely low threshold lasers. Their existence was first theoretically predicted and then experimentally realized [1,16–19]. In contrast, the realization of acoustic wave band-gap crystals is hampered by two difficulties. The first is that at sonic frequencies, the requirement that the lattice constant be comparable to the acoustic wavelength implies an extremely large scale (comparable to large outdoor sculptures) for the required structure, thus putting 3D sonic band-gap crystals essentially out of reach for most laboratories. Second, similar to photonic crystals, the existence of a 3D acoustic/elastic band gap in a two-component composite is predicted to be sensitive to not only the symmetry of the periodic structure, but also to the materials properties.

In this paper, we report a new paradigm for the realization of robust elastic wave band gaps in the sonic frequency range. By considering the idea of localized sonic resonances, we have demonstrated theoretically and experimentally that not only is it possible to realize sonic band-gap crystals with a lattice constant order(s) of magnitude smaller than the acoustic wavelength, but the band gap can also exist for non-periodic structures. These results can lead to practical applications for the sonic

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0921-4526/$ - see front matter
doi:10.1016/S0921-4526(03)00487-3

band-gap materials, such as effective low-frequency sound shield that breaks the mass density law for sound transmission.

2. Theoretical background

To explain the basic idea, it is instructive to first introduce the concept of localized resonances, by drawing an analogy with the atomic electronic levels and their effect on optical properties of an atomic gas. Atoms are known to have discrete electronic energy levels. In the Lorentzian formulation of atom–light interaction, an atomic "resonance" occurs when the light frequency (times Planck's constant) coincides with the difference between two electronic energy levels. Since visible light has wavelength much larger than the atomic dimension, it follows from Rayleigh's law that light generally scatters very weakly from the atomic gas. However, at those light frequencies corresponding to the resonance frequencies, the light can be strongly scattered by the gas, in spite of the small size of the atoms as compared to the wavelength. In other words, the Rayleigh scattering law applies only to scattering from inert objects, i.e., those without internal resonances, or at frequencies away from resonances.

Our localized elastic resonances are the low-frequency mechanical analog to atomic resonances. Here the "atom" is a structural unit in the form of a hard and dense sphere, on the order of a few millimeters to a few centimeters in diameter, coated with a layer of soft rubber/polymer material. When embedded in a hard matrix material, these structural units display sonic resonances in the frequency range of a few hundred to a few thousand Hertz. Elastic waves traveling in the hard matrix material generally have wavelength orders of magnitude larger than the size of the structural units. Therefore away from the resonance frequencies the scattering is weak, and elastic properties of the composite are essentially described by those of the matrix material. However, in the vicinities of the local resonance frequencies the elastic waves would couple strongly to the resonating structural units, owing to the large frequency dispersion of the wave propagation characteristics. In particular, the resonance response has the universal form $1/(\omega_o^2 - \omega^2)$, so that on one side of the resonance frequency ω_o the response function is negative. For the electromagnetic wave, a negative response function (the dielectric constant) implies non-propagating waves (evanescent waves). In complete analogy, a composite containing a minimum threshold amount of the resonant structural units can possess a stop band where no acoustic/elastic wave can propagate. Moreover, the small size of the structural unit as compared to the wavelength implies a manageable size for the sonic band-gap structure. In the argument above no mention is made of any particular periodic symmetry. Thus our material is based on an entirely different principle from the Bragg scattering, and is predicted to possess sonic band gaps when the density of the structural units exceeds a minimum threshold, irrespective of the periodic symmetry.

Most of the previous theoretical studies on acoustic/elastic wave band-gap materials are based on numerical calculations using the plane wave (PW) approach [2–4,6–8]. Theoretical calculations have demonstrated the existence of elastic wave band gap in 2D and 3D structures. Experimental "acoustic crystals" have been made, and stop bands have been observed [9]. While the PW method has the advantage of being able to handle a variety of geometries, it has convergence problem when there is a large contrast in the Lame coefficients and the densities of the components, which unfortunately is what it takes to create complete spectral gaps. In addition, the PW technique does not exploit the symmetry of the scatterers (usually spheres or cylinders) and becomes very inefficient when the building blocks have coatings. Since our systems consist of coated spheres embedded in a matrix, it is more efficient to use a multiple-scattering type of formalism [10]. We note that the theory and experimental realization of elastic wave band gap to date is basically a direct extension of the photonic band-gap idea, in the sense that wave propagation is forbidden by wave interference in a periodic media. This mandates the structural periodicity to be on the same order as the elastic wavelength, and thus low-frequency wave can only be attenuated by acoustic crystals the size of outdoor sculptures [20]. *What we propose here is fundamentally different*.

A considerable amount of work has already been done in testing the idea of localized resonances by using a new type of composite material with a microstructure schematically illustrated in Fig. 1. In the drawing, “A” denotes a solid particle made from a material with reasonably high density and high rigidity, such as steel or tin. “B” denotes a soft elastic material, and “C” denotes the matrix material, which can be any arbitrary structural material, or plastic. Due to the existence of the intervening soft elastic material (“B”) between two high-rigidity materials, there can be a low-frequency resonance due to the center-of-mass motion of the particle relative to the matrix. Higher frequency modes arise from the resonant modes of the soft elastic material “B”. At frequencies just lower than the resonance frequencies the effective elastic constant can be negative, as shown below through numerical calculations. At those frequencies the waves would be unable to propagate, hence strong attenuation is expected in transmission through a slab of this composite material.

3. Experimental

We have realized the structure geometry shown in Fig. 1 by using 1.0 cm diameter lead balls coated with 0.25 cm of silicon rubber. We have measured two types of samples. The first one is a circular slab 9.8 cm in diameter and 2.1 cm in thickness. This sample contains 48 vol % of *randomly* dispersed monolayer of the rubberized metallic particles [21]. The second sample is an $8 \times 8 \times 8$ simple cubic crystal with a lattice constant of 1.55 cm. The matrix is epoxy for both samples. Sonic transmission was measured by using a modified Bruel & Kjaer two-microphone impedance measurement tube, type 4206. The details of the measurements can be found in Ref. [21].

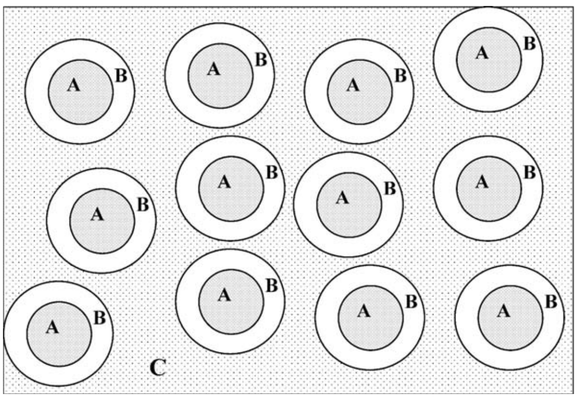

Fig. 1. The schematic structure of our composite material. “A” denotes a lead solid particle 1.0 cm in diameter; “B” denotes a silicone rubber layer; and “C” denotes the matrix material of epoxy.

4. Results and discussion

Fig. 2 shows the measured amplitude transmission (thick solid line) through sample 1. As a reference, the measured amplitude transmission through a 2.1 cm slab of epoxy is also plotted (thin solid line). The striking difference in the transmission coefficient between sample 1 and the pure

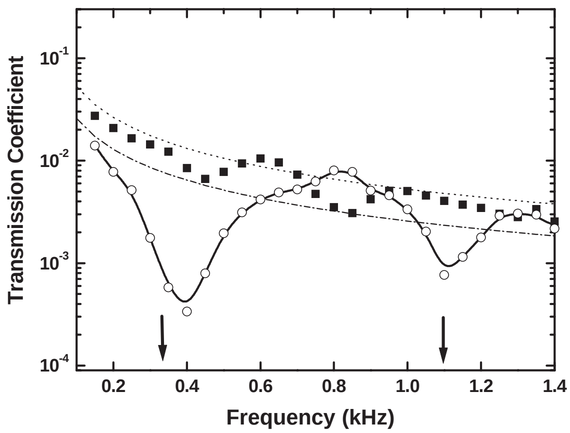

Fig. 2. Transmission coefficient as a function of frequency. Opened dots (with a thick solid line to guide the eye) are the measured transmission amplitude for sample 1. The solid squares are the measured transmission amplitude for the reference sample (pure epoxy slab). The dotted line and the dot–dashed line give, respectively, the calculated transmission amplitudes of a 2.1 cm thick epoxy slab and a 2.1 cm homogeneous slab of the same density as that of the composite material containing the coated spheres. The two arrows indicate the dip frequency positions calculated by using the multiple scattering method [10] for a monolayer of hexagonally arranged locally resonant units.

epoxy slab is the existence of two dips centered at 400 and 1100 Hz for sample 1. These dips originate from the partially developed spectral gaps caused by negative effective elastic constants. It is also found that the measured reflection coefficient for sample 1 and pure epoxy varies between 0.98 and 1 over the whole frequency range, indicating that absorption effect in our sample can be neglected. It is therefore evident that as a potential material candidate for sound shield, our material differs sharply from those sound insulation materials relying on the absorption effect. For comparison, we have calculated the transmission amplitudes for a 2.1 cm thick epoxy slab and a 2.1 cm homogeneous slab of the same density as sample 1. These calculated transmission coefficients, which display the smooth frequency behavior well known for the mass-law (which states that for sound transmission from air through a homogeneous solid slab, the transmitted amplitude is inversely proportional to the slab mass per unit area), are also plotted in Fig. 2 by using dotted line for pure epoxy slab and the dot–dashed line for the other. The most important conclusion drawn from Fig. 2 is that our composite material, with only one layer of coated spheres, breaks the mass-law by at least one order of magnitude at the first-dip frequency. Since the mass-law constitutes a lower bound for sound transmission for a homogeneous solid slab with a given mass per unit area, breaking the mass-law means that if we model our composite material as a homogeneous slab, it cannot have a positive (effective) elastic constant. Indeed, by using the exact formula for sound transmission through a homogeneous slab, we can reproduce the dip regions of the transmission (where it breaks the mass-law) by allowing the (effective) elastic constant parameter to be negative, in agreement with our local resonance model. A direct consequence of negative (effective) elastic constant is that the transmission attenuation increases exponentially with thickness, in contrast to the usual algebraic variation as dictated by the mass-law.

The rigorous calculation for a monolayer of hexagonally arranged coated spheres in an epoxy matrix shows also two dips in the transmission coefficient as a function of frequency. The position of these two dips predicted by the theoretical calculation are indicated by arrows in Fig. 2. A reasonable agreement between the experimental data and the theory is achieved. At the point of transmission minimum ($\sim$400 Hz) only very little energy is transmitted, and this occurs with a thickness of the sample which is only 1/300 the wavelength in epoxy, about 6.4 m.

The results shown in Fig. 3 are the ratio of the amplitude measured at the center of the crystal (sample 2) to the incident wave. The most striking feature of the data is that two dips are clearly visible, with a peak following each dip. In order to understand the experimental results, we have carried out a rigorous multiple-scattering theory [10] calculation of elastic wave propagation and scattering in a composite medium with coated spherical inclusions. The calculation was based on a *infinite slab*, four lattice constants in thickness, of a simple cubic arrangement of coated spheres (in accordance with the structure of sample 2). The concentration, size of the core sphere and coating thickness were fixed at the experimental values: $\rho = 11.6 \times 10^3$ kg/m^3, $\lambda = 4.23 \times 10^{10}$ N/m^2, $\mu = 1.49 \times 10^{10}$ N/m^2 for lead [8], $\rho = 1.3 \times 10^3$ kg/m^3, $\lambda = 6 \times 10^5$ N/m^2, $\mu = 4 \times 10^4$ N/m^2 for the silicone rubber [6], $\rho = 1.23$ kg/m^3, $\kappa = \lambda = 1.42\times 10^5$ N/m^2 for air, and $\rho = 1.18 \times 10^3$ kg/m^3, $\kappa =$

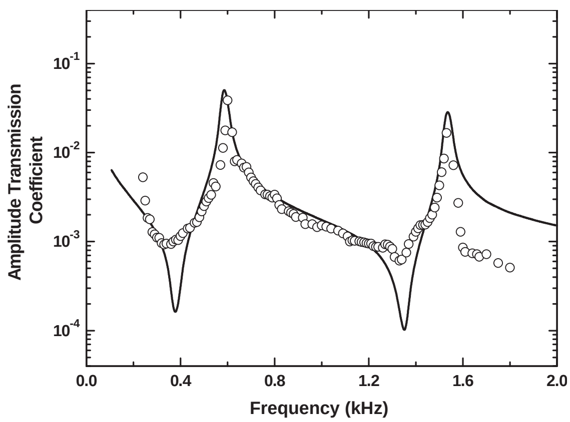

Fig. 3. Measured (open circles) and calculated (solid line) transmission coefficients along the [1 0 0] direction of our simple cubic crystal of locally resonant units (sample 2), plotted as a function of frequency. The calculation is for a four-layer slab of simple cubic arrangement of coated spheres, periodic parallel to the slab.

$\lambda + 2\mu = 7.61 \times 10^9\ \mathrm{N/m^2}$, $\mu = 1.59 \times 10^9\ \mathrm{N/m^2}$ for epoxy [4]. Here λ, μ are the Lame constants, and κ is the longitudinal wave modulus. It is clearly seen that the theoretical results are in good agreement with the experiment, both in terms of the position of the dips (appeared at 380 and 1350 Hz), as well as the qualitative resonance features of the dips. The discrepancy between the two sets of results (experimental and theoretical) can be accounted for by the limitation on the detector sensitivity. The lower frequency dip in transmission is noted to be centered at 400 Hz, corresponding to a 6.4 m longitudinal wavelength in epoxy ($\sim$300 time the lattice constant).

5. Conclusion

Based on simple construction, we have fabricated a novel sonic band-gap material using locally resonant structures. Extension to lower and higher frequency elastic wave systems may lead to applications in seismic wave reflection and ultrasonics.

Acknowledgements

This work is supported in part by the Hong Kong RGC Grant HKUST6145/99P.

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