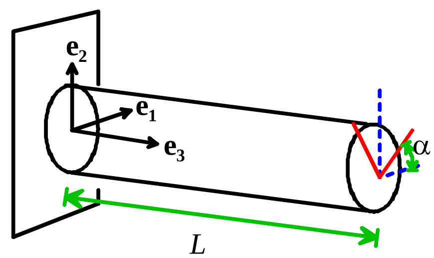
AA 530: SOLID MECHANICS Out: Oct. 5, 2021
HW #1 Due: Oct. 14, 2021

**Due: 9:30 a.m. October 14, Thursday (No late homework accepted)**

1. [20 points] (*Literature survey*) Select one of the following papers and summarize (i) the problem; (ii) objective; (iii) approach; (iv) findings; and (v) conclusions in 200 words.

- R.W. Ogden, Large Deformation Isotropic Elasticity - On the Correlation of Theory and Experiment for Incompressible Rubberlike Solids, *Proc. R. Soc. Lond. A*, vol. 326 no. 1567 565-584, 1972.
- Zhengyou Liu, Xixiang Zhang, Yiwei Mao, Y. Y. Zhu, Zhiyu Yang, C. T. Chan, Ping Sheng, Locally Resonant Sonic Materials, *Science,* Vol. 289 no. 5485 pp. 1734-1736, 2000.
- S. Cai, D. Breid, A.J. Crosby, Z. Suo, J.W. Hutchinson, Periodic patterns and energy states of buckled films on compliant substrates, *Journal of the Mechanics and Physics of Solids*, Volume 59, Issue 5, Pages 1094−1114, 2011.
- James C. Weaver et al., The Stomatopod Dactyl Club: A Formidable Damage-Tolerant Biological Hammer, *Science*, 336 (no. 6086), pp. 1275-1280, 2012.

2. [60 points] The displacement field in a homogeneous, isotropic circular shaft (Radius R) twisted through angle $\alpha$ at one end is given by




$$u_1 = x_1[\cos\left(\frac{\alpha x_3}{L}\right) - 1] - x_2 \sin\left(\frac{\alpha x_3}{L}\right)$$
$$u_2 = x_1 \sin\left(\frac{\alpha x_3}{L}\right) + x_2[\cos\left(\frac{\alpha x_3}{L}\right) - 1]$$
$$u_3 = 0$$

2.1. Calculate the matrix of components of the deformation gradient tensor.

2.2. Calculate the matrix of components of the Lagrange strain tensor. Is the strain tensor a function of $x_3$? Why?

2.3. Find an expression for the increase in length of a material fiber of initial length *dl*, which is on the outer surface of the cylinder and initially oriented in the $\mathbf{e}_3$ direction.

2.4. Show that material fibers initially oriented in the $\mathbf{e}_1$ and $\mathbf{e}_2$ directions do not change their length.

2.5. Calculate the principal values and directions of the Lagrange strain tensor at the point $x_1 = a = L/10,\ x_2 = 0,\ x_3 = 0$ under $\alpha = 5$ degrees. Hence, deduce the orientations of the material fibers that have the largest extension and contraction in length (hint: Use 'eig' function in Matlab).

2.6. Calculate the components of the infinitesimal strain tensor. Show that, for small values of $\alpha$, the infinitesimal strain tensor is identical to the Lagrange strain tensor, but for finite rotations the two measures of deformation differ.

2.7. Use the infinitesimal strain tensor to obtain estimates for the lengths of material fibers initially oriented with the three basis vectors. Where is the absolute error in this estimate greatest? When $R = L/10$, how large can $\alpha$ be before the absolute error in this estimate reaches a 10% strain value?

2.8. [Bonus problem: 20 points] Visualize displacement fields of the given problem using Matlab ($L$ = 1 m, $R$ = 0.1 m, $a = 5$ degrees).

3. [20 points] Consider a strain state described in rectangular Cartesian coordinates:

$$e_{xx} = k\left(x^2 + y^2\right), \quad e_{yy} = k\left(y^2 + z^2\right), \quad e_{xy} = k' xyz$$
$$e_{xz} = e_{yz} = e_{zz} = 0$$

where $k$ and $k'$ are small constants. Is this a possible state of strain for a continuum?