Source Localization using MUSIC in a Multipath Environment

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Abstract — **MUSIC is a popular subspace method because of its superior performance in estimating direction-of-arrival (DOA). However, the performance of the algorithm in a multipath environment has not been fully addressed. This study evaluates the performance of MUSIC algorithm in estimating DOA in such situation. Experimental verification was performed in an enclosed room with varied parameters including the number of sensors in the antenna array and inter-element spacing. Results show that the algorithm localizes the signal with a deviation of 3° for a four-element, λ/2 spaced array.**

Keywords — **direction-of-arrival, MUSIC, multipath.**

I. Introduction

The use of an array of sensors allows many advantages over the use of single receiver. Typically weaker signals can be detected and their localization or directions of arrival estimated. Many popular eigen-subspace approaches to antenna arrays are based on eigen-analysis of the cross spectral matrix of the narrowband receiver outputs and the utilization of the properties of suitably defined projection matrices to calculate direction-of-arrival (DOA) estimate. This is the basis of MUSIC algorithm for narrowband signals in [1].

Most of the studies to estimate signal direction-of-arrivals use simulation [2][3]. The objective of this paper is to investigate the performance of MUSIC algorithms using empirical data in a non-controlled environment. In such case, multipath signals exist that may affect the estimates of MUSIC. We focus on the role of linear array receiver configuration in the performance of the algorithm for direction finding. The algorithm's performance in different experimental parameters is observed. These parameters include the number of antenna elements in the receiving linear array and the spacing between the elements. We also observed the effects of overestimating the number of sources.

This paper is organized as follows. Section II describes the background of subspace methods for DOA estimation. MUSIC algorithm is developed in Section III. Experimental setup is described in Section IV. Finally, Section V contains experimental results highlighting the performance of MUSIC under varied parameters.

II. Subspace Methods for DOA Estimation

All the subspace based methods are based on the eigenvector decomposition of the covariance matrix

$$R_{uu} = E[u(k)^H u(k)] \quad (1)$$

where $u(k)$ is the received signal of the antenna array. If uncorrelated noise is present, then the covariance matrix can be expressed as

$$R_{uu} = A^H E[s(k)^H s(k)]A + E[n(k)^H n(k)] \quad (2)$$

where A is the steering matrix, s(k) is the received signal and n(k) is the noise with power equal to its variance.

The singular value decomposition is then applied to the covariance matrix. If a signal vector is orthogonal to A, then it is an eigenvector of R_{uu} with eigenvalue equal to the variance of noise. Hence the eigenvectors R_{uu} with eigenvalue σ^2 belong to the nullspace of A. From here, the smallest eigenvalues are

$$\lambda_{D+1} = \lambda_{D+2} = \ldots = \lambda_M = \sigma^2 \quad (3)$$

where D is the number of signals and M is the number of sensors.

Therefore it is possible to partition the eigenvectors into noise eigenvectors and signal eigenvectors and thus

$$R[U_s] = R[A] \quad (4)$$

$$R[U_n] = {}^{\perp}R[A^H] \quad (5)$$

$R[U_s]_{MxD}$ is called the signal subspace and $R[U_n]_{Mx(M-D)}$ is called the noise subspace.

The projection operators onto these signal and noise subspaces are defined as

$$P_A = AA^+ = U_s (U^H{}_s U_s)^{-1} U^H{}_s = U_s U_s^H \quad (6)$$

$$P^{\perp}{}_A = I - AA^+ = U_n (U^H{}_n U_n)^{-1} U^H{}_n = U_n U_n^H \quad (7)$$

where A+ is the pseudo-inverse of A.

III. MUSIC Algorithm

The simplest of the algorithms that are based on the above stated subspace decomposition is the MUSIC (Multiple Signal Classification) algorithm. For example, assume L signals are impinging on the sensor array, where L is less than the number of antenna elements. A(θ) is projected into the noise subspace. The projection gives the vector

$$Z_{Mx1} = P^{\perp}_{A\,MxM}\, a(\theta)_{Mx1} \qquad (8)$$

The magnitude squared of z can be written as

$$f(\theta) = z^H z = a(\theta) P^{\perp}{}_A{}^H P^{\perp}{}_A\, a(\theta) = a^H(\theta) U_n U_n^H a(\theta) \qquad (9)$$

Therefore, the array manifold is searched; $f(\theta)$ is evaluated for all θ as DOA estimates the points at which $f(\theta)=0$. The result is plotted as a function of the azimuth θ, called *spatial spectrum*.

$$p(\theta) = \frac{a^H(\theta)a(\theta)}{a^H(\theta)U_n U_n^H a(\theta)} \qquad (10)$$

The peaks in the plot indicate the DOA estimates of the algorithm.

IV. Experimental Setup

The signal used in the study operates at 890 MHz, modulated using binary phase shift keying (BPSK) with data rate of 1.25 Mbps. This is equivalent to a fractional bandwidth of 1.4e-3<<1. Thus the signal used is assumed to be narrowband. The resulting signal was amplified before transmitted.

The receiver block was implemented using an omnidirectional monopole antenna. This antenna was used to create a virtual linear antenna array of 4 elements. Hence, the position of the receiving antenna during data acquisition is such that it will emulate a linear antenna array. The calibrated displacement of the antenna for the next data acquisition will be one wavelength, measured at the center frequency of the operating bandwidth.

The received signal, after being amplified and down converted was sampled using an oscilloscope. The captured data is sent to the personal computer using General Purpose Interface Bus (GPIB) to Universal Serial Bus (USB) device. Data sent by the oscilloscope was stored in the personal computer as a MATLAB® data file for offline processing.

V. Results

Experiments were performed in an enclosed room. Empirical measurements are done using a linear 2-element antenna, and emulating a 4-element antenna array. For the 2-element case, the inter-element spacing were 0.5λ and λ. A BPSK modulated signal is transmitted at an azimuth $\theta=0°$. The receiving testbed is located 7 feet from the transmitter. The far-field distance $2D^2/\lambda$ is 2λ or 2.21 feet. Ten snapshots were taken, equivalent to 1 millisecond of data.

The spatial spectrum for different linear array configurations is shown on figure 1. The peaks in the plot indicate the general DOA estimates of the algorithm. The 4-element array has a closer estimate than the 2-element array with a deviation of 3°. Increasing the number of elements in the receiver increases the capability of MUSIC to more accurately estimate the general DOA. The increase in antenna sensors effectively increases the Signal-to-Noise ratio (SNR). In subspace methods, the ability to discriminate between the desired signal and noise is crucial in determining the signal and noise subspace more accurately. Hence, this will lead to a better estimate of DoA's.

As the algorithm overestimates the number of signal sources as three, see figure 2, three sources are detected, approximately at -40°, 0° and 50°. This is inconsistent since there is only a single source. It even shows that the most probable location of the source is at -40°. Proper estimation of the input covariance matrix and careful classification of the eigenvalues are critical in preventing signal detection errors. The number of impinging signals is assumed based on the multiplicity of the lowest eigenvalue. When overestimation occurs, the remaining dimensions in the subspace are associated with the strong multipaths. Thus, they are incorrectly classified as desired signals.

VI. Conclusions

In this study, experimental verification of MUSIC algorithm has been considered in estimating the direction-of-arrival of a source in a multipath environment. A BPSK signal was transmitted off-the-air at 0° azimuth and was received under varied parameters. The algorithm was then applied on the recorded data.

Experimental results and analyses showed that a 4-element array performs better than a 2-element array. Increasing the number of receiving sensors provides more accurate DOA estimation. The results also showed that the spacing between the elements in the linear array has a significant effect on the algorithms' performance. By increasing the spacing, the effective aperture of the array is also increased which accounts for better signal resolution.

The problem of overestimating the number of impinging signals may occur with MUSIC. Overestimating the number of signals provides sharper peak on the spatial spectrum at the actual source of the signal. But there is a risk that multipath signals will be incorrectly detected. Proper estimation of the input covariance matrix and careful classification of the eigenvalues are critical in preventing signal detection errors. This is done by using a significant amount of data samples. As the number of samples increases, the estimated covariance matrix approaches the theoretical.

Since the experiment was done in an uncontrolled environment, the deviation in DOA estimations may have been due to reflections. If the experiment was done in a controlled room, no reflections, then the direct path of the signal will always be dominant and closely estimated by the algorithm.

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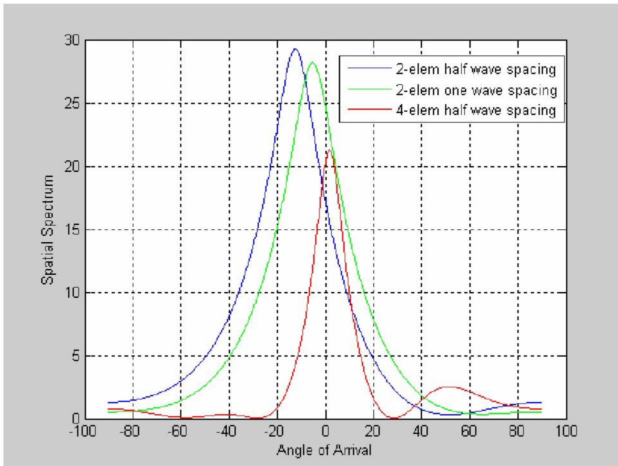

Figure 1. Spatial Spectrum of MUSIC, θ=0 °

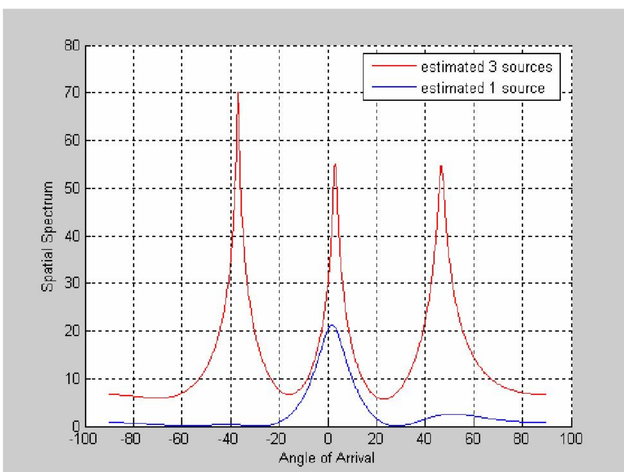

Figure 2. Spatial Spectrum with Overestimated Sources