EIGENSTRUCTURE METHODS FOR DIRECTION FINDING WITH SENSOR GAIN AND PHASE UNCERTAINTIES

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Benjamin Friedlander and Anthony J. Weiss

Saxpy Computer Corporation
255 San Geronimo Way
Sunnyvale, California 94086

ABSTRACT

An eigenstructure-based method for direction finding in the presence of sensor gain and phase uncertainties is presented. The method provides estimates of the Directions of Arrival (DOA) of all the radiating sources as well as calibration of the gain and phase of each sensor in the observing array. The technique is not limited to a specific array configuration and can be implemented in any eigenstructure-based DOA system to improve its performance.

1. Introduction

Direction finding techniques based on eigenstructure methods have been discussed extensively in the literature since the beginning of this decade. Computer simulations and a relatively limited number of experimental systems have demonstrated that in certain cases these techniques have superior performance compared to conventional direction finding techniques.

In spite of the potential advantages of eigenstructure methods their application to real systems has been very limited. One of the main reasons for this situation is the practical difficulties associated with calibrating the data collection system. Eigenstructure-based direction finding techniques such as MUSIC [6] require precise knowledge of the signals received by the sensor array from a standard source located at any direction. The collection of the received signal vectors for all possible directions is often called the *array manifold*. The performance of the eigenstructure based system depends strongly on the accuracy of this array manifold.

A practical approach to alleviating the problems introduced by imprecise array calibration is to use the received signals to adjust or fine-tune the array calibration. Self-calibrating or self-cohering antenna arrays have been developed and tested by Steinberg [8] and others. Rockah and Schultheiss [3]–[4] have studied in detail the self-calibration issue in the context of passive sensor arrays with imprecisely known sensor locations. Self-calibration techniques for eigenstructure based array processing techniques seem to have received little attention. Lo and Marple [5] discussed a calibration technique which requires calibrating sources whose directions are known (at least two sources are required) and therefore their technique is not a true self-calibrating technique. Paulraj and Kailath [1] presented a method for DOA estimation by eigenstructure methods for an array with unknown sensor gains and phases. Their method does not require calibrating sources with known directions, but is limited to uniformly spaced *linear* arrays.

In this paper we address the problem of estimating the direction of arrival of plane waves impinging on a sensor array whose elements have unknown (or imprecisely known) gains and phases. We develop an eigenstructure based method for simultaneously estimating the DOA's and the unknown gain and phase parameters.

The method proposed here is quite different from the one presented in [1]. In particular, our method applies to arrays with arbitrary sensor geometries and is not limited to linear arrays. In fact, a non-linear array geometry is preferred to a linear array when self-calibration techniques are being used.

2. Problem Formulation

Consider N radiating sources observed by an arbitrary array of M sensors. The signal at the output of the mth sensor can be described by

$$x_m(t) = \sum_{n=1}^{N} \alpha_m s_n(t - \tau_{mn} - \psi_m) + v_m(t)\,, \quad -T/2 \le t \le T/2\,, \quad m = 1, 2, \cdots, M\,; \tag{1}$$

where $\{s_n(t)\}_{n=1}^{N}$ are the radiated signals, $\{v_m(t)\}_{m=1}^{M}$ are sample waveforms from additive noise processes and T is the observation interval. The parameters $\{\tau_{mn}\}$ are delays associated with the signal propagation time from the nth source to the mth sensor. These parameters are of interest since they contain information about the source locations relative to the array. Finally, the parameters α_m and ψ_m are the gain and the delay associated with the mth sensor.

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A convenient separation of the parameters to be estimated is obtained by representing the signals using Fourier coefficients defined by

$$X_m(w_l) = \frac{1}{\sqrt{T}} \int_{-T/2}^{T/2} x_m(t) e^{-jw_l t} dt \,, \qquad (2)$$

where $w_l = \frac{2\pi}{T}(l_1 + l)$, $l = 1, 2, \cdots, L$; and l_1 is a constant. In principle the number of coefficients required to capture all the signal information is infinite. However, if we consider signals with energy concentrated in a finite spectral band, we can use only $L < \infty$ coefficients. Moreover, in this work we are interested in narrowband signals with spectrum concentrated around w_0, with a bandwidth that is small compared to $2\pi/T$ and hence $L = 1$. Taking the Fourier coefficients of (1) and suppressing the dependence on w_0 we obtain,

$$X_m = \sum_{n=1}^{N} \alpha_m e^{-jw_0\psi_m} \cdot e^{-jw_0\tau_{mn}} S_n + V_m \; ; \qquad m = 1, 2, \cdots, M \; ; \qquad (3)$$

where S_n and V_m are the Fourier coefficients of $s_n(t)$ and $v_m(t)$, respectively. Equation (3) may be expressed using vector notation as follows,

$$\mathbf{X}(j) = \Gamma \cdot A \cdot \mathbf{S}(j) + \mathbf{V}(j) \quad ; j = 1, 2, \cdots, J \; ; \qquad (4)$$

where j is the index of different (independent) samples and

$$\begin{aligned} \mathbf{X}(j) &= [X_1(j), X_2(j), \cdots, X_M(j)]^T \,, \\ \mathbf{S}(j) &= [S_1(j), S_2(j), \cdots, S_N(j)]^T \,, \\ \mathbf{V}(j) &= [V_1(j), V_2(j), \cdots, V_M(j)]^T \,, \\ \Gamma &= \text{ diag } \left\{1 \,, \alpha_2 e^{-jw_0\psi_2} \cdots, \alpha_M e^{-jw_0\psi_M}\right\} \,, \\ A_{mn} &= e^{-jw_0\tau_{mn}}, \; m = 1, 2, \cdots, M \,; \; n = 1, 2, \cdots, N \,. \end{aligned}$$

To further simplify the exposition we assume that the sensors and sources are coplanar and the sources are far enough from the observing array so that the signal wavefronts are effectively planar over the array. It is easy to verify that the delays τ_{mn} are given by

$$\begin{aligned} \tau_{mn} &= -d_{mn}/c \,, \\ d_{mn} &= \tilde{x}_m \sin\theta_n + \tilde{y}_m \cos\theta_n \,, \end{aligned} \qquad (5)$$

where c is the propagation velocity, d_{mn} is the distance from sensor m to sensor number one (reference sensor) in the direction of the nth source, $(\tilde{x}_m, \tilde{y}_m)$ are the coordinates of sensor m, θ_n is the DOA of the nth source relative to the $\tilde{y}$ axis, and the origin of the cartesian coordinate system coincides with sensor number one.

From (4) and (5) above, it follows that the elements of the matrix A are given by

$$A_{mn} = e^{j(w_0/c)(\tilde{x}_m \sin\theta_n + \tilde{y}_m \cos\theta_n)} \,. \qquad (6)$$

It also follows that $\{A_{1n}\}_{n=1}^{N} = 1$ and that only the nth column of A depends on θ_n.

The problem addressed here is as follows:

Given data $\{x_m(t)\}_{m=1}^{M}$ collected over the observation interval $[-T/2, T/2]$, estimate the unknown directions of arrival $\{\theta_n\}_{n=1}^{N}$, as well as the unknown gains and phases $\{\Gamma_{mm}\}_{m=1}^{M}$. It can be shown [9] that a necessary condition for a unique solution is that the array is nonlinear and $2 \leq N < M$.

3. An Algorithm for Estimating both DOA's and Gain/Phase Parameters

The proposed method is based on the eigendecomposition of the sample covariance matrix of the vector of received signals. We make the standard assumptions underlying the MUSIC algorithm [6] and other eigenstructure based methods for direction finding:

(a) The signals and the noise processes are stationary over the observation interval.

(b) The columns of ΓA are linearly independent.

(c) The signals are not perfectly correlated.

(d) The noise is uncorrelated with the signals and its covariance matrix is full rank and is known except for a multiplicative constant σ^2.

Based on the discussion in the previous section we will add to those the following non-restrictive assumptions:

(e) The array configuration is nonlinear.

(f) The relation between the number of sources N and the number of sensors M is $2 \leq N < M$.

The covariance matrices of the signal, noise and observation vectors are given by

$$\begin{aligned} R_s &= E\{\mathbf{S}\mathbf{S}^H\} \,, \\ \sigma^2\Sigma_0 &= E\{\mathbf{N}\mathbf{N}^H\} \,, \\ R_x &= E\{\mathbf{X}\mathbf{X}^H\} = \Gamma A R_s A^H \Gamma^H + \sigma^2\Sigma_0 \,, \end{aligned} \qquad (7)$$

where $(\cdot)^H$ represents the Hermitian transpose operation. The following theorem forms the basis for the eigenstructure approach.

Theorem: Let λ_i and $\mathbf{u}_i$, $i = 1, 2, \cdots, M$ be the eigenvalues and corresponding eigenvectors of the matrix pencil (R_x, Σ_0), (*i.e.*, the solutions of $R_x\mathbf{u} = \lambda\Sigma_0\mathbf{u}$), where the λ_i's are listed in descending order. Then,

(1) $\lambda_{N+1} = \lambda_{N+2} = \cdots = \lambda_M = \sigma^2$.

(2) Each of the columns of ΓA is orthogonal to the matrix $U = [\mathbf{u}_{N+1}, \mathbf{u}_{N+2}, \cdots, \mathbf{u}_M]$.

Proof: The proof is a straightforward extension of the proof in [2].

This theorem suggests that one should first estimate R_x and use the estimates of λ_i to determine the number of signals. Once N is known reasonable estimates of $\{\theta_n\}_{n=1}^N$ and Γ may be obtained by minimizing:

$$\begin{aligned}\tilde{\mathrm{J}} &= \sum_{n=1}^{N} \|\hat{U}^H \Gamma \mathbf{a}(\theta_n)\|^2 \\ &= \sum_{n=1}^{N} \mathbf{a}(\theta_n)^H \Gamma^H \hat{U}\hat{U}^H \Gamma \mathbf{a}(\theta_n),\end{aligned} \tag{8}$$

where $\hat{U}$ is an estimate of U and $\mathbf{a}(\theta_n)$ is the nth column of A. If $\hat{U}$ were a perfect estimate of U (*i.e.*, $\hat{U} = U$) then the minimum value of $\tilde{\mathrm{J}}$ ($\tilde{\mathrm{J}} = 0$) will be achieved for the true Γ and $\{\theta_n\}_{n=1}^N$. When $\hat{U}$ is an imperfect estimate of U, the minimization of $\tilde{\mathrm{J}}$ will provide estimates $\hat{\Gamma}$ and $\{\hat{\theta}_n\}_{n=1}^N$ of the true DOA's and gain/phase parameters. The accuracy of these estimates has been investigated by simulations (see section 4). A detailed error analysis has not been carried out as yet.

The proposed minimization algorithm is based on a two-step procedure. First, we assume that the gain/phase parameters $\{\Gamma_{mm}\}_{m=2}^M$ are known, and we estimate $\{\theta_n\}$ in the usual way (cf. the description of the MUSIC algorithm in [6]). Given estimates of $\{\theta_n\}_{n=1}^N$ we then minimize $\tilde{\mathrm{J}}$ over the gain/phase parameters $\{\Gamma_{mm}\}_{m=2}^M$. Before presenting the algorithm for minimizing $\tilde{\mathrm{J}}$ note that

$$\Gamma \mathbf{a}(\theta_n) = \tilde{a}(\theta_n)\boldsymbol{\delta}\,, \tag{9}$$

where $\tilde{a}(\theta_n)$ is a diagonal matrix given by

$$\tilde{a}(\theta_n) = \text{ diag } \{\mathbf{a}(\theta_n)\} \tag{10a}$$

and $\boldsymbol{\delta}$ is a vector given by

$$\boldsymbol{\delta} = [\Gamma_{11}, \Gamma_{22}, \cdots, \Gamma_{MM}]^T\,. \tag{10b}$$

The proposed algorithm can then be summarized as follows

(a) Initialization: set $k = 0$; select $\Gamma^{(k)} = \Gamma_0$; Γ_0 may be based on the nominal gain and phase values or on any recent calibration information.

(b) Search for the N highest peaks of the spatial spectrum defined by:

$$P(\theta|\Gamma^{(k)}) = \|\hat{U}^H \Gamma^{(k)} \mathbf{a}(\theta)\|^{-2}\,. \tag{11}$$

These peaks are associated with the N column vectors $\{\mathbf{a}(\theta_n^{(k)})\}_{n=1}^N$

(c) Substituting (9) in (8) we have

$$\tilde{\mathrm{J}} = \boldsymbol{\delta}^H \left\{ \sum_{n=1}^{N} \tilde{a}(\theta_n^{(k)})^H \hat{U}\hat{U}^H \tilde{a}(\theta_n^{(k)}) \right\} \boldsymbol{\delta}\,. \tag{12}$$

Hence, we want to minimize (12) with respect to $\boldsymbol{\delta}$ under the constraint $\boldsymbol{\delta}^H \mathbf{w} = 1$ where $\mathbf{w} = [1, 0, 0, \cdots, 0]^T$. The result of this minimization problem is well known and is given by:

$$\boldsymbol{\delta}^{(k+1)} = Q_k^{-1}\mathbf{w}/(\mathbf{w}^T Q_k^{-1}\mathbf{w})\,, \tag{13}$$

where Q_k is the matrix

$$Q_k = \sum_{n=1}^{N} \tilde{a}(\theta_n^{(k)})^H \hat{U}\hat{U}^H \tilde{a}(\theta_n^{(k)})\,. \tag{14}$$

(d) Compute $\Gamma^{(k+1)}$ from $\boldsymbol{\delta}^{(k+1)}$.

(e) Compute

$$\tilde{\mathrm{J}}_k = \left(\boldsymbol{\delta}^{k+1}\right)^H Q_k \boldsymbol{\delta}^{k+1}$$

If $\tilde{\mathrm{J}}_{k-1} - \tilde{\mathrm{J}}_k > \epsilon$ (a preset threshold) then $k = k+1$, go to (b).
If $\tilde{\mathrm{J}}_{k-1} - \tilde{\mathrm{J}}_k \leq \epsilon$, done.

The algorithm performs the iterations until $\tilde{\mathrm{J}}$ converges. Note that at each updating step (*i.e.*, steps (b) and (c)), we decrease the cost function $\tilde{\mathrm{J}}$ defined in (8). Since $\tilde{\mathrm{J}} \geq 0$ the algorithm will converge at least to a local minimum of $\tilde{\mathrm{J}}$. Depending on the initial estimate of Γ and on the structure of $\tilde{\mathrm{J}}$, the local minimum may or may not coincide with the global minimum.

4. Numerical Examples

In this section we present an example which illustrates the behavior of the proposed algorithm.

Consider a uniform circular array of six sensors separated by half a wavelength of the actual narrowband source signals. Three equal power narrowband sources are located in the far field of the array at directions: $\theta_1 = -30^\circ$, $\theta_2 = -5^\circ$, $\theta_3 = +35^\circ$. Additive uncorrelated sensor noise is injected with SNR of 30 dB referenced to the signal sources. One hundred snapshots of array data are accumulated prior to the application of the algorithm. The nominal gain of the sensors is 1 and the nominal phase is zero. The actual gain is perturbed from the nominal value by up to 80% and the actual phase is perturbed from the nominal value by up to 80 degrees.

Figure 1 shows the performance of the traditional MUSIC algorithm with and without knowledge of the sensor characteristics (gain and phase). We observe that when the MUSIC algorithm has no knowledge of the gain and phase parameters no reliable estimates of the DOA's can be extracted from the plot.

Figure 2 shows the spatial spectrum generated in iterations 1, 3, 7 and 15 of the new technique. Note that iteration one is equivalent to the application of the traditional MUSIC algorithm without any knowledge of the sensor gains and phases. Iteration 15 is already very close to the performance of MUSIC when the gain and phase are known. It is clear that the algorithm has been able to correct DOA errors of more than 10 degrees. The largest phase error after iteration 15 was 1.06°, and the largest gain error was 1.8%.

It should be emphasized that in practice we are not likely to encounter gain and phase errors as large as the initial errors in this example and hence the algorithm will typically converge within 2–6 iterations.

5. Conclusions

In this work the eigenstructure approach has been used to obtain estimates of directions of arrival as well as estimates of gain and phase characteristics of the observing array sensors. We have shown that the basic MUSIC method does not perform well when the array properties are not known accurately. Perhaps the most striking feature of the algorithm is its capability to calibrate the gain and the phase of the sensors without deploying a calibration source at a surveyed location, or alternatively, its ability to accurately estimate DOA's without prior knowledge of the array manifold.

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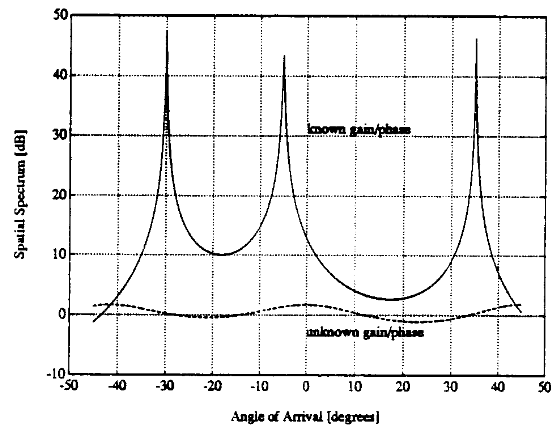

Fig. 1. Spatial spectrum of the MUSIC algorithm.

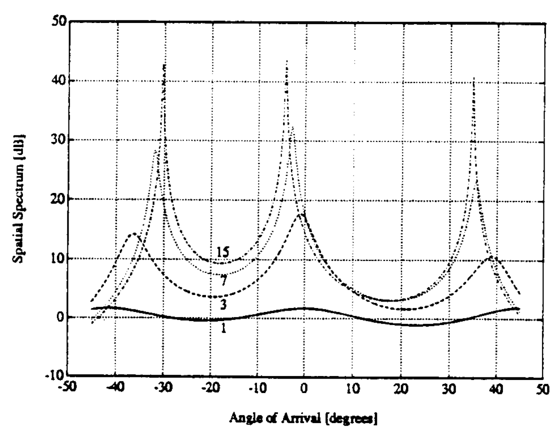

Fig. 2. Spatial spectrum of the proposed algorithm for iterations 1, 3, 7 and 15.