Direction Finding by Time-Modulated Array With Harmonic Characteristic Analysis

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***Abstract*—A novel direction-finding method by the time-modulated array (TMA) is proposed through analyzing the harmonic characteristic of received signal, which requires only two antenna elements and a single RF channel. The signal processing of the proposed method is concise, and its calculation amount concentrates on a two-point discrete Fourier transform (DFT). Numeric simulations are provided to examine the performance of the proposed method, and a simple S band two-element TMA is constructed and tested to verify its effectiveness.**

***Index Terms*—Direction finding, discrete Fourier transform (DFT), time-modulated array (TMA).**

I. Introduction

DIRECTION finding has a very extensive use in wireless communication, sensor network, and military applications. Most existing direction-finding methods, including the Watson–Watt method [1], interferometer [2], and spatial spectrum estimation method [3], generally exploit four or more elements and need a rather complex signal processing. In contrast, the direction-finding function of the time-modulated array (TMA) attracts more and more attention for having simple structure and requiring low signal processing burden [4].

TMA was first proposed by Shanks and Bickmore in the 1950s [5], which added the periodically controlled switches to the RF front end. Because of the periodical modulation, the power of RF signal is distributed along both the fundamental and harmonic components. Recently, the research about TMAs mainly focused on ultra-low sidelobe level (SLL) pattern synthesis [6], the beamforming [7], [8], and the direction finding [9]. The direction-finding property of TMAs was first proposed in [4] by analyzing the relationship between the pattern of the first harmonic component and the mark-to-space ratio of the modulation signal. In 2010, an experimental result of direction finding by a two-element TMA was reported [10], which obtained the direction of an incident plane wave by inserting a coaxial line stretcher into one channel for the phase compensation. The principle is that if the phase shift caused by the stretcher can counteract the phase shift generated in the space, the first harmonic will completely vanish. However, the procedure to adjust the stretcher means searching in the range of $[0, 2\pi]$, which is time-consuming and may not suitable for the real-time direction finding. Actually, the stretcher can be taken out if we make numerical analysis to the harmonic characteristic directly. We deduce the mathematical expression to calculate the incident direction using the fundamental and the first harmonic components. In the digital domain, they can be conveniently calculated out by the discrete Fourier transform (DFT). Then, the proposed method can help to implement direction-finding systems with simple structure and low cost.

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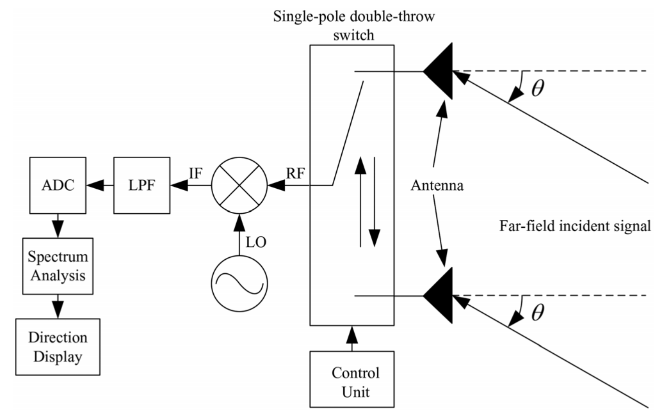

Fig. 1. Block diagram for direction finding of time-modulated array with harmonic characteristic analysis.

The remainder of the letter is as follows. Section II is devoted to the theory of the proposed direction-finding method. In Section III, numeric simulations are given to examine its performance. In Section IV, a simple S-band two-element TMA is constructed and tested to verify its effectiveness. Eventually, some conclusions are drawn in Section V.

II. Theory

The block diagram of the direction finding by the TMA is schematically shown in Fig. 1. Consider a two-element TMA working on the receiving state, with the element spacing D. The RF signals received in the two elements are alternatively selected to enter in the single RF channel by a single-pole double-throw (SPDT) RF switch. The modulation period of the SPDT switch is T_p, and the duty cycle is 50%. Assume that the far-field sinusoidal plane wave at the frequency F_c enters in the array,

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with the incident direction θ. Set the first element as the reference, and the received signal after the SPDT switch is written as

$$s(t) = U(t)e^{j2\pi F_c t} \tag{1}$$

where $U(t)$ is a periodical function, which is expressed as

$$U(t) = \begin{cases} 1, (n-1)T_p < t \le (n-\frac{1}{2})T_p \\ e^{-jKD\sin\theta}, (n-\frac{1}{2})T_p < t \le nT_p \end{cases} \tag{2}$$

where K is the wavenumber and equal to $2\pi F_c/c$, and c is the light speed in the vacuum. Because $U(t)$ is periodical, it can be unfolded by the Fourier series as

$$U(t) = \sum_{k=-\infty}^{\infty} \alpha_k e^{-j2\pi k F_p t} \tag{3}$$

where F_p is the modulation frequency and equal to $1/T_p$, and α_k is the Fourier coefficient of the kth harmonic, which is calculated by

$$\alpha_k = \begin{cases} \frac{1+e^{-jKD\sin\theta}}{2}, & k=0 \\ \frac{j(e^{-jk\pi}-1)}{2k\pi}(1-e^{-jKD\sin\theta}), & k \ne 0. \end{cases} \tag{4}$$

Substitute $k = 0, 1$ into (4) to calculate the fundamental component α_0 and the first harmonic component α_1, respectively. Then, α_0 at the frequency F_c and α_1 at the frequency $F_c + F_p$ are calculated out by

$$\begin{cases} \alpha_0 = \frac{1+e^{-jKD\sin\theta}}{2} \\ \alpha_1 = \frac{-j}{\pi}(1-e^{-jKD\sin\theta}). \end{cases} \tag{5}$$

According to (5), the ratio of the first harmonic component to the fundamental component is

$$\frac{\alpha_1}{\alpha_0} = \frac{2}{\pi}\tan\frac{KD\sin\theta}{2}. \tag{6}$$

Then, the incident direction θ is calculated by the equation

$$\theta = \arcsin\left(\frac{2}{KD}\arctan\frac{\pi\alpha_1}{2\alpha_0}\right). \tag{7}$$

As shown in (7), for the two-channel TMA, the incident direction can be calculated by analyzing the fundamental component α_0 and the first harmonic component α_1 if the element spacing D and the wavenumber K are known beforehand. The Fourier coefficients α_0 and α_1 can be calculated out by the DFT in the digital domain. However, like other RF receivers, before the signal processing in the digital domain, the received signal at the common end of the SPDT switch should commonly enter into the low noise amplifier, mixer, lifter, and analog-to-digital converter (ADC). Assume that the sampling frequency is F_s, and N data points are sampled to calculate α_0 and α_1, which are written as $x(n), n = 1, 2, 3, \cdots, N$. Then, α_0 and α_1 are calculated by

$$\begin{cases} \alpha_0 = \frac{1}{N}\sum_{n=1}^{N} x(n)e^{-j\frac{2\pi F_c}{F_s}(n-1)} \\ \alpha_1 = \frac{1}{N}\sum_{n=1}^{N} x(n)e^{-j\frac{2\pi(F_c+1/T_p)}{F_s}(n-1)}. \end{cases} \tag{8}$$

As shown in (8), the number of the multiply-add operation to calculate α_0 and α_1 is $2N$. Compared to other direction-finding methods, such as the spatial spectrum estimation method, the calculation amount of the proposed method is small. As can be seen from (7), the accuracy of direction finding depends on the estimation results of α_0 and α_1 by the DFT.

The proposed method can be applied to direction finding of multiple sources or the wideband signal. If multiple incoherent RF signals arrive in the two-element TMA simultaneously, their fundamental components and first harmonic components can be distinguished with each other by selecting suitable modulation period T_p. Then, each RF signal's direction can be calculated by (7) independently. For the direction finding of the wideband signal, the fundamental component and the first harmonic component occupy two bandwidths instead of two points in the frequency domain. In this case, the bandwidth of the RF signal can be divided into multiple segments. Each segment can be considered as a frequency point, and the corresponding fundamental component and the first harmonic component can be used to calculate the incident direction by (7). The final estimation of θ is the average of all the calculated results in order to reduce the accident error.

The element spacing D requires careful selection in order to avoid the angle ambiguity. According to (6) and the periodicity of the tangent function, the condition for no angle ambiguity is expressed as

$$-\frac{\pi}{2} < \frac{KD\sin\theta}{2} < \frac{\pi}{2}. \tag{9}$$

If the range of direction finding is $[-\pi/2, \pi/2]$, the value of $\sin\theta$ is located in $[-1, 1]$. Substituting $K = 2\pi/\lambda$ into (9), the aforementioned condition can be satisfied if

$$D < \frac{\pi}{K} = \frac{\lambda}{2}. \tag{10}$$

In consideration of the mutual coupling, the element spacing cannot be very small. Then, the range of the element spacing D is chosen between 0.4λ and 0.5λ.

III. Numeric Results

In this section, two numeric simulations are provided to verify the proposed method. The first numeric simulation examines the performance of the proposed method. Fifteen uncorrelated far-field sources from different directions enter in the two-element TMA proposed in Section II simultaneously. The related simulation parameters are listed in Table I. Assume that all the amplitudes of the incident signals are the same, the system's signal-to-noise ratio (SNR) is 0 dB, and 20 modulation periods' data are sampled to calculate the fundamental component and the first harmonic component using the DFT.

The normalized power spectrum of the received signals are in Fig. 2. As can be seen, the received RF signals contain the fundamental components at the selected carrier frequencies, and the corresponding harmonic components distribute on both their sides, with the frequency step F_p. The results for direction-finding estimations of 15 incident signals are calculated out and plotted in Fig. 3. As is shown, the solid line marked by the down-triangle represents the absolute direction-finding errors of 15 incident sources, and the dashed line marked by the up-triangle indicates their relative errors. As can be seen, the maximum absolute and relative errors are about $0.5°$ and 6%,

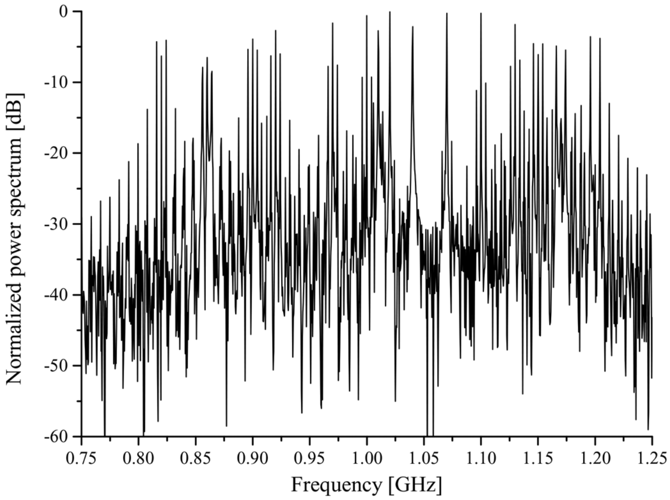

Fig. 2. Normalized power spectrum of received signals.

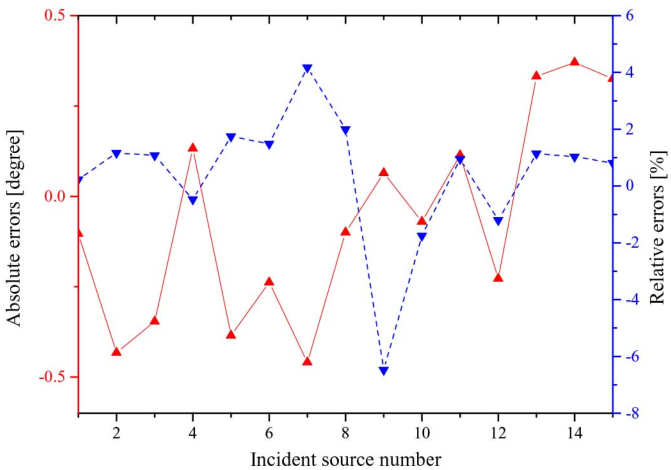

Fig. 3. Absolute and relative errors of direction-finding estimation.

TABLE I
PARAMETERS FOR NUMERIC SIMULATION I

Parameters	*Values*
Incident directions θ_i (°)	-45, -37, -32, -28, -22, -16, -11, -5, -1, 4, 12, 19, 29, 36, 40.
Carrier frequencies F_{ci} (GHz)	0.82, 0.86, 0.9, 0.92, 0.97, 1, 1.01, 1.02, 1.04, 1.07, 1.1, 1.13, 1.15, 1.17, 1.2
Element spacing D	15cm
Modulation frequency T_p	5MHz
Sampling frequency F_s	10GHz
Total sampling time	2μs
Total sampling points	20,000

respectively. As can be calculated, the standard deviations of the absolute and relative errors are 0.28° and 2.3%, respectively.

The second numeric simulation devotes to the standard deviations of direction finding under different SNRs. Assume that the incident direction of the far-field source is $+30^\circ$ and the element spacing is $\lambda/2$. Set the SNRs to increase from -10 to $+20$ dB, with 2-dB step. To calculate the standard deviation of direction finding under each SNR, Monte Carlo simulations are

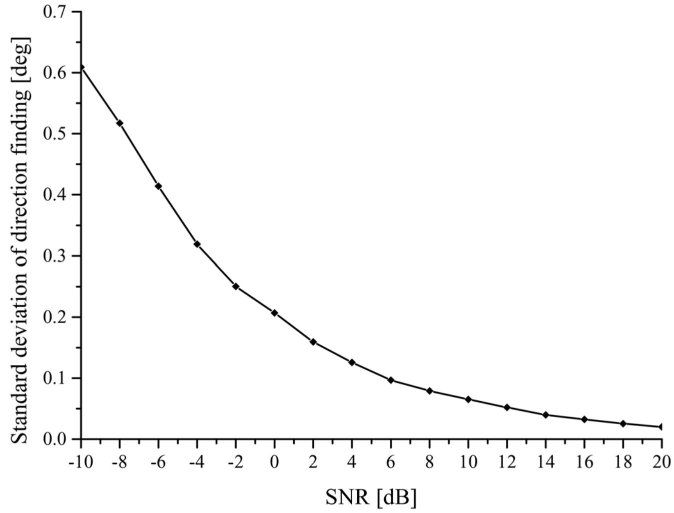

Fig. 4. Standard deviations of direction finding under different SNRs.

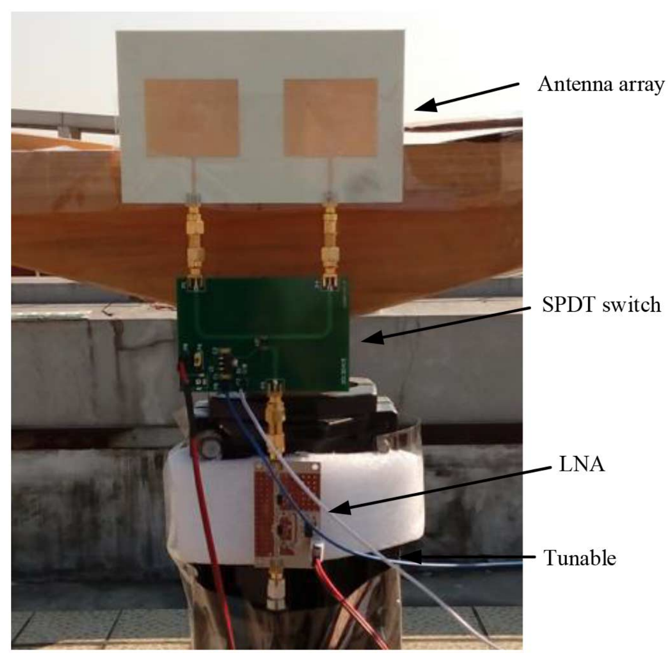

Fig. 5. Experiment for direction finding using the proposed method.

carried out 1000 times. The relationship between the standard deviation of direction finding and the SNR is plotted in Fig. 4. As can be seen, if the SNR is larger than 0 dB, the MSE of direction finding decreases to about 0.2°.

IV. EXPERIMENT

A simple S-band two-element TMA is constructed to verify the proposed method, as shown in Fig. 5. The experiment is finished in the outfield. A far-field sinusoidal signal with 2.525 GHz carrier frequency enters in the TMA. The received signals from the two antenna elements enter into LNA alternatively, under the periodical control of a high-speed SPDT RF switch. The amplified RF signal after LNA is input into an Agilent E4447A spectrum analyzer to measure the fundamental and the first harmonic components. Then, the measurement results are substituted into (7) to calculate incident direction.

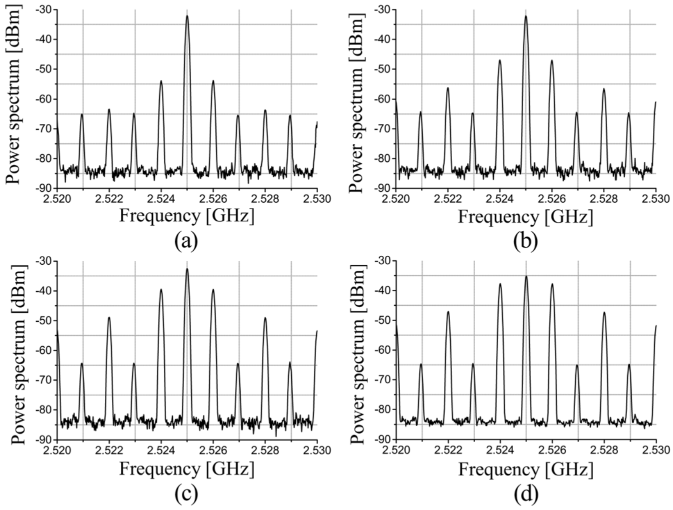

Fig. 6. Power spectrums of received signals from different incident directions. (a) 0° incident direction. (b) 10° incident direction. (c) 20° incident direction. (d) 30° incident direction.

TABLE II
EXPERIMENT RESULTS

Direction [deg]	α_0 [dBm]	α_1 [dBm]	*Estimation* [deg]	*Abso. Error* [deg]	*Rela. Error* [%]
0	-32.02	-53.84	4.7	4.7	-
5	-31.3	-50.36	6.5	1.5	30
10	-32.09	-47.03	10.3	1.3	13
15	-32.16	-41.43	18.9	3.9	16
20	-32.51	-39.48	23.7	3.7	18.5
25	-33.22	-38.91	26.6	1.6	8
30	-35.08	-37.72	34.2	4.2	14

The built TMA is composed of two patch antennas with inset feed. The substrate is 20-mil-thick Rogers 4350B, and the element spacing is 0.48λ. The SPDT switch is ADG918BRM, which is fabricated on the substrate FR4. The LNA module is designed by two series of TQP3M9009, and its substrate is 20-mil-thick Arlon AD1000L020111. The SPDT switch is periodically controlled by a Cyclone IV field-programmable gate array (FPGA), and the modulation frequency is 1 MHz.

The array is fastened to a tunable, which can change the incident direction by a circular scale. Change the incident direction from 0° to $+30^\circ$ with the step 5°. The spectrums of received signals at incident directions $[0^\circ, +10^\circ, +20^\circ, +30^\circ]$ are plotted in Fig. 6. As can be seen, when the angle of incident direction becomes larger, the fundamental component α_0 decreases, and the first harmonic component α_1 increases step by step. The change of the ratio α_0/α_1 reflects the change of the incident direction θ correspondingly.

In Table II, the powers of the α_0 and α_1 under different incident directions are listed, which are measured out by the spectrum analyzer. Then, the estimation results for the incident direction are calculated by (7), and the absolute and relative estimation errors are listed in the last two columns of the table. As can be seen, the estimation errors are less than 5°, and their standard deviation is 3.3°. The direction-finding errors may come from several aspects as follows:

1) the amplitude and phase unbalance, and the isolation characteristic between two RF channels controlled by the SPDT switch;
2) the multipath generated by the surrounding environment;
3) the mutual coupling between two antenna elements;
4) power measurement errors in the spectrum analyzer.

V. CONCLUSION

In this letter, a novel direction-finding method with a two-element TMA is proposed and verified. The proposed method requires only two antenna elements and a single RF channel, and its calculation amount concentrates on a two-point DFT. The proposed method can help to develop simple and low-cost direction-finding systems. It can be applied to multiple incoherent sources' and wideband signal's direction finding. However, the direction finding of multiple coherent sources by the proposed method requires further research.

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