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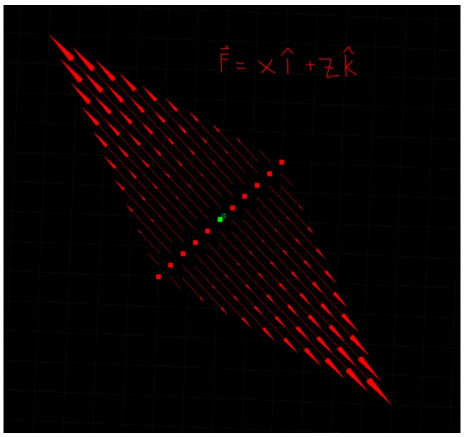
Fundamentals of Computer Graphics

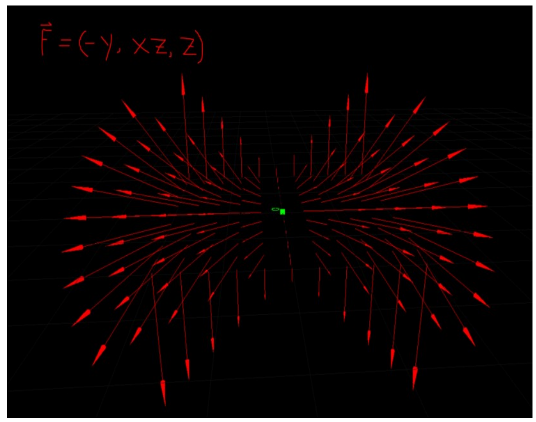
Vector Fields

A vector field is an infinitely continuous space which takes the form of a vector. We can define a field

$$\vec{F} = M\,\hat{i} + N\,\hat{j}$$

where M and N are functions of x and y. That is, the vector in the field depends on the observed location within the field. To visualize a vector field, we can designate a certain interval of sample points and display the resulting vectors at each location in the field. An example of a vector field in life could be a gravitational force. While two bodies in space exhibit forces on each other, the strength of that force depends on its proximity to the other body, and thus the vector fields representing these forces are a function of the planets' sizes and distances to each other.





Vector fields are commonly used to move particles. You might imagine a feather blowing in the wind where the field represents the wind—we could trace the trajectory of the feather by applying force to it at every location as it moves. We may also want to use a vector field to apply forces to many particles at once, such as smoke rising from a bonfire or high-velocity water in a hurricane.

Using Barycentric Coordinates to Determine Constraint-based Fields

Given three constraints in the form of a vector at each vertex, we can construct a linear interpolation using the barycentric coordinates as weighted averages based upon its relative position in the field. Thus:

$$\vec{F}v(x, y, z) = aA + bB + cC$$

where (a, b, c) represent the barycentric coordinates at (x, y, z) and ABC represents the triangle face. To allow the field to be configurable, the user may constrain three vectors at each vertex of the triangle which represent inputs to the field. Then, by using the weighted average above, we can determine any point in the field as an interpolation of the three user constraints. For example, the vector at the point in the field at each vertex will be congruent to the field constraint at that vertex, while the vector at the point at the centroid of the triangle will be a perfect average of the three.

References

1. MIT OpenCourseWare 18.02 Multivariate Calculus, Denis Auroux - https://www.youtube.com/watch?v=xrypSZU8cBE
2. Determining if a point lies on the interior of a polygon – Paul Bourke, 1997. http://www.eecs.umich.edu/courses/eecs380/HANDOUTS/PROJ2/InsidePoly.html