CSC418.

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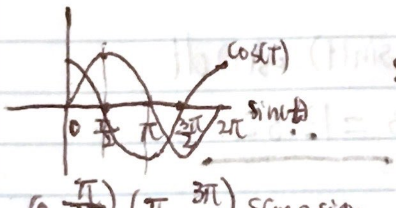
$\cancel{x(t)=}\ x(t) = 2\sin(t) \quad y(t) = 5\sin(t)\cos(t). \quad 0 \leq t \leq 2\pi$

1\.

1). implicit form $f(x,y) = 0$.

$\cos(t) = \pm\sqrt{1-\sin^2(t)}$

$y = \pm 2.5x\sqrt{1-0.25x^2}$



$(0, \frac{\pi}{2}), (\pi, \frac{3\pi}{2})$ same sign

$(\frac{\pi}{2}, \pi), (\frac{3\pi}{2}, 2\pi)$ different sign.

so $\begin{cases} -2.5x\sqrt{1-0.25x^2} + y = 0 & t \in (0, \frac{\pi}{2}), (\pi, \frac{3\pi}{2}) \\ 2.5x\sqrt{1-0.25x^2} + y = 0 & t \in (\frac{\pi}{2}, \pi), (\frac{3\pi}{2}, 2\pi) \end{cases}$

2) tangent $\Rightarrow \frac{dx}{dt} = \frac{d\,2\sin(t)}{dt} = 2\cos(t)$.

$\frac{dy}{dt} = \frac{d\,5\sin(t)\cos(t)}{dt} = \frac{d\,2.5\sin(2t)}{dt} = 5\cos(2t)$

tangent $= \begin{cases} x = 2\cos(t) \\ y = 5\cos(2t) \end{cases}$

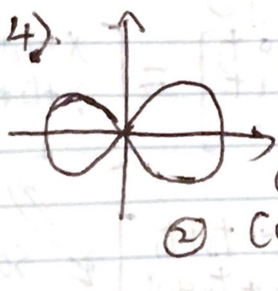
3). normal $\Rightarrow$ ~~if ta~~

$\because$ normal $\perp$ tangent if tangent ~~$t=2x$~~ (x, y) dot product. $= 0$.

normal $(y, -x)$

$\therefore$ normal $= \begin{cases} x = 5\cos(2t) \\ y = -2\cos(t) \end{cases}$

4).



Yes, symetric to both X, & Y axis.

$x = 2\sin(t) \quad y = 2.5\sin(2t)$

① $\sin(t) = -\sin(-t)$ symmetric around origin

② $\cos(t) = -\cos(\pi - t) \quad \sin(t) = \sin(\pi - t)$

so for this interval ~~$\sin(t)$~~ $x(t) = x(\pi - t)\ (2\sin(t))$

$y(t) = -y(\pi - t)\ (5\sin(t)\cos(t))$

$\because$ x is the same sign ~~so~~ symmetric $\because$ symmetric to x and origin

y is opposite $\therefore$ to x $\therefore$ symetric to y axis

5) Area $= \int_0^{2\pi} y\,dx$ $\quad x = 2\sin t \quad dx = 2\cos t\, dt$

$= 4\int_0^{\frac{\pi}{2}} y\,dx = 4\int_0^{\frac{\pi}{2}} 5\sin(t)\cos(t) \cdot 2\cos t\, dt$

$= 4\int_0^{\frac{\pi}{2}} 5\sin(t)2\cos^2 t\, dt$

$= 4\int_0^{\frac{\pi}{2}} 10\sin(t)\cos^2(t)\,dt$

$= 4 \times 3.33 = \underline{13.32}$

6.). Perimeter $= \sqrt{(x'(t)^2 + y'(t)^2)}\,dt$ where $x'(t)$, $y'(t)$ are the tangent.

So $P = 4\int_0^{\frac{\pi}{2}} \sqrt{(x'(t)^2 + y'(t)^2)}\,dt = 4\int_0^{\frac{\pi}{2}} \sqrt{(2\cos t^2 + 5\cos^2 2t)}\,dt$.

To. piece wise linearly approximate the perimeter.

we divide $[0, \frac{\pi}{2}]$ to n interval $[t_0, t_1] \ldots [t_{n-1}, t_n]$

for each pair compute ~~(y(t_1) - y(t_0))~~ $(y(t_{i+1}) - y(t_i))$

and sum them up, the total P is

$$P = \sum_{i=0}^{n-1} |f(t_{i+1}) - f(t_i)|$$

$$= 4\sum_0^{n-1} |f(t_{i+1}) - f(t_i)|$$ where n is $\frac{\pi}{2}$

$n-1$ is almost $\frac{\pi}{2}$.

2. a) Translation and translation. (Commute ✓)

proof: assume 2 translation. $T_1 = \begin{bmatrix} 1 & 0 & t_1 \\ 0 & 1 & t_2 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad T_2 = \begin{bmatrix} 1 & 0 & t_3 \\ 0 & 1 & t_4 \\ 0 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_1 \\ 0 & 1 & t_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} x + t_1 \\ y + t_2 \\ 1 \end{bmatrix} \underset{T}{\Rightarrow} \begin{bmatrix} 1 & 0 & t_3 \\ 0 & 1 & t_4 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} x + t_1 + t_3 \\ y + t_2 + t_4 \\ 1 \end{bmatrix}$$

And if we do $T_2 T_1$

$$\begin{bmatrix} x \\ y' \\ 1 \end{bmatrix} \underset{T_2}{\Rightarrow} \begin{bmatrix} x + t_3 \\ y + t_4 \\ 1 \end{bmatrix} \underset{T_1}{\Rightarrow} \begin{bmatrix} 1 & 0 & t_1 \\ 0 & 1 & t_2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} x + t_3 + t_1 \\ y + t_4 + t_2 \\ 1 \end{bmatrix}$$

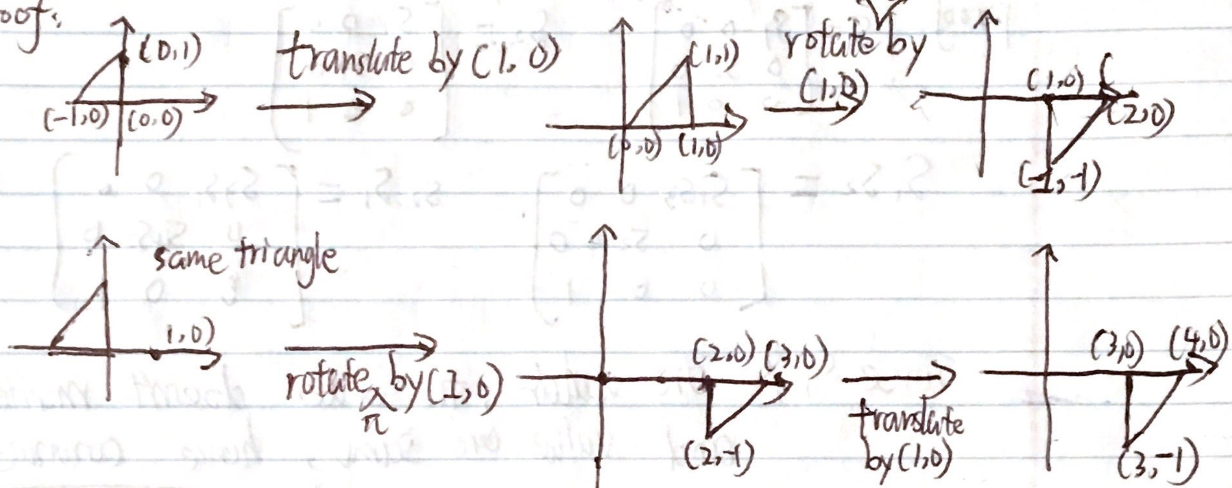
same.

Commute

QED.

b). translation and rotation <u>not commute</u>

proof:



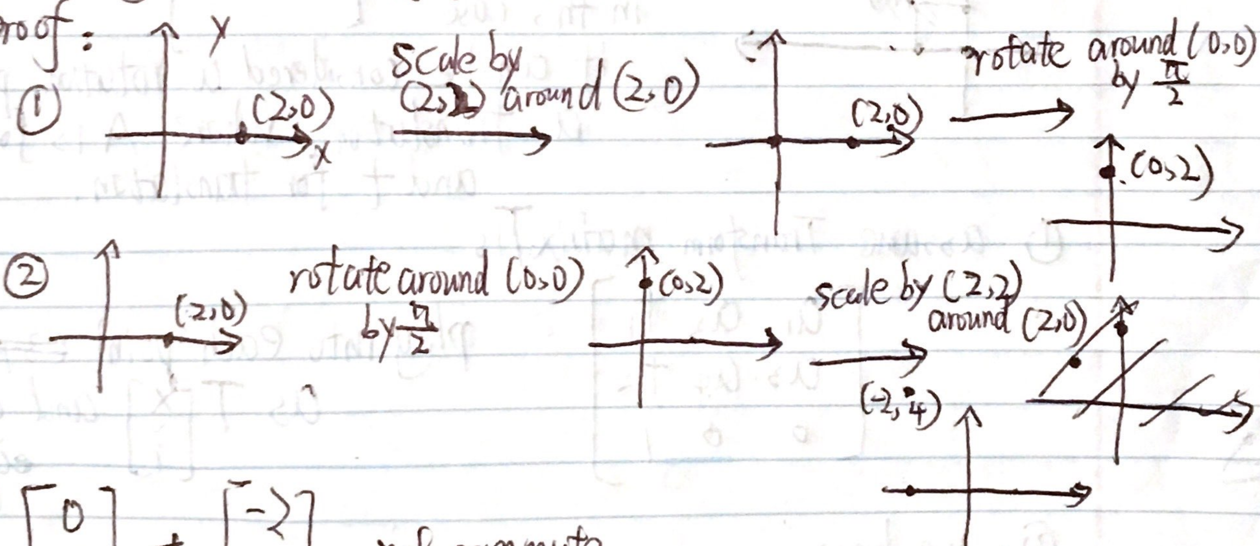
$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix} \neq \begin{bmatrix} 3 & 4 & 3 \\ 0 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

<u>hence not commute</u>

Q.E.D.

c). Scaling, rotation diff fixed point <u>not commute</u>

proof:



$$\begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \neq \begin{bmatrix} -2 \\ 4 \\ 1 \end{bmatrix}$$

<u>not commute</u>

i.e. since in ① the scale is around 'origin', so no effect in that case.

Q.E.D.

D). scale and scale at fixed point commute,

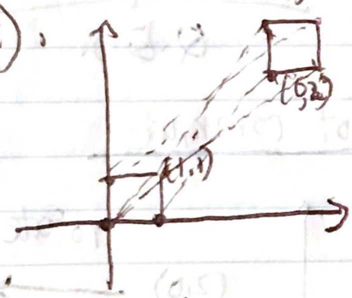
proof: $S_1 = \begin{bmatrix} s_1 & 0 & 0 \\ 0 & s_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad S_2 = \begin{bmatrix} s_3 & 0 & 0 \\ 0 & s_4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$S_1 S_2 = \begin{bmatrix} s_1 s_3 & 0 & 0 \\ 0 & s_2 s_4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad S_2 S_1 = \begin{bmatrix} s_3 s_1 & 0 & 0 \\ 0 & s_4 s_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since S_1, S_2 are scalar, the order doesn't matter and value are same, hence commute

3). (1,0) (0,1) (1,1) (0,0) → (6,2) (7,3) (6,3) (7,2)

a).



Affine transform $\begin{bmatrix} [A] & [t] \\ 0\ 0 & 1 \end{bmatrix}$

in this case

it can be considered a rotation plus a translation. where A is for rotation and t for translation.

① assume Transform matrix T is

$$\begin{bmatrix} a_1 & a_2 & t_1 \\ a_3 & a_4 & t_2 \\ 0 & 0 & 1 \end{bmatrix}$$

plug into each point ~~eq~~ as $T\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ and get the equation

②. we have.

$\begin{cases} a_1 + t_1 = 6 \\ a_3 + t_2 = 2 \end{cases}$ for (1,0) $\qquad \begin{cases} a_2 + t_1 = 7 \\ a_4 + t_2 = 3 \end{cases}$ for (0,1)

$\begin{cases} a_1 + a_2 + t_1 = 6 \\ a_3 + a_4 + t_2 = 3 \end{cases}$ for (1,1) $\qquad \begin{cases} t_1 = 7 \\ t_2 = 2 \end{cases}$ for (0,0).

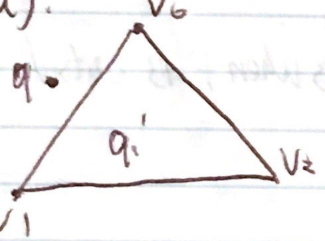
③. compute the equations we get $\begin{bmatrix} -1 & 0 & 7 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$a_1 = -1 \quad a_2 = 0 \quad t_1 = 7$
$a_3 = 0 \quad a_4 = 1 \quad t_2 = 2.$

b). $\begin{bmatrix} p_1' \\ p_2' \\ 1 \end{bmatrix} = T \begin{bmatrix} p_1 \\ p_2 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 7 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} -2+7 \\ 5+2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 1 \end{bmatrix}$

So (2,5) maps to (5,7)

4). a).



If we choose $\overset{V_1V_0}{\cancel{V_0V_1}}$ V_2 is on the right of V_0V_1.
we choose V_0V_2 V_1 is on the right of V_0V_2
we choose V_2V_1 V_0 is on the right of V_2V_1
and this is the same for q: if q is inside a triangle
So the q will be in the ~~convell~~ convex hull of triangle $V_0V_1V_2$.

$q = V_0 + aV_1 + bV_2$

$a = \dfrac{\det(qV_2) - \det(V_0V_2)}{\det(V_1V_2)} \qquad b = -\dfrac{\det(qV_1) - \det(V_0V_1)}{\det(V_1V_2)}.$

We compute the value of a and b.
if $a, b > 0$ and $a+b < 1$, then the point is in the ~~trig~~ triangle

Q.E.D.

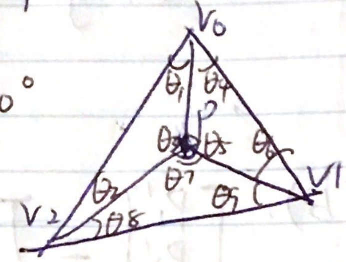
b). if $a, b > 0$ and $a+b = 1$, it is on the edge of the triangle.

More explanation to Q4.

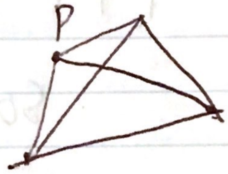
a). My original thought was connect P to V_0, V_1, V_2 and see if

$\theta_1+\theta_2+\theta_3 = \theta_4+\theta_5+\theta_6 = \theta_7+\theta_8+\theta_9 = 180^\circ$

Then it should be in the triangle.



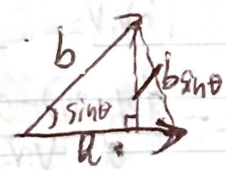
but it's not easy to find ~~the if~~ if it is on the edge. Then I look up online and see the determinat method.



b). Since $a+b<1$ is when P is in the triangle, ~~I~~ we could say if $a+b=1$, is ~~on~~ the point is on ~~a~~ one of the edge, and $a+b>1$ is when P is outside the ~~tri~~ triangle

c). Area $\Rightarrow$ ~~$\frac{\vec{a}\times\vec{b}}{2}$~~

$a\times b = \vec{a}\cdot|a||b|\sin\theta$.



the h is $|b|\sin\theta$

so $\text{Area} = \frac{\vec{a}\times\vec{b}}{2} = \frac{|a||b|\sin\theta}{2} = \frac{|a|\cdot h}{2}$

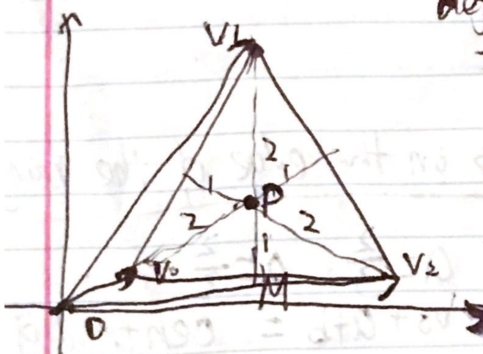
to find centroid.

~~$P = V_0 + aV_1 + bV_2$~~

define an origin O

find ~~$\overrightarrow{OP}$~~ $\overrightarrow{OV_1}$, $\overrightarrow{OV_2}$, $\overrightarrow{OV_3}$

Centre point is P, midpoint between V_0, V_2 is M.



$\overrightarrow{OM} = \frac{1}{2}(\overrightarrow{OV_0}+\overrightarrow{OV_2})$

$\overrightarrow{OP} = \overrightarrow{OV_1} + \frac{2}{3}(\overrightarrow{V_1M})$

$\overrightarrow{V_1M} = \overrightarrow{OM} - \overrightarrow{OV_1} = \frac{1}{2}(\overrightarrow{OV_0}+\overrightarrow{OV_2}) - \overrightarrow{OV_1}$

$\therefore \overrightarrow{OP} = \overrightarrow{OV_1} + \frac{2}{3}(\overrightarrow{V_1M})$

$= \overrightarrow{OV_1} + \frac{1}{3}\overrightarrow{OV_0} + \frac{1}{3}\overrightarrow{OV_2} - \frac{2}{3}\overrightarrow{OV_1}$

$\boxed{= \frac{\overrightarrow{OV_1}+\overrightarrow{OV_2}+\overrightarrow{OV_0}}{3}}$ Q.E.D.