ASTRO 206 Homework 2

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1 Special Relativity (10 points)

a) Time dilation equation: $\Delta t' = \dfrac{\Delta t}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

Solve for v:

$$\Delta t' \sqrt{1 - \frac{v^2}{c^2}} = \Delta t$$

$$(\Delta t')^2 (1 - \frac{v^2}{c^2}) = (\Delta t)^2$$

$$1 - \frac{v^2}{c^2} = \frac{(\Delta t)^2}{(\Delta t')^2}$$

$$\frac{v^2}{c^2} = 1 - \frac{(\Delta t)^2}{(\Delta t')^2}$$

$$v^2 = c^2 \left(1 - \frac{(\Delta t)^2}{(\Delta t')^2}\right)$$

$$v = c\sqrt{1 - \frac{(\Delta t)^2}{(\Delta t')^2}}$$

To live 8 times longer, the ratio $\Delta t' : \Delta t$ would be 8:1. Therefore, $\Delta t' = 8$ and $\Delta t = 1$.

$$v = 3e8\sqrt{1 - \frac{1^2}{8^2}} = 3e8\sqrt{1 - \frac{1}{64}}$$

$v = \mathbf{2.976e8\ m/s}$

b) Time dilation equation (using Lorentz factor): $\Delta t' = \gamma \Delta t$
Solve for γ:

$$\gamma = \frac{\Delta t'}{\Delta t}$$

$$\gamma = 2$$

This conclusion can be reached qualitatively as well considering that the person in motion is experiencing half the time as a person at rest, and that derivation was shown in (a).

Length contraction equation (using Lorentz factor): $L = \dfrac{L_0}{\gamma}$
Solve for L:

$$L = \frac{1m}{2}$$

$L = \mathbf{.5m}$

2 Gravitational Redshift (10 points)

Givens:

Gravitational redshift equation (given): $\dfrac{\lambda_2}{\lambda_1} = \left[\dfrac{1 - \dfrac{2GM}{r_2 c^2}}{1 - \dfrac{2GM}{r_1 c^2}}\right]^{\frac{1}{2}}$.

$G = 6.67\text{e-}11\ m^3 kg^{-1} s^{-2}$
$c = 3\text{e}8\ ms^{-1}$
$M_{sun} = M_{white\_dwarf} = 1.989\text{e}30$ kg
$r_{sun} = 6.95\text{e}8$ m
$r_{white\_dwarf} = (0.01) * r_{sun} = 6.95\text{e}6$ m

Calculations:
Let $\lambda_1 = \lambda_{sun}$ and $\lambda_2 = \lambda_{white\_dwarf}$.

$$\frac{\lambda_2}{\lambda_1} = \left[\frac{1 - \frac{2GM}{r_2 c^2}}{1 - \frac{2GM}{r_1 c^2}}\right]^{\frac{1}{2}}$$

$$= \left[\frac{1 - \frac{2(6.67e{-}11)(1.989e30)}{(6.95e6)(3e8)^2}}{1 - \frac{2(6.67e{-}11)(1.989e30)}{(6.95e8)(3e8)^2}}\right]^{\frac{1}{2}}$$

$$= \left[\frac{1 - \frac{2948.14}{6.95e6}}{1 - \frac{2948.14}{6.95e8}}\right]^{\frac{1}{2}}$$

$$= \left[\frac{1 - 4.24e{-}4}{1 - 4.24e{-}6}\right]^{\frac{1}{2}}$$

$$= \mathbf{.99979}$$

This fact shows us that there is barely any effect on wavelengths from such a white dwarf due to gravitational redshift. Therefore, we would expect the wavelength from the same hydrogen line from the white dwarf to be **656.16 nm**: nearly identical to the original.

3 Sirius Binary System (30 points)

a) $p_{Sirius} = .379$ arc-seconds

$d_{parsecs} = \dfrac{1}{p_{arc-seconds}}$

$d_{parsecs} = \dfrac{1}{.379\, arcseconds} = \mathbf{2.63\ parsecs}$

$d_{lightyear} = 3.26 * d_{parsecs}$
$d_{lightyear} = 3.26 * 2.63\, parsecs = \mathbf{8.60\ light\ years}$

b) Using the scale in the figure, I approximated the semi major axis of Sirius B about Sirius A to be about 7 arcseconds. From there we can use tigonometry to solve for the semimajor axis in astronomical units.
Givens:
$p = 7\,arcseconds = \left(\dfrac{7}{3600}\right)^{\circ} = .00194^{\circ}$
$d = 2.63\,pc$ (From (a))
$tan(p) = \dfrac{a}{d}$

Solve for a:
$a = d * tan(p) = 2.63 * tan(.00194) = 8.91e{-}5\,pc$
$1\,pc = 206264\,AU$
$a = 8.91e{-}5 * 206264 =$ **18.37 AU**

c) $\lambda_{Sirius\ A} = 292$ nm
Wein's Law: $\lambda_{max} \approx \dfrac{2.9e6}{T(kelvin)}$nm
$T \approx \dfrac{2.9e6}{292} \approx$ **9931 K**
Sirius A is much hotter than the sun which has a temperature of 5778 K.
Sirius A is a class-A star.

d) From Figure 2, a line drawn straight up from about the 10,000K marker on the x-axis itersects the white dwarf luminosity curve at about $20 * L_{\odot}$. Therefore we would estimate the luminosity of Sirius A to be approximately $2.11 * M_{\odot} =$ **4.20e30 kg**.

e) Sirius B's period of its orbit around Sirius A is approximately **50 years**.

f) Kepler's 3rd Law Generalized: $P^2 = \dfrac{4\pi^2}{G(M_A + M_B)}a^3$
Solving for M_B:

$$\frac{GP^2}{4a^3\pi^2} = \frac{1}{(M_A + M_B)}$$

$$\frac{4a^3\pi^2}{GP^2} = M_A + M_B$$

$$\frac{4a^3\pi^2}{GP^2} - M_A = M_B$$

$$M_B = \frac{4(18.37AU * 1.5e11\frac{m}{AU})^3\pi^2}{(6.67e{-}11)(50yr * 3.16e7\frac{s}{year})^2} - 4.2e30kg$$

$M_B =$ **7.60e29 kg**

This answer is slightly off from the actual mass of Sirius B, but it is likely to do with the approximation method used in (d); however, it does seem to be the right order of magnitude, and also Sirius A was approximated slightly higher than it should have been.

g) Relationship between magnitude and luminosity: $m_A - m_B = -2.5log_{10}\left(\frac{L_A}{L_B}\right)$

$\Delta m = m_A - m_B = 7.5$

$L_A = 25.4L_{\odot}$

Solve for L_B:

$$\Delta m = -2.5log_{10}\left(\frac{L_A}{L_B}\right)$$

$$\frac{L_A}{L_B} = 10^{\Delta m/2.5}$$

$$L_B = \frac{L_A}{10^{\Delta m/2.5}} = \frac{25.4}{10^3}$$

$L_B = \mathbf{.0254}\ L_{\odot}$

h) Luminosity equation: $L = 4\pi r^2 \sigma T^4$

$\sigma = 5.67 * 10^{-8} \frac{W}{m^2 K^4}$

$L = .0254\, L_{\odot} = 9.77e24\, W$ (From (h))

Solve for r:

$$r = \sqrt{\frac{L}{4\pi\sigma T^4}}$$

$$r = \sqrt{\frac{9.77e24}{4\pi(5.67 * 10^{-8})(25193^4)}}$$

$r = 5.84e6$ m= **5840 km**

i) With this information we can calculate the density of Sirius B.

$m = 7.60e32\ g$ and $V = \frac{4}{3}\pi r^3 = 8.34e26\ cm^3$ (From (f) and (h))

$$\rho = \frac{m}{V}$$

$$\rho = \frac{7.60e32}{8.34e26} = 9.11e5 \frac{g}{cm^3}$$

Knowing that the average density of a neutron star is $2e14 \frac{g}{cm^3}$ and a black hole even more dense, we know that it would be unreasonable to classify Sirius B as such. Additionally, from its small radius we can then conclude that Sirius B is in fact a white dwarf.