Topic Nonlinear Scattering Problems

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1 Bilbiographical searches

Biblio: Lecture Notes on Topological Insulators, in particular Lecture 13. Corresponding JCP paper by B.Hosking,Wang (arxiv version available).
Biblio search: Dirac/Schrödinger equations with cubic nonlinearities. Theories of existence/uniqueness.
Books: Teschl; Reed-Simon; Agmon 1975.

2 Research problem

Consider the linear problem in two space dimensions

$$(H-E)\psi = 0, \qquad H = D_x\sigma_1 + D_y\sigma_2 + y\sigma_3$$

acting on 2-vectors $\psi(x,y)$. We know from lecture 13 that we have a number of solutions

$$\psi(x,y) = e^{i\xi_m(E)x}\phi_m(y)$$

where ϕ_m is a Hermite function and $\xi_m(E)$ a real-valued solution of $E^2 = \xi^2 + 2n$.

Let $w(x,y)$ be a smooth, bounded function. At first, we assume w compactly supported. We want to look at problems of the form

$$(H-E)\psi + w(\psi^* A\psi)\psi = 0,$$

with A a 2×2 matrix, say $A = I_2$ the identity matrix to start with so $\psi^* A\psi = |\psi|^2$. More precisely, we want am *outgoing* solution of the problem

$$\psi + R(E+i0)(w(\psi^* A\psi)\psi) = 0$$

where $R(z) = (H-z)^{-1}$ is the resolvent operator of H. We know that the limits $R(E\pm i0)$ are both defined (and different); see lecture 13.

Question: For A small, can we get an existence result for such solution of Dirac equations with cubic nonlinearity? Do we have a uniqueness result?

Can this be generalized to equations with higher-dimensional scattering theories, such as for instance, for $V(x)$ and $W(x)$ compactly supported,

$$(\Delta + |k|^2 + V(x) + W(x)|u|^2)u = 0.$$

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