

For activities 20.7 and 20.8 we looked at the smallest subgroup of a group containing an element in the group. For activity 20.7 we looked at the smallest subgroup of the group  $(\mathbb{Z}, +)$  containing 5. This subgroup must contain the identity element zero, the inverse of 5, and any element that can be obtained using addition of these three elements (closure property of a group). We proved that the set  $\{5m|m \in \mathbb{Z}\}$  is a subset of this smallest subgroup and went on to prove that the smallest subgroup of  $(\mathbb{Z}, +)$  is a subset of  $\{5m|m \in \mathbb{Z}\}$  and is therefore equal to  $\{5m|m \in \mathbb{Z}\}$ .

In activity 20.8 we generalized what we had done in 20.7 and showed that the smallest subgroup of  $G$  containing  $a$ , denoted  $\langle a \rangle$ , is the set  $\{a^n|n \in \mathbb{Z}\}$ . In this case,  $a^0$  is the identity element in  $G$  and  $a^{-n}$ , where  $n \in \mathbb{N}$ , is the inverse of  $a^n$ .