

For Activity 21.1 we looked at the nontrivial subgroup  $H$  of the cyclic group  $\mathbb{Z}_{100}$ . The smallest number of elements of the subgroup  $H$  containing  $[20]$  is 5. In this case the subgroup  $H = \{[0], [20], [40], [60], [80]\}$ . It should be noted that  $5[20] = [0]$ . For all finite cyclic groups equal to  $\langle a \rangle$  with identity  $e$  and cardinality  $n$ ,  $na = e$ . This is true of  $H$ , because it is also a cyclic group. We will see later that all subgroups of cyclic groups are cyclic groups themselves. There are other subgroups of  $\mathbb{Z}_{100}$  containing  $[20]$  besides the group with cardinality 5. One example is the group  $\{[0], [10], [20], [30], [40], [50], [60], [70], [80], [90]\}$ . This subgroup of  $\mathbb{Z}_{100}$  is equal to  $\langle [10] \rangle$  and  $[20]$  is divisible by  $[10]$ . There is also the subgroup of  $\mathbb{Z}_{100}$  equal to  $\mathbb{Z}_{100}$ , which will also contain  $[20]$ . Overall, the goal of this activity is to look at subgroups of cyclic groups and the properties that these subgroups have. Describing all subgroups of a group is sometimes difficult, but cyclic groups have simpler subgroup structures and can be investigated more thoroughly than other subgroups.