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Reading Assignment 7
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Part (i) in Theorem 21.2 makes sense intuitively after we have proven that there must exist some $k \in \mathbb{Z}^+$ such that $a^k = e$. We could think of the reason for this being true in the following manner. At some point the element a cycles around to itself meaning that $a^i = a$ for some $i \in \mathbb{Z}^+$. Right before this we are at the element a^n in the cycle where n is element a's order. This is because every element after this one is a repeat of the first cycle and so it will not increase the elements order. So, the entire cycle for a is the set $\{a^1, a^2, a^n\}$. After a^n, we repeat the cycle and so $a^{n+1} = a$ and if multiply this equation by a^{-1} on the right side we obtain $a^n = e$. If there were a positive integer j less than n for which $a^j = a$, the magnitude would have to be less than n.

For part (ii) we know that $a^{n \cdot 1} = e$. Using proof by induction we assume that $a^{nm} = e$ for some positive integer m and we can show that $a^{n(m+1)}$ via $a^{n(m+1)} = a^{nm}a^n = e^2 = e$.