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Problem Set #2
January 27, 2013

Activity 18.11
(a) In a group G with identity e, if $ab = e$ for some $a, b \in G$ must it follow that $b = a^{-1}$?
Conjecture. In a group G with identity e, if $ab = e$ for some $a, b \in G$, then $ba = e$ and consequently $b = a^{-1}$.

Proof. Let G be a group with identity e and let $a, b \in G$ such that

$$ab = e. \tag{1}$$

Multiplying a on the right side of (**??**) we obtain

$$(ab)a = ea. \tag{2}$$

Applying the associative property of groups from (**??**) we know that

$$a(ba) = ea. \tag{3}$$

Because e is the identity of G,

$$ea = ae. \tag{4}$$

Applying the transitive property of equality to (**??**) and (**??**) we obtain

$$a(ba) = ae. \tag{5}$$

Applying the group cancellation law to (**??**) we obtain

$$ba = e. \tag{6}$$

In conclusion we have shown that in a group G with identity e, if $ab = e$ for some $a, b \in G$, then $ba = e$. From the fact that $ab = e$ and $ba = e$ we can conclude that b is the inverse of a. □

(a) In a group G with identity e, if $ba = e$ for some $a, b \in G$ must it follow that $b = a^{-1}$?
Conjecture. In a group G with identity e, if $ba = e$ for some $a, b \in G$, then $ab = e$ and consequently $b = a^{-1}$.

Proof. Let G be a group with identity e and let $a, b \in G$ such that

$$ba = e. \tag{7}$$

Multiplying a on the left side of (**??**) we obtain

$$a(ba) = ae. \tag{8}$$

Applying the associative property of groups from (**??**) we know that

$$(ab)a = ae. \tag{9}$$

Because e is the identity of G,

$$ae = ea. \tag{10}$$

Applying the transitive property of equality to (**??**) and (**??**) we obtain

$$(ab)a = ea. \tag{11}$$

Applying the group cancellation law to (**??**) we obtain

$$ab = e. \tag{12}$$

In conclusion we have shown that in a group G with identity e, if $ba = e$ for some $a, b \in G$, then $ab = e$. From the fact that $ab = e$ and $ba = e$ we can conclude that b is the inverse of a. $\square$