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Final Exam
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2.
(a) The operation table for U_{44} is given in Table 1. Explain why $U_44 = \langle[3]\rangle \times \langle[21]\rangle$, the internal direct product of the subgroups $\langle[3]\rangle$ and $\langle[21]\rangle$.

First of all, $\langle[3]\rangle = \{[1],[3],[9],[15],[23],[25],[27],[31],[37]\}$ and $\langle[21]\rangle = \{[1],[21]\}$. From this we see that the intersection of $\langle[3]\rangle$ and $\langle[21]\rangle$ contains only [1]. From Theorem 26.6 (2) we can conclude that each element in $\langle[3]\rangle \times \langle[21]\rangle$ has a unique representation kn where $k \in \langle[3]\rangle$ and $n \in \langle[21]\rangle$. And so, $|\langle[3]\rangle \times \langle[21]\rangle| = |\langle[3]\rangle| \cdot |\langle[21]\rangle| = 10 \cdot 2 = 20$. From the closure property of the group G and the fact that $\langle[3]\rangle$ and $\langle[21]\rangle$ are subgroups of G we know that each element in $\langle[3]\rangle \times \langle[21]\rangle$ is also in G. From this fact and the fact that $\langle[3]\rangle\times\langle[21]\rangle$ and G have the same order, it must be the case that $G = \langle[3]\rangle\times\langle[21]\rangle$

(b)