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Problem Set #1
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3.
The first symmetry is the identity,

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}.$$

The next set of symmetries are reflections. The first is a reflection through the plane containing edge 34 and the midpoint of 12. In permutation notation this is

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 3 & 4 \end{pmatrix} = (12).$$

The next five are the reflections through the plane containing edge 24 and the midpoint of 13, the plane containing edge 23 and the midpoint of 14, the plane containing the edge 14 and the midpoint of 23, the plane containing the edge 13 and the midpoint of 24, and the plane containing the edge 12 and the midpoint of 34. In permutation notation these are

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix} = (13),$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 3 & 1 \end{pmatrix} = (14),$$

$$\begin{pmatrix} 1 & 3 & 2 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} = (23),$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 3 & 2 \end{pmatrix} = (24),$$

and

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 4 & 3 \end{pmatrix} = (34)$$

respectively.
The next set of symmetries are rotations. The first of the three is a 180 degree rotation about the axis through the midpoints of 12 and 34. In permutation notation this is

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} = (12)(34).$$

The next two are all 180 degree rotations about some axis. The first is through the midpoints of 13 and 24 and the second is through the midpoints of 14 and 23. In permutation notation these are

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix} = (13)(24)$$

and

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix} = (14)(23)$$

respectively.
The last set of symmetries are rotations. The first of the three is a 120 degree rotation about the axis through the 1 and the center of $\triangle 123$.