

In Activity 21.4 we proved the following theorem.

Theorem 21.4 *Let G be a finite cyclic group of order n . For each positive divisor m of n , there is exactly one subgroup of G of order m , and these are the only subgroups of G .*

We did this by first showing that there does exist a subgroup of G for some positive divisor m of n . We then showed that two subgroups of order m must be equal to each other. Finally, based on Theorem 21.3 part (ii) the only subgroups of G are ones that have order m .

In Activity 21.5 part (a) we found that all generators of \mathbb{Z}_{30} are of the form $k[n]$ where k is coprime with 30. This satisfied the biconditional statement, a^k is a generator for G if and only if $\gcd(k, n) = 1$. In part (b) we had an example of a group with generator a with order 15 that also satisfies the biconditional statement. Finally, in part (c) we proved this for the general case using Theorem 21.3 part (ii) and Theorem 21.4.