1. Show if $A \subset B$ are non-empty subsets of $\mathbb{R}$ where $B$ is bounded above, then $A$ and $B$ have least upper bounds and
$$\text{lub}(A) \leq \text{lub}(B).$$
Find an example where $\text{lub}(A) = \text{lub}(B)$.

2. Define the greatest lower bound for a set $A \subset \mathbb{R}$. Let $A$ and $B$ be two non-empty subsets of $\mathbb{R}$ which are bounded below. Show
$$\text{glb}(A \cup B) = \min\{\text{glb}(A), \text{glb}(B)\}.$$

3. Suppose that $A$ and $B$ are non-empty subsets of $\mathbb{R}$ that are bounded above. Let
$$A + B = \{a + b \mid a \in A, b \in B\}.$$
Show $A + B$ has a least upper bound and that $\text{lub}(A + B) = \text{lub}(A) + \text{lub}(B)$.

4. Sequences have not been defined in class yet, but for now we will define a sequence $\vec{x}$ as an "infinitely long tuple" of real numbers, i.e.
$$\vec{x} = (x_1, x_2, x_3, \ldots), \text{ where } x_i \in \mathbb{R} \text{ for all } i \in \mathbb{N}.$$
We will say a sequence $\vec{x}$ is bounded if there exists $m, M \in \mathbb{R}$ such that $m \leq x_i \leq M$ for all $i \in \mathbb{N}$. Let $X$ be the set of all bounded sequences. Define a function $d : X \times X \to \mathbb{R}$ by
$$d(\vec{x}, \vec{y}) = \text{lub}\{|x_i - y_i| : \ i \in \mathbb{R}\}.$$
Show that $(X, d)$ is a metric space.

5. Consider the metric space $\mathbb{R}^2$ with the standard Euclidean metric
$$d((x_1, y_1), (x_2, y_2)) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$
Show that the upper half plane,
$$\mathbb{H} = \{(x, y) : y > 0\}$$
is an open set.

**Hint:** You can show this with a roughly geometric argument. Work with the square of the metric using the fact that $f : [0, \infty) \to [0, \infty)$ with $f(x) = x^2$ is an increasing function.