MTH151 **Name:** Solution

Practice Exam 2 **Section:** ____________

This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, ask for an extra sheet of paper to continue the problem on; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	4	
2	4	
3	4	
4	4	
5	4	
6	5	
Total:	25	

1. (4 points) Evaluate the indefinite integrals below.

a) $\int \tan(x)\sec^3(x)\,dx$ Let $u = \sec x$ $du = \sec x \tan x\, dx$

$= \int \sec^2(x) \sec x \tan x\, dx$

$= \int u^2\, du = \frac{u^3}{3} + C = \frac{\sec^3(x)}{3} + C$

b) $\int \sin^5(x)\cos^4(x)\,dx$

$= \int \sin(x) \sin^4(x) \cos^4(x)\, dx$

$= \int \sin(x) \left(1 - \cos^2(x)\right)^2 \cos^4(x)\, dx$ $u = \cos x$
$du = -\sin x\, dx$

$= -\int (1-u^2)^2 u^4\, du$

$= -\int (1-2u^2+u^4)u^4\, du = -\int u^4 - 2u^6 + u^8\, du$

$= -\frac{u^5}{5} + \frac{2u^7}{7} - \frac{u^9}{9} + C = -\frac{\cos^5(x)}{5} + \frac{2\cos^7(x)}{7} - \frac{\cos^9(x)}{9} + C$

2. (4 points) Evaluate the indefinite integrals below.

a) $\int \frac{x}{\sqrt{1-x^4}}\,dx$

Let $u = x^2$ $du = 2x\,dx$

$= \frac{1}{2}\int \frac{du}{\sqrt{1-u^2}}$

Let $u = \sin(w)$ $du = \cos(w)\,dw$

$= \frac{1}{2}\int \frac{\cos w\,dw}{\sqrt{1-\sin^2 w}} = \frac{1}{2}\int dw = \frac{1}{2}w + C$

$= \frac{1}{2}\arcsin(u) + C$

$= \frac{1}{2}\arcsin(x^3) + C$

b) $\int \frac{1}{(x+1)\sqrt{x^2+2x}}\,dx$

Complete the square

$x^2 + 2x + 1 - 1$

$= (x+1)^2 - 1$

$= \int \frac{1}{(x+1)\sqrt{(x+1)^2-1}}\,dx$

Let $(x+1) = \sec\theta$

$dx = \sec\theta\tan\theta\,d\theta$

$\sqrt{(x+1)^2-1} = \tan\theta$

$= \int \frac{\sec\theta\tan\theta}{\sec\theta\cdot\tan\theta}\,d\theta$

$= \int d\theta = \theta = \operatorname{arcsec}(x+1) + C$

3. (4 points) Evaluate the indefinite integrals.

a) $\int \frac{x^2 - x + 4}{x(x^2+4)}\,dx$ $= \int \frac{1}{x} - \frac{1}{x^2+4}\,dx = \ln x - \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$

Partial fraction:

$\frac{x^2-x+4}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$

Note: $\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$

$\Rightarrow \frac{x^2-x+4}{x^2+4} = A + \frac{(Bx+C)x}{(x^2+4)}$ plug in 0

$\Rightarrow A = 1$

$x^2 - x + 4 = 1(x^2+4) + (Bx+C)x$

plug in $x=1$: $4 = 5 + (B+C)$

plug in $x=2$: $6 = 8 + (2B+C)(2)$

$\Rightarrow \begin{cases} B+C=-1 \\ 2B+C=-1 \end{cases} \Rightarrow \begin{matrix} B=0 \\ C=-1 \end{matrix}$

b) $\int \frac{6x^2+13x-5}{(x+1)(x+3)(x-2)}\,dx$ $= \int \frac{2}{x+1} + \frac{1}{x+3} + \frac{3}{x-2}\,dx$

partial fraction

$\frac{6x^2+13x-5}{(x+1)(x+3)(x-2)} = \frac{A}{(x+1)} + \frac{B}{(x+3)} + \frac{C}{(x-2)}$ $= 2\ln|x+1| + \ln|x+3| + 3\ln|x-2| + C$

$A = \frac{6(-1)^2 + (3(-1)) - 5}{(-1+3)(-1-2)} = 2$

$B = \frac{6(-3)^2 + 13(-3) - 5}{(-3+1)(-3-2)} = 1$

$C = \frac{6(2)^2 + 13(2) - 5}{(2+1)(2+3)} = 3$

4. (4 points) Evaluate the improper integral.

a) $\int_0^\infty \frac{e^{-1/x}}{x^2}\,dx$ $= \int_1^\infty \frac{e^{-1/x}}{x^2}\,dx + \int_0^1 \frac{e^{-1/x}}{x^2}\,dx = (1-e^{-1}) + (e^{-1}-0) = 1$

Let $u = -1/x$; $du = 1/x^2\,dx$

$$\int \frac{e^{-1/x}\,dx}{x^2} = \int e^u\,du = e^{-1/x}$$

$\Rightarrow$ $\int_1^\infty \frac{e^{-1/x}}{x^2}\,dx = \lim_{t\to\infty} e^{-1/t} - e^{-1} = 1 - e^{-1}$

$\int_0^1 \frac{e^{-1/x}}{x^2}\,dx = \lim_{t\to 0} e^{-1} - e^{-1/t} = e^{-1} - 0$

b) $\int_0^9 \frac{1}{\sqrt[3]{x-1}}\,dx$ discontinuous at $x=1$

$$\int_0^9 (x-1)^{1/3}\,dx = \int_0^1 (x-1)^{1/3}\,dx + \int_1^9 (x-1)^{1/3}\,dx$$

$$= \lim_{t\to 1} \frac{3(x-1)^{2/3}}{2}\Big|_0^t + \lim_{t\to 1} \frac{3(x-1)^{2/3}}{2}\Big|_t^9$$

$$= \lim_{t\to 1}\left[\frac{3(t-1)^{2/3}}{2} - 3/2\right] + \lim_{t\to 1}\left[\frac{3(9-1)^{2/3}}{2} - \frac{3(t-1)^{2/3}}{2}\right]$$

$$= -3/2 + 12/2 = 9/2$$

5. (4 points) Find the limit of the sequences.

a) $a_n = \dfrac{n^2}{n^3 - 6n + 1}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n^2}{n^3-6n+1}$$

$$= \lim_{n\to\infty} \frac{2n}{3n^2-6}$$

$$= \lim_{n\to\infty} \frac{2}{6n} = 0$$

b) $a_n = n^2 e^{-3n^2}$

$$\lim_{n\to\infty} n^2 e^{-3n^2} = \lim_{n\to\infty} \frac{n^2}{e^{3n^2}}$$

$$= \lim_{n\to\infty} \frac{2n}{6n e^{3n^2}} = 0$$

c) $a_n = \dfrac{\cos^2(n^5)}{3^n}$

$$0 \le \cos^2(n^5) \le 1$$

$$0 \le \lim_{n\to\infty} \frac{\cos^2(n^5)}{3^n} \le \lim_{n\to\infty} \frac{1}{3^n} = 0$$

So $\displaystyle\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\cos^2(n^5)}{3^n} = 0$

6. (5 points) Determine whether or not the series converges.

a) $\sum_{n=2}^{\infty} 5\frac{2^{n-1}}{3^n}$ $= \sum \frac{5}{3}\cdot\left(\frac{2}{3}\right)^{n-1}$ Converges $r = 2/3$

b) $\sum_{n=1}^{\infty} \frac{e^n}{n}$ $\lim_{n\to\infty} \frac{e^n}{n} = \lim_{n\to\infty} e^n = \infty$

Diverges by Test for Divergence.

c) $\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+2}$ $= \sum \left(\frac{1}{n} - \frac{1}{n+1}\right) + \left(\frac{1}{n+1} - \frac{1}{n+2}\right)$ $\underline{\text{Converges}}$

$$
\begin{aligned}
S_n = a_1 &= \left(1 - \cancel{\tfrac{1}{2}}\right) + \left(\tfrac{1}{2} - \cancel{\tfrac{1}{3}}\right) = 1 + \tfrac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2} \\
+ a_2 &+ \left(\cancel{\tfrac{1}{2}} - \cancel{\tfrac{1}{3}}\right) + \left(\cancel{\tfrac{1}{3}} - \cancel{\tfrac{1}{4}}\right) \\
&\vdots \\
+ a_{n-1} &+ \left(\cancel{\tfrac{1}{n-1}} - \cancel{\tfrac{1}{n}}\right) + \left(\cancel{\tfrac{1}{n}} - \cancel{\tfrac{1}{n+1}}\right) \\
+ a_n &+ \left(\cancel{\tfrac{1}{n}} - \frac{1}{n+1}\right) + \left(\cancel{\tfrac{1}{n+1}} - \tfrac{1}{n+2}\right)
\end{aligned}
$$