MATH2210Q

Name: Soln

Exam 1

Date:

This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any unapproved calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, ask for an extra sheet of paper to continue the problem on; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	5	
2	3	
3	4	
4	5	
5	4	
6	4	
Total:	25	

1. (a) (3 points) If possible find the inverse of the following matrix.

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 1 & 4 & 3 & 0 & 1 & 0 \\ 1 & 3 & 4 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 1 & 3 & 4 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 3 & 0 & 4 & 0 & -3 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 7 & -3 & -3 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

(b) (2 points) If possible find the inverse of the following matrices.

a) $B = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

$$B^{-1} = \frac{1}{1(4) - 3(2)} \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix} = -\frac{1}{2} \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$$

b) $C = \begin{bmatrix} 1 & 4 \\ 1 & 4 \end{bmatrix}$

C^{-1} doesnot Exist

$1(4) - 4(1) = 0$

2. (a) (2 points) Let

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 1 & 9 \\ -3 & k \end{bmatrix}.$$

What value(s) of k, if any, will make $AB = BA$.

$$AB = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ -3 & K \end{bmatrix} = \begin{bmatrix} -7 & 18+3K \\ -4 & -9+K \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 9 \\ -3 & K \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 12 \\ -6-K & -9+K \end{bmatrix}$$

Need $-4 = -6 - K$

and $18 + 3K = 12$

Therefore $\boxed{K = -2}$

(b) (1 point) If C is a 5×3 matrix and the product CD is 5×7, what is the size of D?

$$\underset{5 \times 3}{C} \cdot \underset{n \times p}{D} = \underset{5 \times 7}{CD}$$

Need $\boxed{\begin{array}{l} n = 3 \\ p = 7 \end{array}}$

3. (a) (1 point) Find a value k to make the following set of vectors linearly independent.

$$\left\{ \underset{\vec{u}}{\begin{bmatrix} 1 \\ 2 \\ 5 \\ 2 \end{bmatrix}}, \underset{\vec{v}}{\begin{bmatrix} 2 \\ 4 \\ 10 \\ k \end{bmatrix}} \right\}$$

Let $K \neq 4$. Then $\vec{u} \neq c\vec{v}$ for any $c \in \mathbb{R}$.

Determine which of the following sets of vectors are linearly independent.

(b) (1 point) $\left\{ \begin{bmatrix} 4 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} 8 \\ 9 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 7 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix} \right\}$

linearly dependent, more vectors than entries.

(c) (2 points) $\left\{ \begin{bmatrix} 4 \\ -2 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ -3 \end{bmatrix}, \begin{bmatrix} -5 \\ 4 \\ 7 \end{bmatrix}, \right\}$

$$\left[\begin{array}{ccc:c} 4 & 3 & -5 & 0 \\ -2 & -2 & 4 & 0 \\ -2 & -3 & 7 & 0 \end{array}\right] \sim \left[\begin{array}{ccc:c} 2 & 2 & -4 & 0 \\ 4 & 3 & -5 & 0 \\ -2 & -3 & 7 & 0 \end{array}\right] \sim \left[\begin{array}{ccc:c} 2 & 2 & -4 & 0 \\ 0 & -1 & 3 & 0 \\ 0 & -1 & 3 & 0 \end{array}\right] \sim \left[\begin{array}{ccc:c} 1 & 1 & -2 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc:c} 1 & 0 & 1 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

free variables exist, linearly dependent

4. (a) (2 points) Determine if **b** lies in the span of the given vectors.

$$\mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}; \quad \left\{ \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}, \right\}$$

$$\left[\begin{array}{ccc:c} 1 & 0 & 5 & 2 \\ -2 & 1 & -6 & -1 \\ 0 & 2 & 8 & 6 \end{array}\right] \sim \left[\begin{array}{ccc:c} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 2 & 8 & 6 \end{array}\right] \sim \left[\begin{array}{ccc:c} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Consistent, $\vec{b}$ lies in the span.

(b) (3 points) Is the vector $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ contained in the plane generated by the following vectors?

$$\left\{ \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ -2 \end{bmatrix} \right\}$$

$$\left[\begin{array}{cc:c} 3 & 0 & 1 \\ -1 & 5 & 1 \\ 2 & -2 & 1 \end{array}\right] \sim \left[\begin{array}{cc:c} 1 & 0 & 1/3 \\ -1 & 5 & 1 \\ 2 & -2 & 1 \end{array}\right] \sim \left[\begin{array}{cc:c} 1 & 0 & 1/3 \\ 0 & 5 & 4/3 \\ 2 & -2 & 1 \end{array}\right] \sim \left[\begin{array}{cc:c} 1 & 0 & 1/3 \\ 0 & 5 & 4/3 \\ 0 & -2 & 1/3 \end{array}\right] \sim \left[\begin{array}{cc:c} 1 & 0 & 1/3 \\ 0 & 1 & 4/15 \\ 0 & -2 & 1/3 \end{array}\right]$$

$$\sim \left[\begin{array}{cc:c} 1 & 0 & 1/3 \\ 0 & 1 & 4/15 \\ 0 & 0 & 13/15 \end{array}\right]$$

inconsistent system

$\vec{b}$ does not lie in the plane.

5. Consider the following system:

$$\begin{array}{rcrcrcr} x_1 & + & 4x_2 & + & 5x_3 & = & -9 \\ -x_1 & - & 2x_2 & - & x_3 & = & 3 \\ 2x_1 & + & 3x_2 & & & = & -3 \end{array}$$

(a) (2 points) Find the solution set.

$$\left[\begin{array}{ccc:c} 1 & 4 & 5 & -9 \\ -1 & -2 & -1 & 3 \\ 2 & 3 & 0 & -9 \end{array}\right] \sim \left[\begin{array}{ccc:c} 1 & 4 & 5 & -9 \\ 0 & 2 & 4 & -6 \\ 0 & -5 & -10 & 15 \end{array}\right] \sim \left[\begin{array}{ccc:c} 1 & 4 & 5 & -9 \\ 0 & 1 & 2 & -3 \\ 0 & 1 & 2 & -3 \end{array}\right] \sim \left[\begin{array}{ccc:c} 1 & 4 & 5 & -9 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & -3 & 3 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 1 & 0 \end{array}\right] \Leftrightarrow \begin{array}{l} x_1 - 3x_3 = 3 \\ x_2 + 2x_3 = -3 \end{array} \Leftrightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

(b) (2 points) Write the equivalent matrix equation and vector equation.

$$x_1 \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -9 \\ 3 \\ -3 \end{bmatrix}$$

and

$$\begin{bmatrix} 1 & 4 & 5 \\ -1 & -2 & -1 \\ 2 & 3 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -9 \\ 3 \\ -3 \end{bmatrix}$$

6. Consider the following matrix.

$$A = \begin{bmatrix} 4 & -5 & -1 & 8 \\ 3 & -7 & -4 & 2 \\ 5 & -6 & -1 & 4 \\ 9 & 1 & 10 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -5 & -1 & 8 \\ 3 & -7 & -4 & 2 \\ 5 & -6 & -1 & 4 \\ 9 & 1 & 10 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 6 \\ 3 & -7 & -4 & 2 \\ 5 & -6 & -1 & 4 \\ 9 & 1 & 10 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 6 \\ 0 & -13 & -13 & -16 \\ 0 & -16 & -16 & -26 \\ 0 & -17 & -17 & -47 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 6 \\ 0 & -13 & -13 & -16 \\ 0 & -3 & -3 & -10 \\ 0 & -4 & -4 & -31 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 6 \\ 0 & 10 & 10 & 6 \\ 0 & 3 & 3 & 10 \\ 0 & -4 & -4 & -31 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 6 \\ 0 & 10 & 10 & 6 \\ 0 & 3 & 3 & 10 \\ 0 & 1 & 1 & 21 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 6 \\ 0 & 1 & 1 & 21 \\ 0 & 3 & 3 & 10 \\ 0 & 5 & 5 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 6 \\ 0 & 1 & 1 & 21 \\ 0 & 0 & 0 & -53 \\ 0 & 0 & 0 & -102 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) (2 points) If $T(\mathrm{x}) = A\mathrm{x}$ is T a one to one transformation?

$T: \mathbb{R}^4 \longrightarrow \mathbb{R}^4$

No,
free variables exist.

(b) (2 points) If $T(\mathrm{x}) = A\mathrm{x}$ is T onto?

No, only 3 pivots.