

MTH354  
Homework 5

1. Suppose  $f : [a, b] \rightarrow \mathbb{R}$  is Riemann integrable. Prove,

$$\lim_{c \rightarrow b, c < b} \int_a^c f \, dx = \int_a^b f(x) \, dx$$

2. Show that if  $f$  is a continuous real valued function on the interval  $[a, b]$  then

$$\int_a^b f(x) \, dx = f(\xi)(b - a)$$

for some  $\xi \in [a, b]$ .

3. Let  $c < a < b < d$  and define  $\chi_{[a,b]} : [c, d] \rightarrow \mathbb{R}$  as follows:

$$\chi_{[a,b]}(x) = \begin{cases} 1 & x \in [a, b] \\ 0 & x \notin [a, b] \end{cases}$$

Show starting from the definition that  $\chi_{[a,b]}$  is Riemann integrable on  $[c, d]$  and compute

$$\int_c^d \chi_{[a,b]}(x) \, dx.$$

4. Prove that if  $f$  is a continuous real valued function on the interval  $[a, b]$  such that  $f(x) \geq 0$  for all  $x \in [a, b]$  and  $f(x_0) > 0$  for some  $x_0 \in [a, b]$  then  $\int_a^b f(x) \, dx > 0$ .
5. Give an example of a Riemann integrable function  $g : [a, b] \rightarrow [a, b]$  such that  $g(x) \geq 0$  for all  $x \in [a, b]$  and  $g(x_0) > 0$  for some  $x_0 \in [a, b]$  such that  $\int_a^b g(x) \, dx = 0$ .