MTH107   **Name:**_____

Practice Exam 1   **Section:**_____

This exam contains 5 pages (including this cover page) and 7 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books or notes on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.

- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

- If you need more space, ask for an extra sheet of paper to continue the problem on; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	6	
2	2	
3	5	
4	2	
5	3	
6	3	
7	4	
Total:	25	

1. Let
$$U = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$$
and
$$A = \{a, b, c, d, e, f, g, h, i, j, k\} \qquad B = \{a, e, i, o, u\}.$$

(a) (3 points) Mark the statements below as true or false. If the statement is false explain why.

     i) $A \subseteq U$ **T**

     ii) $A \in U$ **F** A is a subset.

     iii) $\emptyset \subseteq B$ **T**

     iv) $B \in A$ **F** B is a set and not an element of A.

     v) $B \subset A$ **F**   The elements o and u are not in A.

     vi) $B \subset A \cup U$ **T**

(b) (3 points) Construct the following sets

     i) $A \cup B = \{a, b, c, d, e, f, g, h, i, j, k, o, u\}$

   ii) $A \cap B = \{a, e, i\}$

     iii) $A \cup B \cup U = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$

   iv) $A \cap U = \{a, b, c, d, e, f, g, h, i, j, k\}$

     v) $A^c = \{l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$

     vi) $B^c = \{b, c, d, f, g, h, j, k, l, m, n, p, q, r, s, t, v, w, x, y, z\}$

2. (2 points) Suppose a number of people were surveyed about the movies they saw last weekend. Of those interviewed 30 people only saw the new action movie, 40 only saw the new romantic comedy, and 20 people claimed they saw both. Let A be the set of people who saw the action movie and R the set of people who saw the romantic comedy. Draw and label a venn diagram to represent the survey results.
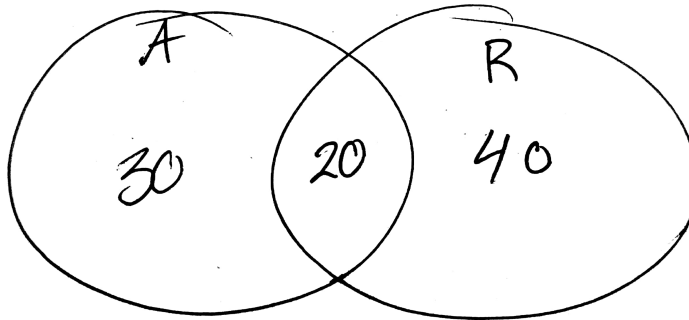
3. (5 points) We are given the following information: $n(U) = 1000$, $n(A) = 290$, $n(B) = 490$, $n(C) = 570$, $n(A \cap B) = 130$, $n(A \cap C) = 140$, $n(B \cap C) = 230$, and $n(A \cap B \cap C) = 80$. Draw a venn diagram to represent this information.
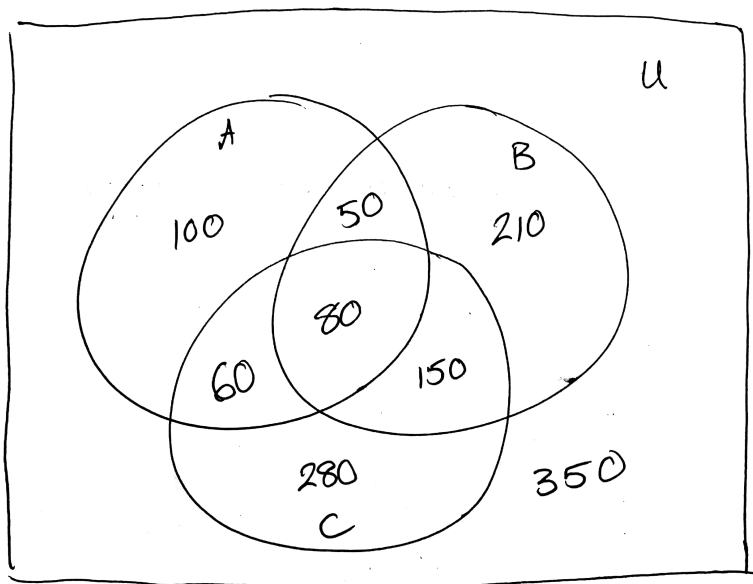
4. In a survey of 200 people, it was found that 190 people own a cat or dog, 145 people own a cat, and 85 people own a dog. Let C be the set of cat owners and D the set of dog owners.

   (a) (1 point) Find $n(C \cap D)$

$$n(C \cap D) = n(C) + n(D) - n(C \cup D) = 145 + 85 - 190 = 40$$
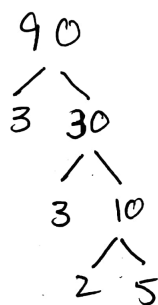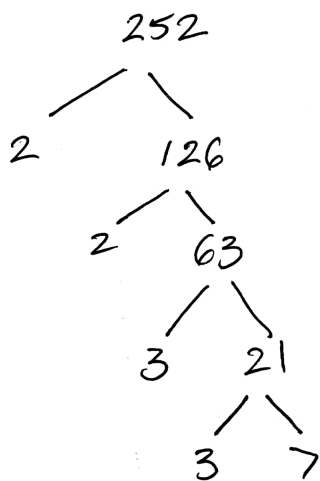
   (b) (1 point) Find $n((C \cup D)^c)$

$$n((C \cup D)^c) = 200 - 190 = 10$$

5. (3 points) Find all the primes under 30 using the Sieve of Eratosthenes.

②, ③, 4̶, ⑤, 6̶, ⑦, 8̶, 9̶, 1̶0̶
⑪, 1̶2̶, ⑬, 1̶4̶, 1̶5̶, 1̶6̶, ⑰, 1̶8̶, ⑲, 2̶0̶
2̶1̶, 2̶2̶, ㉓ 2̶4̶, 2̶5̶, 2̶6̶, 2̶7̶, 2̶8̶, ㉙ 3̶0̶

6. (3 points) Let $a = 252$ and $b = 90$. Find the prime factorizations of the following numbers then use the prime factorizations to find the $\gcd(a,b)$ and $\operatorname{lcm}(a,b)$.

$$252 = 2^2 \cdot 3^2 \cdot 7^1 = 2^2 \cdot 3^2 \cdot 5^0 \cdot 7^1$$
$$90 = 2^1 \cdot 3^2 \cdot 5 = 2^1 \cdot 3^2 \cdot 5^1 \cdot 7^0$$

$$\gcd(252,90) = 2^1 \cdot 3^2 \cdot 5^0 \cdot 7^0 = 18$$

$$\operatorname{lcm}(252,90) = 2^2 \cdot 3^2 \cdot 5^1 \cdot 7^1 = 1260$$

7. (4 points) Solve the following Diophantine equation.

$$65x + 27y = 3$$

$$65 = 2(27) + 11$$
$$27 = 2(11) + 5$$
$$11 = 2(5) + 1$$
$$5 = 5(1) + 0$$

$$\underline{65x + 27y = 1}$$
$$1 = 11 - 2(5)$$
$$1 = 11 - 2(27 - 2(11))$$
$$1 = 5(11) - 2(27)$$
$$1 = 5(65 - 2(27)) - 2(27)$$

$$\underline{1 = 5(65) - 12(27)}$$

$$\Rightarrow 3 = 15(65) - 36(27)$$

$$x_0 = 15 \qquad y_0 = -36$$

$$x = x_0 + \left(\tfrac{b}{d}\right)n \qquad y = y_0 - \left(\tfrac{a}{d}\right)n$$

$$\Rightarrow x = 15 + 27n \qquad y = -36 - 65n$$