For full credit, you must <u>show all work</u> and <u>circle</u> your final answer.

1 | Find the solution set to the following system of equations. (Write it in parametric form.)

$$
\begin{aligned}
x_1 &+ 2x_2 - 3x_3 &= 5 \\
2x_1 &+ x_2 - 3x_3 &= 13 \\
-x_1 &+ x_2 &= -8
\end{aligned}
$$

$$
\begin{bmatrix} 1 & 2 & -3 & 5 \\ 2 & 1 & -3 & 13 \\ -1 & 1 & 0 & -8 \end{bmatrix} \sim
\begin{bmatrix} 1 & 2 & -3 & 5 \\ 0 & -3 & 3 & 3 \\ 0 & 3 & -3 & -3 \end{bmatrix} \sim
\begin{bmatrix} 1 & 2 & -3 & 5 \\ 0 & -3 & 3 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim
\begin{bmatrix} 1 & 2 & -3 & 5 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim
\begin{bmatrix} 1 & 0 & -1 & 7 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}
$$

$$
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad t \text{ is a real } \#
$$

2   Determine which of the following sets of vectors are linearly independent.

(a) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 9 \end{bmatrix}, \begin{bmatrix} 8 \\ 9 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 5 \\ 2 \\ 7 \end{bmatrix} \right\}$

linearly dependent , contains the zero vector

(b) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 7 \\ 9 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} \right\}$

linearly dependent, more vectors than entries

(c) $\left\{ \begin{bmatrix} 0 \\ 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 4 \\ -4 \end{bmatrix}, \begin{bmatrix} 9 \\ -7 \\ -5 \\ -2 \end{bmatrix}, \right\}$

$$\begin{bmatrix} 0 & -3 & 9 & \vdots & 0 \\ 2 & 1 & -7 & \vdots & 0 \\ -1 & 4 & -5 & \vdots & 0 \\ 1 & -4 & -2 & \vdots & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & -2 & 0 \\ 2 & 1 & -7 & 0 \\ -1 & 4 & -5 & 0 \\ 0 & -3 & 9 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & -2 & 0 \\ 0 & 9 & -3 & 0 \\ 0 & 0 & -7 & 0 \\ 0 & -3 & 9 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & -2 & 0 \\ 0 & 3 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 8 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -4 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{array}{l} x_1 = 0 \\ x_2 = 0 \\ x_3 = 0 \end{array}$$ linearly independent

3   Write the following vector equation as a matrix equation

$$x_1 \begin{bmatrix} 4 \\ -1 \\ 7 \\ -4 \end{bmatrix} + x_2 \begin{bmatrix} -5 \\ 3 \\ -5 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 7 \\ -8 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 0 \\ -7 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -5 & 7 \\ -1 & 3 & -8 \\ 7 & -5 & 0 \\ -4 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 0 \\ -7 \end{bmatrix}$$