Exam 2A

Course ID:	**MAC 2312**
Course Title:	Calculus II
Date of Exam:	July 17th 2013
Duration of Exam:	90 minutes

Instructions

A. Sign your scantron sheet in the white area on the back in ink.

B. Write and code in the spaces indicated:

1) Name (last name, first name, middle initial)

2) UF ID number

3) Section number

C. Under "special code" code the test ID numbers 2 (1st row), 1 (2nd row).

1 • 3 4 5 6 7 8 9 0
• 2 3 4 5 6 7 8 9 0

D. Under "form code" code in A.

• B C D E

E. While taking the test, please keep your answer sheet covered or turned over at all times.

F. This test consists of 14 multiple choice questions and 4 free response questions. No calculators are allowed.

G. **When you are finished:**

1) Before turning in your test check for transcribing errors. No changes may be made after submitting your scantron.

2) You must turn in your scantron and tear off sheets to your instructor. Be prepared to show your picture ID with a legible signature.

3) The answers will be posted within one day after the exam.

The following questions are worth 6 points each.

1. Find the radius and interval of convergence for

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)2^{2n+1}} x^{2n+1}.$$

A. $R = 2$, $I = (-2, 2]$

B. $R = 2$, $I = [-2, 2]$

C. $R = 2$, $I = [-2, 2)$

D. $R = 2$, $I = (-2, 2)$

E. $R = 0$, The sum only converges at $x = 2$.

2. Which of the following is true about $\displaystyle\sum_{n=1}^{\infty} \frac{\sin\left(\frac{n\pi}{2}\right)}{\left(\frac{n\pi}{2}\right)}$?

A. Divergent by test for divergence.

B. Convergent by limit comparison test.

C. Convergent by direct comparison test with a p-series.

D. Convergent by alternating series test.

E. Convergent by integral test.

3. Find the sum if possible.

$$\sum_{n=1}^{\infty} 4 \cdot \frac{3^{n+1}}{5^n}$$

A. 10 B. 30 C. 18 D. 5/2 E. The series is divergent.

4. Find the sum of the series.

$$\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{6^{2n+1}(2n+1)!}$$

A. $\frac{\sqrt{3}}{2}$ B. $\frac{1}{2}$ C. 1 D. 0 E. $\frac{6\pi}{36-\pi^2}$

5. Evaluate as a power series.

$$\int \frac{1}{4+x^4}\, dx$$

A. $\sum_{n=0}^{\infty} \frac{(-1)^n}{(4n+1)4^n} x^{4n+1} + C$

B. $\sum_{n=0}^{\infty} \frac{(-1)^n}{(4n+1)4^{n+1}} x^{4n} + C$

C. $\sum_{n=0}^{\infty} \frac{(-1)^n}{(4n)4^{n+1}} x^{4n+1} + C$

D. $\sum_{n=0}^{\infty} \frac{(-1)^n}{(4n+1)4^{n+1}} x^{4n+1} + C$

E. $\sum_{n=0}^{\infty} \frac{(-1)^n}{(4n+2)4^{n+1}} x^{4n+1} + C$

6. According to the alternating series estimation theorem, which one of the following choices gives the upper bound on the error by using the 10-th partial sum s_{10} to approximate the actual value of the sum of the following series?

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2 + 3^n}$$

A. $\dfrac{1}{11^2 + 3^{11}}$ B. $\dfrac{1}{10^2 + 3^{10}}$ C. $\dfrac{-1}{11^2 + 3^{11}}$ D. $\dfrac{1}{9^2 + 3^9}$ E. $\dfrac{-1}{9^2 + 3^9}$

7. Which of the following is true about $\displaystyle\sum_{n=2}^{\infty} \frac{\sqrt{n^4 + 2}}{n^4 + n^2 + 1}$?

A. Converges by root test.

B. Converges by ratio test.

C. Diverges by integral test.

D. Diverges by test for divergence.

E. Converges by limit comparison test.

8. How many of the following are true?

I. If $\sum a_n$ is conditionally convergent, then $\sum |a_n|$ is divergent.

II. If $0 \le a_n \le b_n$ and $\sum b_n$ diverges, then so does $\sum a_n$.

III. If $a_n \to 0$ as $n \to \infty$, then $\sum a_n$ converges.

IV. The sequence $\left\{\frac{1+\sin n}{n^2}\right\}_{n=1}^{\infty}$ converges.

V. If a_n is an increasing sequence and bounded above, then a_n converges.

A. 1 B. 2 C. 3 D. 4 E. 5

9. Find the Taylor series centered at 1 for f given that

$$f^{(n)}(1) = \frac{(-1)^n n!}{3^n n^n}.$$

A. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (x-1)^n}{3^n n^n}$

B. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{3^n n^n}$

C. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (x-1)^n}{n! 3^n n^n}$

D. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n! 3^n n^n}$

E. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n n! (x-1)^n}{3^n n^n}$

10. How many of the following are true?

I. According to the integral test, $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2} = \int_1^{\infty} \frac{1}{x^2}\, dx.$

II. According to the limit comparison test, $\displaystyle\sum_{n=1}^{\infty} \tan\left(\frac{1}{n}\right)$ converges.

III. According to the test for divergence, $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.

IV. According to the alternating series test, $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n+1}$ converges.

A. 0 B. 1 C. 2 D. 3 E. 4

Bonus Questions.

$\boxed{\text{The following questions are worth 2 points each.}}$

11. If $\{a_n\}$ and $\{b_n\}$ are divergent sequences, then $\{a_n + b_n\}$ is divergent.

    A. True   B. False.

12. If $-1 < \alpha < 1$, then $\lim\limits_{n\to\infty} \alpha^n = 0$.

    A. True   B. False.

13. The ratio test can be used to determine whether or not $\sum \dfrac{1}{n^3}$ converges.

    A. True   B. False.

14. If $a_n > 0$ and $\sum a_n$ is convergent, then $\sum(-1)^n a_n$ converges.

    A. True   B. False.

MAC2312 **Name:**______________________

Exam 2A **Section:**______________________

Instructions: You must show all work to receive full credit.

1a. Find the power series representation of

$$f(x) = \frac{1}{(x+2)}.$$

1b. Use part a to find the power series for

$$f(x) = \frac{x^3}{(x+2)^2}.$$

2. Find the Taylor series for $f(x) = \dfrac{1}{x}$ centered at $a = -4$.

Consider the following series,

$$\sum_{n=2}^{\infty} \ln\left(\frac{n}{n+2}\right).$$

3a. Find the n^{th} partial sum S_n.

3b. Determine whether the series converges or diverges.

4a. State the ratio test.

If $\sum a_n$ is a series, and $L = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$, then if,

(1)

(2)

(3)

4b. Determine if the following series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{n^n}{(2n)!}$$

University of Florida Honor Code:

On my honor, I have neither given nor received unauthorized aid in doing this assignment.

Signature:________________________