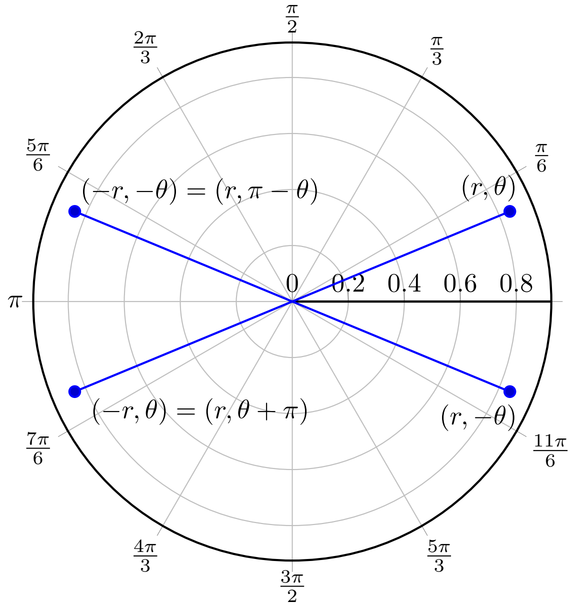
1 Polar Coordinates

Polar coordinates are coordinates of the form (r, θ), where r represents a distance from the origin $(0, 0)$, called the pole, and θ is the angle relative to the polar axis going counter-clockwise. We consider $(-r, \theta)$ to be at angle θ but directly across the pole. This is shown below,



As you can see above, representation of coordinates in polar form is not unique. In fact,

$$(r, \theta) = (r, \theta \pm 2n\pi) = (-r, \pm(2n + 1)\pi)$$

represent the same point. To convert between polar and cartesian coordinates you have the following relations,

$$x = r\cos(\theta); \quad y = r\sin(\theta)$$

$$r^2 = x^2 + y^2; \quad \tan(\theta) = \frac{y}{x}$$

2 Circles.

Some of the basic equations for circles are,

$$r = a$$

$$r = a\cos(\theta)$$

$$r = a\sin(\theta).$$

We note that these are drawn once as θ goes from 0 to 2π.

1) $r = a$ is a circle of radius a centered at the origin.

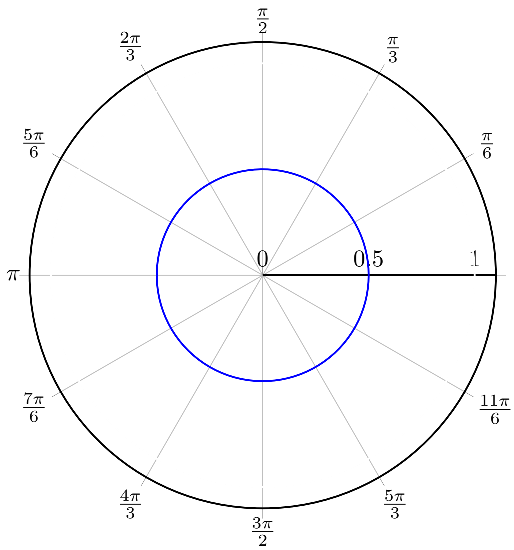


Figure 1: $r = \frac{1}{2}$.

2) $r = a\cos(\theta)$ is a circle of radius $a/2$ centered at the cartesian point $\left(\frac{a}{2}, 0\right)$.

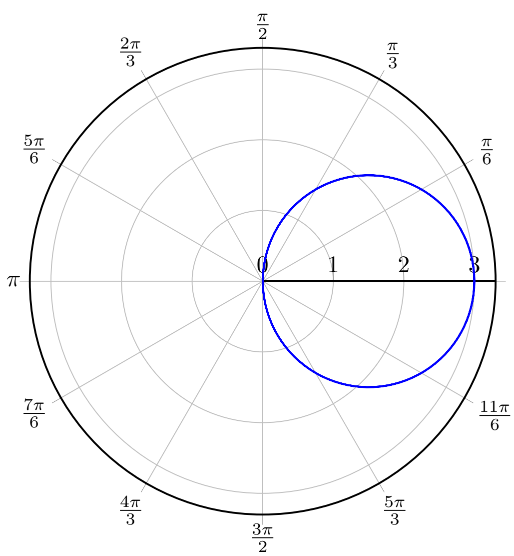


Figure 2: $r = 3\cos(\theta)$.

3) $r = a\sin(\theta)$ is a circle of radius $a/2$ centered at the cartesian point $\left(0, \frac{a}{2}\right)$.

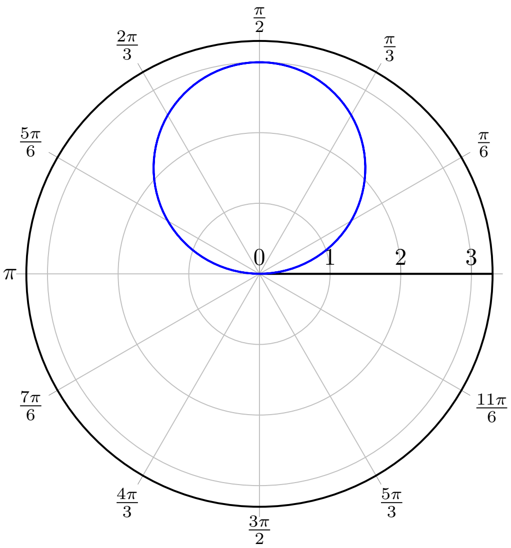


Figure 3: $r = 3\sin(\theta)$.

3 Roses.

Roses have similar equations to the circles. They have the following forms.

$$r = a\sin(n\theta)$$

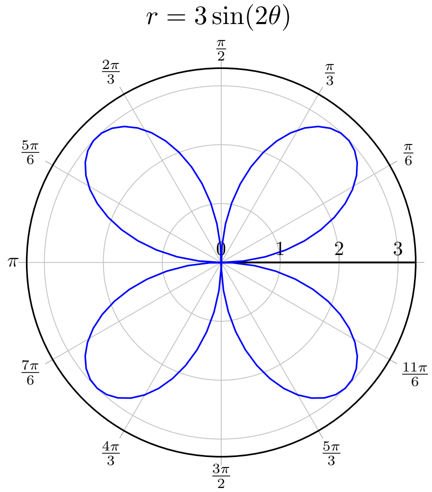
$$r = a\cos(n\theta)$$

Here, "a" is the length of the petals and the number of petals is given by,

$$\#\text{ of petals} = \begin{cases} n & : \text{n is odd} \\ 2n & : \text{n is even} \end{cases}$$

While it is possible to graph these by looking at the cartesian graph of r versus θ it is in general much easier to use a chart of values for the graph. The chart of values should go in intervals of $\dfrac{\pi}{2n}$. Consider the example below.

$r = 3\sin(2\theta)$



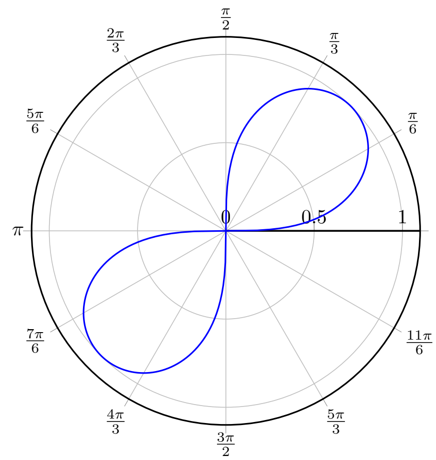
θ	r
0	0
$\pi/4$	3
$\pi/2$	0
$3\pi/4$	-3
π	0
$5\pi/4$	3
$3\pi/2$	0
$7\pi/4$	-3
2π	0

Occasionally, you will encounter equations such as

$$r^2 = \sin(2\theta)$$

Portions of these graphs will not be defined in the real numbers. However, you can still treat them as normal roses as long as you ignore the undefined petals. It is standard notation to let, $i = \sqrt{-1}$.

θ	r^2	r	θ	r^2	r
0	0	0	$\frac{5\pi}{4}$	1	1
$\frac{\pi}{4}$	1	1	$\frac{3\pi}{2}$	0	0
$\frac{\pi}{2}$	0	0	$\frac{3\pi}{2}$	0	0
$\frac{3\pi}{4}$	-1	i	$\frac{7\pi}{4}$	-1	i
π	0	0	2π	0	0



4 Cardioids and Limaçons.

Another category of polar graphs you will encounter are limaçons and cardioids. A cardioid is a type of limaçon. They have the following equations.

$$r = a + b\sin(\theta)$$

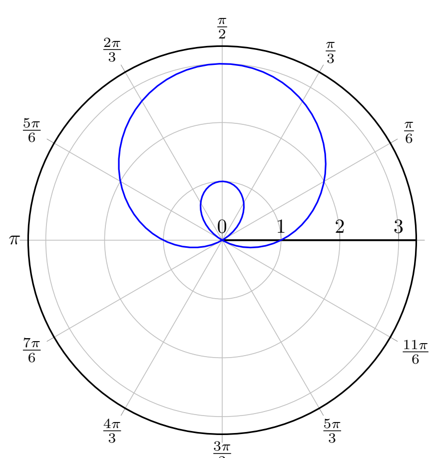
$$r = a + b\cos(\theta)$$

In general, these graphs have a heart shape and can be divided into three groups.

4.1 $a < b$

If $a < b$ in either of the above equation, then the graph will have an inner loop and an outer loop. The inner loop comes from the value of r becoming negative for some range of angles. By continuity the graph must always hit zero before the inner loop starts and hence must cross the pole.

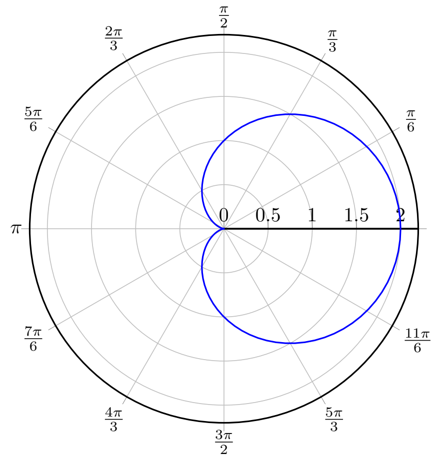
$$r = 1 + 2\sin(x)$$



4.2 $a = b$

If $a = b$ then this is a cardioid and the graph hits zero at some angle.

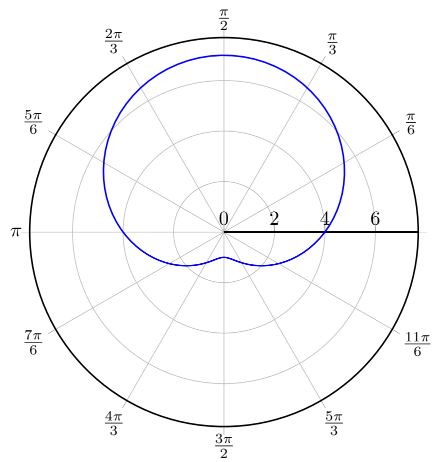
$$r = 1 + \cos(x)$$



4.3 $a > b$

If $a > b$ then this is a dimpled limaçon and the graph never hits zero.

$$r = 4 + 3\sin(x)$$



Note that the above could be graphed by the same charting method used for roses. This is possible since we know what the general shape of each graph should be.