MATH 3150 Your name
Homework 1 August 29, 2017

1. Let $\mathbb{C}$ denote the complex numbers with the standard addition and multiplication. Show that there is no order relation $>$ such that $\mathbb{C}$ is an ordered field. As a reminder:

   Definition 0.1. An *ordered field* $\mathbb{F} = (\mathbb{F}, +, \cdot, <)$ consists of a field $(\mathbb{F}, +, \cdot)$ together with a relation $<$ on $\mathbb{F}$, called an *order*, satisfying

   (i) (*trichotomy*) for each $x, y \in S$, exactly one of the following hold,
   $$x < y, \quad y < x, \quad x = y;$$
   (ii) (*transitivity*) for $x, y, z \in S$, if $x < y$ and $y < z$, then $x < z$.
   (iii) if $x, y, z \in \mathbb{F}$ and $x < y$, then $x + z < y + z$;
   (iv) if $x, y \in \mathbb{F}$ and $x, y > 0$, then $xy > 0$.

2. Find the supremum and infimum of the following set: $S = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\}$. Prove your claim.

3. Show if $A \subset B$ are subsets of $\mathbb{R}$ where B is bounded above, then A and B are least upper bounds and
   $$\sup(A) \leq \sup(B).$$
   Find an example where $\sup(A) = \sup(B)$.

4. Let A and B be two non-empty subsets of $\mathbb{R}$ which are bounded below. Show
   $$\inf(A \cup B) = \min\{\inf(A), \inf(B)\}.$$

5. Suppose that A and B are non-empty subsets of $\mathbb{R}$. Let
   $$A + B = \{a + b \mid a \in A, b \in B\}.$$
   Show $A + B$ has a supremum and that $\sup(A + B) = \sup(A) + \sup(B)$.