MATH3210 **Name:**________________________

Exam 1 **Date:**________________________

This exam contains 3 pages (including this cover page). Check to see if any pages are missing. Enter all requested information on the top of this page, and put your name on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any unapproved calculator on this exam.

The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Write in complete sentences.
- **Mysterious or unsupported claims will not receive full credit**. Unreasonably large gaps in logic or an argument will receive little credit. You may quote theorems from class or the book.
- If you need more space, ask for an extra sheet of paper to continue the problem on; clearly indicate when you have done this.

Do not write in the table to the right.

Definitions	
Question 1	
Question 2	
Question 3	
Question 4	
Question 5	

1. State the definitions of the following concepts:

   (a) (1 point) Basis

   (b) (1 point) Linear independence of a set of vectors

   (c) (1 point) Linear transformation

   (d) (1 point) Null space of a transformation

   (e) (1 point) Range of a transformation

Choose 5 of the 6 questions below to turn in for your exam. Please label the questions and solutions clearly. They are worth 4 points each.

- Suppose that V is a finite dimensional vector space and U is a subspace of V. Prove that $\dim(U) = \dim(V)$ if and only if $U = V$.

- Prove that there does not exist a linear map $T : \mathbb{R}^5 \to \mathbb{R}^5$ such that
$$\operatorname{ran}(T) = \operatorname{null}(T).$$

- Consider $C[0,1]$; the space of continuous real valued functions on $[0,1]$. Show that
$$U = \{f \in C[0,1] \mid f(0) = f(1)\}$$
is a subspace of $C[0,1]$.

- Suppose that V is finite dimensional and that $T \in \mathcal{L}(V, W)$. Prove that T is surjective if and only if there exists an $S \in \mathcal{L}(W, V)$ such that TS is the identity map on W.

- Let V be a finite dimensional vector space and U be a subspace of V. Suppose T is a linear transformation defined on U. Show that T can be extended to a linear transformation $\hat{T}$ on V, i.e.
$$\hat{T}(u) = T(u) \quad \text{for } u \in U.$$

- Suppose $v_1, \ldots, v_m$ is linearly independent in V and $w \in V$. Prove that if $v_1+w, \ldots, v_m+w$ is linearly dependent, then $w \in \operatorname{span}\{v_1, \ldots, v_m\}$.