MAP 2302 - Quiz #3 **Name:**

For full credit, you must show all work and circle your final answer.

1 *(2.5 point)* Find the Laplace transform of the periodic function below.

$$f(t) = e^t, \quad 0 < t < 1, \text{ and } f(t) \text{ has period } 1.$$

$$f_T(t) = e^t \Pi_{0,1}(t) = e^t[u(t) - u(t-1)]$$

$$F_T(s) = \mathcal{L}\{f_T(s)\} = \mathcal{L}\{e^t u(t)\} - \mathcal{L}\{e^t u(t-1)\}$$

Note: $\mathcal{L}\{g(t)u(t-a)\} = e^{-as}\mathcal{L}\{g(t+a)\}$

$$\Rightarrow F_T(s) = e^{-0s}\mathcal{L}\{e^t\} - e^{-s}\mathcal{L}\{e^{t+1}\}$$

$$F_T(s) = \frac{1}{s-1} - \frac{e^{-s+1}}{s-1}$$

$$\left.\begin{array}{l} F(s) = \dfrac{F_T(s)}{1-e^{-sT}} \\ \\ T = 1 \end{array}\right\} \Rightarrow F(s) = \frac{1-e^{-s+1}}{(s-1)(1-e^{-s})}$$

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2 (a) *(1.5 points)* Show that

$$2\sum_{n=0}^{\infty} a_n x^{n+1} + \sum_{n=1}^{\infty} n b_n x^{n-1} = b_1 + \sum_{n=1}^{\infty} [2a_{n-1} + (n+1)b_{n+1}] x^n$$

let $K=n+1$ $K=n-1$

$$2\sum_{K=1}^{\infty} a_{K-1} X^K + \sum_{K=0}^{\infty} (K+1) b_{K+1} X^K$$

$$= b_1 + \sum_{K=1}^{\infty} (K+1) b_{K+1} X^K + 2\sum_{K=1}^{\infty} a_{K-1} X^K$$

$$= b_1 + \sum_{K=1}^{\infty} [(K+1) b_{K+1} + 2a_{K-1}] X^K$$

(b) *(1 points)* Given

$$\sin(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1},$$

find a power series for $\cos(x)$. (Show work, do not simply write an answer.)

$$\cos(X) = \frac{d}{dX} \sum_{K=0}^{\infty} \frac{(-1)^K}{(2K+1)!} X^{2K+1} = \frac{d}{dX}\left(X - \frac{X^3}{3!} + \frac{X^5}{5!} + \cdots\right)$$

$$= \sum_{K=0}^{\infty} \frac{(-1)^K}{(2K+1)!} (2K+1) X^{2K}$$

$$= \sum_{K=0}^{\infty} \frac{(-1)^K}{(2K)!} X^{2K}$$

Since the series starts with "X", no index shift is necessary

University of Florida Honor Code:

On my honor, I have neither given nor received unauthorized aid in doing this assignment.

Signature