MATH2210Q **Name:**________________________

Exam 1 **Date:**________________________

This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any unapproved calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, ask for an extra sheet of paper to continue the problem on; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	4	
2	4	
3	4	
4	5	
5	4	
6	4	
Total:	25	

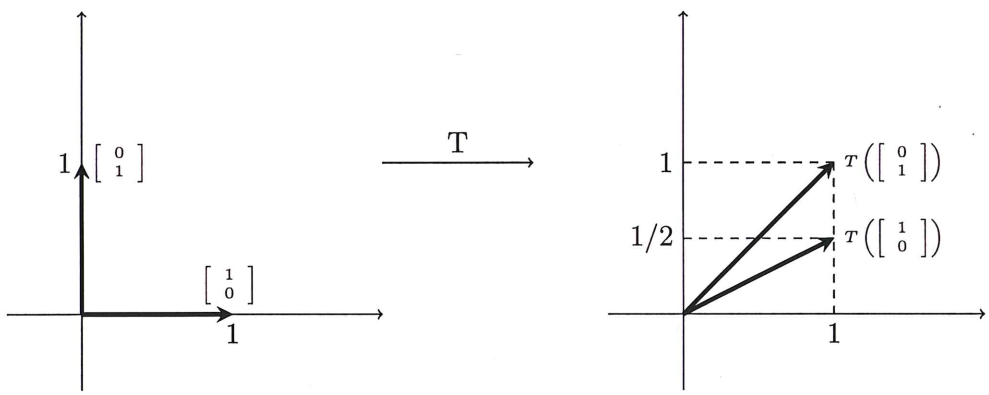
1. (a) (2 points) Consider the following linear transformation.

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2x_3 \\ 2x_2 - 3x_3 \\ 4x_1 - x_2 + x_3 \end{bmatrix}$$

Find the standard matrix for T.

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & -3 \\ 4 & -1 & 1 \end{bmatrix}$$

(b) (2 points) Consider the following diagram:



Find the standard matrix for T.

$$A = \begin{bmatrix} 1 & 1 \\ 1/2 & 1 \end{bmatrix}$$

2. Suppose

$$A = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 1 & 1 & 0 & 2 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ -1 & 1 \\ 1 & -2 \\ -1 & 3 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 1 & 0 & 3 & 1 \\ -1 & 1 & 0 & 1 \end{bmatrix}$$

Compute the following multiplications if possible, otherwise write undefined.

(a) (1 point) $A \cdot B$

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 1 & 1 & 0 & 2 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & 1 \\ 1 & -2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ -2 & 9 \\ 0 & 2 \\ 0 & 2 \end{bmatrix}$$

(b) (1 point) $B \cdot A$

undefined.

(c) (1 point) $C \cdot B$

$$\begin{bmatrix} 1 & 0 & 3 & 1 \\ -1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & 1 \\ 1 & -2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -3 & 2 \end{bmatrix}$$

(d) (1 point) $A \cdot C$

undefined.

3. Determine which of the following sets of vectors are linearly independent.

(a) (1 point) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 5 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 10 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ 8 \\ 20 \\ 8 \end{bmatrix} \right\}$

linearly dependent ; $2\begin{bmatrix} 1 \\ 2 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 10 \\ 4 \end{bmatrix}$

(b) (1 point) $\left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 9 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 8 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 7 \\ 2 \end{bmatrix} \right\}$

linearly dependent ; more vectors than entries.

(c) (2 points) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$ linearly independent

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 1 & -1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & -1 \\ 0 & -2 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$x_1 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

has only the trivial solution

4. Determine if $\mathbf{b}$ lies in the span of the given vectors.

(a) (1 point) $\mathbf{b} = \begin{bmatrix} 3 \\ 7 \\ 2 \end{bmatrix}$; $\left\{ \begin{bmatrix} 3 \\ 7 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ yes

$$\begin{bmatrix} 3 \\ 7 \\ 2 \end{bmatrix} = 1\begin{bmatrix} 3 \\ 7 \\ 2 \end{bmatrix} + 0\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 0\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

(b) (2 points) $\mathbf{b} = \begin{bmatrix} -3 \\ 8 \\ 1 \end{bmatrix}$; $\left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ -13 \\ -3 \end{bmatrix} \right\}$ No

$$x_1\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} + x_2\begin{bmatrix} 5 \\ -13 \\ -3 \end{bmatrix} = \begin{bmatrix} -3 \\ 8 \\ 1 \end{bmatrix}$$

$\Leftrightarrow$

$$\left[\begin{array}{cc|c} 1 & 5 & -3 \\ -2 & -13 & 8 \\ 3 & -3 & 1 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 5 & -3 \\ 0 & -3 & 2 \\ 0 & -18 & 10 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 5 & -3 \\ 0 & -3 & 2 \\ 0 & -9 & 5 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 5 & -3 \\ 0 & -3 & 2 \\ 0 & 0 & 11 \end{array}\right] \leftarrow \text{inconsistent}$$

(c) (2 points) Does every vector $\mathbf{b}$ in $\mathbb{R}^3$ lie in the span of the set below?

$$\left\{ \begin{bmatrix} 1 \\ -4 \\ -3 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} 4 \\ -6 \\ -7 \end{bmatrix} \right\}$$

No; $\begin{bmatrix} 1 & 3 & 4 \\ -4 & 2 & -6 \\ -3 & -2 & -7 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 4 \\ 0 & 14 & 10 \\ 0 & -7 & -5 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 4 \\ 0 & 7 & 5 \\ 0 & 0 & 0 \end{bmatrix}$ $\leftarrow$ would give inconsistent system

Not every row has a pivot

5. Consider the following Matrix Equation:

$$\begin{bmatrix} 1 & -2 & -1 \\ -2 & 2 & 0 \\ 4 & -1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(a) (2 points) Find the solution set.

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 0 \\ -2 & 2 & 0 & 0 \\ 4 & -1 & 3 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -2 & -1 & 0 \\ 0 & -2 & -2 & 0 \\ 0 & 7 & 7 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -2 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\Leftrightarrow \begin{array}{l} x_1 = -x_3 \\ x_2 = -x_3 \\ x_3 = x_3 \text{ free} \end{array} \Leftrightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \quad t \in \mathbb{R}$$

(b) (2 points) Given that

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

is a particular solution to

$$\begin{bmatrix} 1 & -2 & -1 \\ -2 & 2 & 0 \\ 4 & -1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 \\ +2 \\ 2 \end{bmatrix}$$

construct the entire solution set.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \quad t \in \mathbb{R}$$

6. Consider the following matrix.

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 4 & 0 & 1 \\ 3 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 4 & 0 & 1 \\ 3 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 1 \\ 3 & 2 & 1 \\ 1 & -2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & -8 & -11 \\ 0 & -4 & -8 \\ 0 & -4 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 2 \\ 0 & 4 & 8 \\ 0 & 8 & 11 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 2 \\ 0 & 0 & 6 \\ 0 & 0 & 7 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

(a) (2 points) If $T(\mathbf{x}) = A\mathbf{x}$ is T a one to one transformation?

$$x_1 \begin{bmatrix} 1 \\ 4 \\ 3 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 0 \\ 2 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \iff \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 4 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \\ 1 & 2 & 3 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \iff x_1 = x_2 = x_3 = 0$$

only trivial soln exists

Linearly indep. so it is 1 to 1.

(b) (2 points) If $T(\mathbf{x}) = A\mathbf{x}$ is T onto?

Not Every row has a pivot

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & c_1 \\ 0 & 1 & 0 & c_2 \\ 0 & 0 & 1 & c_3 \\ 0 & 0 & 0 & c_4 \end{array}\right] \leftarrow \text{inconsistent system}$$

T is not onto.