MATH2710   **Name:**_____

Exam 1   **Date:**_____

_____

This exam contains 7 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any unapproved calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.

- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

- If you need more space, ask for an extra sheet of paper to continue the problem on; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	6	
2	4	
3	5	
4	4	
5	6	
Total:	25	

1. (a) (1 point) State the definition of the lcm(a, b) for positive integers a and b.

   (b) (2 points) State the GCD Characterization theorem.

   (c) (3 points) Complete the following theorem:

      Theorem.

      (i) *The linear Diophantine equation*

      $$ax + by = c$$

      has a solution _____

      (ii) *If* gcd$(a, b) = d \neq 0$ *and* $x = x_0$ *and* $y = y_0$ *is one particular solution, then the complete integer solution is*

      _____ , _____ _____

2. (4 points) Solve the following linear Diophantine equation.

$$44x + 16y = 20$$

3. (5 points) If $3p^2 = q^2$ for some integers p and q show that 3 is a common divisor of p and q.

4. (4 points) Prove the following theorem

Theorem. *If p is a prime and $p \mid ab$, then $p \mid a$ or $p \mid b$.*

5. (6 points) Prove the following distributive law.

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

BONUS 2pts: Find all the prime numbers under 60. (all or nothing)