MATH2710 **Name:**__________________________

Exam 1 **Date:**__________________________

This exam contains 7 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any unapproved calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, ask for an extra sheet of paper to continue the problem on; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	6	
2	4	
3	5	
4	4	
5	6	
Total:	25	

1. (a) (1 point) State the definition of the lcm(a, b) for positive integers a and b.

The least common multiple of two positive integers a, b is the smallest positive integer that is divisible by both a and b.

(b) (2 points) State the GCD Characterization theorem.

If d is a non-negative common divisor of integers a and b and there exists integers x and y such that $ax+by=d$ then $d=\gcd(a,b)$

(c) (3 points) Complete the following theorem:

Theorem.

(i) The linear Diophantine equation

$$ax + by = c$$

has a solution $\gcd(a,b) \mid c$

(ii) If $\gcd(a, b) = d \neq 0$ *and* $x = x_0$ *and* $y = y_0$ *is one particular solution, then the complete integer solution is*

$x = x_0 + n(b/d)$, $y = y_0 - n(a/d)$ for all $n \in \mathbb{Z}$

2. (4 points) Solve the following linear Diophantine equation.

$$44x + 16y = 20$$

1	0	44	–	–
0	1	16	–	–
1	-2	12	2	44 = 2(16) + 12
-1	3	4	1	16 = 1(12) + 4
4	-11	0	3	12 = 3(4) + 0

$44(-1) + 16(3) = 4$

$44(-5) + 16(15) = 20$

$x_0 = -5 \quad y_0 = 15$ is a solution

3. (5 points) If $3p^2 = q^2$ for some integers p and q show that 3 is a common divisor of p and q.

Since $3p^2 = q^2$ then $3 | q^2$ so $3 | q$.

If $3 | q$, $\exists\ m \in \mathbb{Z}$ such that $3m = q$

So $3p^2 = 9m^2$ i.e. $p^2 = 3m^2$.

So $3 | p^2$ and thus $3 | p$.

4. (4 points) Prove the following theorem

Theorem. *If p is a prime and $p \mid ab$, then $p \mid a$ or $p \mid b$.*

Suppose $p \mid ab$ but $p \nmid a$.

The only divisors of p are p and 1

Since $p \nmid a$ the $\gcd(a,p) = 1$.

So $p \mid ab$ and $\gcd(a,p) = 1$ so $p \mid b$.

5. (6 points) Prove the following distributive law.

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

We must show $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ and $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$. To show $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ suppose $x \in A \cup (B \cap C)$ is arbitrary. Then $x \in A$ or $x \in B \cap C$. If $x \in A$ then $x \in A \cup B$ and $x \in A \cup C$. If $x \in B \cap C$ then $x \in B$ and $x \in C$. So $x \in A \cup B$ and $x \in A \cup C$. In either case $x \in (A \cup B) \cap (A \cup C)$. So $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$, since x was arbitrary.

To show $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$, let $y \in (A \cup B) \cap (A \cup C)$ be arbitrary. So $y \in (A \cup B)$ and $y \in A \cup C$. If $y \in A \cup B$ then $y \in A$ or $y \in B$. If $y \in A$ then ~~$y \in A \cup B$~~ $y \in A \cup (B \cap C)$. If $y \notin A$ then $y \in B$. In addition ~~$y \in A \cup C$~~ $y \in A \cup C$. If $y \notin A$ then $y \in C$. So $y \in B \cap C$ and thus $y \in A \cup (B \cap C)$.

Since y was arbitrary $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$.

BONUS 2pts: Find all the prime numbers under 60. (all or nothing)

~~1~~, $\boxed{2}$, $\boxed{3}$, ~~4~~, $\boxed{5}$ ~~6~~, $\boxed{7}$ ~~8~~, ~~9~~, ~~10~~,

$\boxed{11}$, ~~12~~, $\boxed{13}$, ~~14~~, ~~15~~, ~~16~~, $\boxed{17}$ ~~18~~, $\boxed{19}$, ~~20~~,

~~21~~, ~~22~~, $\boxed{23}$, ~~24~~, ~~25~~, ~~26~~, ~~27~~, ~~28~~, $\boxed{29}$ ~~30~~,

$\boxed{31}$, ~~32~~, ~~33~~, ~~34~~, ~~35~~, ~~36~~, $\boxed{37}$ ~~38~~, ~~39~~, ~~40~~

$\boxed{41}$, ~~42~~, $\boxed{43}$ ~~44~~, ~~45~~, ~~46~~, $\boxed{47}$, ~~48~~, ~~49~~, ~~50~~,

~~51~~, ~~52~~, $\boxed{53}$ ~~54~~, ~~55~~ ~~56~~, ~~57~~, ~~58~~, $\boxed{59}$ ~~60~~

Primes under 60

2, 3, 5, 7, 11, 13, 17, 19,

23, 29, 31, 37, 41, 43, 47,

53, 59.