MATH2210Q - Quiz #2 Name:

For full credit, you must show all work and circle your final answer.

1 Find the inverses of the following matrices.

(a) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$\det A = 4 - 6 = -2$

$A^{-1} = -\frac{1}{2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & -1 \\ 0 & 2 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 2 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1/2 & -1/2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1/2 & -1/2 \\ 0 & 0 & 1 \end{bmatrix}$$

MATH2210Q - Quiz #2 **Name:**

2 Calculate the determinants for the following matrices.

(a) $A = \begin{bmatrix} 1 & 0 & -1 & 4 & -1 \\ 3 & 0 & 8 & 3 & 0 \\ 1 & 0 & 2 & -2 & -2 \\ 0 & 0 & 3 & 8 & 1 \\ 6 & 0 & 5 & 3 & -1 \end{bmatrix}$

$\det(A) = 0$

since there is a zero column

(b) $B = \begin{bmatrix} 1 & 2 & 2 & 5 \\ 0 & 5 & -8 & 5 \\ 0 & 0 & 2 & 5 \\ 0 & 0 & 0 & 3 \end{bmatrix}$

$\det(B) = 1 \cdot 5 \cdot 2 \cdot 3 = 30$

B. upper triangular

3 Find a basis for the null space of the matrix.

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$$

given that

$$[A \mid 0] = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 & 0 \\ 1 & -2 & 2 & 3 & -1 & 0 \\ 2 & -4 & 5 & 8 & -4 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 0 & -1 & 3 & 0 \\ 0 & 0 & 1 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

$\Leftrightarrow X_1 - 2X_2 - X_4 + 3X_5 = 0$

$X_3 + 2X_4 - 2X_5 = 0$

$$\Rightarrow \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} t + \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} -3 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix} r$$

$t, s, r \in \mathbb{R}$

Basis : $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}$