**MTH130** **Name:** Soln

**Practice Exam 2** **Section:**

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This exam contains 6 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books or notes on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, ask for an extra sheet of paper to continue the problem on; clearly indicate when you have done this.

**Do not write in the table to the right.**

| Problem | Points | Score |
|---|---|---|
| 1 | 5 | |
| 2 | 5 | |
| 3 | 5 | |
| 4 | 5 | |
| 5 | 5 | |
| Total: | 25 | |

1. (5 points) Consider the implicitly defined function below.

$$\sqrt{xy} = 3x + 2y^2$$

a) Find $\frac{dy}{dx}$ via implicit differentiation.

$$\sqrt{xy} = 3x + 2y^2$$

$$\tfrac{1}{2}(xy)^{-1/2}\left(x\frac{dy}{dx} + y\right) = 3 + 4y\frac{dy}{dx}$$

$$\Rightarrow \frac{1}{2\sqrt{xy}}\left(x\frac{dy}{dx} + y\right) = 3 + 4y\frac{dy}{dx}$$

$$\Rightarrow \frac{x}{2\sqrt{xy}}\frac{dy}{dx} + \frac{y}{2\sqrt{xy}} = 3 + 4y\frac{dy}{dx}$$

$$\Rightarrow \left(\frac{x}{2\sqrt{xy}}\right)\frac{dy}{dx} - (4y)\frac{dy}{dx} = 3 - \frac{y}{2\sqrt{xy}}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{x}{2\sqrt{xy}} - 4y\right) = 3 - \frac{y}{2\sqrt{xy}}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{x - 8y\sqrt{xy}}{2\sqrt{xy}}\right) = \frac{6\sqrt{xy} - y}{2\sqrt{xy}}$$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{6\sqrt{xy} - y}{2\sqrt{xy}}\right)\left(\frac{2\sqrt{xy}}{x - 8y\sqrt{xy}}\right)$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{6\sqrt{xy} - y}{x - 8y\sqrt{xy}}}$$

b) Find the equation of the tangent line at $(-2, -2)$.

@ $(-2,2)$ $$\frac{dy}{dx} = \frac{6\sqrt{(-2)(-2)} - (-2)}{(-2) - 8(-2)\sqrt{(-2)(-2)}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{6(2) + 2}{-2 + (32)} = \frac{14}{30} = \frac{7}{15}$$

Tangent line:

$$y - (-2) = 7/15\,(x - (-2))$$

$$\Rightarrow \boxed{y + 2 = 7/15\,(x + 2)}$$

2. (5 points) a) Find the differential of $f(x) = \frac{2x^2+1}{3x+2}$

$$dy = f'(x)\,dx$$

$$\Rightarrow dy = \left(\frac{(3x+2)(4x) - (2x^2+1)(3)}{(3x+2)^2}\right) dx$$

$$\Rightarrow dy = \left(\frac{12x^2+8x-6x^2-3}{(3x+2)(3x+2)}\right) dx$$

$$\Rightarrow \boxed{dy = \left(\frac{6x^2+8x-3}{4x^2+12x+4}\right) dx}$$

b) Find the differential of $f(x) = \sqrt{\frac{x^2}{2x+1}}$

$$dy = \frac{1}{2}\left(\frac{x^2}{2x+1}\right)^{-\frac{1}{2}} \left(\frac{(2x+1)(2x) - x^2(2)}{(2x+1)^2}\right) dx$$

$$\Rightarrow dy = \frac{1}{2\sqrt{\frac{x^2}{2x+1}}} \left(\frac{4x^2+2x-2x^2}{(2x+1)(2x+1)}\right) dx$$

$$\Rightarrow \boxed{dy = \frac{1}{2\sqrt{\frac{x^2}{2x+1}}} \left(\frac{2x^2+2x}{4x^2+4x+1}\right) dx}$$

c) Use the differential of $f(x) = \sqrt{x}$ to approximate $\sqrt{26}$. Hint: $\sqrt{25} = 5$.

$$dy = \frac{1}{2}x^{-\frac{1}{2}}\,dx$$

$$\Rightarrow dy = \frac{1}{2\sqrt{x}}\,dx$$

Note $\Delta y \approx dy$ & $dx = \Delta x$

$$\sqrt{26} = \sqrt{25} + \Delta y$$

So

$$\sqrt{26} \approx \sqrt{25} + dy$$

$$= 5 + \frac{1}{2\sqrt{25}} \cdot (1)$$

$$= 5 + \frac{1}{10}$$

$$= 5.1$$

Note $\sqrt{26} \approx 5.09902\ldots$ by calculator

3. (5 points) A sphere's volume is expanding at a rate of 2 in$^3$/sec, where $V(r) = \frac{4\pi}{3}r^3$.

a) Find the rate of change of the radius when $r = 4$.

$$V'(r) = \left(\frac{4\pi}{3}\right) 3r^2 \frac{dr}{dt}$$

$$\therefore \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$2 = 4\pi(16)\frac{dr}{dt}$$

$$\Rightarrow \frac{dr}{dt} = \frac{1}{32\pi} \approx 0.00995 \text{ in/sec}$$

b) Find the rate of change of the surface area when $r = 4$, given $S(r) = 4\pi r^2$.

$$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$$

$$\therefore \frac{dS}{dt} = 8\pi \cdot 4 \cdot \frac{1}{32\pi} = 1 \text{ in}^2\text{/sec}$$

4. (5 points) Consider the function below

$$f(x) = x^3 - 3x + 1$$

a) Find the intervals when the function is increasing and decreasing.

$f'(x) = 3x^2 - 3$

zeros: $3x^2 - 3 = 0$

$\Rightarrow x = \pm 1$

| $+$ | $-1$ | $-$ | $1$ | $+$ | $f'(x)$ |
|---|---|---|---|---|---|

Increasing: $(-\infty, -1)$ and $(1, \infty)$

Decreasing: $(-1, 1)$

b) Find the intervals when the function is concave up and concave down.

$f''(x) = 6x$

zeros: $6x = 0$

$\Rightarrow x = 0$

| $\frown$ $-$ | $0$ | $\smile$ $+$ | $f''(x)$ |
|---|---|---|---|

Concave up: $(0, \infty)$

concave down: $(-\infty, 0)$

c) Find the inflection points of the function.

Inflection pt is @ $x = 0$ by part (b)

5. (5 points) Consider the function below

$$f(x) = x^4 - 2x^2 + 1.$$

a) Find the local maxima and minima via the 1st derivative test.

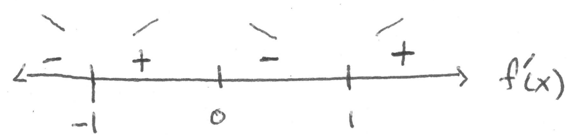
Critical points:

$f'(x) = 4x^3 - 4x$

zeros: $4x^3 - 4x = 0$

$\Rightarrow 4x(x^2 - 1) = 0$

$x = 0, \pm 1$




local min @ $x = -1$ & $x = 1$

local max @ $x = 0$

b) Find the local maxima and minima via the 2nd derivative test.

From above the critical points are: $x = 0, \pm 1$

$f''(x) = 12x^2 - 4$

$f''(0) = -4 < 0$ local max @ $x = 0$

$f''(1) = 8 > 0$ local min @ $x = 1$

$f''(-1) = 8 > 0$ local min @ $x = -1$

c) Find the absolute maximum on $[0, 2]$.

We test the critical points and the end points.

$f(0) = 1$

$f(1) = 0$

$f(-1) = 0$

$f(2) = 9$ ← absolute max