MAP2302 **Name:**____________________

Practice Exam 1 **Section:**____________________

This exam contains 6 pages (including this cover page) and 7 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, ask for an extra sheet of paper to continue the problem on; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	2	
2	2	
3	3	
4	5	
5	5	
6	3	
7	5	
Total:	25	

1. (2 points) Determine whether the given relation is an implicit solution to the given differential equation. Assume that the relationship does define y as a function of x.

$$y - \ln y = x^2 + 1, \quad \frac{dy}{dx} = \frac{2xy}{y-1}$$

We use implicit differentiation

$$\frac{dy}{dx} - \frac{1}{y}\frac{dy}{dx} = 2x$$

$$\frac{dy}{dx}\left(1 - \frac{1}{y}\right) = 2x$$

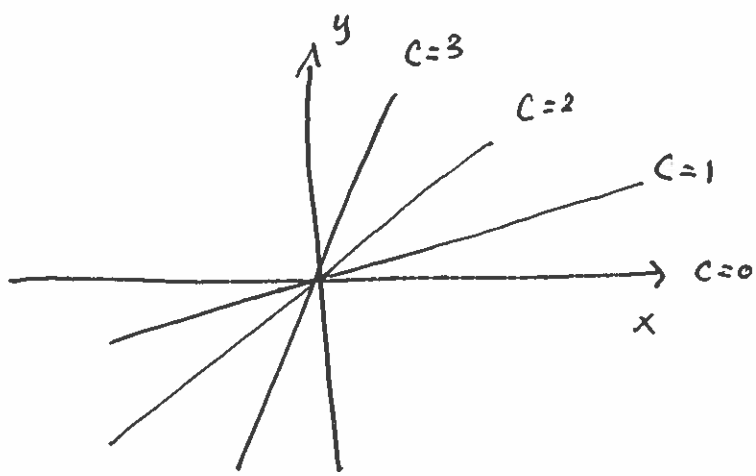
$$\frac{dy}{dx}\left(\frac{y-1}{y}\right) = 2x$$

$$\frac{dy}{dx} = \frac{2xy}{y-1} \quad \checkmark$$

2. (2 points) Draw and label the isoclines for the differential equation for the listed values.

$$\frac{dy}{dx} = \frac{2y}{x} \qquad c = 0,\ 1,\ 2,\ 3$$

c	isocline
0	$y = 0$
1	$y = \frac{x}{2}$
2	$y = x$
3	$y = \frac{3x}{2}$



3. (3 points) Find a general solution to the given differential equation.

$$y'' + 2y' + 5y = 0.$$

Aux eqn : $r = \dfrac{-2 \pm \sqrt{4 - 4(5)(1)}}{2(1)}$

$r = -1 \pm 2i \Rightarrow r_1 = -1 + 2i \quad r_2 = -1 - 2i$

$$\boxed{y(t) = C_1 e^{-t} \cos 2t + C_2 e^{-t} \sin 2t}$$

4. (5 points) Solve the differential equation.

$$\frac{dy}{dx} + \frac{3y}{x} + 2 = 3x$$

$\dfrac{dy}{dx} + \left(\dfrac{3}{x}\right) y = 3x - 2$

$x^3 \dfrac{dy}{dx} + 3x^2 y = 3x^4 - 2x^3$

$\dfrac{d}{dx}[x^3 y] = 3x^4 - 2x^3$

$x^3 y = \int 3x^4 - 2x^3 \, dx$

$x^3 y = \frac{3}{5} x^5 - \frac{1}{2} x^4 + C$

$$\boxed{y = \frac{3}{5} x^2 - \frac{1}{2} x + C x^{-3}}$$

side work

$P(x) = 3/x$

$e^{\int P(x) dx} = e^{\int 3/x \, dx} = e^{3 \ln x} = x^3$

5. (5 points) Solve the following Bernoulli equation.

$$\frac{dy}{dx} + \frac{y}{x} = x^2 y^2.$$

sub

Let $v = y^{-1}$

$$\frac{dv}{dx} = -y^{-2}\frac{dy}{dx}$$

$$y^{-2}\frac{dy}{dx} + y^{-1}\left(\frac{1}{x}\right) = x^2$$

$$\Rightarrow \quad -\frac{dv}{dx} + \left(\frac{1}{x}\right)v = x^2$$

$$\frac{dv}{dx} + \left(\frac{-1}{x}\right)v = -x^2$$

$$P = \frac{-1}{x}$$

$$e^{\int P(x)dx} = e^{\int -1/x \, dx} = e^{-\ln x} = x^{-1}$$

$$x^{-1}\frac{dv}{dx} - x^{-2}v = -x$$

$$\frac{d}{dx}\left[x^{-1}v\right] = -x$$

$$x^{-1}v = -\int x \, dx$$

$$x^{-1}v = -\frac{x^2}{2} + C$$

$$v = -\frac{x^3}{3} + Cx$$

$\boxed{y^{-1} = \frac{-x^3}{3} + Cx \text{ is an implicit solution}}$

6. (3 points) Give an implicit solution to the initial value problem. (Hint: Try factoring.)

$$x^2\frac{dy}{dx} = \frac{x^2+3x+2}{(x+1)(y+2)}; \qquad y(1) = 2$$

$$x^2\frac{dy}{dx} = \frac{(x+2)\cancel{(x+1)}}{\cancel{(x+1)}(y+2)} = \frac{(x+2)}{(y+2)}$$

$$\Rightarrow \quad y+2\ dy = \frac{x+2}{x^2}\ dx$$

$$\Rightarrow \quad y+2\ dy = x^{-1} + 2x^{-2}\ dx$$

$$\int y+2\ dy = \int x^{-1}+2x^{-2}\ dx$$

$$\frac{y^2}{2} + 2y = \ln x - 2x^{-1} + C$$

initial cond: $\frac{2^2}{2} + 2(2) = \ln(1) - 2(1) + C$

$$6 = -2 + C$$

$$C = 8$$

implicit solution: $\boxed{\frac{y^2}{2} + 2y = \ln x - 2x^{-1} + 8}$

7. (5 points) Find an integrating factor, then solve the differential equation.

$$(x^4 - x + y)\,dx - x\,dy = 0$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{1-(-1)}{-x} = -\frac{2}{x}$$ ← a function of x

$$\mu(x) = e^{\int \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}\,dx} = e^{-\int \frac{2}{x}\,dx} = x^{-2}$$ ← integrating factor

$$\mu M\,dx + \mu N\,dy = \underbrace{(x^2 - x^{-1} + x^{-2}y)\,dx + (-x^{-1})\,dy = 0}$$

↳ exact eqn.

$$F(x,y) = \int -x^{-1}dy + h(x)$$

$$F(x,y) = -x^{-1}y + h(x)$$

If $M = \frac{\partial F}{\partial x}$, then....

$$x^4 - x + y = y + h'(x)$$

$$\Rightarrow h'(x) = x^4 - x$$

$$\Rightarrow h(x) = \frac{1}{5}x^5 - \frac{x^2}{2} + C$$

$$\Rightarrow \boxed{F(x,y) = -x^{-1}y + \frac{1}{5}x^5 - \frac{x^2}{2} + C}$$