MATH2210Q **Name:**________________________

Practice Exam 1 **Date:**________________________

This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any unapproved calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, ask for an extra sheet of paper to continue the problem on; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	5	
2	3	
3	4	
4	5	
5	4	
6	4	
Total:	25	

1. (a) (1 point) Consider the following linear transformation.

$$T(\mathbf{x}) = A\mathbf{x} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

where A is the standard matrix for T i.e.

$$A = [\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3] = [T(\mathbf{e}_1)\ T(\mathbf{e}_2)\ T(\mathbf{e}_3)]$$

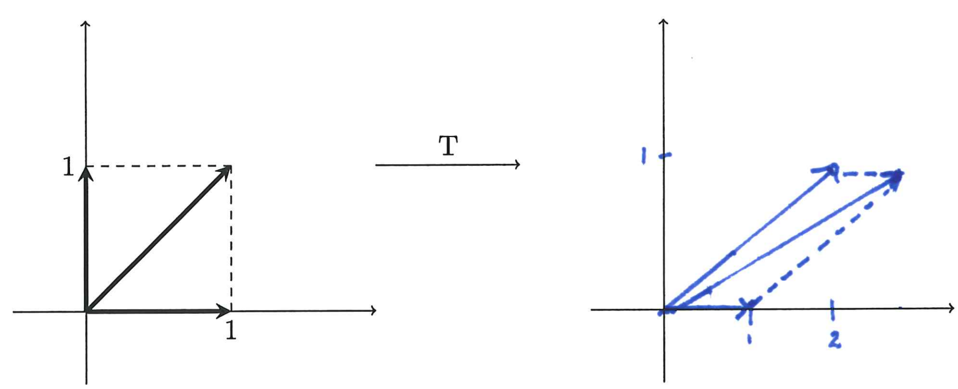
In terms of the columns of A write $T\left(\begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}\right)$.

$$T\left(\begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}\right) = 2\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + 1\begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} + 5\begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix}$$

Define the following two transformations

$$T(\mathbf{x}) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\mathbf{x}; \quad S(\mathbf{x}) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\mathbf{x}$$

(b) (2 points) Draw the image of the unit square under T



(c) (2 points) Find the standard matrix for $S(T(\mathbf{x}))$ and $T(S(\mathbf{x}))$.

$$ST = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$$

$$TS = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$$

2. Suppose

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$

(a) (1 point) Compute $2A + B$

$$2A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 2 & 4 & 2 & 0 \\ 2 & 6 & 6 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$

$$2A + B = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 3 & 2 & 3 & 0 \\ 1 & 9 & 3 & 3 \end{bmatrix}$$

(b) (2 points) Compute $A \cdot B$

$$A \cdot B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. Determine which of the following sets of vectors are linearly independent.

(a) (1 point) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 5 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 9 \\ 0 \\ -1 \end{bmatrix} \right\}$ Linearly indep. $\begin{bmatrix} 1 \\ 2 \\ 5 \\ 2 \end{bmatrix} \neq c \begin{bmatrix} 2 \\ 9 \\ 0 \\ -1 \end{bmatrix}$

for any $c \in \mathbb{R}$.

(b) (1 point) $\left\{ \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 7 \\ 9 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 7 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 7 \\ 5 \end{bmatrix} \right\}$

Linearly dependent, More vectors than entries.

(c) (2 points) $\left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -4 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ -10 \\ -8 \end{bmatrix}, \right\}$ Linearly dependent,

$$\begin{bmatrix} 2 & 3 & 5 \\ -1 & -4 & -10 \\ 1 & -2 & -8 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -8 \\ -1 & -4 & -10 \\ 2 & 3 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -8 \\ 0 & -6 & -18 \\ 0 & 7 & 21 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -8 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

X_3 is a free variable.

4. Determine if $\mathbf{b}$ lies in the span of the given vectors.

(a) (1 point) $\mathbf{b} = \begin{bmatrix} 3 \\ 7 \\ 2 \end{bmatrix}$; $\left\{ \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix} \right\}$

$$-3\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} + 7\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - 1\begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ 2 \end{bmatrix}$$

yes, $\vec{b}$ lies in the span.

(b) (2 points) $\mathbf{b} = \begin{bmatrix} 4 \\ 1 \\ -4 \end{bmatrix}$; $\left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 8 \\ -2 \end{bmatrix}, \begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix} \right\}$

$$\begin{bmatrix} 2 & 0 & 6 & 4 \\ -1 & 8 & 5 & 1 \\ 1 & -2 & 1 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 1 & -4 \\ -1 & 8 & 5 & 1 \\ 2 & 0 & 6 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 1 & -4 \\ 0 & 6 & 6 & -3 \\ 0 & 4 & 4 & 12 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 1 & -4 \\ 0 & 3 & 3 & -1 \\ 0 & 1 & 1 & 3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & 1 & -4 \\ 0 & 1 & 1 & 3 \\ 0 & 3 & 3 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 1 & -4 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & -10 \end{bmatrix}$$

inconsistent.

$\vec{b}$ does not lie in the span.

(c) (2 points) Is it possible for $\mathbb{R}^3$ to be contained in

$$\text{span}\left\{ \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ -2 \end{bmatrix} \right\}$$

No, the above span is a plane in $\mathbb{R}^3$.

5. Consider the following system:

$$\begin{aligned} x_1 &+ x_2 &+ x_3 &= 0 \\ x_1 &- 2x_2 &+ 2x_3 &= 4 \\ x_1 &+ 2x_2 &- x_3 &= 2 \end{aligned}$$

(a) (2 points) Find the solution set.

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & -2 & 2 & 4 \\ 1 & 2 & -1 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & -3 & 1 & 4 \\ 0 & 1 & -2 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & -2 & 2 \\ 0 & -3 & 1 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & -5 & 10 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -2 \end{bmatrix} \iff \begin{aligned} x_1 &= 4 \\ x_2 &= -2 \\ x_3 &= -2 \end{aligned}$$

(b) (2 points) Write the equivalent matrix equation and vector equation.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 2 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}$$

$$x_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}$$

6. Consider the following matrix.

$$A = \begin{bmatrix} 4 & 5 & 9 & -2 \\ 6 & 5 & 1 & 12 \\ 3 & 4 & 8 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 5 & 9 & -2 \\ 6 & 5 & 1 & 12 \\ 3 & 4 & 8 & -3 \end{bmatrix} \sim \begin{bmatrix} 4 & 5 & 9 & -2 \\ 3 & 4 & 8 & -3 \\ 0 & -3 & 15 & 18 \end{bmatrix} \sim \begin{bmatrix} 4 & 5 & 9 & -2 \\ 0 & 1 & 5 & -6 \\ 0 & 1/4 & 5/4 & -6/4 \end{bmatrix} \sim \begin{bmatrix} 4 & 5 & 9 & -2 \\ 0 & 1 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 4 & 4 & 4 & 4 \\ 0 & 1 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -4 & 7 \\ 0 & 1 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) (2 points) If $T(\mathbf{x}) = A\mathbf{x}$ is T a one to one transformation?

No, there is a free variable

(b) (2 points) If $T(\mathbf{x}) = A\mathbf{x}$ is T onto?

No, T is onto if and only if the columns span $\mathbb{R}^3$ which is equivalent to there being a pivot in every row.