MAP 2302 - Quiz #3 **Name:**

For full credit, you must show all work and circle your final answer.

1 *(2.5 point)* Write the following piecewise function in terms of step and window functions then find the Laplace transform

$$f(t) = \begin{cases} \cos(t) & 0 < t < 2\pi \\ t & 2\pi < t < 8 \\ t^2 & t > 8 \end{cases}$$

Solution:
We have,

$$f(t) = \cos(t) \cdot \Pi_{0,2\pi} + t \cdot \Pi_{2\pi,8} + t^2 \cdot u(t-8).$$

So,

$$f(t) = \cos(t) \cdot (u(t) - u(t-2\pi)) + t \cdot (u(t-2\pi) - u(t-8)) + t^2 \cdot u(t-8).$$

Since $\mathscr{L}\{g(t)u(t-a)\}(s) = e^{-as}\mathscr{L}\{g(t+a)\}$ we have,

$$\mathscr{L}\{f(t)\} = \mathscr{L}\{\cos(t)\} - e^{-2\pi s}\mathscr{L}\{\cos(t+2\pi)\} + e^{-2\pi s}\mathscr{L}\{(t+2\pi)\} - e^{-8s}\mathscr{L}\{(t+8)\} + e^{-8s}\mathscr{L}\{(t+8)^2\}.$$

Hence,

$$\mathscr{L}\{f(t)\} = \frac{s}{s^2+1} - \frac{e^{-2\pi s}s}{s^2+1} + \frac{e^{-2\pi s}}{s^2} + \frac{2\pi e^{-2\pi s}}{s} - \frac{e^{-8s}}{s^2} - \frac{8e^{-8s}}{s} + \frac{2e^{-8s}}{s^3} + \frac{16e^{-8s}}{s^2} + \frac{64e^{-8s}}{s}$$

2 (a) *(1.5 points)* Find the power series expansion of $f+g$ in the form $\sum a_n x^n$ given

$$f = \sum_{n=3}^{\infty} \frac{2^n}{n!} x^{n-2}, \quad g(x) = \sum_{n=1}^{\infty} \frac{n^2}{2^n} x^{n-1}$$

Solution:

After an index shift we have

$$f = \sum_{k=1}^{\infty} \frac{2^{k+2}}{(k+2)!} x^k$$

and

$$g = \sum_{k=0}^{\infty} \frac{(k+1)^2}{2^{k+1}} x^k$$

We take the first term off of g

$$g = \frac{1}{2} + \sum_{k=1}^{\infty} \frac{(k+1)^2}{2^{k+1}} x^k.$$

So,

$$f + g = \frac{1}{2} + \sum_{k=1}^{\infty} \left[\frac{(k+1)^2}{2^{k+1}} + \frac{2^{k+2}}{(k+2)!} \right] x^k$$

(b) *(1.5 points)* Find the power series expansion of $g'(x)$ in the form $\sum a_n x^n$. **Solution:**

$$g(x) = \sum_{k=0}^{\infty} \frac{(k+1)^2}{2^{k+1}} x^k$$

So

$$g'(x) = \sum_{n=1}^{\infty} \frac{k(k+1)^2}{2^{k+1}} x^{k-1}.$$

University of Florida Honor Code:

On my honor, I have neither given nor received unauthorized aid in doing this assignment.

Signature