1. Let A be a set and define $P(A)$ to be the set of all subsets of A. Let C be a fixed subset of the set A and define relation R on the set $P(A)$ by XRY if and only if $X \cap C = Y \cap C$. Prove that this is an equivalence relation.

2. Let A be a set and let P be a partition of the set A i.e. $P = \{A_1, A_2, \ldots A_n\}$ where

   i) $A_i \subset A$,

   ii) $\emptyset \notin P$

   iii) $A_1 \cup A_2 \cup \ldots \cup A_n = A$

   iv) $A_i \cap A_j = \emptyset$ when $i \neq j$.

   For $x, y \in A$ we say that xRy if and only if $x \in A_i$ and $y \in A_i$ for the same i. Prove this is an equivalence relation.

3. Prove or disprove: The relation R defined on the set $\mathbb{Z}$ by xRy if and only if $xy > 0$ is an equivalence relation.

4. Find all the x that satisfy the following equation. (Hint: Use Fermat's Little theorem and notice that if x_0 is a solution then it's entire residue class is a solution.)

$$x^{86} \equiv 2 \pmod 7$$

5. Prove that every integer of the form $5n + 3$ for $n \in \mathbb{Z}$, $n \geq 1$, cannot be a perfect square.

Bonus Question: (+3 Points added to exam)

Jim is looking to have a easy life and make a lot of money. Jim goes looking for employment and finds a mysterious man. The man points to a bridge and says the following to Jim: "The work I have for you is light and you will get rich. Do you see the bridge? Each time you cross it I will double the money in your pocket. But since I am so generous you must give me back \$ 24 after each crossing." Jim accepts and walks across the bridge. Miraculously the money in his pocket doubled! He threw \$ 24 dollars to the mystery man for the first crossing and crossed again. Amazingly his money doubled! He paid the mystery man \$ 24 again for the second crossing. He crossed a third time, again his money doubles. He goes to pay the mystery man, but the mystery man laughs because Jim only had \$ 24 dollars in his pocket and had to give it all away. How much money did Jim start with ?