MATH2710 **Name:**_________________________

Exam 1 **Date:**_________________________

This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any unapproved calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, ask for an extra sheet of paper to continue the problem on; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	4	
2	4	
3	5	
4	3	
5	4	
6	5	
Total:	25	

1. (a) (1 point) State a definition of the $\gcd(a, b)$ for $a, b \in \mathbb{Z}$.

   (b) (2 points) State the Unique Factorization Theorem.

   (c) (1 point) Complete the following theorem:

   Theorem. *If a and b are integers, with b positive, then there exist ____________________ integers q and r such that*

   $$a = \underline{\hspace{4cm}}, \quad \textit{where } \underline{\hspace{4cm}}$$

2. (4 points) Solve the following linear Diophantine equation.

$$21x + 15y = 12$$

3. (5 points) Let a and b be two positive even integers. Prove that

$$\gcd(a, b) = 2\gcd\left(\frac{a}{2}, \frac{b}{2}\right)$$

4. True or False. Briefly justify your answers

   (a) (1 point) The linear Diophantine equation $ax = b$ has a solution if and only if $a \nmid b$

   (b) (1 point) The only even prime number is 2.

   (c) (1 point) The integer d is the $\gcd(a, b)$ if $d = ax + by$ for some integers x and y.

5. (4 points) Prove the following theorem

Theorem. *An integer $x > 1$ is either prime or contains a prime factor $p \leq \sqrt{x}$.*

6. (5 points) Let A, B and U be sets such that $A \subseteq U$ and $B \subseteq U$. The complement of the set A denoted A^c is the following set

$$A^c = \{x \in U \mid x \notin A\}.$$

The complement of B is defined similarly. Prove the following identity

$$(A \cap B)^c = A^c \cup B^c$$