MATH2210Q   Name:_____

Exam 1   Date:_____

This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any pages
are missing. Enter all requested information on the top of this page, and put your initials on the
top of every page, in case the pages become separated.

You may *not* use your books, notes, or any unapproved calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and
  coherent way, in the space provided. Work scat-
  tered all over the page without a clear ordering
  will receive very little credit.

- **Mysterious or unsupported answers will not
  receive full credit.** A correct answer, unsup-
  ported by calculations, explanation, or algebraic
  work will receive no credit; an incorrect answer
  supported by substantially correct calculations and
  explanations might still receive partial credit.

- If you need more space, ask for an extra sheet of
  paper to continue the problem on; clearly indicate
  when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	4	
2	4	
3	5	
4	2	
5	4	
6	6	
Total:	25	

1. (4 points) Find the inverse of the following matrix.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 3 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 1 & 3 & 1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 3 & 1 & -1 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & 5 & -3 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 5 & -3 & 1 \end{bmatrix}$$

2. (a) (2 points) Find the solution set of the following system.

$$\begin{aligned} x_1 &+ 2x_2 &- 3x_3 &= 0 \\ 2x_1 &+ x_2 &- 3x_3 &= 0 \\ -x_1 &+ x_2 & &= 0 \end{aligned}$$

$$\begin{bmatrix} 1 & 2 & -3 & 0 \\ 2 & 1 & -3 & 0 \\ -1 & 1 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -3 & 0 \\ 2 & 1 & -3 & 0 \\ 0 & 3 & -3 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -3 & 0 \\ 2 & 1 & -3 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -3 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x_1 = x_3$$
$$x_2 = x_3$$
$$x_3 \text{ free}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

t is a real #.

(b) (2 points) Can the following matrix be invertible? (your answer from part (a) is relevant)

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & -3 \\ -1 & 1 & 0 \end{bmatrix}$$

No, $A\vec{x} = \vec{0}$ has more than just the trivial solution.

3. Suppose

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$

(a) (1 point) Compute $2A + B$

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 2 & 4 & 2 & 0 \\ 2 & 6 & 6 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 3 & 2 & 3 & 0 \\ 1 & 9 & 3 & 3 \end{bmatrix}$$

(b) (2 points) Compute $A \cdot B$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(c) (2 points) What is A^{-1}

$$A^{-1} = B$$

4. (2 points) Consider the following linear transformation.

$$T(\mathbf{x}) = A\mathbf{x} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

where A is the standard matrix for T i.e.

$$A = [\mathbf{a_1}\ \mathbf{a_2}\ \mathbf{a_3}] = [T(\mathbf{e_1})\ T(\mathbf{e_2})\ T(\mathbf{e_3})]$$

In terms of the columns of A write $T\left(\begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}\right)$.

$$T\left(2\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 1\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 5\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right)$$

$$= 2T\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 1T\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 5T\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= 2\vec{a_1} + \vec{a_2} + 5\vec{a_3}$$

5. Consider the following matrix equation.

$$\begin{bmatrix} 1 & 0 & -4 \\ 0 & 3 & -2 \\ -2 & 6 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ -4 \end{bmatrix}$$

(a) (2 points) Find the solution set.

$$\begin{bmatrix} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ -2 & 6 & 3 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ 0 & 6 & -5 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -4 & 4 \\ 0 & 3 & 0 & -3 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$
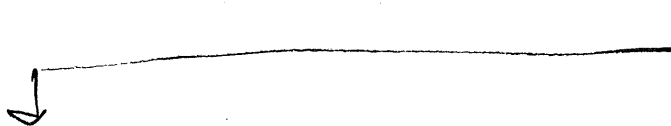
$$\sim \begin{bmatrix} 1 & 0 & -4 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 \end{bmatrix} \qquad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4 \\ -1 \\ -2 \end{bmatrix}$$

(b) (2 points) Write the equivalent linear system and vector equation.

$$\begin{aligned} x_1 \quad -4x_3 &= 4 \\ 3x_2 - 2x_3 &= 1 \\ -2x_1 + 6x_2 + 3x_3 &= -4 \end{aligned}$$

$$x_1 \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix} + x_3 \begin{bmatrix} -4 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 4 \end{bmatrix}$$

6. Consider the following matrix and one of its echelon forms.

$$A = \begin{bmatrix} 3 & -6 & 9 & 0 \\ 2 & -4 & 7 & 2 \\ 3 & -6 & 6 & -6 \end{bmatrix} \sim E = \begin{bmatrix} 1 & -2 & 5 & 4 \\ 0 & 0 & 3 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) (2 points) Find a basis for Null(A).

$$[A|0] \approx [E|0] = \begin{bmatrix} 1 & -2 & 5 & 4 & 0 \\ 0 & 0 & 3 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = t\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} 6 \\ 0 \\ -2 \\ 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & 5 & 4 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \text{Null}(A) \text{ basis} = \left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \\ -2 \\ 1 \end{bmatrix} \right\}$$

$$\sim \begin{bmatrix} 1 & -2 & 0 & -6 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(b) (2 points) Find a basis for Col(A).

Pivot columns are 1 and 3

$$\text{Col}(A) \text{ basis} = \left\{ \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 9 \\ 7 \\ 6 \end{bmatrix} \right\}$$

(c) (1 point) If $T(\mathbf{x}) = A\mathbf{x}$ is T a one to one transformation?

No, $A\vec{x} = \vec{0}$ has more than just the trivial solution

(d) (1 point) If $T(\mathbf{x}) = A\mathbf{x}$ is T onto?

No, $T: \mathbb{R}^4 \to \mathbb{R}^3$ col(A) $\neq \mathbb{R}^3$