

For full credit, you must show all work and circle your final answer.

- 1 Find the solution set to the following system of equations. (Write it in parametric form.)

$$\begin{array}{rrcr} x_1 & + & 2x_2 & - & 3x_3 & = & 5 \\ 2x_1 & + & x_2 & - & 3x_3 & = & 13 \\ -x_1 & + & x_2 & & & = & -8 \end{array}$$

$$\begin{bmatrix} 1 & 2 & -3 & 5 \\ 2 & 1 & -3 & 13 \\ -1 & 1 & 0 & -8 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -3 & 5 \\ 0 & -3 & 3 & 3 \\ 0 & 3 & -3 & -3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -3 & 5 \\ 0 & -3 & 3 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -3 & 5 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -1 & 7 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad t \text{ is a real \#}$$

2 Let T be the following linear transformation.

$$T: \mathbb{R}^3 \rightarrow \mathbb{R}^3; \quad T \left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) = \begin{bmatrix} 2x_1 - 4x_2 \\ x_1 - x_2 \\ -x_1 + 3x_2 \end{bmatrix}$$

(a) Find the standard matrix for T .

$$A = \begin{bmatrix} 2 & -4 & 0 \\ 1 & -1 & 0 \\ -1 & 3 & 0 \end{bmatrix} \quad \text{Note!} \quad \begin{bmatrix} 2 & -4 & 0 \\ 1 & -1 & 0 \\ -1 & 3 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2x_1 - 4x_2 \\ x_1 - x_2 \\ -x_1 + 3x_2 \end{bmatrix}$$

(b) Determine if T is a one to one linear transformation.

No, A linear transformation is one to one if and only if the columns are linearly indep.

The columns of A are not linearly indep. there is a zero column.

3 (a) Compute $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 0 & 1 \\ 1 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 2 & 0 \\ 1 & 1 & 2 \\ 2 & 0 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 11 & 4 & 4 \\ 11 & 6 & 0 \\ 9 & 4 & 4 \end{bmatrix}$$

(b) Find the inverse of $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$

$$\det = 10 - 12 = -2$$

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^{-1} = \frac{-1}{2} \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$$