MTH130 **Name:**________________________

Practice Final **Section:**________________________

This exam contains 8 pages (including this cover page) and 7 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books or notes on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, ask for an extra sheet of paper to continue the problem on; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	5	
2	5	
3	5	
4	5	
5	5	
6	5	
7	5	
Total:	35	

1. (5 points) Find the limits of the following functions

a) $\lim_{x\to\infty} \frac{x^2+1}{x^3+1}$

$$\lim_{x\to\infty} \frac{x^2+1}{x^3+3} = 0 \quad \deg(\text{top}) < \deg(\text{bot})$$

b) $\lim_{x\to\infty} \frac{x^5+2x^4+3x^2+1}{x^3+3x^2+1}$

$$\lim_{x\to\infty} \frac{x^5+2x^4+3x^2+1}{x^3+3x^2+1} = \infty,\ \text{D.N.E}$$

$\deg(\text{top}) > \deg(\text{bot})$

c) $\lim_{x\to\infty} \frac{2x^2+5x}{18x}$

$$\lim_{x\to\infty} \frac{2x^2+5x}{18x} = \infty \ \text{D.N.E}$$

$\deg(\text{top}) > \deg(\text{bot})$

d) $\lim_{x\to-4} \frac{x^2-16}{x+4}$

$$\lim_{x\to-4} \frac{x^2-16}{(x+4)} = \lim_{x\to-4} \frac{(x+4)(x-4)}{(x+4)} = \lim_{x\to-4} (x-4) = -8$$

e) $\lim_{x\to 2} x^2+2x+1$

$$\lim_{x\to 2} x^2+2x+1 = 2^2+2(2)+1 = 9$$

since x^2+2x+1 is cont at $x=2$

2. (5 points) Differentiate the following functions

a) $f(x) = x^5 + 2x^4 + 3x^2 + x - 1$

$$f'(x) = 5x^4 + 8x^3 + 6x + 1 - 0$$

$$\Rightarrow f'(x) = 5x^4 + 8x^3 + 6x + 1$$

b) $f(x) = (x^2 + 3x + 1)\cos(x^2)$

$$f(x) = (x^2 + 3x + 1)\cos(x^2)$$

$$\Rightarrow f'(x) = (x^2 + 3x + 1)\cdot \tfrac{d}{dx}(\cos(x^2)) + \cos(x^2)\cdot \tfrac{d}{dx}(x^2 + 3x + 1)$$

$$= (x^2 + 3x + 1)\left(-\sin(x^2)\cdot \tfrac{d}{dx}(x^2)\right) + \cos(x^2)\cdot(2x + 3)$$

$$= -(x^2 + 3x + 1)(2x)\sin(x^2) + (2x + 3)\cos(x^2)$$

c) $f(x) = \sqrt{\sin(2x + 3)}$

$$f(x) = (\sin(2x + 3))^{\frac{1}{2}}$$

$$f'(x) = \tfrac{1}{2}(\sin(2x + 3))^{-\frac{1}{2}} \tfrac{d}{dx}(\sin(2x + 3))$$

$$= \frac{1}{2}(\sin(2x + 3))^{-\frac{1}{2}} \cdot \cos(2x + 3)\cdot \tfrac{d}{dx}(2x + 3)$$

$$= \frac{1}{2}(\sin(2x + 3))^{-\frac{1}{2}} \cdot \cos(2x + 3)\cdot(2)$$

$$= \frac{\cos(2x + 3)}{\sqrt{\sin(2x + 3)}}$$

continued...

d) $f(x) = e^{\ln(x)}$

$$f(x) = e^{\ln x}$$

$$f'(x) = e^{\ln x} \cdot \tfrac{d}{dx} \ln x = \frac{e^{\ln x}}{x}$$

e) $f(x) = \frac{e^{2x+1}+5x}{\tan x}$

$$f'(x) = \frac{\tan(x) \cdot \frac{d}{dx}\left(e^{2x+1} + 5x\right) - \left(e^{2x+1} + 5x\right) \frac{d}{dx} \tan x}{\tan^2(x)}$$

$$= \frac{\tan x \cdot \left[e^{2x+1} \cdot \frac{d}{dx}(2x+1) + 5\right] - \left(e^{2x+1} + 5x\right)\left(\sec^2 x\right)}{\tan^2(x)}$$

$$= \frac{\tan x\left(2e^{2x+1} + 5\right) - \left(e^{2x+1} + 5x\right)\left(\sec^2 x\right)}{\tan^2 x}$$

3. (5 points) Consider the function below

$$f(x) = \frac{(x-1)(x+1)}{(x+2)(x-1)} = \frac{x^2-1}{x^2+x-2}$$

a) Find the...

points of discontinuities:

$x = -2$, $x = 1$

vertical asymptotes:

$x = -2$

horizontal asymptotes:

$\lim_{x \to \infty} \frac{x^2-1}{x^2+x-2} = 1 \Rightarrow y = 1$ is a H.A.

b) Find the tangent line at the point $x = 0$.

$$f'(x) = \frac{(x^2+x-2)(2x) - (x^2-1)(2x+1)}{(x^2+x-2)^2}$$

@ $x=0$ $f'(0) = \frac{0 - (-1)(1)}{(-2)^2} = 1/4$, $f(0) = \frac{-1}{-2} = 1/2$

Tangent line: $(y - 1/2) = 1/4\,(x - 0) \Rightarrow y = 1/4\,x + 1/2$

4. (5 points) Consider the function below.

$$f(x) = \frac{x^2+1}{2x+1}$$

a) Compute the differential of $f(x)$ at $x = 2$.

$$dy = f'(x)\,dx$$

$$dy = \left[\frac{(2x+1)(2x) - (x^2+1)2}{(2x+1)^2}\right] dx$$

$@x=2 \quad dy = \left(\frac{20-10}{25}\right) dx$

$\Rightarrow dy = \frac{2}{5}\,dx$

b) Compute the linearization of $f(x)$ at $x = 2$.

linearization $@x=2 \Leftrightarrow$ Tangent line $@ x=2$

$f(2) = \frac{2^2+1}{2(2)+1} = 1$; $f'(2) = 2/5$

$L(x) = 2/5\,(x-2) + 1$

$L(x) = 2/5\,x + 1/5$

c) Given $f(2) = 1$ approximate the value of $f(2.1)$.

$f(2.1) \approx f(2) + dy$ $\qquad dx = 2.1 - 2 = .1 = 1/10$

$= 1 + (2/5)\,dx$

$= 1 + (2/5)(1/10)$

$= 1 + 2/50$

$= 1.04$

5. (5 points) A cube's volume is expanding at a rate of 2 in^3/sec, where $V(x) = x^3$ and x is the side length.

   a) Find the rate of change of the side-length when $x = 2$.

   See the Exam 2 solution Question #3

   b) Find the rate of change of the surface area when $x = 2$, given $S(x) = 6x^2$.

   See the Exam 2 solution Question #3

6. (5 points) Consider the function below

$$f(x) = x^4 - 8x^2 + 16$$

a) Find the intervals when the function is increasing and decreasing.

Critical #'s

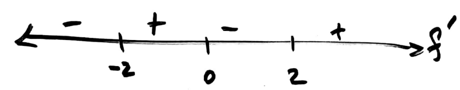
$f'(x) = 4x^3 - 16x$

$f'(x) = 4x(x^2-4)$

$f'(x)=0$ when $4x=0$

and $x^2-4=0$

$\Rightarrow x=0$; $x=\pm 2$



dec : $(-\infty,-2)$ and $(0,2)$

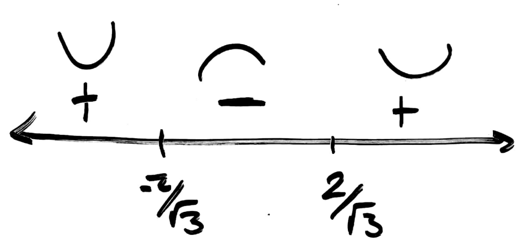
inc : $(-2,0)$ and $(2,\infty)$

b) Find the intervals when the function is concave up and concave down.

$f''(x) = 12x^2 - 16$

$f''(x) = 0$ when $12x^2-16=0$

i.e. when $x = \pm \frac{2}{\sqrt{3}}$



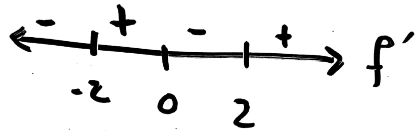
Concave up : $(-\infty, -2/\sqrt{3})$ and $(2/\sqrt{3}, \infty)$

Concave down: $(-2/\sqrt{3}, 2/\sqrt{3})$

continued...

c) Find the local maxima and minima via the 1st derivative test.

by part (a)



local max at $x=0$

local min at $x=2, x=-2$

d) Find the local maxima and minima via the 2nd derivative test.

by part (b) $f''(x)=12x^2-16$

$f''(0)=-16<0$ local max

$f''(2)=48-16>0$ local min

$f''(-2)=48-16>0$ local min.

e) Find the absolute maximum on $[0, 3]$.

$f(0)=16$

$f(2)=0$

$f(3)=25 \leftarrow$ absolute max

7. (5 points) Compute the net area under the given curve on the indicated interval.

a) $f(x) = 2x^2 + 3x + 1$ on $[0, 2]$.

$$\int_0^2 2x^2+3x+1\,dx$$

$$= \frac{2x^3}{3} + \frac{3x^2}{2} + x \Big|_0^2$$

$$= \left[\frac{2(2)^3}{3} + \frac{3(2)^2}{2} + 2\right] - \left[\frac{2}{3}(0)^3 + \frac{3(0)^2}{2} + 0\right]$$

$$= \left[16/3 + \frac{6}{2} + 2\right]$$

$$= \left[\frac{32+18+12}{6}\right]$$

$$= 62/6$$

b) $f(x) = 2\cos(x)$ on $[0, \pi]$

$$\int_0^\pi 2\cos(x)\,dx$$

$$= 2\sin(x)\Big|_0^\pi$$

$$= 2\sin(\pi) - 2\sin(0)$$

$$= 0$$

continued...
Compute the indefinite integrals

a) $f(x) = 1/x$.

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

b) $f(x) = \sin(2x)$.

$$\int \sin(2x)\,dx = \int \tfrac{1}{2}\sin(u)\,du = -\frac{1}{2}\cos(u) + C$$

$$= -\tfrac{1}{2}\cos(2x) + C$$

$u = 2x$

$du = 2dx$

$\Rightarrow \tfrac{1}{2}du = dx$

c) $f(x) = 2xe^{x^2}$.

$$\int 2xe^{x^2}\,dx = \int e^u\,du = e^u + C = e^{x^2} + C$$

$u = x^2$

$du = 2x\,dx$