

For full credit, you must show all work and circle your final answer.

- 1 Use the fundamental theorem of calculus to find the derivative of the given function.

$$h(x) = \int_1^{\sqrt{x}} \frac{z^2}{z^4 + 1} dz$$

Let $u = \sqrt{x}$

$$\begin{aligned} \frac{d}{dx} \left[\int_1^{\sqrt{x}} \frac{z^2}{z^4 + 1} dz \right] &= \frac{d}{dx} \left[\int_1^u \frac{z^2}{z^4 + 1} dz \right] = \frac{d}{du} \left[\int_1^u \frac{z^2}{z^4 + 1} dz \right] \cdot \frac{du}{dx} \\ &= \frac{u^2}{u^4 + 1} \cdot \frac{du}{dx} = \frac{x}{x^2 + 1} \cdot \frac{1}{2} x^{-3/2} \end{aligned}$$

- 2 Find the general indefinite integral.

$$\begin{aligned} \int \left(\frac{1+r}{r} \right)^2 dr &= \int \left(\frac{1}{r} + 1 \right)^2 dr = \int \left(\frac{1}{r^2} + \frac{2}{r} + 1 \right) dr \\ &= -\frac{1}{r} + 2 \ln|r| + r + C \end{aligned}$$

- 3 Use a substitution to evaluate the following indefinite integral.

$$\int \cos^3(\theta) \sin(\theta) d\theta$$

Let $u = \cos \theta$ $du = -\sin \theta d\theta$

$$\begin{aligned} \int \cos^3(\theta) \sin \theta d\theta &= -\int u^3 du = -\frac{u^4}{4} + C \\ &= -\frac{\cos^4(\theta)}{4} + C \end{aligned}$$