

Definitions to know:

- 1) connected metric space (pg. 57 notes)
- 2) continuous function (pg. 61 notes)
- 3) limit of a function (pg. 63 notes)
- 4) limit of a function at infinity (homework 3)
- 5) uniformly continuous function (pg. 66 notes)
- 6) differentiable function (pg. 78 notes)
- 7) Riemann integral of a function (pg. 87 notes)
- 8) refinement of a partition (pg. 88 notes)
- 9) common refinement of two partitions (pg. 88 notes)
- 10) point-wise convergence of sequence of functions (pg. 72 notes)
- 11) uniform convergence of a sequence of functions (pg. 73 notes)
- 12) uniform metric (pg. 74 notes)
- 13) convergent series (pg. 103 notes)

Proofs to know:

1) Pg. 61 notes

Theorem 1. *A function $f : X \rightarrow Y$ is continuous if and only if $f^{-1}(U) \subset X$ is open for every open set $U \subset Y$.*

2) Pg. 65 notes

Theorem 2. *If $f : X \rightarrow Y$ is continuous and X is compact then $f(X)$ is compact.*

3) Pg. 65 notes

Theorem 3. *If $f : X \rightarrow \mathbb{R}$ is continuous and X is non-empty and compact then there exists an $x_0 \in X$ such that $f(x_0) \geq f(x)$ for all $x \in X$, i.e. f has a maximum on X .*

4) Pg. 66 notes

Theorem 4. *If $f : X \rightarrow Y$ is continuous on X and X is compact then f is uniformly continuous on X .*

5) Pg. 73 notes

Theorem 5. *Suppose X and Y are metric spaces and (f_n) is a sequence of functions $f_n : X \rightarrow Y$. If each f_n is continuous and if (f_n) converges uniformly to f then f is continuous.*

6) Pg. 81 notes

Theorem 6. *Suppose $f : (a, b) \rightarrow \mathbb{R}$ is differentiable. The function f is increasing if and only if $f' \geq 0$.*

7) Pg. 90 notes

Theorem 7. *If f is continuous on $[a, b]$ then $f \in \mathcal{R}([a, b])$.*

8) Proved in class

Theorem 8. *If $f_n : [a, b] \rightarrow \mathbb{R}$ is a sequence of Riemann integrable functions which converge uniformly to a function $f : [a, b] \rightarrow \mathbb{R}$, then $f \in \mathcal{R}([a, b])$ and*

$$\lim_{n \rightarrow \infty} \int_a^b f_n dx = \int_a^b f dx$$

Know the statements of the following theorems:

1) Fundamental Theorem of Calculus I

2) Fundamental Theorem of Calculus II