MAP 2302 - Quiz #2 Name: Key.

For full credit, you must <u>show all work</u> and <u>circle</u> your final answer.

1a (*1 point*) Use the method of undermined coefficients to find a particular solution to the differential equation.

$$y'' - y = e^t$$

Aux eqn: $r^2 - 1 = 0$

$r = \pm 1$

$y_p = Ate^t$

$y_p' = A(te^t + e^t)$

$y_p'' = A(te^t + e^t + e^t)$

$y_p'' - y_p = \cancel{Ate^t} + 2Ae^t - \cancel{Ate^t} = e^t$

$\Rightarrow 2Ae^t = e^t$

$\Rightarrow A = \frac{1}{2}$

So $y_p = \frac{1}{2}te^t$

1b (*1.5points*) Write the form of the particular solution.

(i) $y'' - y = (2t+1)e^t$

(ii) $y'' + 2y' + y = (5t^2 + 3t + 1)e^t$

(iii) $y'' + y = \cos(t)$

(i) $r = \pm 1$ as above

$y_p = t(At + B)e^t$

(ii) Aux eqn: $r^2 + 2r + 1 = 0$

$(r+1)^2 = 0$

$r = -1 \Rightarrow s = 0$

$y_p = (A_2t^2 + A_1t + A_0)e^t$

(iii) Auxeqn: $r^2 + 1 = 0$

$r = \pm i$

$\Rightarrow s = 1$

$y_p = t(A\cos t + B\sin t)$

2 *(2.5 points)* Use variation of parameters to find a particular solution to the differential equation.

$$y'' - 2y' + y = t^{-1}e^t$$

Aux eqn: $r^2 - 2r + 1 = 0$

$(r-1)^2 = 0$

$r = 1$ double root

$y_1 = e^t \quad y_2 = te^t$

$W[y_1, y_2] = y_1 y_2' - y_2 y_1' = [e^t(te^t + e^t) - e^t(te^t)]$

$= e^{2t}$

$y_p = V_1 y_1 + V_2 y_2$

$V_1 = -\int \frac{e^t te^t}{t e^{2t}} dt = -\int dt = -t$

$\boxed{y_p = -te^t + (te^t)\ln|t|}$

$V_2 = \int \frac{e^t e^t}{t e^{2t}} = \int \frac{1}{t} dt = \ln|t|$

University of Florida Honor Code:

On my honor, I have neither given nor received unauthorized aid in doing this assignment.

Signature