$$\begin{array}{c} \text{MTH354} \\ \text{Homework 5} \end{array}$$

1. Suppose $f:[a,b]\to\mathbb{R}$ is Riemann integrable. Prove,

$$\lim_{c \to b, c < b} \int_{a}^{c} f \, dx = \int_{a}^{b} f(x) \, dx$$

2. Show that if f is a continuous real valued function on the interval [a, b] then

$$\int_{a}^{b} f(x) dx = f(\xi)(b - a)$$

for some $\xi \in [a, b]$.

3. Let c < a < b < d and define $\chi_{[a,b]} : [c,d] \to \mathbb{R}$ as follows:

$$\chi_{[a,b]}(x) = \begin{cases} 1 & x \in [a,b] \\ 0 & x \notin [a,b] \end{cases}$$

Show starting from the definition that $\chi_{[a,b]}$ is Riemann integrable on [c,d] and compute

$$\int_{c}^{d} \chi_{[a,b]}(x) \, dx.$$

- 4. Prove that if f is a continuous real valued function on the interval [a, b] such that $f(x) \ge 0$ for all $x \in [a, b]$ and $f(x_0) > 0$ for some $x_0 \in [a, b]$ then $\int_a^b f(x) \, dx > 0$.
- 5. Give an example of a Riemann integrable function $g:[a,b]\to [a,b]$ such that $g(x)\geq 0$ for all $x\in [a,b]$ and $g(x_0)>0$ for some $x_0\in [a,b]$ such that $\int_a^b g(x)\,dx=0$.