Power Series

Find the radius of convergence and the interval of convergence.

1. $\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}}(x+3)^n$
2. $\sum_{n=1}^{\infty} \frac{n^2 x^n}{2 \cdot 4 \cdot 6 \cdot \cdots \cdot (2n)}$
3. $\sum_{n=1}^{\infty} \sqrt{n} x^n$
4. $\sum_{n=1}^{\infty} \frac{n(x-4)^n}{n^3+1}$
5. $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^n}$

Representations of Functions as Power Series

Find a power series representation for the function and determine the interval of convergence.

6. $f(x) = \dfrac{x}{4x+1}$
7. (a) $f(x) = \ln(1+x)$
   (b) $f(x) = x\ln(1+x)$
   (c) $f(x) = x\ln(x^2+1)$ (Hint: Use part (a).)
8. (a) Evaluate $\displaystyle\int \frac{1}{1+x^7}dx$
   (b) Use part (a) to approximate $\displaystyle\int_{-.5}^{0} \frac{1}{1+x^7}dx$
9. Evaluate $\int \arctan(2x^3)\, dx$ using a power series.

Taylor and Maclaurin Series

10. Find the Taylor series for f centered at $x = 1$ and the radius of convergence given that $f^{(n)}(1) = \dfrac{(-1)^n n! n}{2^n}$
11. Use the Maclaurin series for e^x to calculate $e^{-0.2}$ correct to five decimal places.
12. Find the Taylor series for $f(x) = \cos x$ centered at $a = \pi$.
13. Find the Maclaurin series for $f(x) = e^{-x/2}$ and the radius of convergence.
14. Use the Taylor series for $f(x) = x^4 - 2x^3 + 1$ centered at $x = -1$ to evaluate $f(-1.1)$.
15. Find the sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{4^{2n+1}(2n+1)!}$
16. Use a Maclaurin series to evaluate the following limit:

$$\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}.$$

17. Evaluate the indefinite integrals using infinite series.
   (a) $\displaystyle\int \frac{\sin x}{x}dx$ (b) $\displaystyle\int \frac{x - \sin x}{x^3}dx$

Parametric Equations

18. Eliminate the parameter to find a Cartesian equation of the curve. $x = e^{2t}$, $y = e^t - t$

19. $x = 4\cos\theta$, $y = 5\sin\theta$, $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

    (a) Eliminate the parameter to find a Cartesian equation of the curve.

    (b) Sketch the curve and indicate with an arrow the direction in which the curve is traced.

20. Find an equation of the tangent line to the curve $x = \cos\theta + \sin(2\theta)$,$y = \sin\theta + \cos(2\theta)$ at the point $\theta = 0$.

21. Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ for the curve $x = t + \ln t$, $y = t - \ln t$. For what values of t is the curve concave up?

22. Find the length of the curve $x = e^t + e^{-t}x$, $y = 5 - 2t$, for $0 \leq t \leq 3$.

23. Find the length of the curve $x = \dfrac{t}{1+t}$, $y = \ln(1+t)$, for $0 \leq t \leq 2$.