**MTH322** **Name:**______________________

**Practice Exam 2** **Section:**______________________

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This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, ask for an extra sheet of paper to continue the problem on; clearly indicate when you have done this.

**Do not write in the table to the right.**

| Problem | Points | Score |
|---|---|---|
| 1 | 4 | |
| 2 | 4 | |
| 3 | 5 | |
| 4 | 5 | |
| 5 | 4 | |
| 6 | 3 | |
| Total: | 25 | |

1. (4 points) Find the solution to the equation below.

$$y'' - 2y' + 5y = 0, \quad y(0) = 1, \; y'(0) = 1$$

Aux Eqn: $r^2 - 2r + 5 = 0$

$r = \dfrac{2 \pm \sqrt{4 - 4(1)(5)}}{2}$

$r = \dfrac{2 \pm \sqrt{-16}}{2}$

$r = \dfrac{2 \pm 4i}{2} = 1 \pm 2i$

General Solution: $y(t) = C_1 e^{t} \cos(2t) + C_2 e^{t} \sin(2t)$

$\Rightarrow y'(t) = -2C_1 e^{t} \sin(2t) + C_1 e^{t} \cos(2t) + 2C_2 e^{t} \cos(2t) + C_2 e^{t} \sin(2t)$

Initial Data: $\left.\begin{array}{l} 1 = y(0) = C_1 1 + C_2 0 \\ 1 = y'(0) = -2C_1 \cdot 0 + C_1 \cdot 1 + 2C_2 \cdot 1 + C_2 \cdot 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 1 = C_1 \\ 1 = C_1 + 2C_2 \end{array}\right\} \Rightarrow \begin{array}{l} C_1 = 1 \\ C_2 = 0 \end{array}$

Solution: $y(t) = e^{t} \cos(2t)$

2. (4 points) Determine the form of the particular solution for the differential equations below. DO NOT FIND THE COEFFICIENTS.

a) $y'' + 9y = 4t^3 \sin(3t)$

$r^2+9=0 \Rightarrow r=\pm 3i$

$\alpha=0 \quad \beta=3 \; ; \; \alpha+i\beta=3i \Rightarrow s=1$

$y_p(t) = t^1(A_3t^3+A_2t^2+A_1t+A_0)\sin(3t) + t^1(B_3t^3+B_2t^2+B_1t+B_0)\cos(3t).$

b) $y'' - 6y' + 9y = 5t^6e^{3t}$

$r^2-6r+9=0 \Rightarrow (r-3)^2=0$

$r=3$ doubleroot

$s=2$

$y_p(t) = t^2(A_6t^6+A_5t^5+A_4t^4+A_3t^3+A_2t^2+A_1t+A_0)e^{3t}$

c) $2y'' + 3y' - 4y = 2t + \sin(t) + 3$

$2r^2+3r-4=0 \Rightarrow r = \dfrac{-3\pm\sqrt{9-4(2)(-4)}}{2(2)}$

$r = \dfrac{-3\pm\sqrt{41}}{4} \; ; \; s=0$

$y_p(t) = (A_1t+A_0) + B_1\sin(t) + C_1\cos(t)$

d) $y'' - 5y' + 6y = 2\sin(3t) + (5t+2)\cos(2t)$

$r^2-5r+6=0$

$(r-3)(r-2)=0$

$r=3, \; r=2$

$2\sin(3t) \Rightarrow \alpha+i\beta=3i \Rightarrow s=0$

$(5t+2)\cos(2t) \Rightarrow \alpha+i\beta=2i \Rightarrow s=0$

$y_p(t) = A\sin(3t) + B\cos(3t) + (C_1t+C_0)\sin(2t) + (D_1t+D_0)\cos(2t)$

3. (5 points) Use the method of undetermined coefficients to find the solution to the I.V.P.

$$y'' - 2y' + y = 2e^{t}; \qquad y(0) = 1,\ y'(0) = [redacted]$$

Aux Eqn: $r^2 - 2r + 1 = 0$

$(r-1)^2 = 0$

$r = 1$ double root

Homogeneous Soln: $y_H(t) = C_1 e^t + tC_2 e^t$

Particular Soln: $y_p(t) = At^2e^t \Rightarrow y_p(t) = t^2e^t$

General Soln: $y(t) = At^2e^t + C_1e^t + tC_2e^t$

$\Rightarrow y(t) = t^2e^t + C_1e^t + tC_2e^t$

Side Work:

$y_p(t) = At^2e^t$

$y_p'(t) = At^2e^t + 2Ate^t$

$y_p''(t) = At^2e^t + 2Ate^t + 2Ate^t + 2Ae^t$

$y_p'' - 2y_p' + y_p \stackrel{?}{=} 2e^t$

$(At^2 + 4At + 2A)e^t - 2At^2e^t - 4Ate^t + At^2e^t = 2e^t$

$\Rightarrow 2Ae^t = 2e^t$

$\Rightarrow A = 1$

Fitting Data:

$1 = y(0) = C_1$

$y'(t) = t^2e^t + 2te^t + C_1e^t + C_2te^t + C_2e^t$

$1 = y'(0) = C_1 + C_2$

$\Rightarrow \begin{array}{l} C_1 = 1 \\ C_1 + C_2 = 1 \end{array} \Rightarrow \begin{array}{l} C_1 = [redacted] \\ C_2 = 0 \end{array}$

$\boxed{\text{Solution: } y(t) = (t^2+1)e^t}$

4. (5 points) Find the Laplace transform of the following functions.

a) $\mathscr{L}\left\{t^2 + e^t \sin(2t)\right\}(s)$ $= \frac{2}{s^3} + \frac{4}{(s-1)^2+4}$

b) $\mathscr{L}\left\{3t^2 - e^{2t}\right\}(s)$ $= 6/s^3 - \frac{1}{s-2}$

c) $\mathscr{L}\left\{3t^4 - 2t^2 + 1\right\}(s)$ $= \frac{72}{s^5} - \frac{4}{s^3} + \frac{1}{s}$

d) $\mathscr{L}\left\{t^2 e^{-t} - t + \cos(4t)\right\}(s)$ $= \frac{2}{(s+1)^3} - \frac{1}{s^2} + \frac{s}{s^2+16}$

5. (4 points) Find the Laplace transform of the solution to the differential equation below.

$$y'' - 5y' + 6y = 5e^{2t} + 3\sin(3t), \qquad y(0) = 1,\ y'(0) = 2$$

$$(s^2 Y - s - 2) - 5(sY - 1) + 6Y = \frac{5}{s-2} + \frac{9}{s^2+9}$$

$$(s^2 - 5s + 6)Y - s + 3 = \frac{5}{s-2} + \frac{9}{s^2+9}$$

$$(s^2 - 5s + 6)Y = \frac{5}{s-2} + \frac{9}{s^2+9} + s - 3$$

$$Y(s) = \frac{5}{(s-2)(s^2-5s+6)} + \frac{9}{(s^2+9)(s^2-5s+6)} + \frac{s-3}{s^2-5s+6}$$

6. (3 points) Use the following trigonometric identity

$$\sin(\alpha)\cos(\beta) = \frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{2}$$

to find the Laplace transform of the function

$$f(t) = \sin(4t)\cos(3t).$$

$f(t) = \sin(4t)\cos(3t) = \frac{1}{2}\sin(7t) + \frac{1}{2}\sin(t)$

$$\boxed{\mathcal{L}\{f(t)\}(s) = \frac{1}{2}\cdot\frac{7}{s^2+49} + \frac{1}{2}\cdot\frac{1}{s^2+1}}$$