MATH2210Q - Quiz #2 Name:

For full credit, you must show all work and circle your final answer.

1 Determine the rank and nullity of the following matrices. Are either of the two matrices invertible?

(a) $A = \begin{bmatrix} 1 & 2 & 3 & 4 & -1 \\ 0 & 1 & 1 & 3 & 0 \\ 0 & 0 & 0 & -2 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

rank(A) = 3
nullity(A) = 2
not invertible

(b) $B = \begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & 5 & 0 & 3 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 3 \end{bmatrix}$

rank(B) = 4
nullity (B) = 0
invertible

2 Let

$$\mathcal{E} = \{\vec{e}_1, \vec{e}_2\}, \quad \mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}, \quad \mathcal{C} = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right\}$$

where $\mathcal{E}$ is the standard basis for $\mathbb{R}^2$. Compute the following change of basis matrices.

(a) $P_{\mathcal{B}}$

$$P_B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

(b) $\underset{\mathcal{C} \leftarrow \mathcal{B}}{P}$

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 1 & 3 & 2 & 1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & 0 & -1 & -2 \\ 0 & 1 & 1 & 1 \end{array}\right]$$

$$\underset{C \leftarrow B}{P} = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$$

(c) $\underset{\mathcal{B} \leftarrow \mathcal{C}}{P}$

$$\underset{B \leftarrow C}{P} = \underset{C \leftarrow B}{P}^{-1} = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$$

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$\boxed{3}$ Suppose A is a 2×3 matrix and B is a 4×2 matrix.

a) What is the minimum nullity of A?

nullity$(A) \geq 1$

b) If $T(\vec{x}) = A \cdot \vec{x}$ is this a one to one transformation?

No, nullity$(A) \neq 0$

c) Is it possible for B to be an invertible matrix?

No, only square matrices are invertible.