MATH2210Q - Quiz #1 Name: Soln

For full credit, you must show all work and circle your final answer.

$\boxed{1}$ (a) Find the solution set to the following system of equations. (Write it in parametric form.)

$$\begin{array}{rcrcrl} x_1 & + & 2x_2 & - & 3x_3 & = 3 \\ 2x_1 & + & x_2 & - & 3x_3 & = 3 \\ -x_1 & + & x_2 & & & = 0 \end{array}$$

$$\begin{bmatrix} 1 & 2 & -3 & 3 \\ 2 & 1 & -3 & 3 \\ -1 & 1 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -3 & 3 \\ 0 & -3 & 3 & -3 \\ 0 & 3 & -3 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -3 & 3 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\Leftrightarrow$ $x_1 - x_3 = 1$ $\Leftrightarrow$ $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $t \in \mathbb{R}$

$x_2 - x_3 = 1$

$x_3 = $ free

(b) Find the solution set to the following matrix equation. (Hint: Compare to the above.)

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & -3 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix}$$

This is equivalent to the above system

Soln : $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $t \in \mathbb{R}$

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2 Determine which of the following sets of vectors are linearly independent.

(a) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 5 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 9 \\ 0 \\ -1 \end{bmatrix} \right\}$

linearly independent $\begin{bmatrix} 1 \\ 2 \\ 5 \\ 2 \end{bmatrix} \neq c \begin{bmatrix} 2 \\ 9 \\ 0 \\ -1 \end{bmatrix}$ for any $c \in \mathbb{R}$.

(b) $\left\{ \begin{bmatrix} 2 \\ -2 \\ 3 \\ 9 \end{bmatrix}, \begin{bmatrix} 7 \\ 9 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 7 \\ 2 \\ 5 \end{bmatrix} \right\}$

linearly dependent

contains the zero vector

3 Determine if **b** lies in the span of the given vectors.

(a) $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix}$; $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

yes; $\begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 6 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

(b) $\mathbf{b} = \begin{bmatrix} 4 \\ 1 \\ -4 \end{bmatrix}$; $\left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 8 \\ -2 \end{bmatrix}, \begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix} \right\}$

$\vec{b}$ is not in the span

$$\left[\begin{array}{ccc|c} 2 & 0 & 6 & 4 \\ -1 & 8 & 5 & 1 \\ 1 & -2 & 1 & -4 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -2 & 1 & -4 \\ 2 & 0 & 6 & 4 \\ -1 & 8 & 5 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -2 & 1 & -4 \\ 0 & 4 & 4 & 12 \\ 0 & 6 & 6 & -3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -2 & 1 & -4 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 15 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \leftarrow \text{inconsistent system}$$