MATH2210Q - Quiz #3 **Name:**

For full credit, you must show all work and circle your final answer.

1 Determine if the following set of vectors is orthogonal.

$$\left\{ \underset{u_1}{\begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix}}, \underset{u_2}{\begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix}}, \underset{u_3}{\begin{bmatrix} 3 \\ -4 \\ -7 \end{bmatrix}} \right\}$$

No, $u_1 \cdot u_3 = (-1)(3) + 4(-4) + (-3)(-7) \neq 0$.

2 (a) Verify that $\{\mathbf{u}_1, \mathbf{u}_2\}$ is an orthogonal set then compute the orthogonal projection of $\mathbf{y}$ onto $\text{span}\{\mathbf{u}_1, \mathbf{u}_2\}$.

$$\mathbf{y} = \begin{bmatrix} 6 \\ 3 \\ -2 \end{bmatrix}, \; \mathbf{u}_1 = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}, \; \mathbf{u}_2 = \begin{bmatrix} -4 \\ 3 \\ 0 \end{bmatrix}$$

$\vec{u}_1 \cdot \vec{u}_2 = -12 + 12 = 0$

$\vec{u}_1 \cdot \vec{u}_1 = 9 + 16 = 25$

$\vec{u}_2 \cdot \vec{u}_2 = 9 + 16 = 25$

$\vec{y} \cdot \vec{u}_1 = 18 + 12 = 30$

$\vec{y} \cdot \vec{u}_2 = -24 + 9 = -15$

Let $W = \text{span}\{\vec{u}_1, \vec{u}_2\}$

$\text{Proj}_W y = \hat{y} = \left(\frac{30}{25}\right)\vec{u}_1 + \left(\frac{-15}{25}\right)\vec{u}_2$

$\Rightarrow \hat{y} = \frac{6}{5}\vec{u}_1 - \frac{3}{5}\vec{u}_2$

$\Rightarrow \hat{y} = \begin{bmatrix} 18/5 \\ 24/5 \\ 0 \end{bmatrix} - \begin{bmatrix} -12/5 \\ 9/5 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 0 \end{bmatrix}$

(b) What is the distance between $\mathbf{y}$ and the plane formed from $\mathbf{u}_1$ and $\mathbf{u}_2$?

$\text{dist} = \|y - \hat{y}\| = \left\| \begin{bmatrix} 6 \\ 3 \\ -2 \end{bmatrix} - \begin{bmatrix} 6 \\ 3 \\ 0 \end{bmatrix} \right\| = \left\| \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix} \right\| = 2$

$\boxed{3}$ Suppose we have the following

$$\left\{ \mathbf{u}_1 = \begin{bmatrix} 3/\sqrt{11} \\ 1/\sqrt{11} \\ 1/\sqrt{11} \end{bmatrix}, \ \mathbf{u}_2 = \begin{bmatrix} -1/\sqrt{6} \\ 2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}, \ \mathbf{u}_3 = \begin{bmatrix} -1/\sqrt{66} \\ -4/\sqrt{66} \\ 7/\sqrt{66} \end{bmatrix} \right\}$$

as an orthonormal basis for $\mathbb{R}^3$.

Let $\mathbf{y} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$. Find c_1, c_2, and c_3 such that $y = c_1\mathbf{u}_1 + c_2\mathbf{u}_2 + c_3\mathbf{u}_3$.

$$c_1 = y \cdot u_1 = \frac{3}{\sqrt{11}} + \frac{1}{\sqrt{11}} = \frac{4}{\sqrt{11}}$$

$$c_2 = y \cdot u_2 = -\frac{1}{\sqrt{6}} + \frac{1}{\sqrt{6}} = 0$$

$$c_3 = y \cdot u_3 = -\frac{1}{\sqrt{66}} + \frac{7}{\sqrt{66}} = \frac{6}{\sqrt{66}}$$