MAP 2302 - Quiz #1 **Name:** Solution

For full credit, you must show all work and circle your final answer.

1 *(2 points)* Solve the following IVP.

$$(e^x y + 1)\,dx + (e^x - 1)\,dy = 0, \qquad y(1) = 1$$

Let $M(x,y) = e^x y + 1 \qquad N(x,y) = e^x - 1$

$\frac{\partial M}{\partial y} = e^x \qquad \frac{\partial N}{\partial x} = e^x$ Equation is exact

$F(x,y) = \int (e^x - 1)\,dy + h(x)$

$\boxed{F(x,y) = e^x y - y + h(x)}$

$\frac{\partial F}{\partial x} = e^x y + h'(x) = M(x,y) = e^x y + 1$

$\Rightarrow h'(x) = 1 \Rightarrow h(x) = \int 1\,dx$

so $\boxed{h(x) = x}$

Solution is of the form: $F(x,y) = C$

$e^x y - y + x = C$

initial conditions: $y(1) = 1 \Rightarrow e^1(1) - (1) + (1) = C$

So $\boxed{C = e}$

$\boxed{\text{Final solution: } e^x y - y + x = e \quad \text{or} \quad y = \frac{e - x}{e^x - 1}}$

2 *(3 points)* Solve the following IVP.

$$(x^2+1)\frac{dy}{dx} + xy - x = 0, \quad y(0) = 1$$

Standard Form: $\frac{dy}{dx} + \frac{x}{x^2+1} y = \frac{x}{x^2+1}$

$P(x) = \frac{x}{x^2+1}$ $\quad Q(x) = \frac{x}{x^2+1}$

integrating Factor: $\mu(x) = e^{\int P(x)dx} = \exp\left[\int \frac{x}{x^2+1} dx\right]$ let $u = x^2+1$, $du = 2x\,dx$

$= \exp\left[\frac{1}{2}\int \frac{du}{u}\right]$

$= \exp\left[\frac{1}{2}\ln|x^2+1|\right]$

$= \sqrt{x^2+1}$

$\mu(x)\frac{dy}{dx} + \mu(x)p(x)y = \mu(x)Q(x)$

$\Rightarrow \sqrt{x^2+1}\frac{dy}{dx} + \frac{x}{\sqrt{x^2+1}} y = \frac{x}{\sqrt{x^2+1}}$

$\Rightarrow \frac{d}{dx}\left(\sqrt{x^2+1}\, y\right) = \frac{x}{\sqrt{x^2+1}}$ So $y = \frac{1}{\sqrt{x^2+1}}\left[\int \frac{x}{\sqrt{x^2+1}} dx + C\right]$

$\boxed{y = 1 + \frac{C}{\sqrt{x^2+1}}}$

initial conditions: $1 = 1 + \frac{C}{\sqrt{0+1}} \Rightarrow \boxed{C=0}$ Final Soln: $\boxed{y = 1}$

University of Florida Honor Code:

On my honor, I have neither given nor received unauthorized aid in doing this assignment.

Note: $\int \frac{x}{\sqrt{x^2+1}} dx$ let $u = x^2+1$, $du = 2x\,dx$

$= \int \frac{1}{2} u^{-1/2}\, du$

$= u^{1/2} = \sqrt{x^2+1}$

Signature