1. Let X and Y be sets of positive real numbers which are bounded above. Define

$$XY = \{xy \mid x \in X,\, y \in Y\}.$$

Show that $\text{lub}(XY) = \text{lub}(X) \cdot \text{lub}(Y)$.

2. Show, that the sequence

$$a_n = \frac{2n-3}{n+5} \qquad n \geq 1$$

converges.

3. Prove that $\{n^2+2\}_{n=1}^{\infty}$ diverges to infinity.

4. Let $\{x_n\}$ and $\{y_n\}$ be convergent sequences with limits x and y respectively. Prove

   (a) $\{cx_n\}$ converges to cx where $c \in \mathbb{R}$.

   (b) $\{x_n + y_n\}$ converges to $x + y$.

5. Use the monotone convergence theorem to show the sequence $\{x_n\}$ defined by

$$x_1 = \sqrt{2}, \quad x_{n+1} = \sqrt{2 + x_n} \quad \text{for } n > 1$$

converges.

Hint: Show by induction that the sequence is increasing and bounded above by 2.