MATH 3210
Exam 2

1. Find a formula for the determinant of a matrix A whose entries are given by $A_{i,j} = \frac{1}{\min\{i,j\}}$. (You must prove your formula works.)

2. Let $P(\mathbb{R})$ be the vector space of polynomials and $\mathbb{R}^\infty$ be the vector space of sequences of real numbers. Show that $P(\mathbb{R})^*$ and $\mathbb{R}^\infty$ are isomorphic.

Hint: Construct an explicit isomorphism between $P_n(\mathbb{R})'$ and $\mathbb{R}^{n+1}$ and extend this in the natural way to $P(\mathbb{R})$ and $\mathbb{R}^\infty$.

3. Let $M_{2\times 2}(\mathbb{R})$ denote the vector space of 2×2 matrices with real entries. Define the trace of a matrix A as the sum of the diagonal entries, i.e.

$$\operatorname{tr}(A) = a_{1,1} + a_{2,2}$$

Show that

$$\langle A, B \rangle = \operatorname{tr}(B^\top A)$$

is an inner product on this space.

4. Let $n \in \mathbb{N}$ be an odd number. Show that every $n \times n$ matrix with real entries has at least one real eigenvalue.

5. Let V be a finite dimensional vector-space and $P \in \mathcal{L}(V)$ be an operator such that $P^2 = P$. Show that the only eigenvalues are 1 and 0.