

1. A sequence of integers x_1, x_2, x_3, \dots is defined recursively by $x_1 = 3$, $x_2 = 7$ and

$$x_k = 5x_{k-1} - 6x_{k-2} \quad \text{for all } k \geq 3$$

Prove by induction that $x_n = 2^n + 3^{n-1}$ for all positive integers n .

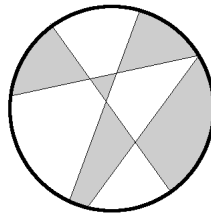
2. Prove by induction that a set of n elements contains 2^n subsets (including the set itself and \emptyset).
3. Prove by induction that if n points lie in a plane and no three are colinear, prove that there are $\frac{1}{2}n(n-1)$ lines joining these points.

Example:



4. Suppose that n chords are drawn in a circle, dividing the circle into different regions. Prove that every region can be colored one of two colors such that adjacent regions are different colors.

Example:



5. Prove that multiplication is a well defined operation on \mathbb{Q} .
6. Prove that $\sqrt{3}$ is irrational.