

MTH151

Name: Soln

Exam 2

Section: _____

This exam contains 6 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, ask for an extra sheet of paper to continue the problem on; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	5	
2	5	
3	5	
4	5	
5	5	
Total:	25	

1. (5 points) Evaluate the indefinite integrals below.

a) $\int \tan^2(x) \sec^4(x) dx$ Note! $1 + \tan^2 x = \sec^2(x)$

and $\frac{d}{dx}(\tan(x)) = \sec^2(x)$

$$= \int \tan^2(x) \sec^2(x) \sec^2(x) dx$$

$$= \int \tan^2(x) (1 + \tan^2(x)) \sec^2(x) dx \quad \text{let } u = \tan(x) \quad du = \sec^2(x) dx$$

$$= \int u^2(1-u^2) du$$

$$= \int u^2 - u^4 du = \frac{u^3}{3} - \frac{u^5}{5} + C$$

$$= \frac{\tan^3(x)}{3} - \frac{\tan^5(x)}{5} + C$$

b) $\int \sin^2(x) \cos^3(x) dx$ Note! $\cos^2(x) + \sin^2(x) = 1$

$\frac{d}{dx}(\sin(x)) = \cos(x)$

$$= \int \sin^2(x) \cos^2(x) \cos(x) dx$$

$$= \int \sin^2(x) (1 - \sin^2(x)) \cos(x) dx \quad \text{let } u = \sin(x) \quad du = \cos(x) dx$$

$$= \int u^2(1-u^2) du$$

$$= \frac{u^3}{3} - \frac{u^5}{5} + C$$

$$= \frac{\sin^3(x)}{3} - \frac{\sin^5(x)}{5} + C$$

2. (5 points) Evaluate the indefinite integrals below using a trig substitution.

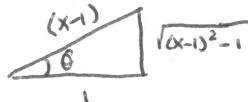
$$\text{a) } \int (x-1)\sqrt{x^2-2x} dx$$

$$= \int (x-1)\sqrt{x^2-2x+1-1} dx$$

$$= \int (x-1)\sqrt{(x-1)^2-1} dx \quad \text{let } (x-1) = \sec\theta \quad \sqrt{(x-1)^2-1} = \tan\theta \\ dx = \sec\theta \tan\theta d\theta$$

$$= \int \sec^2\theta \tan^2\theta d\theta \quad \text{let } u = \tan\theta \quad du = \sec^2\theta d\theta$$

$$= \int u^2 du$$



$$= \frac{u^3}{3} + C$$

$$= \frac{\tan^3\theta}{3} + C$$

$$= \frac{1}{3} (\sqrt{(x-1)^2-1})^3 + C$$

$$\text{b) } \int \frac{x}{\sqrt{1-x^2}} dx \quad \text{let } x = \sin\theta \quad dx = \cos\theta d\theta$$

$$\sqrt{1-x^2} = \cos\theta$$

$$= \int \frac{\sin\theta \cdot \cos\theta}{\cos\theta} d\theta$$

$$= \int \sin\theta d\theta$$

$$= -\cos\theta + C$$

$$= -\sqrt{1-x^2} + C$$

3. (5 points) Evaluate the indefinite integrals.

$$\text{a)} \int \frac{x^2 + 2x + 6}{(x+1)(x^2+4)} dx = \int \frac{A}{(x+1)} + \frac{Bx+C}{(x^2+4)} dx$$

$$x^2 + 2x + 6 = A(x^2+4) + (Bx+C)(x+1)$$

$$\text{plug in } x = -1 : 1 - 2 + 6 = 5A \Rightarrow A = 1$$

$$\text{plug in } x = 0 : 6 = 4 + C \Rightarrow C = 2$$

$$\text{plug in } x = 1 : 9 = 5 + (B+2)(2) \Rightarrow B = 0$$

$$\int \frac{x^2 + 2x + 6}{(x+1)(x^2+4)} dx = \int \frac{1}{(x+1)} dx + \int \frac{2}{x^2+4} dx \quad \text{let } \frac{x}{2} = \tan \theta \\ dx = 2\sec^2 \theta d\theta$$

$$= \ln|x+1| + \int \frac{4\sec^2 \theta}{14\tan^2 \theta + 4} d\theta$$

$$= \ln|x+1| + 2 \int d\theta = \ln|x+1| + 2\theta + C \\ = \ln|x+1| + 2 \arctan\left(\frac{x}{2}\right) + C$$

$$\text{b)} \int \frac{4x^2 - 4x - 2}{(x-2)(x-1)(x+1)} dx = \int \frac{A}{(x-2)} + \frac{B}{(x-1)} + \frac{C}{(x+1)} dx$$

$$4x^2 - 4x - 2 = A(x-1)(x+1) + B(x-2)(x+1) + C(x-2)(x-1)$$

$$\text{plug in } x = 2 : 16 - 8 - 2 = A(1)(3) \Rightarrow A = 2$$

$$\text{plug in } x = 1 : 4 - 4 - 2 = B(-1)(2) \Rightarrow B = 1$$

$$\text{plug in } x = -1 : 4 + 4 - 2 = C(-3)(-2) \Rightarrow C = 1$$

$$\int \frac{4x^2 - 4x - 2}{(x-2)(x-1)(x+1)} dx = \int \frac{2}{(x-2)} + \frac{1}{(x-1)} + \frac{1}{(x+1)} dx$$

$$= 2 \ln|x-2| + \ln|x-1| + \ln|x+1| + C$$

4. (5 points) Consider the following integral

$$\int_0^4 2x^3 - 1 \, dx$$

a) Estimate the integral with the Trapezoid rule with 4 sub-intervals.

$$\Delta x = \frac{4-0}{4} = 1 \quad ; \quad x_0 = 0, x_1 = 1, x_2 = 2, x_3 = 3, x_4 = 4$$

$$\begin{aligned} \int_0^4 2x^3 - 1 \, dx &\approx \frac{\Delta x}{2} [f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)] \\ &= \frac{1}{2} [-1 + 2(1) + 2(15) + 2(53) + 127] \\ &= 132 \end{aligned}$$

b) Estimate the integral with Simpson's Rule with 4 sub-intervals.

$$\begin{aligned} \int_0^4 2x^3 - 1 \, dx &\approx \frac{\Delta x}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)] \\ &= \frac{1}{3} [-1 + 4(1) + 2(15) + 4(53) + 127] \\ &= 124 \end{aligned}$$

5. (5 points) Evaluate the improper integral.

$$\text{a)} \int_e^\infty \frac{1}{x(\ln(x))^2} dx$$

$$= \lim_{t \rightarrow \infty} \int_e^t \frac{1}{x(\ln(x))^2} dx \quad \text{let } u = \ln x \\ du = \frac{1}{x} dx$$

$$= \lim_{t \rightarrow \infty} \left[-\frac{1}{\ln(x)} \right]_e^t$$

$$= \lim_{t \rightarrow \infty} -\frac{1}{\ln(t)} + 1$$

$$= 1$$

$$\text{b)} \int_5^{10} \frac{1}{\sqrt{x-5}} dx \quad \text{discont @ } x=5$$

$$= \lim_{t \rightarrow 5} \int_t^{10} \frac{1}{\sqrt{x-5}} dx$$

$$= \lim_{t \rightarrow 5} 2\sqrt{x-5} \Big|_t^{10}$$

$$= \lim_{t \rightarrow 5} 2\sqrt{5} - 2\sqrt{t-5}$$

$$= 2\sqrt{5}$$