MTH150 **Name:**__________________________

Practice Exam 2 **Section:**__________________________

This exam contains 8 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, ask for an extra sheet of paper to continue the problem on; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	4	
2	4	
3	4	
4	5	
5	4	
6	4	
Total:	25	

1. (4 points) Differentiate the following functions

a) $f(x) = \ln(x^4 \sin^2(x))$

$$f'(x) = \frac{1}{x^4 \sin^2(x)} \cdot (x^4 \cdot 2\sin(x)\cdot\cos(x) + \sin^2(x)\cdot 4x^3)$$

$$f'(x) = \frac{2x^4 \sin(x)\cos(x) + 4x^3\sin^2(x)}{x^4\sin^2(x)}$$

b) $f(x) = (1 + x^2)\arctan(x)$

$$f'(x) = (1+x^2)\cdot\frac{1}{1+x^2} + 2x\arctan(x)$$

$$f'(x) = 1 + 2x\arctan(x)$$

c) $f(x) = \arcsin(2x + 1)$

$$f'(x) = \frac{1}{\sqrt{1-(2x+1)^2}} \cdot 2$$

$$f'(x) = \frac{2}{\sqrt{-4x^2-4x}} = \frac{1}{\sqrt{-x^2-x}}$$

d) $f(x) = \sqrt{\tan^{-1}(x)}$

Note: $\tan^{-1}(x) = \arctan(x)$.

$$f'(x) = \frac{1}{2}(\arctan(x))^{-\frac{1}{2}} \cdot \frac{1}{1+x^2}$$

$$f'(x) = \frac{1}{(2+2x^2)\sqrt{\arctan(x)}}$$

2. (4 points) Use implicit differentiation to find the tangent line to the curve

$$xe^{xy} = 2x - y$$

at the point $(0, 0)$.

$$xe^{xy} = 2x - y$$

implicitly differentiate: $x \cdot \frac{d}{dx}(e^{xy}) + e^{xy} = 2 - \frac{dy}{dx}$

$$\Rightarrow x\left[e^{xy} \cdot \frac{d}{dx}(xy)\right] + e^{xy} = 2 - \frac{dy}{dx}$$

$$\Rightarrow x\left[e^{xy}\left(x\frac{dy}{dx} + y\right)\right] + e^{xy} = 2 - \frac{dy}{dx}$$

$$\Rightarrow x^2 e^{xy} \frac{dy}{dx} + xy + e^{xy} = 2 - \frac{dy}{dx}$$

$$\Rightarrow x^2 e^{xy} \frac{dy}{dx} + \frac{dy}{dx} = 2 - xy - e^{xy}$$

$$\frac{dy}{dx} = \frac{2 - xy - e^{xy}}{x^2 e^{xy} + 1}$$

$$\frac{dy}{dx} = \frac{2-1}{1} = 1 \quad @ (0,0)$$

Tangent line: $y - 0 = 1 \cdot (x - 0)$

$$\Rightarrow y = x$$

3. (4 points) Use logarithmic differentiation to find the derivatives of the following functions.

a) $f(x) = (2x)^{x^2}$

$$\ln(f(x)) = \ln\left(2x^{x^2}\right) = x^2 \ln(2x)$$

$$\Rightarrow \frac{f'(x)}{f(x)} = x^2 \cdot \frac{1}{2x} \cdot 2 + \ln(2x) \cdot 2x$$

$$\Rightarrow \frac{f'(x)}{f(x)} = x + 2x \ln(2x)$$

$$\Rightarrow f'(x) = [x + 2x \ln(2x)]\,(2x)^{x^2}$$

b) $f(x) = (\ln(x))^{\cos(9x)}$

$$\ln(f(x)) = \ln\left(\ln(x)^{\cos(9x)}\right) = \cos(9x)\ln(\ln(x))$$

$$\Rightarrow \frac{f'(x)}{f(x)} = \cos(9x) \cdot \frac{d}{dx}\left(\ln(\ln(x))\right) + \ln(\ln(x)) \cdot \frac{d}{dx}(\cos(9x))$$

$$\Rightarrow \frac{f'(x)}{f(x)} = \frac{\cos(9x)}{x \ln(x)} + \ln(\ln(x)) \cdot (-9\sin(9x))$$

$$\Rightarrow \frac{f'(x)}{f(x)} = \left[\frac{\cos(9x)}{x \ln(x)} - 9\sin(9x)\ln(\ln(x))\right]$$

$$\Rightarrow f'(x) = \left[\frac{\cos(9x)}{x \ln(x)} - 9\sin(9x)\ln(\ln(x))\right] \cdot (\ln(x))^{\cos(9x)}$$

4. (a) (2 points) Find the linearization $L(x)$ of the function below at $a = 0$

$$f(x) = \sqrt{9 - x}$$

$L(x) = f(a) + f'(a)(x-a)$

$f(x) = \sqrt{9-x}$

$f'(x) = -\frac{1}{2}(9-x)^{-1/2} = \frac{-1}{2\sqrt{9-x}}$ @ $x=0$ $\Rightarrow f'(0) = \frac{-1}{6}$

$\boxed{L(x) = 3 + (-\frac{1}{6})(x) = \frac{-x}{6} + 3}$

(b) (2 points) Use the linearization found about to estimate the value of $\sqrt{8.99}$.

$\sqrt{8.99} = \sqrt{9-0.01} = f(0.01)$

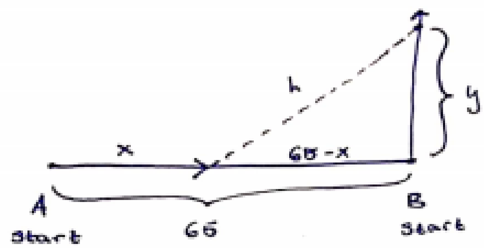
$f(0.01) \approx L(0.01) = \frac{-(0.01)}{6} + 3 = \frac{-1}{600} + 3 = \frac{1799}{600}$

(c) (1 point) Find the differential for the function below.

$$f(x) = \cos(x)$$

$dy = f'(x)\,dx \Rightarrow dy = -\sin(x)\,dx$

5. (4 points) At noon, ship A is 65 km west of ship B. Ship A is sailing east at 15 km/h and ship B is sailing north at 7 km/h. How fast is the distance between the ships changing at 3:00 PM?



$$\frac{dy}{dt} = 7 \quad ; \quad \frac{dx}{dt} = 15$$

$$h = \sqrt{(65-x)^2 + y^2}$$

$$\frac{dh}{dt} = \frac{1}{2}\left((65-x)^2 + y^2\right)^{-\frac{1}{2}} \cdot \left[-2(65-x)\frac{dx}{dt} + 2y\frac{dy}{dt}\right]$$

$$\frac{dh}{dt} = \frac{2y\frac{dy}{dt} - 2(65-x)\frac{dx}{dt}}{2\sqrt{(65-x)^2 + y^2}}$$

3 hrs passed

$x = 45$ km @ 15 km/hr

$y = 21$ km @ 7 km/hr

$$\frac{dh}{dt} = \frac{2(21)(7) - 2(20)(15)}{2\sqrt{20^2 + 21^2}} = -\frac{153}{29} \text{ km/hr} \approx -5.28 \text{ km/hr}$$

Negative indicates the ships are getting closer!

6. (4 points) Find the absolute extrema for the function below in the interval $[-1, 1]$.

$$f(x) = \ln(x^2 + 4)$$

$f(x) = \ln(x^2+4)$

Critical #'s: $f'(x) = \dfrac{2x}{x^2+4}$

$f'(x) = 0$ when $x = 0$

$f'(x)$ Exists for all x.

Test: $f(-1) = \ln(5)$ max

$f(1) = \ln(5)$

$f(0) = \ln(4)$ min