Basic Truth Tables You Should Know

(1) **Negation**

p	$\sim p$	
		The truth value of $\sim p$ takes the opposite value of p .
F	Т	

(2) Conjunction

p	q	$p \wedge q$	
\overline{T}	Т	Т	-
${ m T}$	F	F	Only true when both p and q are true.
F	Т	F	
F	F	F	

(3) **Disjunction**

p	q	$p \lor q$	
Т	Т	Т	
${ m T}$	F	T	True when either p or q are true.
\mathbf{F}	T	T	
\mathbf{F}	F	F	

(4) Conditional

p	q	$p \rightarrow q$	
Т	Т	Т	-
${ m T}$	F	F	Always true except for when p is true and q is false
F	Γ	Γ	
F	F	Γ	

(5) **Biconditional**

p	q	$p \rightarrow q$	
\overline{T}	Т	Т	
T	F	F	True only when p and q share the same truth value.
F	Т	F	
F	F	Т	

How to Read a Truth Table.

A truth table is essentially a chart of the different possible truth values of a statement given the truth value of the simpler statements. For example, the truth value of $p \lor q$ is true when p is true and q is false, as shown below;

p	q	$p \lor q$
T	Τ	Т
\mathbf{T}	F	\mathbf{T}
F	Т	Т
F	F	F

This makes sense, since it's an "or" statement, like "It is raining or it is snowing" and thus is true when either part of the statement p: "It is raining" or q: "it is snowing" is true.

Constructing a Truth Table.

We will show by example how to construct a truth table. We construct the truth table for the statement;

$$p \lor \sim q$$
.

We first note that the truth value of the above statement depends on the truth value for p and q. So we need both a p column, a q column and a column for the statement itself.

p	q	$p \lor \sim q$

Since we are aiming to make "a chart of the different possible truth values of a statement given the truth value of the simpler statements", we first need to know the possible different truth values for p and q. So we fill this in the chart, as shown below

$$\begin{array}{c|ccc} p & q & p \lor \sim q \\ \hline T & T & \\ T & F & \\ F & T & \\ F & F & \\ \end{array}$$

We note, that by filling in the first columns in with this pattern, we get all the possible combinations of truth values for p and q. Also, notice that the statement " $p \lor \sim q$ " does not really depend on p and q, but rather it depends on the truth value of p and $\sim q$. So we add a column to show this.

	p	q	$\sim q$	$p \lor \sim q$
_	Т	Τ		
- 1	Τ	\mathbf{F}		
	F	Τ		
	F	\mathbf{F}		

From the rule on negation, in section on basic truth tables, we know that the truth value of $\sim q$ takes the opposite value of the truth value of q. We show this below,

p	q	$\sim q$	$p \lor \sim q$
Т	\mathbf{T}	F	
${ m T}$	F		
\mathbf{F}	Т		
F	F		

We now fill in the rest of the column for $\sim q$.

p	q	$\sim q$	$p \lor \sim q$
\overline{T}	Т	F	
${\rm T}$	\mathbf{F}	${ m T}$	
\mathbf{F}	Τ	F	
\mathbf{F}	\mathbf{F}	Τ	

So since this statement is overall a disjunction, we note the rule for disjunction;

"True when either p or q are true."

and fill in the table accordingly. Consider the first row, p is true, and $\sim q$ is false, so the disjunction $p \vee \sim q$ is true, since at least one is true.

p	q	$\sim q$	$p \lor \sim q$
\mathbf{T}	Т	F	\mathbf{T}
Τ	F	Т	
\mathbf{F}	Т	\mathbf{F}	
\mathbf{F}	F	${ m T}$	

We continue on with this in the next row, p is true and $\sim q$ is true so the disjunction $p \vee \sim q$ is true.

p	q	$\sim q$	$p \lor \sim q$
Τ	Т	F	Т
\mathbf{T}	F	\mathbf{T}	\mathbf{T}
F	Т	F	
\mathbf{F}	F	${ m T}$	

We continue on and fill in the remainder of the column.

p	q	$\sim q$	$p \lor \sim q$
\overline{T}	Т	F	T
\mathbf{T}	\mathbf{F}	Τ	Τ
\mathbf{F}	Τ	F	F
F	F	Τ	T

One question remains, I said above that "[the] statement is overall a disjunction". How did I decide this? The answer is simple, it is decided by the dominance of connectives. The statement is overall a disjunction since disjunction was the most dominate connective in the statement, the other being negation. We used the type of statement it was, in the case a disjunction, to help us fill out the final column of the chart.