MTH151 **Name:** Solution

Practice Exam 1 **Section:** ____________

This exam contains 5 pages (including this cover page) and 7 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, ask for an extra sheet of paper to continue the problem on; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	3	
2	3	
3	5	
4	3	
5	4	
6	3	
7	4	
Total:	25	

1. (3 points) Use the fundamental theorem of calculus to find the derivatives of the following functions

   a) $g(x) = \int_0^x \sqrt{t + t^3}\, dt$ $\Rightarrow g'(x) = \sqrt{x + x^3}$

   b) $h(x) = \int_1^{e^x} \ln(t)\, dt$ $\Rightarrow h'(x) = \ln(e^x) \cdot e^x = e^x$

   c) $f(x) = \int_1^{\sqrt{x}} \frac{t^2}{t^4 + 1}\, dt$ $\Rightarrow f'(x) = \frac{(\sqrt{x})^2}{(\sqrt{x})^4 + 1} \cdot \frac{1}{2\sqrt{x}} = \frac{x}{x^2+1} \cdot \frac{1}{2\sqrt{x}}$

2. (3 points) Evaluate the definite integrals

   a) $\int_{\pi/2}^{\pi} \sin(\theta)\, d\theta$ $= -\cos(\theta)\Big|_{\pi/2}^{\pi} = \left[-\cos(\pi)\right] - \left[-\cos(\pi/2)\right] = 1 - 0 = 1$

   b) $\int_0^1 x^e + e^x\, dx$ $= \frac{x^{e+1}}{e+1} + e^x \Big|_0^1 = \left[\frac{1}{e+1} + e\right] - [1] = \frac{e^2}{e+1}$

   c) $\int_0^4 (4 - t)\sqrt{t}\, dt$ $= \int_0^4 4t^{1/2} - t^{3/2}\, dt = \frac{8}{3} t^{3/2} - \frac{2}{5} t^{5/2} \Big|_0^4$

   $= \frac{8}{3}(4)^{3/2} - \frac{2}{5}(4)^{5/2}$

   $= \frac{128}{5}$

3. (5 points) Evaluate the indefinite integrals.

a) $\int \frac{(\ln x^2)^3}{x}\,dx$

$= \frac{1}{2}\int u^3\,du$

$= \frac{u^4}{8} + C = \frac{[\ln x^2]^4}{8} + C$

Let $u = \ln x^2$

$du = \frac{1}{x^2} \cdot 2x\,dx$

$\Rightarrow \frac{1}{2}du = \frac{1}{x}dx$

b) $\int \sin^2(5t)\cos(5t)\,dt$

$= \frac{1}{5}\int u^2\,du$

$= \frac{1}{15}u^3 + C = \frac{1}{15}\sin^3(5t) + C$

Let $u = \sin(5t)$

$du = 5\cos(5t)\,dt$

$\Rightarrow \frac{1}{5}du = \cos(5t)\,dt$

c) $\int \frac{x^2e^{x^2}+1}{x}\,dx$ $= \int xe^{x^2} + \frac{1}{x}\,dx$

$= \int xe^{x^2}\,dx + \int \frac{1}{x}\,dx$

$= \frac{1}{2}e^{x^2} + \ln|x| + C$

For the first integral

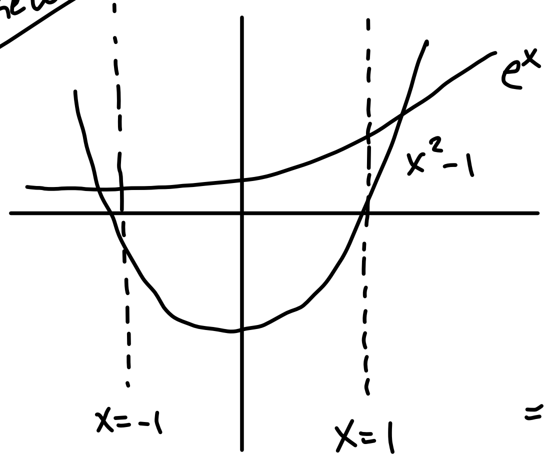
let $u = x^2$

$\frac{1}{2}du = x\,dx$

4. (3 points) Find the area between the curves on the given interval .

$$y = e^x; \qquad y = x^2 - 1; \qquad \text{on the interval } [-1, 1].$$

Sketch



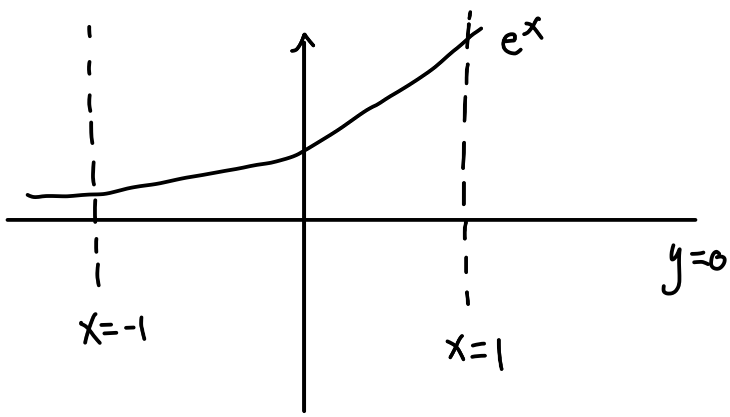
$$\text{Area} = \int_{-1}^{1} e^x - x^2 + 1 \, dx$$

$$= e^x - \frac{x^3}{3} + x \Big|_{-1}^{1}$$

$$= \left[e - \tfrac{1}{3} + 1\right] - \left[\tfrac{1}{e} + \tfrac{1}{3} - 1\right] = \tfrac{4}{3} + \frac{e^2 - 1}{e}$$

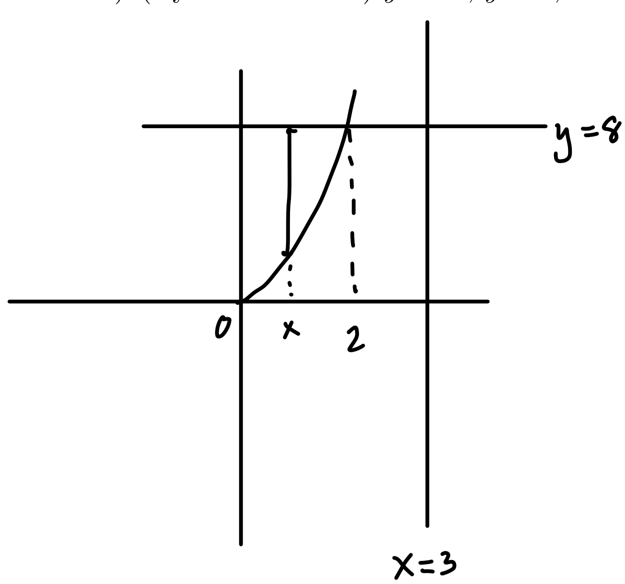
5. (4 points) Set up an integral to represent the volume of the solid of revolution given by the curves about the specified axis (DO NOT EVALUATE).

a) (Washer/Disc) $y = e^x$, $y = 0$, $x = -1$, $x = 1$ about the x-axis.



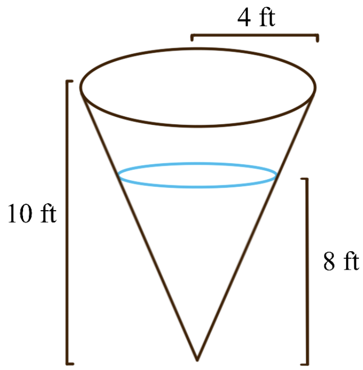
$$V = \pi \int_{-1}^{1} (e^x)^2 \, dx$$

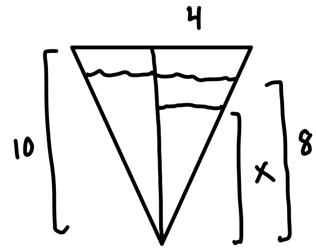
b) (Cylindrical Shell) $y = x^3$, $y = 8$, $x = 0$ about $x = 3$.



$$V = 2\pi \int_{0}^{2} (3-x)(8-x^3) \, dx$$

6. (3 points) A tank has the shape of an inverted circular cone with height 10 ft and a radius of 4 ft. It is filled with a liquid, weighing 1 lb/ft^3, to a height of 8ft. Write an integral equation for the work necessary to pump all the liquid out of the top of the tank.





Radius of slice:

$$\frac{4}{10} = \frac{r}{x}$$

$$\Rightarrow r = \frac{4x}{10}$$

Volume of slice:

$$V = \pi\left(\tfrac{4}{10}x\right)^2 dx$$

$$= \frac{4\pi}{25}x^2 dx$$

Since the liquid weighs 1 lb/ft^3

Force per slice $= \dfrac{4\pi}{25}x^2 dx$ & Each slice moves a distance of $(10-x)$

$$W = \int_0^8 \tfrac{4\pi}{25} x^2 (10-x)\, dx$$

7. (4 points) Use integration by parts to find the indefinite integral.

a) $\int x\cos(5x)\,dx$

$u = x$ $\quad dv = \cos(5x)\,dx$

$du = dx$ $\quad v = \frac{1}{5}\sin(5x)$

$$= \frac{x}{5}\sin(5x) - \tfrac{1}{5}\int \sin(5x)\,dx = \tfrac{x}{5}\sin(5x) + \tfrac{1}{25}\cos(5x) + C$$

b) $\int t^2\sin(\beta t)\,dt$ $= -\frac{t^2}{\beta}\cos(\beta t) + \frac{2t}{\beta^2}\sin(\beta t) + \frac{2}{\beta^3}\cos(\beta t) + C$

u	dv
t^2 $+$	$\sin(\beta t)$
$2t$ $-$	$-\frac{1}{\beta}\cos(\beta t)$
2 $+$	$-\frac{1}{\beta^2}\sin(\beta t)$
0	$\frac{1}{\beta^3}\cos(\beta t)$