

MTH107

Name: _____

Practice Final

Section: _____

This exam contains 8 pages (including this cover page) and 12 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books or notes on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, ask for an extra sheet of paper to continue the problem on; clearly indicate when you have done this.

Do not write in the table to the right.

| Problem | Points | Score |
|---------|--------|-------|
| 1 | 4 | |
| 2 | 4 | |
| 3 | 3 | |
| 4 | 3 | |
| 5 | 3 | |
| 6 | 3 | |
| 7 | 3 | |
| 8 | 2 | |
| 9 | 3 | |
| 10 | 2 | |
| 11 | 3 | |
| 12 | 2 | |
| Total: | 35 | |

1. (4 points) Let

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

and

$$A = \{1, 2, 3, 4, 5\} \quad B = \{2, 4, 6, 8, 10\}.$$

Construct the following sets

i) $A \cup B$

ii) $A \cap B$

iii) A^c

iv) B^c

2. (4 points) We are given the following information: $n(U) = 70$, $n(A) = 45$, $n(B) = 43$, $n(C) = 47$, $n(A \cap B) = 20$, $n(A \cap C) = 30$, $n(B \cap C) = 25$, and $n(A \cap B \cap C) = 10$.
Draw a venn diagram to represent this information.

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3. (3 points) Let $a = 222$ and $b = 72$. Use the Euclidean algorithm to find $\gcd(a, b)$.
4. (3 points) Find the prime factorization and compute $D(n)$ for each of the numbers below.
- a) 27
- b) 82
- c) 242

5. (3 points) Find the prime factorization and compute $S(n)$ for each of the numbers below and determine if the number is deficient, abundant, or perfect.

a) 42

b) 93

c) 202

6. (3 points) Find all numbers equivalent to the given number for the stated modulus.

a) $342 \bmod 2$

b) $7234 \bmod 5$

c) $3236 \bmod 10$

7. (3 points) Compute the following for the given modulus.

a) $[2] + [5] + [4] \bmod 2$

b) $[8] \cdot [341] \bmod 5$

c) $([21] \cdot [14]) + [42] \bmod 7$

8. (2 points) Complete the Cayley table for the given modular group.

$\{[0], [1], [2], [3]\}$ under addition mod 4

9. (3 points) Find the product of the following matrices

a) $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 0 \\ 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 2 & 3 \end{bmatrix}$

10. (2 points) Given $A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$ compute its determinant and its inverse.

11. (3 points) Compute the composition of the following permutations

a) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{bmatrix}$

c) $(2, 3, 4, 1, 5) \cdot (2, 1, 4)(3, 5)$

12. (2 points) Compute the Euler Characteristic of the tetrahedron below.

