MATH 3210 Your name
Homework 3 date

1. (§3.A #7) Show that every linear map from a 1-dimensional vector space to itself is multiplication by some scalar. More precisely, prove that if $\dim(V) = 1$ and $T \in \mathcal{L}(V)$, then there exists a $\lambda \in \mathbb{F}$ such that $Tv = \lambda v$ for all $v \in V$.

2. (§3.B # 9) Suppose that $T \in \mathcal{L}(V, W)$ is injective and $v_1, \ldots, v_n$ is linearly independent in V. Prove that $Tv_1, \ldots, Tv_n$ is linearly independent in W.

3. (§3.B # 10) Suppose that $v_1, \ldots, v_n$ spans V and $T \in \mathcal{L}(V, W)$. Prove that $Tv_1, \ldots, Tv_n$ spans $\mathrm{ran}(T)$.

4. (§3.B #12) Suppose that V is finite dimensional and that $T \in \mathcal{L}(V, W)$. Prove that there exists a subspace U of V such that $U \cap \mathrm{null}(T) = \{0\}$ and $\mathrm{ran}(T) = \{Tu : u \in U\}$.

5. (§3.D # 8) Suppose V is finite dimensional and $T : V \to W$ is a surjective linear map of V onto W. Prove that there is a subspace U of V such that $T\mid_U$ is an isomorphism of U onto W. (Here $T\mid_U$ means the function T restricted to U. In other words, $T\mid_U$ is the function whose domain is U, with $T\mid_U$ defined by $T\mid_U(u) = T(u)$ for every $u \in U$.)

6. (§3.D # 18) Show that V and $\mathcal{L}(\mathbb{F}, V)$ are isomorphic vector spaces.