Final Exam A

Course ID:	**MAC 2312**
Course Title:	Calculus II
Date of Exam:	August 8th, 2013
Duration of Exam:	120 minutes

Instructions

A. Sign your scantron sheet in the white area on the back in ink.

B. Write and code in the spaces indicated:

1) Name (last name, first name, middle initial)
2) UF ID number
3) Section number

C. Under "special code" code the test ID numbers 4 (1st row), 1 (2nd row).

1 2 3 • 5 6 7 8 9 0
• 2 3 4 5 6 7 8 9 0

D. Under "form code" code in A.

• B C D E

E. While taking the test, please keep your answer sheet covered or turned over at all times.

F. This test consists of 26 multiple choice questions. No calculators are allowed.

G. **When you are finished:**

1) Before turning in your test check for transcribing errors. No changes may be made after submitting your scantron.
2) You must turn in your scantron and tear off sheets to your instructor. Be prepared to show your picture ID with a legible signature.
3) The answers will be posted within one day after the exam.

The following questions are worth 6 points each.

1. How many of the following statements are true?

   I. If the sequence a_n is bounded above and increasing, then a_n converges.

   II. If $\lim_{n\to\infty} a_n = 0$, then the sum $\sum a_n$ converges.

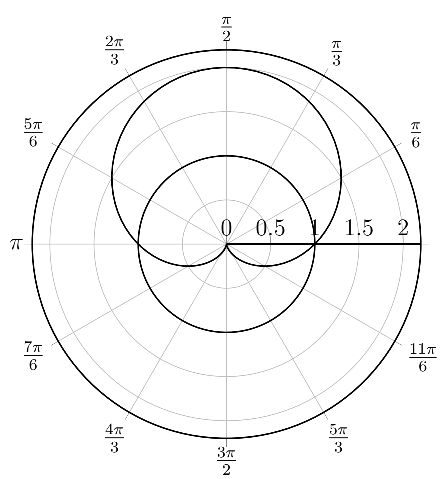
   III. The sequence $\left\{ \dfrac{\sin(n)}{n^2+1} \right\}_{n=1}^{\infty}$ converges.

   IV. If $\lim_{n\to\infty} |a_n| = 2$, then $\lim_{n\to\infty} a_n = 2$

   A. 0 B. 1 C. 2 D. 3 E. 4

2. Find the area of the region inside the first curve and outside the second curve:

$$r_1 = 1 + \sin(\theta); \quad r_2 = 1.$$



   A. $\dfrac{2+\pi}{4}$ B. $2+\dfrac{\pi}{4}$ C. $\dfrac{1+\pi}{4}$ D. $1+\dfrac{\pi}{4}$ E. $2-\dfrac{\pi}{4}$

3. A force of 20 N is required to stretch a spring from its natural length of 1 m to a length of 3 m. How much work is done in stretching the spring from 2 m to 5 m?

A. 150 J B. 45 J C. 75 J D. 105 J E. 210 J

4. Find the volume of the solid generated by rotating the region bounded by $y = \cos(x)$, $y = 2$, $x = 0$, and $x = \pi$ about the line $y = 2$.

A. $\frac{9}{2}$ B. $\frac{9\pi^2}{4}$ C. $\frac{9}{4}$ D. $\frac{9\pi}{2}$ E. $\frac{9\pi^2}{2}$

5. Which of the following statements are false?

I. If $\sum a_n$ and $\sum b_n$ both diverge, then so does $\sum(a_n + b_n)$.

II. If $\lim\limits_{n\to\infty} |a_n| = 0$, then $\lim\limits_{n\to\infty} a_n = 0$

III. $\sum\limits_{n=2}^{\infty} \frac{1}{n\ln(n)}$ converges by the integral test.

IV. $\sum\limits_{n=1}^{\infty} \frac{1}{n^2+1}$ converges by the ratio test.

V. $\sum\limits_{n=1}^{\infty} \frac{1}{n^2+1}$ converges by the direct comparison test.

A. I, II, and IV B. I and IV C. I, III, and IV

D. II and IV E. III only

6. If the partial fraction decomposition of $\frac{8}{x(x^2+4)}$ is $\frac{A}{x} + \frac{Bx+C}{x^2+4}$, then what is $A + B$?

A. $\frac{1}{4}$ B. 4 C. 0 D. 2 E. -2

7. Find the vertical and horizontal tangent lines for the following parametric curve:

$$x = 2\sin(t); \quad y = \sin(2t).$$

A. VTL: $x = \pm 1$; HTL: $y = \pm 2$.

B. VTL: $x = 2$; HTL: $y = \pm 1$.

C. VTL: $x = \pm 2$; HTL: $y = 1$.

D. VTL: $x = -2$; HTL: $y = \pm 1$.

E. VTL: $x = \pm 2$; HTL: $y = \pm 1$.

8. What is the interval of convergence for the following series?

$$\sum_{n=1}^{\infty} (-1)^n \frac{x^{2n}}{n4^n}$$

A. $[-2, 2]$

B. $(-2, 2)$

C. $[-2, 2)$

D. $(-3, 3]$

E. $[-3, 3]$

9. Find the length of the following parametric curve on the interval $-1 \le t \le 0$:

$$x = 2 + 3t^2; \quad y = 1 + 2t^3.$$

A. $2 - 2(2)^{3/2}$

B. $3\sqrt{2}$

C. $2\sqrt{2}$

D. $\sqrt{2}$

E. $-2(2)^{3/2}$

10. Integrate $\displaystyle\int_0^{\pi/4} \sec^4 x \, dx$.

A. $\dfrac{2^{5/2} - 1}{5}$

B. $\dfrac{4\sqrt{2} - 4}{3}$

C. $\dfrac{4\sqrt{2}}{3}$

D. $\dfrac{4}{3}$

E. $\dfrac{2^{5/2}}{5}$

11. Which of the following is the best substitution to integrate $\int \frac{x+1}{\sqrt{x^2-2x+5}}\,dx$?

A. $x-1=4\tan\theta$

B. $x^2-2x=\tan\theta$

C. $x=\sqrt{5}\tan\theta$

D. $x-2=\sqrt{5}\tan\theta$

E. $x-1=2\tan\theta$

12. A chain that weighs 400 lbs is 200 ft long and hangs vertically from the top of a tall building. How much work is required to lift half the chain to the top of the building?

A. 30,000 ft-lb B. 10,000 ft-lb C. 20,000 ft-lb D. 3,000 ft-lb E. 1,000 ft-lb

13. Which of the following can not be shown to converge by the ratio test?

A. $\sum_{n=1}^{\infty} \frac{1}{3^n+1}$ B. $\sum_{n=1}^{\infty} \frac{n^3}{2^n}$ C. $\sum_{n=1}^{\infty} \frac{1\cdot 4\cdot 7\cdot \ldots (3n-2)}{(2n)!}$

D. $\sum_{n=3}^{\infty} \frac{\sqrt{n}}{n^4+1}$ E. $\sum_{n=3}^{\infty} \frac{n!}{n^n}$

14. The Taylor series for $f(x)=x^2+x+1$ centered at $a=1$ is equal to

$$f(x)=c_0+c_1(x-1)+c_2(x-1)^2.$$

Find the value of c_1+c_2.

A. 5 B. 4 C. 3 D. 2 E. 1

15. Integrate $\int e^x \sin(x)\ dx$.

A. $e^x(\sin(x) - \cos(x)) + C$

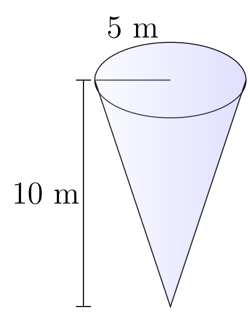
B. $\frac{1}{2}e^x(\sin(x) - \cos(x)) + C$

C. $\frac{1}{2}e^x(\sin(x) + \cos(x)) + C$

D. $-e^x \cos(x) + C$

E. $e^x(\sin(x)\cos(x)) + C$

16. A tank has the shape of an inverted cylindrical cone with a height of 10 m and a radius of 5 m. It is completely filled with a liquid which has a density of 12 kg/m^3. Find the work required to empty the tank by pumping all of the liquid to the top of the tank. Assume the acceleration due to gravity is 10 m/s^2.



A. 25,000 J   B. 2,500 J   C. 10,000π J   D. 2,500π J   E. 25,000π J

17. Evaluate the following.

$$\int_0^\infty xe^{\frac{-x^2}{2}}\ dx$$

A. 0   B. 1   C. $\frac{1}{2}$ D. 2   E. -1

18. Find the sum of the series, if it converges.

$$\sum_{n=0}^{\infty} \frac{(-9)^n}{(2n)!}$$

A. The series diverges. B. $\sin(3)$ C. $\sin(9)$ D. $\cos(3)$ E. $\cos(9)$

19. Which of the following is the correct partial fraction decomposition for

$$\frac{1}{(x^2+1)^2(x-1)^2(x+1)} \ ?$$

A. $\dfrac{Ax+B}{x^2+1} + \dfrac{Cx+D}{(x^2+1)^2} + \dfrac{E}{x-1} + \dfrac{Fx+G}{(x-1)^2} + \dfrac{H}{x+1}$

B. $\dfrac{Ax+B}{x^2+1} + \dfrac{Cx+D}{(x^2+1)^2} + \dfrac{Ex+F}{x-1} + \dfrac{Gx+H}{(x-1)^2} + \dfrac{Ix+J}{x+1}$

C. $\dfrac{Ax+B}{(x^2+1)^2} + \dfrac{Cx+D}{(x-1)^2} + \dfrac{E}{x+1}$

D. $\dfrac{Ax+B}{x^2+1} + \dfrac{Cx+D}{(x^2+1)^2} + \dfrac{E}{x-1} + \dfrac{F}{(x-1)^2} + \dfrac{G}{x+1}$

E. $\dfrac{A}{x^2+1} + \dfrac{B}{(x^2+1)^2} + \dfrac{C}{x-1} + \dfrac{D}{(x-1)^2} + \dfrac{E}{x+1}$

20. How many of the following series converge?

$$\sum_{n=1}^{\infty} \frac{1}{n^3} \qquad \sum_{n=1}^{\infty} \frac{1}{n} \qquad \sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$$

$$\sum_{n=1}^{\infty} \frac{2^n+1}{3^n+1} \qquad \sum_{n=2}^{\infty} \left(\frac{4}{3}\right)^n$$

A. 1 B. 2 C. 3 D. 4 E. 5

Bonus Questions.

The following questions are worth 2 points each.

21. The graph of the polar equation $r = 3\sin(3\theta)$ has 3 "petals".

A. True

B. False

22. Suppose the integral test is valid for $\sum_{n=0}^{\infty} f(n)$. Then according to the integral test

$$\sum_{n=0}^{\infty} f(n) = \int_0^{\infty} f(x)\ dx.$$

A. True

B. False

23. $\int \ln(x)\ dx = x\ln(x) - \int \frac{\ln(x)}{x}\ dx.$

A. True

B. False

24. Polar coordinates are unique.

A. True

B. False

25. The following integral converges: $\int_1^{\infty} \frac{1}{x^{\pi}}\ dx.$

A. True

B. False

26. Taking Calc 2 over the summer is the best thing ever.

A. True