

MTH322

Name: Sohn

Exam 1

Section: _____

This exam contains 5 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, ask for an extra sheet of paper to continue the problem on; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	3	
2	3	
3	4	
4	5	
5	5	
6	5	
Total:	25	

1. (3 points) Determine whether the given relation is an implicit solution to the given differential equation. Assume that the relationship does define y as a function of x .

$$e^{xy} + y = x - 1; \quad \frac{dy}{dx} = \frac{e^{-xy} - y}{e^{-xy} + x}$$

$$e^{xy} + y = x - 1$$

$$\Rightarrow e^{xy} \left(x \frac{dy}{dx} + y \right) + \frac{dy}{dx} = 1$$

$$\Rightarrow x e^{xy} \frac{dy}{dx} + \frac{dy}{dx} + y e^{xy} = 1$$

$$\Rightarrow \frac{dy}{dx} (x e^{xy} + 1) = 1 - y e^{xy}$$

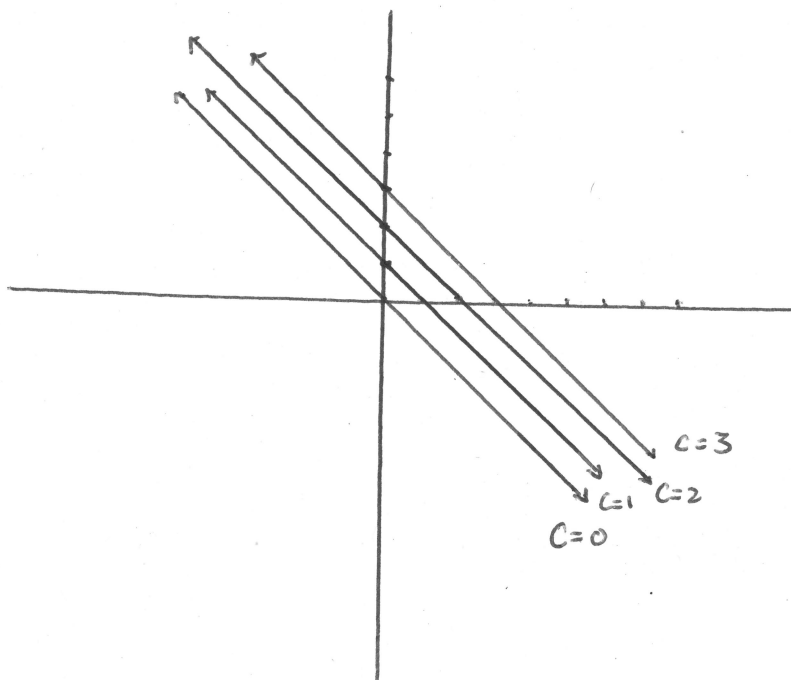
$$\Rightarrow \frac{dy}{dx} = \frac{1 - y e^{xy}}{1 + x e^{xy}} = \frac{e^{-xy} - y}{e^{-xy} + x}$$

The above equation is
an implicit solution.

2. (3 points) Draw and label the isoclines for the differential equation for the listed values.

$$\frac{dy}{dx} = x + y \quad c = 0, 1, 2, 3$$

c	isocline
0	$y = -x$
1	$y = -x + 1$
2	$y = -x + 2$
3	$y = -x + 3$



3. (4 points) Find a general solution to the given differential equation.

$$y'' + 6y' + 9y = 0.$$

$$\text{Aux Egn: } r^2 + 6r + 9 = 0$$

$$(r+3)^2 = 0$$

$$r = -3 \text{ repeated root}$$

$$y(t) = C_1 e^{-3t} + C_2 t e^{-3t}$$

General soln.

4. (5 points) Find a general solution to the given differential equation.

$$\frac{dy}{dx} - \frac{y}{x} = x e^x$$

$$P(x) = -\frac{1}{x} \Rightarrow e^{\int P(x) dx} = e^{-\int \frac{1}{x} dx} = e^{-\ln|x|} = \frac{1}{x}$$

$$\text{Let } \mu(x) = \frac{1}{x}$$

$$\frac{1}{x} \frac{dy}{dx} - \frac{1}{x^2} y = e^x$$

$$\frac{d}{dx} \left[\frac{1}{x} y \right] = e^x$$

$$\frac{y}{x} = \int e^x dx + C$$

$$\boxed{y = x e^x + C x}$$

5. (5 points) Fill out the table using Euler's Method with step size $h = 1$ using the I.V.P below.

$$\frac{dy}{dx} = x + y, \quad y(0) = 1$$

n	x_n	y_n
0	0	1
1	1	2
2	2	5
3	3	12
4	4	27

$$x_{n+1} = x_n + h$$

$$y_{n+1} = y_n + h f(x_n, y_n)$$

$$y_1 = y_0 + 1(0+1) = 2$$

$$y_2 = 2 + 1(1+2) = 5$$

$$y_3 = 5 + 1(2+5) = 12$$

$$y_4 = 12 + 1(3+12) = 27$$

6. (5 points) Give an implicit solution to the initial value problem.

$$\frac{1}{2} \frac{dy}{dx} = \sqrt{y+1} \cos(x); \quad y(\pi) = 0$$

$$\Rightarrow \frac{dy}{\sqrt{y+1}} = 2 \cos(x) dx$$

$$\int (y+1)^{-\frac{1}{2}} dy = \int 2 \cos(x) dx$$

$$\Rightarrow 2\sqrt{y+1} = 2\sin(x) + C$$

$$@ (\pi, 0)$$

$$2 = 2\sin(\pi) + C$$

$$\Rightarrow \boxed{C=2}$$

$$\text{Implicit Soln: } 2\sqrt{y+1} = 2\sin(x) + 2$$