MTH150 **Name:** Solution

Practice Exam 1 **Section:** ______

This exam contains 6 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, ask for an extra sheet of paper to continue the problem on; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	5	
2	4	
3	4	
4	3	
5	5	
6	4	
Total:	25	

1. (5 points) Find the indicated limits if they exist.

a) $\lim_{x\to 4} \frac{x^2-4x}{x^2-x-12}$ $\left(\frac{0}{0}\right)$ indeterminate form

$$\lim_{x\to 4} \frac{x^2-4x}{x^2-x-12} = \lim_{x\to 4} \frac{x(x-4)}{(x-4)(x+3)} = \lim_{x\to 4} \frac{x}{(x+3)} = 4/7$$

b) $\lim_{x\to 0} \frac{\sqrt{9+x}-3}{x}$ $\left(\frac{0}{0}\right)$ indeterminate form

$$= \lim_{x\to 0} \frac{\sqrt{9+x}-3}{x} \cdot \left(\frac{\sqrt{9+x}+3}{\sqrt{9+x}+3}\right) = \lim_{x\to 0} \frac{(9+x)-9}{x(\sqrt{9+x}+3)} = \lim_{x\to 0} \frac{1}{\sqrt{9+x}+3} = 1/6$$

c) $\lim_{x\to\infty} \frac{2x^2+3x+1}{3x^2+2x+1} = 2/3$ since $\deg(\text{top}) = \deg(\text{bot})$

d) $\lim_{x\to\infty} e^{-x^2+2}\sin(x)$

Note: $-1 \leq \sin(x) \leq 1$ and $e^{-x^2+2} > 0$ for all x.

So $\lim_{x\to\infty} -e^{-x^2+2} \leq \lim_{x\to\infty} e^{-x^2+2}\sin(x) \leq \lim_{x\to\infty} e^{-x^2+2}$

Since $\lim_{x\to\infty} e^{-x^2+2} = \lim_{x\to\infty} -e^{-x^2+2} = 0$ we have $\lim_{x\to\infty} e^{-x^2+2}\sin(x) = 0$.

2. (4 points) Using the limit definition find the derivative of the function below.

$$f(x) = (x+2)^2$$

$$f(x) = x^2 + 4x + 4$$

$$\lim_{x\to a} \frac{f(x)-f(a)}{x-a} = \lim_{x\to a} \frac{(x^2+4x+4)-(a^2+4a+4)}{x-a} = \lim_{x\to a} \frac{x^2-a^2+4(x-a)}{x-a}$$

$$= \lim_{x\to a} \frac{(x-a)(x+a)+4(x-a)}{(x-a)}$$

$$= \lim_{x\to a} (x+a)+4$$

$$= 2a+4$$

So $f'(x) = 2x+4$

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or

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2+4(x+h)+4-(x^2+4x+4)}{h}$$

$$= \lim_{h\to 0} \frac{(\cancel{x^2}+2xh+h^2)+(\cancel{4x}+4h)+\cancel{4}-\cancel{x^2}-\cancel{4x}-\cancel{4}}{h}$$

$$= \lim_{h\to 0} \frac{2xh+h^2+4h}{h}$$

$$= \lim_{h\to 0} 2x+4+h = 2x+4$$

So $f'(x) = 2x+4$

3. (4 points) Consider the function below.

$$f(x) = \frac{(x-1)(x+2)(2x-3)}{(x+2)(x-3)(x+1)}$$

Find the vertical asymptotes, horizontal asymptotes, and the points of discontinuities.

Vertical Asymptotes: Get a V.A. when bot $=0$ unless it cancels

$X=3$; $x=-1$

Horizontal Asymptotes: $\lim\limits_{x \to \pm\infty} \frac{2x^3+\dots}{x^3+\dots} = 2 \Rightarrow$ HA $y=2$

Points of Discontinuities:

the function is discontinuous @ $x=-2$, $x=3$, $x=-1$.

4. (3 points) Find where the following functions are continuous.

a) $f(x) = \ln(x^2+2)$ $\quad$ $\ln(x)$ is continuous when $x>0$

$x^2+2>0$ for all x, so $f(x)$ is continuous Everywhere.

$(-\infty, \infty)$

b) $f(x) = \sqrt{2+x}$ $\quad$ Continuous when $x+2 \geq 0$ i.e. when $x \geq -2$

$[-2, \infty)$

c) $f(x) = 8x^2 + \frac{1}{x^5}$ $\quad$ Discontinuous when $x=0$

$(-\infty, 0) \cup (0, \infty)$

5. (5 points) Find the derivative of the following functions.

a) $f(x) = \sqrt{5x} + \frac{\sqrt{7}}{x}$ $= \sqrt{5}\, x^{1/2} + \sqrt{7}\, x^{-1}$

$f'(x) = \frac{\sqrt{5}}{2} x^{-1/2} - \sqrt{7}\, x^{-2} = \frac{\sqrt{5}}{2\sqrt{x}} - \frac{7}{x^2}$

b) $f(x) = 3e^x + \frac{2}{\sqrt[4]{x}}$ $= 3e^x + 2x^{-1/4}$

$f'(x) = 3e^x + 2\left(-\frac{1}{4}\right) x^{-5/4} = 3e^x - \frac{1}{2x^{5/4}}$

c) $f(x) = (x^2 + 5x + 1)\sin(x)$

$f'(x) = (x^2+5x+1)\cos(x) + (2x+5)\sin(x)$

d) $f(x) = \frac{e^x}{\tan(x)}$

$f'(x) = \frac{e^x \tan(x) - e^x \sec^2(x)}{\tan^2(x)} = e^x \cot(x) - e^x \csc^2(x)$

e) $f(x) = \frac{3e^x \sin(x)}{(3x+1)}$

$f'(x) = \frac{(3x+1)\left[3e^x\cos(x) + 3e^x \sin(x)\right] - 3e^x \sin(x)(3)}{(3x+1)^2}$

$f'(x) = \frac{9xe^x(\cos(x)+\sin(x)) + 3e^x\cos(x) - 6e^x\sin(x)}{(3x+1)^2}$

$f'(x) = \frac{(9x+3)e^x\cos(x) + (9x-6)e^x\sin(x)}{(3x+1)^2}$

6. (4 points) Consider the function below.

$$f(x) = e^x(x^2 + 2x + 1)$$

a) Find the average rate of change of the function over $[1, 2]$

$$\text{Avg rate} = \frac{\Delta f}{\Delta x} = \frac{f(2) - f(1)}{2-1} = \frac{e^2(9) - e^1(4)}{1} = 9e^2 - 4e$$

b) Find the tangent line at the point $(0, 1)$.

$$f'(x) = e^x(2x+2) + (x^2+2x+1)e^x$$

$$f'(0) = e^0(0+2) + (0+0+1)e^0$$

$$\Rightarrow f'(0) = 3 \leftarrow \text{slope}$$

$$(0,1) \leftarrow \text{point}$$

Tangent line: $y - 1 = 3(x - 0)$

$$\Rightarrow \boxed{y = 3x + 1}$$