MATH 3150 Your name
Homework 1 August 29, 2017

1. Let $\mathbb{C}$ denote the complex numbers with the standard addition and multiplication. Show that there is no relation $>$ such that $\mathbb{C}$ is an ordered field.

2. Let A and B be two subsets of $\mathbb{R}$ which are bounded below. Show

$$\inf(A \cup B) = \min\{\inf(A), \inf(A)\}.$$

3. Find the supremum and infimum of the following set: $S = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\}$. Prove your claim.

4. Let $a \in \mathbb{R}$ such that $a > 1$. Show that $A = \{a, a^2, a^3, \ldots\}$ is not bounded above.

   Hint: Do not forget about induction!

   (i) Show that there exists an integer n_0 such that $a > 1 + \frac{1}{n_0}$.

   (ii) Show that $\left(1 + \frac{1}{n}\right)^n \geq 2$ for all natural numbers n.

   (iii) Show $2^n > n$ for all natural numbers n.

   (iv) Since $a > 1 + \frac{1}{n_0}$ then $a^{n_0} > \left(1 + \frac{1}{n_0}\right)^{n_0} > 2$. Use (i) and (ii) to reduce the problem to showing $\{2, 2^2, 2^3, \ldots\}$ is unbounded and then show this is true by item (iii)