MATH2210Q

Exam 1

Name:________________________

Date:________________________

This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any unapproved calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, ask for an extra sheet of paper to continue the problem on; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	4	
2	4	
3	3	
4	4	
5	4	
6	6	
Total:	25	

1. (4 points) Find the inverse of the following matrix.

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ -3 & 1 & 4 & 0 & 1 & 0 \\ 2 & -3 & 4 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & -3 & 8 & -2 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & 0 & 1 & 7/2 & 3/2 & 1/2 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 1 & 7/2 & 3/2 & 1/2 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 8 & 3 & 1 \\ 10 & 4 & 1 \\ 7/2 & 3/2 & 1/2 \end{bmatrix}$$

2. (a) (2 points) Determine if the following matrix is inveritble. Justify your answer.

$$B = \begin{bmatrix} 3 & 4 & 7 & 4 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & 2 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Yes, B is a 4x4 matrix with 4 pivotpositions

therefore B is invertible

(Theorem 8c, §2.3)

(b) (2 points) Can a square matrix with two identical rows be invertible ? Why or why not?

No, with two identical rows it cannot

be row equivalent to the identity matrix.

(Theorem 8 b, §2.3)

3. (3 points) Suppose

$$\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \cdot \begin{bmatrix} I & 0 \\ X & Y \end{bmatrix} = \begin{bmatrix} 0 & I \\ Z & 0 \end{bmatrix}$$

Where A, B, C , X, Y, Z are square $n \times n$ matrices and I is the $n \times n$ identity. Find a formula for X, Y and Z in terms of A, B and C.

$$\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} I & 0 \\ X & Y \end{bmatrix} = \begin{bmatrix} A+BX & BY \\ C & 0 \end{bmatrix} = \begin{bmatrix} 0 & I \\ Z & 0 \end{bmatrix}$$

So $A + BX = 0$

$BY = I$

$\boxed{C = Z}$ *

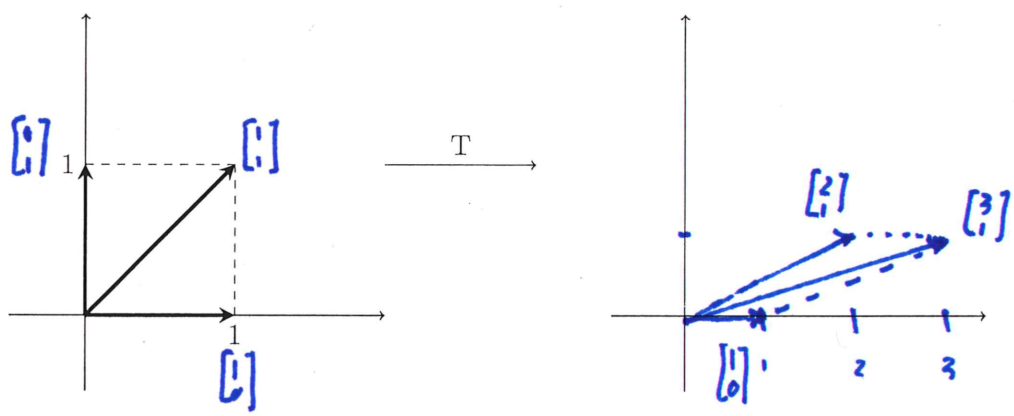
Since $BY = I$, B is invertible and $\boxed{Y = B^{-1}}$ *

Since $A = -BX$ then $\boxed{-B^{-1}A = X}$ *

4. Define the following two transformations

$$T(\mathbf{x}) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\mathbf{x}; \quad S(\mathbf{x}) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\mathbf{x}$$

(a) (2 points) Draw the image of the unit square under T



$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

(b) (2 points) Find the standard matrix for $S(T(\mathbf{x}))$ and $T(S(\mathbf{x}))$.

$$S(T(\vec{x})) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\vec{x} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}\vec{x}$$

$$T(S(\vec{x})) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\vec{x} = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}\vec{x}$$

5. Consider the following system:

$$\begin{array}{ccccccl} x_1 & + & x_2 & + & x_3 & = 0 \\ x_1 & - & 2x_2 & + & 2x_3 & = 4 \\ x_1 & + & 2x_2 & - & x_3 & = 2 \end{array}$$

(a) (2 points) Find the solution set.

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & -2 & 2 & 4 \\ 1 & 2 & -1 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & -3 & 1 & 4 \\ 0 & 1 & -2 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & -2 & 2 \\ 0 & -3 & 1 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & -5 & 10 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$$x_1 = 4$$
$$x_2 = -2$$
$$x_3 = -2$$

(b) (2 points) Write the equivalent matrix equation and vector equation.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 2 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}$$

and

$$x_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}$$

6. Consider the following matrix and one of its echelon forms.

$$A = \begin{bmatrix} 4 & 5 & 9 & -2 \\ 6 & 5 & 1 & 12 \\ 3 & 4 & 8 & -3 \end{bmatrix} \sim E = \begin{bmatrix} 1 & 2 & 6 & -5 \\ 0 & 1 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) (2 points) Find a basis for Null(A).

$$A \sim E \sim \begin{bmatrix} 1 & 0 & -4 & 7 \\ 0 & 1 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

To find a basis for Null A

We solve $A\vec{x} = 0$

$$[A \vdots 0] = \begin{bmatrix} 4 & 5 & 9 & 2 & 0 \\ 6 & 5 & 1 & 12 & 0 \\ 3 & 4 & 8 & -3 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -4 & 7 & 0 \\ 0 & 1 & 5 & -6 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Solns: $\vec{x} = t\begin{bmatrix} 4 \\ -5 \\ 1 \\ 0 \end{bmatrix} + s\begin{bmatrix} -7 \\ 6 \\ 0 \\ 1 \end{bmatrix}$

Basis Null A $= \left\{ \begin{bmatrix} 4 \\ -5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ 6 \\ 0 \\ 1 \end{bmatrix} \right\}$

(b) (2 points) Find a basis for Col(A).

E has the 1st and 2nd Columns as pivot columns

So a basis for Col(A) are the first two columns of A

$$\left\{ \begin{bmatrix} 4 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ 4 \end{bmatrix} \right\}$$

(c) (1 point) If $T(\mathbf{x}) = A\mathbf{x}$ is T a one to one transformation?

No, The equation $A\vec{x} = \vec{0}$ has more than just the trivial soln. (§1.7 pg 57)

(d) (1 point) If $T(\mathbf{x}) = A\mathbf{x}$ is T onto?

No, $T : \mathbb{R}^4 \longrightarrow \mathbb{R}^3$

$$\mathrm{Col}(A) = \mathrm{span}\left\{ \begin{bmatrix} 4 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ 4 \end{bmatrix} \right\} \neq \mathbb{R}^3$$

(Theorem 12 a §1.9)