①

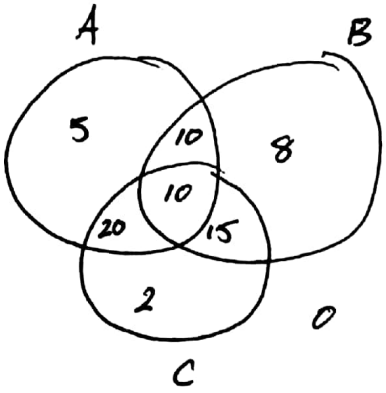
i) $A \cup B = \{1, 2, 3, 4, 5, 6, 8, 10\}$

ii) $A \cap B = \{2, 4\}$

iii) $A^c = \{6, 7, 8, 9, 10\}$

iv) $B^c = \{1, 3, 5, 7, 9\}$

②



③

$222 = 3(72) + 6$

$72 = 6(12) + 0$

$\gcd(222, 72) = 6$

④

a) $27 = 3^3$

$D(27) = (3+1) = 4$

b) $82 = 2^1 \cdot 41^1$

$D(82) = (1+1)(1+1) = 4$

c) $242 = 2^1 \cdot 11^2$

$D(242) = (1+1)(2+1) = 6$

5) a) $42 = 2^1 \cdot 3^1 \cdot 7^1$

$S(42) = \left(\frac{2^2-1}{1}\right)\left(\frac{3^2-1}{2}\right)\left(\frac{7^2-1}{6}\right) = 3 \cdot 4 \cdot 8 = 96$

$p(42) = 96 - 42 = 54$ abundant

b) $93 = 3^1 \cdot 31^1$

$S(93) = \left(\frac{3^2-1}{2}\right)\left(\frac{31^2-1}{30}\right) = 4 \cdot 32 = 128$

$p(93) = 128 - 93 = 35$ deficient

c) $202 = 2^1 \cdot 101^1$

$S(202) = \left(\frac{2^2-1}{1}\right) \cdot \left(\frac{101^2-1}{100}\right) = 3 \cdot 102 = 306$

$p(202) = 306 - 202 = 104$ deficient

6) a) $2\overline{)342}$ quotient 171, 342, 0 $\quad [342] = \{0 + 2n \mid n \in \mathbb{Z}\}$

b) $5\overline{)7234}$ quotient 1446, 7230, 4 $\quad [7234] = \{4 + 5n \mid n \in \mathbb{Z}\}$

c) $10\overline{)3236}$ quotient 323, 3230, 6 $\quad [3236] = \{6 + 10n \mid n \in \mathbb{Z}\}$

(7) a) $[2] + [5] + [4] \mod 2$

$= [0] + [1] + [0]$

$= [1]$

b) $[8] \cdot [341] \mod 5$

$= [3] \cdot [1]$

$= [3]$

c) $([21] \cdot [14]) + [42] \mod 7$

$= ([0] \cdot [0]) + [0]$

$= [0]$

(8)

	[0]	[1]	[2]	[3]
[0]	[0]	[1]	[2]	[3]
[1]	[1]	[2]	[3]	[0]
[2]	[2]	[3]	[0]	[1]
[3]	[3]	[0]	[1]	[2]

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a) $\left[\begin{array}{cc|c} 1 & 2 & \\ 2 & 3 & \end{array}\right]\left[\begin{array}{c|c} 1 & 1 \\ 0 & 1 \end{array}\right] = \begin{bmatrix} 1\cdot1+0\cdot2 & 1\cdot1+1\cdot2 \\ 1\cdot2+0\cdot3 & 1\cdot2+1\cdot3 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$

b) $\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 2 & 3 & 0 \\ \hline 1 & 1 & 2 \end{array}\right]\left[\begin{array}{c|c|c} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 2 & 3 \end{array}\right] = \begin{bmatrix} 1\cdot1+0\cdot2+1\cdot1 & 1\cdot1+1\cdot2+2\cdot1 & 0\cdot1+0\cdot2+3\cdot1 \\ 1\cdot2+0\cdot3+1\cdot0 & 1\cdot2+1\cdot3+2\cdot0 & 0\cdot2+0\cdot3+3\cdot0 \\ 1\cdot1+0\cdot1+1\cdot2 & 1\cdot1+1\cdot1+2\cdot2 & 0\cdot1+0\cdot1+3\cdot2 \end{bmatrix}$

$$= \begin{bmatrix} 2 & 5 & 3 \\ 2 & 5 & 0 \\ 3 & 4 & 6 \end{bmatrix}$$

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$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} \frac{d}{\det A} & \frac{-b}{\det A} \\ \frac{-c}{\det A} & \frac{a}{\det A} \end{bmatrix}$$

where $\det A = ad - bc$

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$$

$\det A = 1\cdot3 - 2\cdot1 = 1$

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a) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 3 & 2 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{bmatrix}$

c) $(2,3,4,1,5)\ (2,1,4)\ (3,5) = (1)\ (2543)$

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$$\chi = V - E + F$$

$$= 4 - 6 + 4$$

$$= 2$$