MATH 3210 Your name
Homework 1 January 18, 2017

1. Let $\mathbb{Q}(\sqrt{2}) = \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$. Note that $\mathbb{Q}(\sqrt{2})$ is field and more specifically it is known as an algebraic number field. The binary operations on $\mathbb{Q}(\sqrt{2})$ are the standard addition and multiplication of numbers. Verify for all $\alpha \neq 0$ in $\mathbb{Q}(\sqrt{2})$ that there exists a $\beta \in \mathbb{Q}(\sqrt{2})$ such that $\alpha \cdot \beta = 1$.

For the next two problems let $\mathbb{F}$ be an arbitrary field. We define the following vector space over $\mathbb{F}$. Let

$$\mathbb{F}^n = \{(x_1, x_2, \ldots, x_n) : x_j \in \mathbb{F},\ j = 1, \ldots n\}$$

where scalar multiplication and vector addition is defined thusly,

$$\lambda \cdot (x_1, \ldots, x_n) = (\lambda x_1, \ldots, \lambda x_n)$$

$$(x_1, \ldots, x_n) + (y_1, \ldots, y_n) = (x_1 + y_1, \ldots x_n + y_n).$$

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2. (#13 §1.A) Show that $(ab)x = a(bx)$ for all $x \in \mathbb{F}^n$ and all $a, b \in \mathbb{F}$.

3. (# 15§1.A) Show that $\lambda \cdot (x + y) = \lambda x + \lambda y)$ for all $\lambda \in \mathbb{F}$ and all $x, y \in \mathbb{F}^n$.

For the next two problems let $\mathbb{F}$ be an arbitrary field and V a vector space over $\mathbb{F}$.

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4. (#1 §1.B) Prove that $-(-v) = v$ for every $v \in V$.

5. (#2 §1.B) Suppose $a \in \mathbb{F}$, $v \in V$, and $av = 0$. Prove $a = 0$ or $v = 0$.