

For full credit, you must show all work and circle your final answer.

1 Calculate the determinants for the following matrices.

$$(a) A = \begin{bmatrix} 1 & 2 & 0 & 4 & -1 \\ 3 & 1 & 0 & 3 & 0 \\ 2 & 2 & 0 & -2 & 2 \\ 6 & -3 & 0 & 3 & -1 \end{bmatrix}$$

$A$  is  $4 \times 5$   $\det A$  is undefined

$$(b) B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 5 & 0 & 0 \\ 2 & 2 & 2 & 0 \\ 8 & -3 & 0 & 3 \end{bmatrix}$$

$$\det B = 1 \cdot 5 \cdot 2 \cdot 3 = 30$$

2 (a) Compute  $\det \left( \begin{bmatrix} 1 & 0 & 0 \\ 3 & 2 & 0 \\ 1 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix} \right)$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 2 & 0 \\ 1 & 2 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\det A = 4 \quad \det B = 3 \det \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} = -3$$

$$\det(AB) = -12$$

(b) Is the following matrix invertible?

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$A$  has a zero row  $\det A = 0$

so  $A$  is not invertible

3 Consider the vector space  $\mathbb{P}_2(t)$ .

(a) Are  $p_1(t) = 3$ ,  $p_2(t) = 2 - 4t$ , and  $p_3(t) = 5t$  linearly independent?

No,  $\vec{p}_2(t) = \frac{2}{3} \vec{p}_1(t) - \frac{4}{5} \vec{p}_3(t)$

(b) Write a basis for the subspace  $H = \text{span}\{p_1(t), p_2(t), p_3(t)\}$ .

$\vec{p}_1$  &  $\vec{p}_3$  are  
linearly indep

$$\text{Basis for } H = \left\{ \vec{p}_1(t), \vec{p}_3(t) \right\}$$