1. Suppose $f : [a, b] \to \mathbb{R}$ is Riemann integrable. Prove,

$$\lim_{c \to b, c<b} \int_a^c f\,dx = \int_a^b f(x)\,dx$$

2. Show that if f is a continuous real valued function on the interval $[a, b]$ then

$$\int_a^b f(x)\,dx = f(\xi)(b-a)$$

   for some $\xi \in [a, b]$.

3. Let $c < a < b < d$ and define $\chi_{[a,b]} : [c, d] \to \mathbb{R}$ as follows:

$$\chi_{[a,b]}(x) = \begin{cases} 1 & x \in [a, b] \\ 0 & x \notin [a, b] \end{cases}$$

   Show starting from the definition that $\chi_{[a,b]}$ is Riemann integrable on $[c, d]$ and compute

$$\int_c^d \chi_{[a,b]}(x)\,dx.$$

4. Prove that if f is a continuous real valued function on the interval $[a, b]$ such that $f(x) \geq 0$ for all $x \in [a, b]$ and $f(x_0) > 0$ for some $x_0 \in [a, b]$ then $\int_a^b f(x)\,dx > 0$.

5. Give an example of a Riemann integrable function $g : [a, b] \to [a, b]$ such that $g(x) \geq 0$ for all $x \in [a, b]$ and $g(x_0) > 0$ for some $x_0 \in [a, b]$ such that $\int_a^b g(x)\,dx = 0$.