Polar Coordinates

1. Consider the point $(r, \theta) = (-2, -2\pi/3)$.

    (a) Find 2 other pairs of polar coordinates that represent the same point, one with $r > 0$ and one with $r < 0$.

    (b) Convert the point from polar coordinates to Cartesian coordinates.

2. Sketch the graphs of the following polar equations:

    (a) $r = \sin(3\theta)$

    (b) $r = 1 - \cos\theta$

    (c) $r = 2 + \sin\theta$

    (d) $r = 2 - 3\cos\theta$

    (e) $r = 6\cos(4\theta)$

    (f) $r^2 = 2\sin(2\theta)$.

3. (a) Find the slope of the tangent line to the polar curve $r = \cos\theta + \sin\theta$ at the value $\theta = 0$.

    (b) Given the polar curve $r = 1 + \cos\theta$, find the values of θ at which the curve has a horizontal tangent line or a vertical tangent line.

Areas in Polar Coordinates

4. Sketch the curve $r = 3 + 3\cos\theta$ and find the area that it encloses.

5. Sketch the curve $r = 2\cos(3\theta)$ and find the area that it encloses.

6. Find the area inside the inner loop of the curve $r = 1 + 2\sin\theta$.

7. Find the area that lies inside the curve $r = 1 - \sin\theta$ and outside the curve $r = 1$.

8. Find the area that lies inside the curve $r = 1 + \cos\theta$ and outside $r = 3\cos\theta$.

9. Find the area that lies inside the curve $r^2 = 2\sin(2\theta)$ and outside $r = 1$.

10. Find the area inside the larger loop and outside the smaller loop of the curve $r = \frac{1}{2} + \cos\theta$.

11. Find the polar coordinates with $0 \le \theta < 2\pi$ of all points of intersection of the curves $r = \sin\theta$ and $r = \sin(2\theta)$.

Area
Sketch the region enclosed by the given curves. Then find the area of the region.

12. $y = 1 + \sqrt{x}$, $y = \frac{3+x}{3}$

13. $y = \sin x$, $y = \sin(2x)$, $x = 0$, $x = \pi/2$

14. $x = 2y^2$, $x + y = 1$

Volumes

In questions 15- 19, find the volume of the solid by rotating the region bounded by the given curves about the specified line.

15. $y = x$, $y = \sqrt{x}$, about $y = 1$

16. $y = e^x$, $y = 0$, $x = 0$, $x = 1$, about the x-axis

17. $y = x^2$, $0 \le x \le 2$, $y = 4$, $x = 0$, about the y-axis

18. $y = x$, $y = \sqrt{x}$ about $x = 2$

19. $y = 0$, $y = \sin x$, $0 \le x \le \pi$, about $y = -2$

    In questions 20- 24, use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

20. $x = \sqrt{y}$, $x = 0$, $y = 1$, about the x-axis

21. $y = x^2$, $y = 0$, $x = 1$, $x = 2$, about $x = 1$

22. $y = x^2$, $y = 0$, $x = 1$, $x = 2$, about $x = 4$

23. $y = \sqrt{x - 1}$, $y = 0$, $x = 5$, about $y = 3$

24. $y = \frac{1}{1+x^2}$, $y = 0$, $x = 0$, $x = 2$, about $x = 2$

25. Find the volume of a frustrm of a right circular cone with height h, lower base radius R, and top radius r.

26. Find the volume of a cap of a sphere with radius r and height h.

27. The base of a solid S is a circular disk with radius r. Parallel cross sections perpendicular to the base are squares. Find the volume of S.

Work

28. A rope that weighs 0.5 lb/ft is 100 feet long and hangs over the edge of a building 150 feet high.

  (a) How much work is done in pulling the rope to the top of the building?

  (b) How much work is done in pulling the rope up 80 feet?

29. How much work is done in lifting a 1.4 kg book off the floor to put it on a desk that is 0.8 m high? (Use the fact that acceleration due to gravity is 9.8m/s^2.)

30. Suppose a force of 7 J of work is needed to stretch a spring from its natural length of 15 cm to a length of 20 cm. How much work is done in stretching the spring from 20 cm to 25 cm?

31. If 6 J of work is needed to stretch a spring from 10 cm to 12 cm and another 10 J is needed to stretch it from 12 cm to 14 cm, what is the natural length of the spring?

32. A circular swimming pool has a diameter of 24 ft, the sides are 5 ft high, and the depth of the water is 4 ft. How much work is required to pump all of the water out over the side. (Use 62.5 lb/ft^3 for the weight of water.)

33. An 8 ft long rough, with vertical cross-sections in the shape of a semicircle with a 4 ft diameter is full of water. Find the amouunt of work required to pump the water out of a vertical spout which is 1 foot above the top of the tank.