

1. Let $f : X \rightarrow Y$ and $g : Y \rightarrow X$ be functions so that $g \circ f = 1_X$. Prove that f is injective and that g is surjective.
2. Prove that if $|A| = |B|$ then $|A \times A| = |B \times B|$.

3. If $f(x + y) = f(x)f(y)$ and f is a bijection, show that the inverse satisfies

$$f^{-1}(xy) = f^{-1}(x) + f^{-1}(y)$$

4. A card shuffling machine always rearranges cards in the same way relative to the order in which they were given to it. All of the hearts arranged in order from ace to king were put into the machine, and then the shuffled cards were put into the machine again to be shuffled. If the cards emerged in the order 10, 9, Q, 8, K, 3, 4, A, 5, J, 6, 2, 7, in what order were the cards after the first shuffle?
5. Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ and suppose that $g \circ f$ is onto. Prove that g is onto then prove or disprove that f is onto.