Telescoping Series

- 1. Find the sum of the series $\sum_{n=1}^{\infty} (\arctan(n+1) \arctan(n))$.
- 2. Find the sum of the series $\sum_{n=1}^{\infty} \frac{2}{x^2 + 6x + 8}.$

Integral Test

Determine whether the series converges or diverges.

3.
$$\sum_{n=0}^{\infty} \frac{n}{n^2 + 4}$$

$$4. \sum_{n=1}^{\infty} \frac{\ln n}{n^2}$$

$$5. \sum_{n=1}^{\infty} \frac{n}{n^4 + 1}$$

Find the values of p for which the series is convergent

$$6. \sum_{n=3}^{\infty} \frac{(\ln n)^p}{n}$$

Comparison Tests

Determine whether the series converges or diverges

$$7. \sum_{n=2}^{\infty} \frac{\sqrt{n}}{n-1}$$

8.
$$\sum_{n=0}^{\infty} \frac{1 + \cos n}{n^5}$$

9.
$$\sum_{n=1}^{\infty} \frac{n+5}{\sqrt[3]{n^7+n^2}}$$

10.
$$\sum_{n=1}^{\infty} \frac{n+2^n}{n+3^n}$$

Alternating Series Test

Determine whether the series converges or diverges

11.
$$\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n^{3/4}}$$

12.
$$\sum_{n=1}^{\infty} (-1)^n \frac{n!}{n^n}$$

13.
$$\sum_{n=2}^{\infty} (-1)^n \frac{n}{\ln n}$$

14.
$$\sum_{n=1}^{\infty} \left(-\frac{n}{5} \right)^n$$

Alternating Series Test (continued)

Show that each of the following sequences are convergent, and answer the indicated questions.

- 15. Approximate $\sum_{n=1}^{\infty} \frac{(-1)^n}{3^n n!}$ correct to four decimal places. (|error| < 0.0001)
- 16. How many terms of $\sum_{n=1}^{\infty} \frac{(-1)^n n}{4^n}$ do we need to add in order to find the sum with |error| < 0.002?

Absolute Convergence and the Ratio and Root Tests

Determine whether the series is absolutely convergent, conditionally convergent, or diverges

$$17. \sum_{n=1}^{\infty} \frac{n^n}{n!}$$

18.
$$\sum_{n=0}^{\infty} \frac{(-10)^n}{n!}$$

19.
$$\sum_{n=1}^{\infty} e^{-n} n!$$

20.
$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$$

21.
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(\arctan n)^n}$$

22.
$$\sum_{1}^{\infty} \left(\frac{n^2 + 1}{2n^2 + 1} \right)^n$$

23.
$$1 - \frac{1 \cdot 3}{3!} + \frac{1 \cdot 3 \cdot 5}{5!} - \cdots + (-1)^n \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{(2n-1)!}$$

Strategy for Testing Series

Determine whether the series is absolutely convergent, conditionally convergent, or diverges

24.
$$\sum_{n=1}^{\infty} \tan\left(\frac{1}{n}\right)$$

25.
$$\sum_{n=1}^{\infty} \frac{n!}{e^{n^2}}$$

26.
$$\sum_{n=1}^{\infty} \frac{n \ln n}{(n+1)^3}$$

27.
$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n}-1}$$

28.
$$\sum_{n=1}^{\infty} \frac{1 \cdot 2 \cdot 5 \cdot \dots \cdot (2n-1)}{5^n n!}$$

$$29. \sum_{n=1}^{\infty} \frac{7^n}{5^n + 6^n}$$