MATH 3150 Your name
Homework 3 November 6, 2017

1. Assuming the elementary properties of the trigonometric functions show on the interval $(0, \pi/2)$ that the function $\tan(x) - x$ is strictly increasing and $\frac{\sin(x)}{x}$ is strictly decreasing.

2. We first define limits at infinity.

   Definition 0.1. Given a metric space Y, a point $L \in Y$ and $f : [0, \infty) \to Y$ *has limit* $L \in Y$ *at infinity*, written
   $$\lim_{x \to \infty} f(x) = L,$$
   if for every $\varepsilon > 0$ there is a $C > 0$ such that if $x > C$ then $d_Y(f(x), L) < \varepsilon$.

   Warning: This is now a definition you will be expected to know

   Show that if $f : [0, \infty) \to Y$ is continuous and has a limit at infinity then f is uniformly continuous.

3. Let $f : [0, 1] \to [0, 1]$ be a continuous function. Show that f has a fixed point, i.e. there is a point $x \in [0, 1]$ such that $f(x) = x$.

4. Formulate and prove a squeeze theorem for functions.

5. We start with the following definition

   Definition 0.2. Let X and Y be metric spaces. We call a function $f : X \to Y$ *Lipschitz continuous* if there exists a $K > 0$ such that
   $$d_Y(f(p), f(q)) \leq K d_X(p, q)$$
   for all $p, q \in X$.

   Let U be an open interval of $\mathbb{R}$. Prove that if f is differentiable and $f' : U \to \mathbb{R}$ is bounded, then f is Lipschitz continuous.