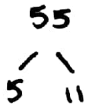
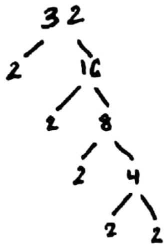
1. (5 points) Find the prime factorization and compute $D(n)$ for each of the numbers below.

a) 55 $= 5^1 \cdot 11^1$



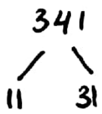
$D(55) = D(5^1) \cdot D(11^1) = (1+1) \cdot (1+1) = 4$

b) 32 $= 2^5$



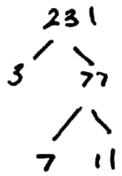
$D(32) = D(2^5) = (5+1) = 6$

c) 341 $= 11^1 \cdot 31^1$



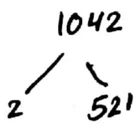
$D(341) = D(11^1) \cdot D(31^1) = (1+1)(1+1) = 4$

d) 231 $= 3^1 \cdot 7^1 \cdot 11^1$



$D(231) = D(3^1) \cdot D(7^1) \cdot D(11^1) = (1+1)(1+1) \cdot (1+1) = 8$

e) 1042 $= 2^1 \cdot 521^1$



$D(1042) = D(2^1) \cdot D(521^1) = (1+1)(1+1) = 4$

2. (4 points) Find the prime factorization and compute $S(n)$ for each of the numbers below and determine if the number is deficient, abundant, or perfect.

a) 237

237

3 79

$S(237) = \left(\frac{3^2-1}{2}\right)\left(\frac{79^2-1}{78}\right) = 320$

$P(n) = 320 - 237 = 83$ deficient

b) 93

93

3 31

$S(93) = \left(\frac{3^2-1}{2}\right)\left(\frac{31^2-1}{2}\right) = 128$

$P(93) = 128 - 93 = 35$ deficient

c) 308

$308 = 2^2 \cdot 7 \cdot 11$

$S(308) = \left(\frac{2^3-1}{2-1}\right)\left(\frac{7^2-1}{7-1}\right)\cdot\left(\frac{11^2-1}{11-1}\right) = 7 \cdot 8 \cdot 12 = 672$

$P(308) = 672 - 308 = 364$ Abundant

d) 467

467 is prime!

$S(467) = \frac{467^2-1}{467-1} = 468$

$P(467) = 468 - 467 = 1$ deficient

3. (4 points) Find all numbers equivalent to the given number for the stated modulus.

a) 43 mod 2

$[43] = \{1 + 2n \mid n \in \mathbb{Z}\}$

b) 782 mod 5

$[782] = \{2 + 5n \mid n \in \mathbb{Z}\}$

c) 381 mod 17

$[381] = \{7 + 17n \mid n \in \mathbb{Z}\}$

d) 738 mod 23

$[738] = \{2 + 23n \mid n \in \mathbb{Z}\}$

e) 811 mod 6

$[811] = \{1 + 6n \mid n \in \mathbb{Z}\}$

f) 939 mod 92

$[939] = \{19 + 92n \mid n \in \mathbb{Z}\}$

g) 122 mod 19

$[122] = \{8 + 19n \mid n \in \mathbb{Z}\}$

h) 303 mod 8

$[303] = \{7 + 8n \mid n \in \mathbb{Z}\}$

4. (4 points) Compute the following for the given modulus.

a) $[2] + [2] + [3] \bmod 2$

$= [0] + [0] + [1] = [1]$

b) $[8] \cdot [12] \bmod 5$

$[8] \cdot [12] = [96] = [1]$

c) $([21] \cdot [15]) + [21] \bmod 17$

$([21] \cdot [15]) + [21] = ([4] \cdot [15]) + [4] = [60] + [4] = [9] + [4] = [13]$

d) $[-19] + [22] \bmod 6$

$[-19] + [22] = [5] + [4] = [9] = [3]$

e) $[24] - [233] \bmod 21$

$[24] - [233] = [3] - [2] = [1]$

f) $[24] + [1] + [-18] \bmod 9$

$[24] + [1] + [-18] = [6] + [1] + [0] = [7]$

g) $[93] + [2] + [31] \bmod 2$

$[93] + [2] + [31] = [1] + [0] + [1] = [2] = [0]$

h) $[10] \cdot [2] \cdot [18] \bmod 5$

$[10] \cdot [2] \cdot [18] = [20] [18] = [0] \cdot [3] = [0]$

5. (4 points) Evaluate whether or not the statement is true or false. If they are false give a counter example or state why they are false.

a) The integers are closed under addition.

True Ex// $2+3=5$

b) The additive identity for the integers $\mathbb{Z}$ is 1.

False : 0 is the additive identity

Ex// $2+0=2$

c) For mod 5 arithmetic the additive inverse of [2] is [3].

True $[2]+[3]=[5]=[0]$

d) The odd numbers are closed under addition.

False $3+3=6$ 6 is Even!

6. (4 points) Complete the Cayley table for the given modular group.

$\{[0],[1],[2],[3]\}$ under addition mod 4

	[0]	[1]	[2]	[3]
[0]	[0]	[1]	[2]	[3]
[1]	[1]	[2]	[3]	[0]
[2]	[2]	[3]	[0]	[1]
[3]	[3]	[0]	[1]	[2]