MTH151 **Name:**________________________

Practice Final Exam **Section:**________________________

This exam contains 8 pages (including this cover page) and 7 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books or notes on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, ask for an extra sheet of paper to continue the problem on; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	5	
2	5	
3	5	
4	5	
5	5	
6	5	
7	5	
Total:	35	

1. (5 points) Evaluate the integrals.

a) $\int (3x^2 + 2x + 1)\sin(2x)\,dx$

$u = 3x^2+2x+1$		$dv = \sin(2x)$
$3x^2+2x+1$	$+$	$\sin(2x)$
$6x+2$	$-$	$-\frac{1}{2}\cos(2x)$
6	$+$	$-\frac{1}{4}\sin(2x)$
0		$\frac{1}{8}\cos(2x)$

$$\int (3x^2+2x+1)\sin(2x)dx = -\frac{1}{2}(3x^2+2x+1)\cos(2x) + \frac{1}{4}(6x+2)\sin(2x) + \frac{6}{8}\cos(2x) + C$$

b) $\int \sin(x) e^{2x}\,dx$

$u = e^{2x}$		$dv = \sin(x)$
e^{2x}	$+$	$\sin(x)$
$2e^{2x}$	$-$	$-\cos(x)$
$4e^{2x}$	$+$	$-\sin(x)$

$$\int \sin(x)e^{2x}dx = -e^{2x}\cos(x) + 2e^{2x}\sin(x) - 4\int e^{2x}\sin(x)dx$$

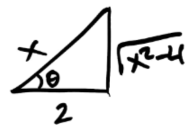
$$\Rightarrow \int \sin(x)e^{2x}dx = \frac{1}{5}\left[-e^{2x}\cos(x) + 2e^{2x}\sin(x)\right] + C$$

2. (5 points) Evaluate the integrals.

a) $\int \frac{\sqrt{x^2-4}}{x}\,dx$

Let $x = 2\sec\theta$, $dx = 2\sec\theta\tan\theta\,d\theta$

$\sqrt{x^2-4} = \sqrt{4\sec^2\theta - 4} = 2\tan\theta$



$$\int \frac{\sqrt{x^2-4}}{x}\,dx = \int \frac{2\tan\theta}{2\sec\theta}\cdot 2\sec\theta\tan\theta\,d\theta$$

$$= 2\int \tan^2\theta\,d\theta = 2\int \sec^2\theta - 1\,d\theta$$

$$= 2\tan\theta - 2\theta + C$$

$$= \sqrt{x^2-4} - 2\sec^{-1}\left(\frac{x}{2}\right) + C$$

b) $\int \frac{x^2-29x+5}{(x-4)^2(x^2+3)}\,dx$

$$\frac{x^2-29x+5}{(x-4)^2(x^2+3)} = \frac{A}{(x-4)} + \frac{B}{(x-4)^2} + \frac{Cx+D}{x^2+3}$$

$x^2-29x+5 = A(x-4)(x^2+3) + B(x^2+3) + (Cx+D)(x-4)^2$

Find B by plugging in $x=4$

$16 - 29(4) + 5 = B(16+3) \Rightarrow B = -5$

$\Rightarrow x^2-29x+5 = A(x-4)(x^2+3) - 5(x^2+3) + (Cx+D)(x-4)^2$

$\Rightarrow 6x^2-29x+20 = A(x-4)(x^2+3) + (Cx+D)(x-4)^2$

Plug in $x=-1, 1, 0$

$x=-1$: $55 = -20A - 25C + 25D$

$x=1$: $-3 = -12A + 9C + 9D$

$x=0$: $20 = -12A + 16D \Rightarrow A = \frac{4D-5}{3}$ (**) plug in

$\Rightarrow$ $165 = -20(4D-5) - 75C + 75D$

$-3 = -4(4D-5) + 9C + 9D$

$\Rightarrow$ $65 = -75C - 5D$

$-23 = 9C - 7D$

$\Rightarrow -91 = 105C + 7D$ (*)

$+\ \underline{-23 = 9C - 7D}$

$\Rightarrow -114 = 114C$

$\Rightarrow \boxed{C=-1} \overset{\text{by } (*)}{\Rightarrow} \boxed{D=2} \overset{\text{by } (**)}{\Rightarrow} \boxed{A=1}$

The above is the partial fraction decomposition

Continued...

$$\int \frac{x^2-29x+5}{(x-4)^2(x^2+3)}dx = \int \frac{1}{(x-4)} - \int \frac{5}{(x-4)^2} + \int \frac{-x+2}{x^2+3}\,dx$$

$$= \ln|x-4| + 5(x-4)^{-1} - \int \frac{x}{x^2+3}dx + 2\int \frac{1}{x^2+3}dx$$

$$= \ln|x-4| + 5(x-4)^{-1} - \tfrac{1}{2}(x^2+3)^{-1} + \frac{2}{\sqrt{3}}\arctan\left(\frac{x}{\sqrt{3}}\right) + C$$

3. (5 points) Evaluate the integrals.

a) $\int_0^\infty \frac{1}{\sqrt{5-x}}\,dx$

$$\int_0^\infty \frac{1}{\sqrt{5-x}}\,dx = \int_0^6 \frac{1}{\sqrt{5-x}}\,dx + \int_6^\infty \frac{1}{\sqrt{5-x}}\,dx$$

discont @ $x=5$

$$= \int_0^5 \frac{1}{\sqrt{5-x}}\,dx + \int_5^6 \frac{1}{\sqrt{5-x}}\,dx + \int_6^\infty \frac{1}{\sqrt{5-x}}\,dx$$

$$= \lim_{t\to 5}\int_0^t \frac{1}{\sqrt{5-x}}\,dx + \lim_{t\to 5}\int_t^6 \frac{1}{\sqrt{5-x}}\,dx + \lim_{t\to\infty}\int_6^\infty \frac{1}{\sqrt{5-x}}\,dx$$

$$= \lim_{t\to 5}\left[-2\sqrt{5-x}\,\Big|_0^t\right] + \lim_{t\to 5}\left[-2\sqrt{5-x}\,\Big|_t^6\right] + \lim_{t\to\infty}\left[\cancel{-2\sqrt{5-x}\,\Big|_6^t}\right] \to \infty$$

integral does not converge!

b) $\int_1^2 \frac{4x}{\sqrt[3]{x^2-4}}dx$

$$\int_1^2 \frac{4x}{\sqrt[3]{x^2-4}}\,dx = \lim_{t\to 2}\int_1^t \frac{4x}{\sqrt[3]{x^2-4}}\,dx$$

discont @ $x=2$

$$= \lim_{t\to 2}\left[3(x^2-4)^{2/3}\,\Big|_1^t\right]$$

$$= -3(1-4)^{2/3} = -3(-3)^{2/3}$$

$$= -3\sqrt[3]{9}$$

4. (5 points) Determine which of the series below converge or diverge.

a) $\displaystyle\sum_{n=0}^{\infty} \frac{3n^3+5}{5n^4\cos^2(n)}$

$$\frac{3n^3+5}{5n^4\cos^2(n)} > \frac{3n^3}{5n^4\cos^2(n)} \geq \frac{3n^3}{5n^4} = \frac{3}{5}\,\frac{1}{n}$$

since $3n^3+5 > 3n^3$

since $\cos^2(n) \leq 1$

So $\displaystyle\sum^{\infty} \frac{3n^3+5}{5n^4\cos^2(n)} \geq \sum^{\infty} \frac{3}{5}\left(\frac{1}{n}\right)$

diverges by Comparison test

b) $\displaystyle\sum_{n=3}^{\infty} \frac{e^{4n}}{(n-2)!}$

Ratio test:

$$\lim_{n\to\infty} \left| \frac{e^{4(n+1)}}{((n+1)-2)!} \cdot \frac{(n-2)!}{e^{4n}} \right|$$

$$= \lim_{n\to\infty} \left| e^4 \frac{(n-2)!}{(n-1)!} \right| = \lim_{n\to\infty} \frac{e^4}{n-1} = 0 < 1$$

Converges by Ratio test.

c) $\displaystyle\sum_{n=2}^{\infty} \frac{n^n}{n!}$

$\displaystyle\sum_{n=2}^{\infty} \frac{n^n}{n!}$ diverges by Test for divergence $\displaystyle\lim_{n\to\infty} \frac{n^n}{n!} = \infty$

alternatively by Ratio test,

$$\lim_{n\to\infty} \left| \frac{(n+1)^{n+1}}{(n+1)!} \cdot \frac{n!}{n^n} \right| = \lim_{n\to\infty} \left| \frac{(n+1)^{n+1}}{n+1} \cdot \frac{1}{n^n} \right| = \lim_{n\to\infty} \left| \left(1+\tfrac{1}{n}\right)^n \right|$$

$$= e > 1$$

Diverges.

5. (5 points) Determine the interval of convergence for the following power series.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n(4x-8)^n}{n}$

Ratio test: $\displaystyle\lim_{n\to\infty} \left| \frac{2^{n+1}(4x-8)^{n+1}}{n+1} \cdot \frac{n}{2^n(4x-8)^n} \right| = |2(4x-8)| < 1$

$\Rightarrow |8(x-2)| < 1$

$\Rightarrow |x-2| < \frac{1}{8}$

End pts: $x = 2 + \frac{1}{8} = \frac{17}{8} \Rightarrow \displaystyle\sum_{n=1}^{\infty} \frac{2^n(\frac{1}{2})^n}{n} = \sum_{n=1}^{\infty} \frac{1}{n}$ diverges Harmonic series

$x = 2 - \frac{1}{8} = \frac{15}{8} \Rightarrow \displaystyle\sum_{n=1}^{\infty} \frac{2^n(-\frac{1}{2})^n}{n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges A.S.T

I.O.C $= [\frac{15}{8}, \frac{17}{8})$ centered at $x=2$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n(x-2)^n}{(n+1)^n}$

Root test: $\displaystyle\lim_{n\to\infty} \left| \frac{3^n(x-2)^n}{(n+1)^n} \right|^{1/n} = \lim_{n\to\infty} \frac{3|x-2|}{n+1} = 0 < 1$

$\Rightarrow$ Radius of convergence $R = \infty$

So I.O.C $= (-\infty, \infty)$

6. (5 points) Find a powerseries represntation at $x = 0$ for the functions below.

a) $f(x) = \dfrac{3}{2+2x^2}$

$$\frac{3}{2+2x^2} = \frac{3}{2}\left(\frac{1}{1-(-x^2)}\right)$$

if $\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ then $\dfrac{3}{2+2x^2} = \sum_{n=0}^{\infty} \frac{3}{2}(-x^2)^n = \sum_{n=0}^{\infty} \frac{(-1)^n 3x^{2n}}{2}$

b) $f(x) = (x^2+1)\sin(5x^3)$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\Rightarrow (x^2+1)\sin(5x^3) = \sum_{n=0}^{\infty} \frac{(-1)^n 5^{2n+1}(x^2+1)x^{6n+3}}{(2n+1)!}$$

c) $f(x) = (2x^2+5x+1)e^{-3x^2}$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$(2x^2+5x+1)e^{-3x^2} = \sum_{n=0}^{\infty} \frac{(-3)^n(2x^2+5x+1)x^{2n}}{n!}$$

Continued...

d) $f(x) = 5\cos(3x^2)$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$5\cos(3x^2) = \sum_{n=0}^{\infty} \frac{(-1)^n 5 \cdot (3)^{2n} x^{4n}}{(2n)!}$$

e) $f(x) = (3x+1)\ln(1-3x^2)$

$$\ln(1-x) = -\sum_{n=1}^{\infty} \frac{x^n}{n}$$

$$(3x+1)\ln(1-3x^2) = -\sum_{n=1}^{\infty} \frac{(3x+1) \cdot 3^n \cdot x^{2n}}{n}$$

7. (5 points) Find a power-series representation of the function below at $x = 1$

$$f(x) = 5e^{-6x}$$

$$f(x) = 5e^{-6x}$$

$$f'(x) = -6 \cdot 5e^{-6x}$$

$$f''(x) = -6 \cdot -6 \cdot 5e^{-6x}$$

$$f'''(x) = -6 \cdot -6 \cdot -6 \cdot 5e^{-6x}$$

$$\vdots$$

$$f^{(n)}(x) = (-6)^n \cdot 5 \cdot e^{-6x}$$

So $\dfrac{f^{(n)}(1)}{n!} = \dfrac{(-6)^n 5}{n!} e^{-6}$

$$\Rightarrow f(x) = 5e^{-6x} = \sum_{n=0}^{\infty} \frac{(-6)^n 5}{n!} e^{-6} (x-1)^n$$