1 Exercise 1.13

$$\sin x = 3\sin\frac{x}{3} - 4\sin^3\frac{x}{3} \tag{1}$$

 $x=rac{x}{3}+rac{x}{3}+rac{x}{3}$ であるので , 加法定理より

$$\sin x = \sin\left(\frac{x}{3} + \frac{x}{3} + \frac{x}{3}\right) \tag{2}$$

$$= \sin\left(\left(\frac{x}{3} + \frac{x}{3}\right) + \frac{x}{3}\right) \tag{3}$$

$$= \sin\left(\frac{x}{3} + \frac{x}{3}\right)\cos\frac{x}{3} + \cos\left(\frac{x}{3} + \frac{x}{3}\right)\sin\frac{x}{3} \tag{4}$$

$$= \sin\left(\left(\frac{x}{3} + \frac{x}{3}\right) + \frac{x}{3}\right)$$

$$= \sin\left(\frac{x}{3} + \frac{x}{3}\right)\cos\frac{x}{3} + \cos\left(\frac{x}{3} + \frac{x}{3}\right)\sin\frac{x}{3}$$

$$= \sin\frac{x}{3}\cos\frac{x}{3}\cos\frac{x}{3} + \cos\frac{x}{3}\sin\frac{x}{3}\cos\frac{x}{3} + \cos^2\frac{x}{3}\sin\frac{x}{3} - \sin^2\frac{x}{3}\sin\frac{x}{3}$$
(5)

$$= \sin\frac{x}{3}\cos^2\frac{x}{3} + \sin\frac{x}{3}\cos^2\frac{x}{3} + \sin\frac{x}{3}\cos^2\frac{x}{3} - \sin^3\frac{x}{3}$$
 (6)

$$= 3\sin\frac{x}{3}\cos^2\frac{x}{3} - 3\sin^3\frac{x}{3} \tag{7}$$

$$= 3\sin\frac{x}{3}(1-\sin^2\frac{x}{3}) - 3\sin^3\frac{x}{3} \tag{8}$$

$$= 3\sin\frac{x}{3} - 4\sin^3\frac{x}{3} \tag{9}$$

加法定理 2

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y \tag{10}$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y \tag{11}$$