Final Sheet

December 2021

1 Data

1.a Types of Variables

Qualitative/Categorical

- Outcomes fall into different categories
- Categories can be ordered

Quantitative

- Measured on a numeric scale

1.b Summarizing Data Visually

Qualitative/Categorical Data

- Frequency tables - displays all categories of a single categorical variable with associated frequencies
- Contingency tables - display two categorical variables simultaneously
- Marginal distributions - display distribution of one of the two variables only
- Conditional distributions - display distribution of one variable, satisfying a condition of the other variable
- Bar charts
- Pie charts

Quantitative Data

- **Graphically**
  - Histogram
  - Stem-and-leaf displays
  - Boxplots
- **Shape of the Distribution**
  - Modality (number of peaks):
    - unimodal
    - bimodal
    - multimodal
  - Symmetry of distribution:
    - unimodal

    - skewed to right (long right tail)
    - skewed to left (long left tail)
  - Presence of outliers

- **Numerically**
  - Measures of center:
    - mean
    - median
  - Measures of spread:
    - variance: $s^2 = \frac{\Sigma_{i=1}^{n}(y_1 - \overline{y})^2}{n-1}$
    - standard deviation: $s = \sqrt{\frac{\Sigma_{i=1}^{n}(y_1 - \overline{y})^2}{n-1}}$
    - interquartile range $IQR = Q3 - Q1$
  - Percentiles (also called quantiles)
  - 5-number summary:
    - minimum
    - first quartile (Q1)
    - second quartile (Q2)
    - third quartile (Q3)
    - maximum

- **Sensitivity to Outliers**
  - Sensitive to outliers:
    - mean
    - range, variance, standard deviation
  - Not sensitive to outliers
    - median
    - IQR

2 Normal Distribution

Characteristics of the Normal Model

- bell-shaped; unimodal
- perfectly symmetric about the mean
- spread of distribution determined by value of standard deviation
- mean μ and the standard deviation σ are parameters(numerical characteristics of a model)
- mean $\overline{y}$ and standard deviation s are statistics (numerical characteristics of a sample)

The 68-95-99.7 Rule

- 68% of data falls within 1 σ of μ
- 95% of data falls within 2 σ of μ
- 99.7% of data falls within 3 σ of μ

Finding Areas Under the Normal Model

Algorithm

- Identify the:
  μ - mean of the model
  σ - standard deviation of the model
  y - observed value
- Construct the normal model: N(μ,σ)
- Calculate the z-score (z): $z = \frac{y-\mu}{\sigma}$
- Using R compute the p-value:
  - Area below y: pnorm(z)
  - Area above y: pnorm(z , lower.tail = F)
  - Area in between y_1 and y_2 (where $y_1 > y_2$): pnorm(z_1) - pnorm(z_2)
- **Finding Z-Score from the Area Under the Normal Model**
  - Area above unknown y: qnorm(p, lower.tail = F)
  - Area below unknown y: qnorm(p)

3 Probability and Random Variables

Determining Independence of Events
The Binomial Model

- Used for discrete random variables
- Binomial Model:
  - Experiment must consist of n identical trials (number of trials is fixed in advance)
  - Outcomes of each trial are either success or failure
  - Probability of success p is constant
  - Probability of failure is q = 1 - p
  - The trials are independent
  - The random variable X represents the number of successes out of n trials

Algorithm for the Probability of Binomials

- Identify the parameters:
  n - number of trials
  p - probability of success
- Construct the binomial model: $X \sim Bin(n, p)$
- Calculate the probability:
  Where the probability that X will take on value x is given by:
  $P(X = x) = ({}_nC_x)p^x * (1 - p)^{n-x}, x = 0, 1, 2, ..., n$
  Where: $({}_nC_x) = \frac{n!}{x!(n-x)!}$

Mean, Variance and Standard Deviation for a Random Binomial Variable

- Mean: np
  Interp. average number of successes if you were to repeat experiment many times

- Variance: np(1 - p)
  Interp. measure of variablility of numbers of successes you were to repeat experiment many times
- Standard deviation: $\sqrt{np(1-p)}$

  Uniform Random Variable

4 Correlation and Association

Scatterplots

- Direction:
  - Positive (x and $($y) values tend to go in the same direction)
  - Negative (x and y values tend to go in the opposite direction)
- Form:
  - Linear
  - Non-linear
- Point relationship:
  - Strong relationship between points
  - weak or no relationship between points (randomly scattered)
- Outliers

Correlation (r)

- Positive correlation: large x values are linearly associated with large y values (r is positive)
- Negative correlation: large x values are linearly associated with small y values (r is negative)
- r has a value between 1 and -1, and has no units
- $r = \frac{\Sigma z_x * z_y}{n-1}$

Association vs Causality

- Association does not imply causation. There may be a lurking variable

5 Regression Analysis

The Regression Line

- Equation for regression line: $\hat{y} = intercept + (slope * x)$
- Equation for slope: $slope = r * \frac{s_y}{s_x}$
  (where s_y and s_x are the standard deviations of y and x respectively)
- Equation for intercept: $intercept = \overline{y} - (slope * \overline{x})$
  (where $\overline{y}$ and $\overline{x}$ are the mean y and x values respectively)

‘‘ **The Residuals**

- The residual (e) is the difference between observed value y and the predicted value $\hat{y}$. Therefore:
  e = y (from data) - $\hat{y}$ (from model)
- The sum of residuals is equal to zero
- Linear model is obtained by minimizing the sum of the squared residuals. Therefore, also referred to as the least squares regression line
- To assess appropriateness of regression model, we use the residual plot (plots residuals against explanatory variable data). If plot shows no pattern, model is appropriate.

6 Experiments and Observational Studies

Types of Studies

- Observational Studies
  - Investigators have no control over either variable
  - No deliberate human intervention
  - Retrospective study: based on information from events that have taken place in the past
  - Prospective study: data and information is gathered in real time
- Experiments
  - Involves planned intervention on the exposure to a condition suspected of altering the response outcome
  - Most often control group(s) will be used

Randomized, Comparative Experiments

- Involves assessing the effect of an explanatory variable, called a factor, on a response variable
- Compares the response variable between different levels of the factor
- Experimenters control what type of treatment individuals receive, the treatment assignment is random
- Participants referred to as subjects or experimental units
- The treatment a subject receives will be a combination of the levels from different factors

Principles of Experimental Design

- Randomize
  - Treatments are randomly assigned to subjects
- Replicate
  - Comparison between different treatment groups will not be reliable unless more individuals receive each treatment
- Blocking
  - May be beneficial to control for variables that are not factors but are believed to have some influence on the response variable
  - Subjects are divided into blocks (ex. male and female groups). Treatment assignment and comparisons are done within each block separately

Blinding and Placebo

- Single Blind: either the subjects or the evaluators are blinded as to treatment assignment
- Double Blind: neither the subjects nor the evaluators knows the treatment assignments
- Blinding is usually done using a placebo which is designed to look like the treatment but has no real treatment value

7 Types of Sampling

Sampling Methods

- Simple Random Sampling
  - Consists of n individuals sampled at random from the population
  - Each individual has an equal chance of being selected
  - Each possible sample size n is equally likely
- Stratified Sampling
  - Population is divided into strata (a stratum is a subset of the population that shares a particular characteristic)
  - Simple random sample is drawn from each stratum
  - Stratified sample has smaller variability across samples and hence give more reliable results
- Cluster Sampling
  - Can be used when natural groups in a population exist
  - Population is divided into those groups/clusters
  - Simple random sample from all clusters is obtained
  - If all individuals in a selected cluster are included, final sample is a one-stage cluster sample
  - If additional simple random sample is drawn from selected clusters, final sample is a two-stage cluster sample
  - This method is used for the sake of convenience, practicality, and cost-efficiency
- Multistage Sampling
  - Involves more than one stage or more than one sampling procedure in obtaining a sample
- Systematic Sampling
  - Obtained by selecting every kth individual from the sampling frame
  - Method can be used as long as list being sampled from does not contain a hidden order

Bad Sampling Procedures and Biases

- Undercoverage
  - When sampling frame or sampling procedure excludes or under-represents certain types of individuals from the population
- Convenience Sampling
  - Selecting individuals from a population based on availability and access
- Voluntary Response Bias
  - If responses are voluntary, those with strong opinions tend to be over-represented
- Non-response Bias
  - Individuals who do not respond in a survey might differ from the respondents in certain aspects
  - Including only the respondents in a sample will result in non-response bias
- Response Bias
  - Subject's response is influenced by how the question was phrased or asked, or due to misunderstanding of a question, or unwillingness to disclose the truth

8 Sampling Distribution Models

Basic Information

- Population: all individuals who want to be studied
- Sample: a subset of individuals selected from a population
- Parameter: a numerical summary of a population
- Statistic: a numerical summary of the sample

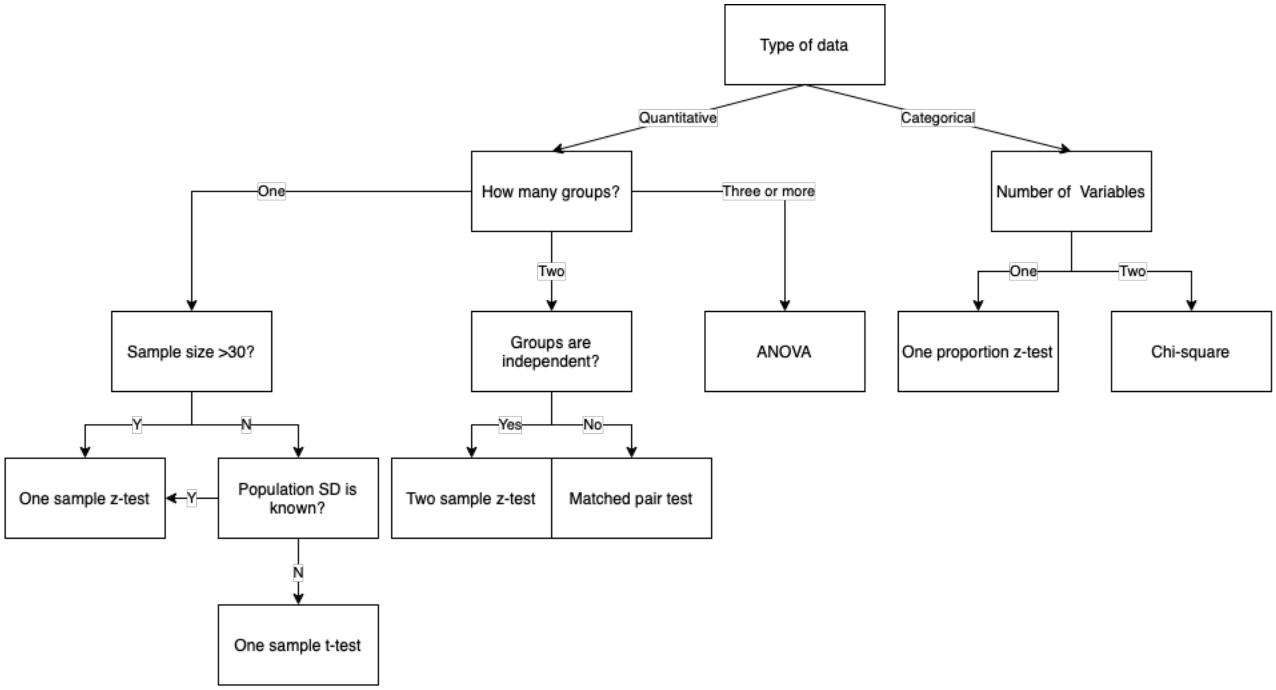
Sampling Distribution of Proportions

- The sample proportion (a) statistic) is given by:
  $\hat{p} = \frac{\text{number of individuals sampled who have the characteristic}}{\text{sample size n}}$
- Value of population proportion p is fixed, usually unknown. Therefore, sample proportion $\hat{p}$ used to estimate
- Sampling distribution of $\hat{p}$:
  - mean $\mu(\hat{p})$: mean of $\hat{p}$ = mean of p
  - standard deviation $\sigma(\hat{p})$: $\sqrt{\frac{p(1-p)}{n}}$
  - Sampling distribution of $\hat{p}$ approximately normal when:
    * Sample is random
    * Individual values are independent (sample size $\leq 10\%$ of population)
    * Sample size is large ($np \geq 10$ and $n(1-p) \geq 10$)

Samplng Distribution of Means

- The sample mean (a statistic) is given by:
  $\bar{y} = \frac{y_1 + y_2 + \ldots + y_n}{n}$
- Population mean μ is a parameter, fixed and usually unknown
- Sampling distribution of means:
  - mean $\mu(\bar{y}) = \mu$
  - standard deviaton $\sigma(\bar{y}) = \frac{\sigma}{\sqrt{n}}$
- Central limit theorm (CLT)
  - For sufficiently large samples, sample mean approximately follows the normal model
  - Assumption for CLT are:
    * Sample is random
    * Individual values are independent (sample size $\leq 10\%$ of population)
    * Sample size is sufficiently large (generally $n \geq 30$)

9 Hypothesis Testing



9.a One sample z-test

Algorithm

- Idenitify parameter of interest. Find the null and alternative hypotheses.
  s - The standard deviation of the sample.
  n - The sample size.
  μ - Hypothesized population mean.
  $\mathbf{SE}(\bar{y}) = \frac{s}{\sqrt{n}}$ - Standard error of the statistic.
- Construct the null-model: $\mathbf{N}(\mu, \frac{s}{\sqrt{n}})$
- Find the test-statistic(t): $\mathbf{Z} = \frac{x-\mu}{\mathbf{SE}(\bar{y})}$
- Using R compute the p-value:
  - One-sided hypothesis : pnorm(t)
  - Two-sided hypothesis : $2 \cdot$ pnorm(t)
- If the p-value is less than α - reject the null-hypothesis. Otherwise, you fail to reject the null-hypothesis.

9.b One proportion z-test

Algorithm

- Idenitify parameter of interest. Find the null and alternative hypotheses.
  n - The sample size.
  p_0 - Hypothethised proportion.
  $\mathbf{SD} = \sqrt{\frac{p_0(1-p_0)}{n}}$ - Standard error of the statistic.
- Construct the null-model: $\mathbf{N}(p_0, \sqrt{\frac{p_0(1-p_0)}{n}})$

- Find the test-statistic(t): $\mathbf{Z} = \frac{x-p_0}{\mathbf{SD}}$
- Using R compute the p-value:
  - One-sided hypothesis : pnorm(t)
  - Two-sided hypothesis : 2 · pnorm(t)
- If the p-value is less than α - reject the null-hypothesis. Otherwise, you fail to reject the null-hypothesis.

9.c Two sample z-test

Algorithm

- Idenitify parameter of interest. Find the null and alternative hypotheseses.
  s - The standard deviation of the sample.
  n_1 - The sample size of the first sample.
  n_2 - The sample size of the second sample.
  Δ_0 - Hypothethised mean of difference between two populations.
  $\mathbf{SD}(\bar{y_1} - \bar{y_2}) = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ - Standard error of the statistic.
  $df = \mathbf{min}(n_1 - 1, n_2 - 1)$
- Construct the null-model: $\mathbf{N}(\mu, \frac{s}{\sqrt{n}})$
- Using R compute the p-value:
  - One-sided hypothesis : pnorm(t)
  - Two-sided hypothesis : 2 · pnorm(t)
- If the p-value is less than α - reject the null-hypothesis. Otherwise, you fail to reject the null-hypothesis.

9.d Matched pair

Algorithm

- Idenitify parameter of interest. Find the null and alternative hypotheseses.
  Δ_0 - Hypothethised population mean difference (usually 0)
  $\bar{d}$ - The mean of the differences.
  $s_d = \sqrt{\frac{\sum_{i=1}^{n}(d_i - \bar{d})}{n-1}}$ = The standard deviation of the sample differences.
  n - The sample size.
- Find the test-statistic(t): $t = \frac{\bar{d} - \Delta_0}{\frac{s_d}{\sqrt{n}}}$
- Using R compute find the p-value using pt function:
  - For one sided tests use: pt(t, n-1)
  - For two sided tests use: 2 · pt(t, n-1)
- If the p-value is less than α - reject the null-hypothesis. Otherwise, you fail to reject the null-hypothesis.

9.e One sample t-test

Algorithm

- Idenitify parameter of interest. Find the null and alternative hypotheses.
  s - The standard deviation of the sample.
  n - The sample size.
  μ - Hypothesized population mean.
  $\mathbf{SE}(\bar{y}) = \frac{s}{\sqrt{n}}$ - Standard error of the statistic.
- Construct the null-model: $\mathbf{N}(\mu, \frac{s}{\sqrt{n}})$
- Find the test-statistic(t): $\mathbf{Z} = \frac{x-\mu}{\mathbf{SE}(\bar{y})}$
- Using R compute find the p-value using pt function:
  - For one sided tests use: pt(t, n-1)
  - For two sided tests use: $2 \cdot$ pt(t, n-1)

9.f ANOVA

¡¡¡¡¡¡ HEAD

- k = number of groups.
- N = number of subjects in total.
- SSTo = Sum of Squares Total.
- SSTo = SSTr + SSEr.

Source of Variation	df	Sum of Squares (SS)	Mean Sum of Squares (MSS)	F-test	p-value
Treatment	k-1	SSTr	MSTr=SSTr/(k-1)	F=MSTr/MSE	
Error	N-k	SSE	MSE=SSE/(N-k)		
Total	N-1	SSTo			

- After filling the table with the values and finding F statistic, you can decide to reject the null-hypothesis or not based on two methods:
  - Method 1:
    i. Compute critical value using R : qf($1-\alpha$, k - 1, N - k, lower.tail=TRUE)
    ii. If F-statistic is greater than critical value - reject the null-hypothesis. Otherwise, you fail to reject the null-hypothesis.
  - Method 2:
    i. Compute the pvalue using R: pf(F, k - 1, N - k, lower.tail=FALSE)
    ii. If pvalue is less than α - reject the null-hypothesis. Otherwise, you fail to reject the null-hypothesis.

9.g Chi-Square Test

- n = number of groups.
- c(kj...) = all observations from the table given where you enter the data for each row from left to right.
- Enter your data: matrix($c(k_ij\ldots)$, ncol=n, byrow=TRUE)
- Using R compute the p-value and test-statistic: chisq.test(sample.data, correct=FALSE)
- If the p-value is less than α - reject the null-hypothesis. Otherwise, you fail to reject the null-hypothesis.
- Example:

	Heart disease	No heart disease	Total
High cholesterol diet	(i) 11	(iii) 4	15
Low cholesterol diet	(ii) 2	(iv) 6	8
	13	10	23

i. Enter data into R: matrix(c(11,4,2,6), ncol=2, byrow=TRUE)
ii. Using chisq function compute test-statistic: chisq.test(sample.data, correct=FALSE)

```
	Pearson's Chi-squared test

data:  sample_data
X-squared = 4.9597, df = 1, p-value = 0.02594
```

iii. At 5% confidence level we would reject the null-hypothesis, because the p-value is less than 5%.

=======

9.h Chi-Square Test

¿¿¿¿¿¿¿ main