Middle East Technical University
Department of Mechanical Engineering
ME 310 Numerical Methods
Spring 2023 (Dr. Cuneyt Sert)
Study Set 6

For Homework 6 submit the answers of questions 2, 4, 8 and 9. Their grade percentages are not known at this point. It will be decided later.

Assigned: 02/06/2023 – Due: 17/06/2023, 23:59

Homework Rules and Suggestions:

- This is an individual assignment. Everything in your report should be the result of your own work. You are allowed to discuss its solution with your classmates and teaching staff up to a certain detail on ODTUClass. You are not allowed to use a solution manual. Put the following honor pledge at the top of your homework report and behave accordingly.

  "I understand that this is an individual assignment. I affirm that I have not given or received any unauthorized help on this assignment, and that this work is my own."

  If you have exchanged ideas with other students outside ODTUClass, you need to put their names and the extent of your discussion at the beginning of your report.

- Homework submission will be allowed until 5 minutes past the due time. Late submission is not allowed unless you have a valid excuse. In such a case, it is better if you let us know about this before the submission deadline.

- Upload your report as a PDF document (not a Word document) together with all other files (such as codes) to ODTUClass. Name your MATLAB files properly. Follow MATLAB file naming rules such as "File names cannot start with a number", "They cannot contain special characters or spaces", etc.

- Also put your MATLAB codes in your report, but in doing that make sure that you format them properly to avoid line wrapping. Codes with wrapped long lines become unreadable and we cannot understand them. If the codes are very long, you can shorten them by omitting noncritical parts.

- In writing your codes, follow good programming practices such as "use explanatory header lines", "explain inputs and outputs of functions", "use self-explanatory variable names", "use comments", "use empty lines and spaces for readability", "use indentation for code blocks", etc.

- Pay attention to the format of your report. Your report should look like a serious academic work, not like a high school student work. Font types and sizes, page margins, empty spaces on pages, equations, figures, tables, captions, colors, etc. are all important to give the desired "academic work feeling". Language used is also important. Reports with poor use of English cannot get a good grade.

- Do not provide an unnecessarily long report, with useless details or wasted spaces in pages. The shorter your report, the better it is, as long as it answers the questions properly. There are about 100 students, and we can spend only about 10 minutes to grade each report. For this, your report should be easy to read and understand. Also we should be able to find the results and judge their correctness easily. We should not get lost in your report. The more we struggle to understand your report, the lower your grade will be. Use figures and tables cleverly for this purpose.

- Reports with only figures, tables and codes, but no comments or discussions will not get a good grade. Even when a question does not specifically ask for a discussion, you better write some comments on its key points and your key learnings.

- Figures and tables should be numbered and should have captions (at the bottom for figures and at the top for tables). Their titles should be self-explanatory, i.e., we should be able understand everything about the table or figure just by reading its title. They should all be referred properly in the written text (such as "… as shown in Fig. 3" or "… (See Table 2)").

- Do not use Appendices in your report. Do not put your codes in Appendices.

- You can have a numbered reference list at the end of your report. In that case, you need to refer to the references in the text.

- If you are inexperienced in programming, converting an algorithm into a code and writing it in a bug-free way can be time consuming and frustrating. This is not something that can be done at the last minute. You are advised to start working on the assignments as soon as they are assigned.

Reading Assignments:

Self-learning is an important skill. Not everything can be discussed in lectures. You need to learn certain things by yourself.

For those of you who haven't got a chance to buy the textbook yet, I extracted the pages of the following reading assignments and uploaded them to ODTUClass.

R1) Read Box 21.2 "Derivation and Error Estimate of the Trapezoidal Rule" at page 617 and Box 21.3 "Derivation and Error Estimate of the Simpson's 1/3 Rule" at page 625 of the textbook.

R2) Study Table 21.2 at page 632 of the textbook to learn what Boole's rule is.

R3) Read Section 21.5 "Multiple Integrals" at page 636 of the textbook.

R4) Read Section 22.6 "Monte Carlo Integration" at page 662 of the textbook.

R5) Read Section 23.4 "Derivatives and Integrals for Data with Errors" at page 673 of the textbook.

R6) Read the Epilogue section of Part 6 (pages 708-710 of 8th edition). Part 6 includes 4 chapters and its epilogue is at the end of Chapter 23. What we call "Chapter 6 Numerical Integration and Differentiation" in our lectures is this 6th part of the textbook.

Questions:

Q1. Speed of a race car during the first 6 seconds of a race is given by

$t\ [s]$:	0	1	2	3	4	5	6
v [km/h]:	0	29	66	101	133	158	179

Determine the distance traveled during the first 6 seconds using **a)** trapezoidal rule, **b)** Simpson's 1/3 rule, **c)** Simpson's 3/8 rule.

Q2. a) Evaluate the following integral

$$\int_0^{2\pi} x \sin(x)\, dx$$

using Gauss quadrature integration with 4 points with the following coordinates and weights.

	Coordinates	Weights
Point 1:	-0.8611363115940526	0.3478548451374539
Point 2:	-0.3399810435848563	0.6521451548625461
Point 3:	0.3399810435848563	0.6521451548625461
Point 4:	0.8611363115940526	0.3478548451374539

Provide the result and the true error. You can use MATLAB's quad function to calculate the exact value.

b) Write a MATLAB code that calculates the integral using multiple segment trapezoidal rule and determine the number of segments necessary to have roughly the same accuracy.

c) Write a MATLAB code that calculates the integral using multiple segment Simpson's 1/3 rule and determine the number of segments necessary to have roughly the same accuracy.

d) Write a MATLAB code that calculates the integral using multiple segment Simpson's 3/8 rule and determine the number of segments necessary to have roughly the same accuracy.

e) Read MATLAB's documentation for the quad function and briefly explain how it works.

Q3. Defibrillators are devices that send an electric pulse or shock to the heart to restore a normal heartbeat. The figure shows the typical output pulse from an MDS (monophasic damped sine) defbrillator. The voltage (in Volt) as a function time (in second) is given by

$$v(t) = 3500\,\sin(140\pi t)\, e^{-63\pi t}$$

The energy (in Joule) delivered to a patient by this pulse is calculated by

$$E = \int_0^t \frac{v(t)^2}{R}\, dt$$

where $R = 50\ \Omega$ is the impedance of the patient. Using the code developed in part (c) of the previous question for Simpson's 1/3 Rule, determine the energy delivered to the patient during the 0.001 s interval.

Q4. The upward velocity of a rocket can be computed by the following formula:

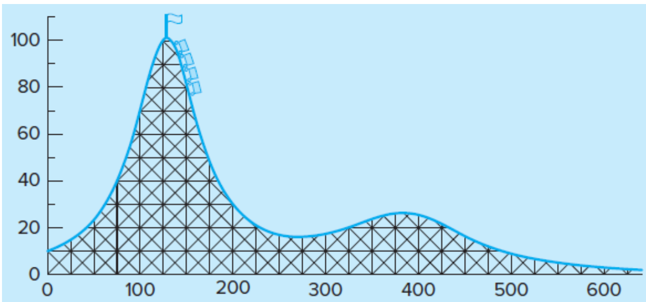
$$v = v_f \ln\left(\frac{m_0}{m_0 - qt}\right) - gt$$

where v = upward velocity, v_f = velocity at which fuel is expelled relative to the rocket, m_0 = initial mass of the rocket at time $t = 0$, q = fuel consumption rate, and g = downward acceleration due to gravity (9.81 m/s^2). If v_f = 1500 m/s, $m_0 = 1.5\times10^5$ kg, and q = 2300 kg/s, use $O(h^8)$ Romberg integration to determine how high the rocket will fly in 30 s. <u>Show all calculation details</u>.

Q5. This question is about the <u>reading assignment R4</u>. Write a MATLAB code to evaluate the following integral using the rectangular Monte Carlo algorithm. Employ 1000, 2500, 5000, 10000 and 20000 points and compare the results with the analytical solution.

$$\int_{1.5}^{15} (-0.0125x^3 + 0.115x^2 + 1.18x - 6.3)\, dx$$

Q6. A useful application of integration is to find the arc length of a curve. If a curve in Cartesian (x-y) coordinates is given by the function $y = f(x)$, the length of the curve between $x = a$ and b can be computed as



$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

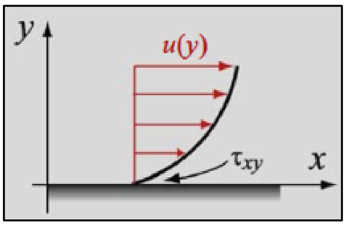
As shown, a two-dimensional roller coaster track's x-y coordinates are defined by the following function

$$f(x) = \frac{1}{(0.0025x - 0.3)^2 + 0.01} + \frac{1}{(0.0025x - 0.9)^2 + 0.04} - 1.14$$

After differentiating this equation, use numerical integration to estimate the length of track from $a = 0$ to $b = 600$.

Q7. The distribution of the x component of the velocity u of a fluid near a flat surface is measured as a function of the distance y fom the surfce as follows

y [mm]	0	2	4	6	8
u [mm/s]	0	5	8	17	22



The shear stress applied by the fluid on the surface is given by

$$\tau = \mu \frac{\partial u}{\partial y}\bigg|_{y=0}$$

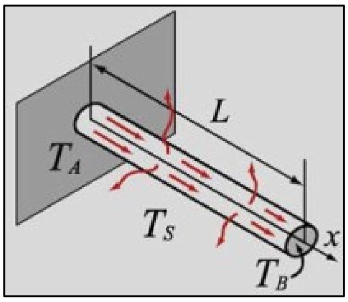
where μ is the coeffcient of viscosity. Calculate the shear stress at the surface using **a)** two points, **b)** three points, and **c)** four points. Take $\mu = 0.005\ \mathrm{Ns/m^2}$.

Q8. Measurement of the one-dimensonal temperature distribution along a fin gives the following data.

x [cm]	0	1	2	3	4	5	6	7	8	9	10
T [K]	473	446.3	422.6	401.2	382.0	364.3	348.0	332.7	318.1	304.0	290.1

The fin has a length $L = 10$ cm, constant cross-sectional area of $1.6 \times 10^{-5}\ \mathrm{m^2}$, and thermal conductivity $k = 240\ \mathrm{W/(mK)}$. The heat flux $(\mathrm{W/m^2})$ is given by

$$q_x = -k\frac{dT}{dx}$$



a) Determine the heat flux at $x = 0$ using 2, 3 and 4 points. Show all calculation details.

b) Determine the heat flux at $x = L$ using 2, 3 and 4 points. Show all calculation details.

Q9. A radar station is tracking the motion of an aircraft. The recorded distance to the aircraft, r, and the angle θ during a period of 48 s is given as follows.

t [s]	0	4	8	12	16	20	24	28	32	36	40	44	48
r [km]	18.80	18.86	18.95	19.04	19.15	19.26	19.38	19.50	19.62	19.74	19.87	19.99	20.12
θ [rad]	0.785	0.779	0.770	0.759	0.748	0.735	0.722	0.707	0.693	0.677	0.661	0.645	0.628

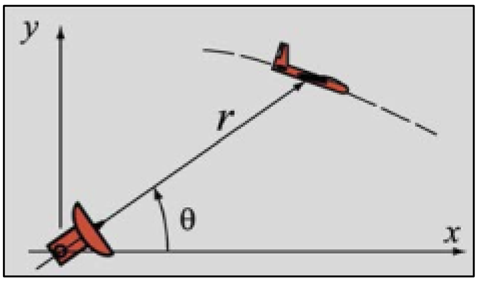
The magnitude of the instantaneous velocity and acceleration of the aircraft can be calculated by

$$v = \sqrt{\left(\frac{dr}{dt}\right)^2 + \left(r\frac{d\theta}{dt}\right)^2} \quad , \quad a = \sqrt{\left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right]^2 + \left[r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt}\right]^2}$$

Write a MATLAB code that calculates and plots the velocity and acceleration of the aircraft. Use $O(h^2)$ accurate central approximations for the first and second order derivatives whenever possible, else use $O(h^2)$ accurate forward or backward approximations.

Provide the formulas that you need to use for different parts of the data set.

Provide the plots of $v(t)$ and $a(t)$.



Q10. Using the given data set of 4 unequally spaced points, derive forward difference formulas for the first and second derivatives, i.e. df/dx and d^2f/dx^2, at $x = 0$. Evaluate the derivatives using the derived formulas.

x	0	1	3	6
f	0	0.1	2.6	21.5