Middle East Technical University
Department of Mechanical Engineering
ME 310 Numerical Methods
Spring 2023 (Dr. Cuneyt Sert)
Study Set 4

For Homework 4 submit the answers of questions 2, 3 and 4. Their grade percentages are not known at this point. It will be decided later. If you want, you can also work on the bonus question Q1.

Assigned: 30/04/2023 – Due: 13/05/2023, 23:59

Homework Rules and Suggestions:

- This is an individual assignment. Everything in your report should be the result of your own work. You are allowed to discuss its solution with your classmates and teaching staff up to a certain detail on ODTUClass. You are not allowed to use a solution manual. Put the following honor pledge at the top of your homework report and behave accordingly.

  "I understand that this is an individual assignment. I affirm that I have not given or received any unauthorized help on this assignment, and that this work is my own."

  If you have exchanged ideas with other students outside ODTUClass, you need to put their names and the extent of your discussion at the beginning of your report.

- Homework submission will be allowed until 5 minutes past the due time. Late submission is not allowed unless you have a valid excuse. In such a case, it is better if you let us know about this before the submission deadline.
- Upload your report as a PDF document (not a Word document) together with all other files (such as codes) to ODTUClass. Name your MATLAB files properly. Follow MATLAB file naming rules such as "File names cannot start with a number", "They cannot contain special characters or spaces", etc.
- Also put your MATLAB codes in your report, but in doing that make sure that you format them properly to avoid line wrapping. Codes with wrapped long lines become unreadable and we cannot understand them. If the codes are very long, you can shorten them by omitting noncritical parts.
- In writing your codes, follow good programming practices such as "use explanatory header lines", "explain inputs and outputs of functions", "use self-explanatory variable names", "use comments", "use empty lines and spaces for readability", "use indentation for code blocks", etc.
- Pay attention to the format of your report. Your report should look like a serious academic work, not like a high school student work. Font types and sizes, page margins, empty spaces on pages, equations, figures, tables, captions, colors, etc. are all important to give the desired "academic work feeling". Language used is also important. Reports with poor use of English cannot get a good grade.
- Do not provide an unnecessarily long report, with useless details or wasted spaces in pages. The shorter your report, the better it is, as long as it answers the questions properly. There are about 100 students, and we can spend only about 10 minutes to grade each report. For this, your report should be easy to read and understand. Also we should be able to find the results and judge their correctness easily. We should not get lost in your report. The more we struggle to understand your report, the lower your grade will be. Use figures and tables cleverly for this purpose.
- Reports with only figures, tables and codes, but no comments or discussions will not get a good grade. Even when a question does not specifically ask for a discussion, you better write some comments on its key points and your key learnings.
- Figures and tables should be numbered and should have captions (at the bottom for figures and at the top for tables). Their titles should be self-explanatory, i.e., we should be able understand everything about the table or figure just by reading its title. They should all be referred properly in the written text (such as "… as shown in Fig. 3" or "… (See Table 2)").
- Do not use Appendices in your report. Do not put your codes in Appendices.
- You can have a numbered reference list at the end of your report. In that case, you need to refer to the references in the text.
- If you are inexperienced in programming, converting an algorithm into a code and writing it in a bug-free way can be time consuming and frustrating. This is not something that can be done at the last minute. You are advised to start working on the assignments as soon as they are assigned.

Reading Assignments:

Self-learning is an important skill. Not everything can be discussed in lectures. You need to learn certain things by yourself.

For those of you who haven't got a chance to buy the textbook yet, I extracted the pages of the following reading assignments and uploaded them to ODTUClass.

R1) Read page 366 of the 8th edition to learn an alternative way of calculating ε_a for the golden section search method.

R2) Read page 384 of the 8th edition to learn how the derivatives of the Hessian matrix can be evaluated approximately when exact derivatives are not available for a 2D problem.

R3) Read the Epilogue section of Part 4 (page 445 of 8th edition). Part 4 includes 4 chapters and its epilogue is at the end of Chapter 16. What we call "Chapter 4 Optimization" in our lectures is this 4th part of the textbook.

Questions:

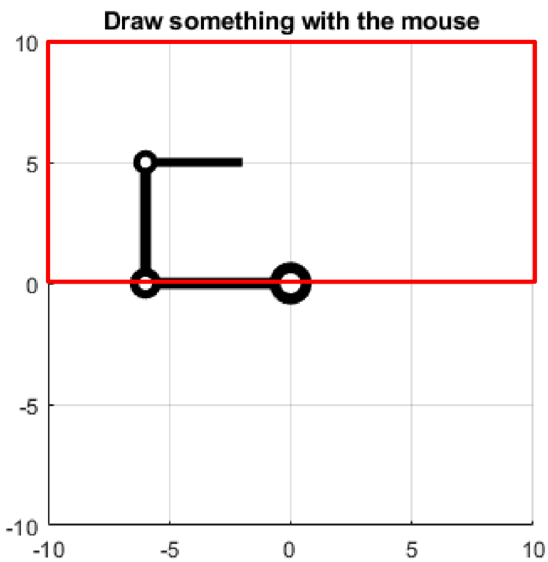
Q0. We believe that some students are using ChatGPT to solve homework questions and prepare reports, without mentioning about it. We think that this is not ethical. What do you think?

Q1. (BONUS QUESTION) Download Salvador.m MATLAB code and watch the related tutorial video. Especially pay attention to the part where it is explained how a root finding problem is converted to a least squares minimization problem.

In this code, there are no constraints on the rotation of the links, i.e. both links can make full 360° rotations, which is not necessarily the case for actual robots. Consider the 3-linked robot shown on the right, with link lengths of 6, 5 and 4 units. The links need to stay in the red region shown, i.e. they cannot pass to the bottom half of the xy plane.

Unfortunately, we cannot provide any constraints to the `fsolve` command used in Salvador.m code. Therefore, formulate the problem in a different way and use a different optimization function which works with constraints. Or you can totally avoid the optimization idea.

Modify the code and explain your changes. Run the code with a few different paths drawn inside the allowed red region and share the screen recordings of how it works when its motion is constrained as explained above, and when not constrained.



Q2. The specific growth rate g of a yeast that produces an antibiotic is a function of the food concentration c.

$$g = \frac{5c}{5 + 0.8c + c^2 + 0.2c^3}$$

The growth rate goes to zero at very low concentrations due to food limitation. It also goes to zero at high concentrations due to toxicity effects.

a) Use golden section search to find the value of c at which the growth rate is maximized. Start with an interval of [0, 10]. Show all the details of the calculations for the first 3 iterations. You can continue to do hand calculations for the rest, or write a MATLAB code. Perform iterations until the interval width reduces down to 0.01. Tabulate x_L, x_U, x_1 and x_2 values of all the iterations.

b) Repeat part (a) using quadratic interpolation.

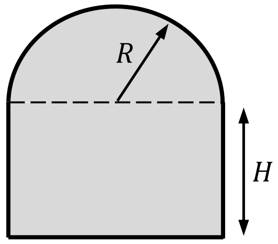
c) Repeat part (a) using Newton's method. Use an initial guess of zero. Perform iterations until ε_a reduces below 0.1 %.

d) Repeat part (c), but using the following approximations for the derivatives

$$f'(x_i) \approx \frac{f(x_i + \delta x_i) - f(x_i - \delta x_i)}{2\delta x_i} \quad , \quad f''(x_i) \approx \frac{f(x_i + \delta x_i) - 2f(x_i) + f(x_i - \delta x_i)}{(\delta x_i)^2}$$

where δ is a small perturbation parameter. Select a proper value for it.

Q3. A fence is to be constructed to enclose land in the shape of a semicircle adjacent to a rectangle. If the length of the fence is restricted to 300 m, formulate the problem of determining the dimensions R and H that correspond to the maximum enclosed land area.



a) Derive equations for the objective function f and the constraint. Plot the contours of f and the constraint equation together to obtain a graphical solution. See Handout 6 for plotting contour lines.

b) Write a MATLAB code that performs random search to solve the problem.

c) Study Handout 6 and use one of MATLAB's built-in optimization functions to solve the problem. Provide the code used and the result obtained.

Q4. Write a MATLAB code to solve Rosenbrock's "banana function" test problem (see Handout 6) using a simplified version of the steepest descent method. Instead of moving in ∇f direction by an amount $\alpha \nabla f$ or solving a 1D optimization problem at each step, use a fixed step size of h. In other words, after determining the ∇f direction at each step, travel in that direction a constant distance of h. Your code should generate a contour plot of the function, and on top of it show the solution of each step with a dot. When the iterations are over, all the dots will give us the path followed from the initial guess to the final solution.

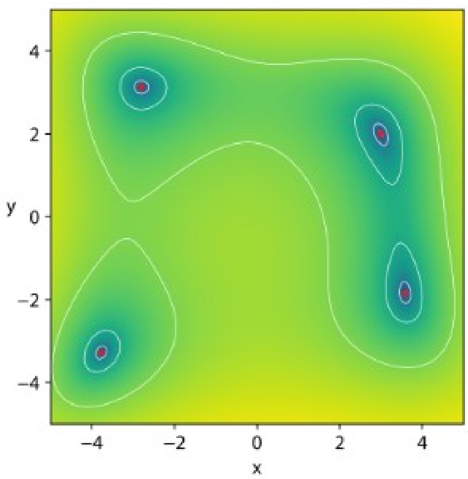
Perform solutions starting from (-1.9, 2). Use a number of different step sizes and discuss how the step size affects accuracy and speed of convergence. You can also make comparisons with the solution provided in Handout 6.

Q5. Write a MATLAB code to solve Rosenbrock's "banana function" test problem using Newton's method. Try different starting points and discuss the convergence characteristics.

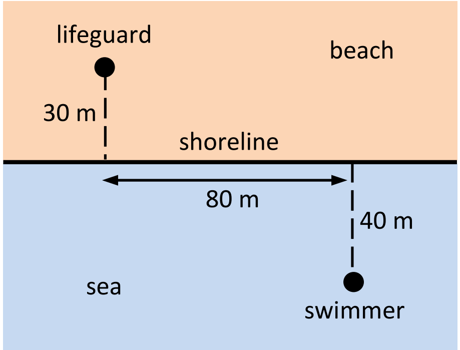
Q6. Following web site lists some well-known test functions that are used to compare the convergence characteristics of different optimization methods.

https://en.wikipedia.org/wiki/Test_functions_for_optimization

As an example, the one shown on the right is called Himmelblau's function. Select a few of these functions and find their minimum points with MATLAB's built-in routines and with the codes you've developed in Questions 4 and 5.



Q7. A lifeguard sees a swimmer in distress, with the two positioned as shown. If the lifeguard runs at a speed of 5 m/s and swims at 1 m/s, what is the optimal path for the lifeguard to reach the swimmer in the shortest time? Formulate an optimization problem whose solution is the point along the shoreline at which the lifeguard should enter the water. Using the optimal path, how long will it take the lifeguard to reach the swimmer?



Q8. (This is the problem discussed in the recorded video of Week 8. But the numbers are modified) A manufacturer produces two types of products (A and B) using three different types of machines; lathes, milling machines and grinding machines. The amount of machining time required and the profit of each product are as follows.

Product	Machining time required (hours)			Profit per unit (TL)
	On lathe	On milling machine	On grinding machine	
A	14	8	9	1800
B	9	15	7	2300

The maximum machining times available per day on lathes, milling machines and grinding machines are 130, 100 and 90 hours, respectively. Determine the number of units of products A and B to be manufactured per day for maximum profit.

a) Solve the problem graphically. Do NOT draw by hand. Instead work on a computer generated scaled graph.

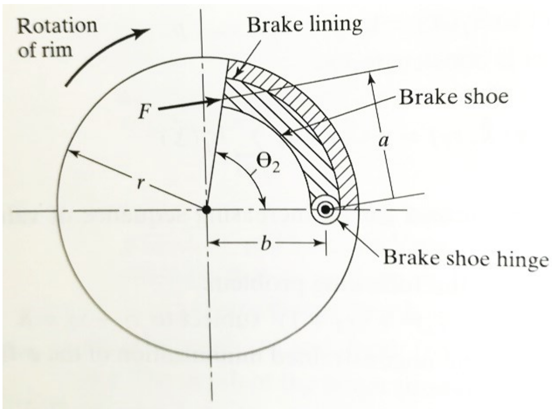
b) Solve the problem using MATLAB's `linprog` function. Study Handout 6 first.

Q9. The actuation force (F) necessary to apply the brake shown is given by

$$F = \frac{M_n - M_f}{a}$$

$$M_n = p_a t r b \left[\frac{\theta_2}{2} - \frac{1}{4}\sin(2\theta_2)\right]$$

$$M_f = p_a t r f \left[r - r\cos(\theta_2) \; - \frac{b}{2}\sin^2(\theta_2)\right]$$



where p_a is the maximum pressure exerted on the brake lining, t is the face width of the brake shoes, r is the inner radius of the brake drum, b is the radius of the brake shoe, f is the coefficient of friction, a is the linear distance between the brake shoe hinge and the line of application of force F, and θ_2 is the angle between the brake-shoe hinge and the point of application of force F. Formulate and solve the problem of determining the values of r, b, t and θ_2 for minimizing the volume of the brake lining. The maximum actuation force is limited to $F_{max} = 80$ N. Use $p_a = 10^5$ Pa, $f = 0.35$, $a = 20$ cm.

Q10. Automobile doors are often reinforced by aluminum beams to help absorb side impact energy and reduce injury to passengers from side impacts. Formulate and solve the problem of designing a minimum weight beam of rectangular cross section with the following constraints.

a) The bending stress induced in the beam, assumed to be simply supported, during the impact should not exceed 140 MPa. The impact energy absorbed by the beam can be approximated as

$$E = \frac{1}{2}\frac{m_2(v_1 - v_2)^2}{(1 + m_2/m_1)}$$

where m_1 and m_2 are the masses of the two impacting vehicles and v_1 and v_2 are their speeds at the time of impact. The impact force induced (F_i) at the middle of the beam is given by $F_i = 2E/s$, where s is the net crush distance during impact. The bending stress induced in a simply supported beam due to central concentrated load (F_i) can be expressed as

$$\sigma_b = \frac{3F_i l}{2bh^2}$$

where l, b, and h are the length, width and depth of the beam. Assume that $v_1 = 0$, $v_2 = 16\ \mathrm{km/h}$, $m_1 = m_2 = 1250\ \mathrm{kg}$, $s = 20\ \mathrm{cm}$, and $l = 60\ \mathrm{cm}$.

b) The width and depth of the beam should not be less than 0.6 cm.

c) The depth of the beam should not exceed the width of the beam.