Middle East Technical University
Department of Mechanical Engineering
ME 310 Numerical Methods
Spring 2023 (Dr. Cuneyt Sert)
Study Set 5

For Homework 5 submit the answers of questions 2, 6, 8 and 13. Their grade percentages are not known at this point. It will be decided later.

Assigned: 15/05/2023 – Due: 27/05/2023, 23:59

Homework Rules and Suggestions:

- This is an individual assignment. Everything in your report should be the result of your own work. You are allowed to discuss its solution with your classmates and teaching staff up to a certain detail on ODTUClass. You are not allowed to use a solution manual. Put the following honor pledge at the top of your homework report and behave accordingly.

  "I understand that this is an individual assignment. I affirm that I have not given or received any unauthorized help on this assignment, and that this work is my own."

  If you have exchanged ideas with other students outside ODTUClass, you need to put their names and the extent of your discussion at the beginning of your report.

- Homework submission will be allowed until 5 minutes past the due time. Late submission is not allowed unless you have a valid excuse. In such a case, it is better if you let us know about this before the submission deadline.
- Upload your report as a PDF document (not a Word document) together with all other files (such as codes) to ODTUClass. Name your MATLAB files properly. Follow MATLAB file naming rules such as "File names cannot start with a number", "They cannot contain special characters or spaces", etc.
- Also put your MATLAB codes in your report, but in doing that make sure that you format them properly to avoid line wrapping. Codes with wrapped long lines become unreadable and we cannot understand them. If the codes are very long, you can shorten them by omitting noncritical parts.
- In writing your codes, follow good programming practices such as "use explanatory header lines", "explain inputs and outputs of functions", "use self-explanatory variable names", "use comments", "use empty lines and spaces for readability", "use indentation for code blocks", etc.
- Pay attention to the format of your report. Your report should look like a serious academic work, not like a high school student work. Font types and sizes, page margins, empty spaces on pages, equations, figures, tables, captions, colors, etc. are all important to give the desired "academic work feeling". Language used is also important. Reports with poor use of English cannot get a good grade.
- Do not provide an unnecessarily long report, with useless details or wasted spaces in pages. The shorter your report, the better it is, as long as it answers the questions properly. There are about 100 students, and we can spend only about 10 minutes to grade each report. For this, your report should be easy to read and understand. Also we should be able to find the results and judge their correctness easily. We should not get lost in your report. The more we struggle to understand your report, the lower your grade will be. Use figures and tables cleverly for this purpose.
- Reports with only figures, tables and codes, but no comments or discussions will not get a good grade. Even when a question does not specifically ask for a discussion, you better write some comments on its key points and your key learnings.
- Figures and tables should be numbered and should have captions (at the bottom for figures and at the top for tables). Their titles should be self-explanatory, i.e., we should be able understand everything about the table or figure just by reading its title. They should all be referred properly in the written text (such as "... as shown in Fig. 3" or "... (See Table 2)").
- Do not use Appendices in your report. Do not put your codes in Appendices.
- You can have a numbered reference list at the end of your report. In that case, you need to refer to the references in the text.
- If you are inexperienced in programming, converting an algorithm into a code and writing it in a bug-free way can be time consuming and frustrating. This is not something that can be done at the last minute. You are advised to start working on the assignments as soon as they are assigned.

Reading Assignments:

Self-learning is an important skill. Not everything can be discussed in lectures. You need to learn certain things by yourself.

For those of you who haven't got a chance to buy the textbook yet, I extracted the pages of the following reading assignments and uploaded them to ODTUClass.

R1) Read page 516 of the 8th edition to learn the Newton-Gregory formula that is used to find interpolating polynomials for equispaced data points. Understand its similarity to the Taylor series expansion.

R2) Read pages 529-531 of the 8th edition to learn multidimensional interpolation.

R3) Read about the Runge's phenomenon. You can find it on Wikipedia.

R4) Read the Epilogue section of Part 5 (pages 592-595 of 8th edition). Part 5 includes 4 chapters and its epilogue is at the end of Chapter 20. What we call "Chapter 5 Curve Fitting" in our lectures is this 5th part of the textbook. Note that we've skipped the Fourier Approximation chapter. You are not responsible for that chapter.

Questions:

Q1. The number of transistors on Intel central processing units since the early 1970s is given below. Fit the model $y = Ae^{Bt}$ to the data. Plot the data points together with the model curve. Comment on the quality of the fit. Use the model curve to estimate the number of transistors in year 2023.

CPU	Year	Transistors
4004	1971	2,250
8008	1972	2,500
8080	1974	5,000
8086	1978	29,000
286	1982	120,000
386	1985	275,000
486	1989	1,180,000
Pentium	1993	3,100,000
Pentium II	1997	7,500,000
Pentium III	1999	24,000,000
Pentium 4	2000	42,000,000
Itanium	2002	220,000,000
Itanium 2	2003	410,000,000

Q2. The measured level of a drug in a patient's blood is given below.

Time [h]	1	2	3	4	5	6	7	8
Concentration [ng/ml]	8.0	12.3	15.5	16.8	17.1	15.8	15.2	14.0

a) Fit the model $y = Ate^{Bt}$ to the data by applying linearization and least squares regression. Plot the model curve together with the data points. Estimate the concentration at $t = 16\text{ h}$. Comment on the quality of the fit. Show all calculation details.

b) Repeat part (a) without linearizing the data, i.e. by directly performing non-linear regression. Show all calculation details.

Q3. The following data is given

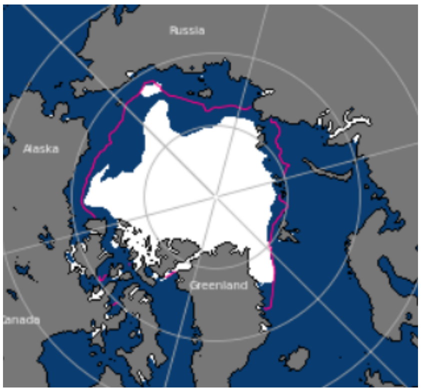
x:	-1	-0.3	0.2	1.0	2.0
y:	2.8	4.4	4.6	2.7	1.2

We want to express this data with the equation $y = [1/(Ax^2 + B)]^2$

a) Linearize this relation and determine the constants A and B using least-squares regression.

b) Plot the data points and the regression curve. Comment on the quality of the fit.

Q4. The ice extent at the North Pole (in million km²) during the years of 2015 - 2016 is shown below. Fit the model $y = c_1 + c_2 t + c_3 \sin(2\pi t) + c_4 \cos(2\pi t)$ to the data, where y denotes the ice extent and t is time in years beginning Jan. 2015. Plot the data points together with the fitted curve. Comment on the quality of the fit.



Month	**2015**	**2016**
Jan.	13.75	13.64
Feb.	14.51	14.32
Mar.	14.49	14.53
Apr.	13.98	13.83
May	12.69	12.08
Jun.	11.05	10.60
Jul.	8.83	8.13
Aug.	5.66	5.60
Sept.	4.68	4.72
Oct.	7.79	6.45
Nov.	10.11	9.08
Dec.	12.33	12.09

Q5. The thermal conductivity of iron is found to vary with temperature as follows.

Temperature [K]	70	100	200	300	400	500	600
Conductivity [W/(mK)]	215	134	93	81	70	61	55

a) Plot the data and suggest a relation that you think will fit to it.

b) Perform least squares regression and calculate the constants of the suggested regression. Plot the regression curve and the data points. Comment on the quality of the fit.

Q6. The efficiency of a reaction type hydraulic turbine is found to vary with the output power and the available water head as follows. Develop a least-squares fit of the form $\eta = a_0 + a_1\mathcal{P} + a_2 h$. Comment on the quality of the fit. <u>Show all calculation details.</u>

Power ($\mathcal{P}$ [MW])	16.4	22.4	29.8	23.9	32.1	41.0	38.8	46.2	59.7
Water head (h [ft])	80	80	80	110	110	110	145	145	145
Efficiency (η [%])	75	80	85	75	80	85	75	80	85

Q7. An experiment provided a data set of y vs. x. According to the theory, the relation between these variables is of the form $y = A\left(1 - e^{-Bx^2}\right)/x$

a) Is this relation in the "general linear form"?

b) If your answer to part (a) is "No", can you linearize the given relation?

Q8. An experiment produced the following data points

x	-1	-0.96	-0.86	-0.79	0.22	0.5	0.93
y	-1	-0.151	0.894	0.986	0.895	0.5	-0.306

a) Construct the finite divided difference table and determine the Newton's interpolating polynomial that passes through all the points. Plot the function together with the data points. Interpolate for $y(-0.2)$. <u>Show all calculation details.</u>

b) Repeat part (a), but this time determine the Lagrange interpolating polynomial. <u>Show all calculation details.</u>

c) Determine the natural cubic splines that interpolate the data set. Interpolate for $y(-0.2)$. <u>Show all calculation details.</u>

d) Compare the results and discuss.

Q9. The power generated by a windmill varies with the wind speed as follows

Wind speed [km/h]	22	35	48	61	74
Power [W]	320	490	540	500	480

a) Form the FDD table of the data and interpolate for the power at 42 km/h wind speed using 2, 3, 4 and 5 points.

b) Using MATLAB's spline function determine the power at 42 km/h.

Q10. Apply the following world population data to estimate the 1980 population, using

a) the straight line through the 1970 and 1990 estimates

b) the parabola through the 1960, 1970, and 1990 estimates

c) the cubic curve through all four data points.

Compare each with the 1980 estimate of 4,452,584,592.

Year	1960	1970	1990	2000
Population	3,039,585,530	3,707,475,887	5,281,653,820	6,079,603,571

Q11. The estimated mean atmospheric concentration of carbon dioxide in earth's atmosphere is given below, in parts per million by volume. Find the 3rd degree interpolating polynomial of the data and use it to estimate the CO_2 concentration in 1950 and 2050 (The actual concentration in 1950 was 310 ppm).

Year	1800	1850	1900	2000
CO_2 [ppm]	280	283	291	370

Q12. The expected lifetime (in 1000 hours) of an industrial fan when operated at the listed temperature is given below. Estimate the lifetime at 70 °C by using

a) the parabola from the last three data points

b) the 3rd degree curve using all four points.

Temp [°C]	25	40	50	60
Lifetime [×1000 h]	95	75	63	54

Q13. (Work on the reading assignment R2 first) Temperatures are measured at 25 points on a heated square plate. Estimate the temperature at x = 3, y = 3.2, and x = 4.3, y = 2.7 by performing bi-quadratic interpolation. Show all the calculation details.

	x = 0	**x = 2**	**x = 4**	**x = 6**	**x = 8**
y = 0	100.00	90.00	80.00	70.00	60.00
y = 2	85.00	64.49	53.50	48.15	50.00
y = 4	70.00	48.90	38.43	35.03	40.00
y = 6	55.00	38.78	30.39	27.07	30.00
y = 8	40.00	35.00	30.00	25.00	20.00