Credit Markets

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Agenda: Single-Name Credit

Credit Default Swaps

- A CDS consists of **two legs** (fee and protection leg).
- During the lifetime of a CDS the buyer of a CDS (called the protection buyer) pays a fee (called the CDS premium) for a protection against default risk to the protection seller.
- This **fee** is **paid quarterly or semiannually in arrear** and is fixed at the time when the CDS is issued.
- It is chosen such that the **initial value** of the CDS is **zero**.
- This payment stream stops at the maturity of the CDS or at default, whichever occurs first.
- If **default** occurs during the lifetime of the CDS, the **protection buyer** is **compensated for the loss** that a typical bondholder would suffer.

Credit Default Swaps

- Consider a CDS which starts at time 0 and has maturity T
- The time-0 CDS spread is denoted by $S = S_0 = S_0(T)$
- The time-t CDS spread is denoted by $S_t = S_t(T)$.
- During the lifetime of the CDS fee payments are made at times t_j, $j = 1, \ldots, n$ if default has not occurred before t_j.
- Note that $T = t_n$.
- Payments are made at equidistant points in time, i.e. $\delta = t_j - t_{j-1}$ for all $j = 1, \ldots, n$.
- Since we look at spot CDS contracts, we have $t_0 = 0$.
- For forward CDS contracts we would have $0 < t_0$ (see below).
- The notional is normalized to one.
- The default time is modeled via a stopping time τ.

CDS: Payment Streams

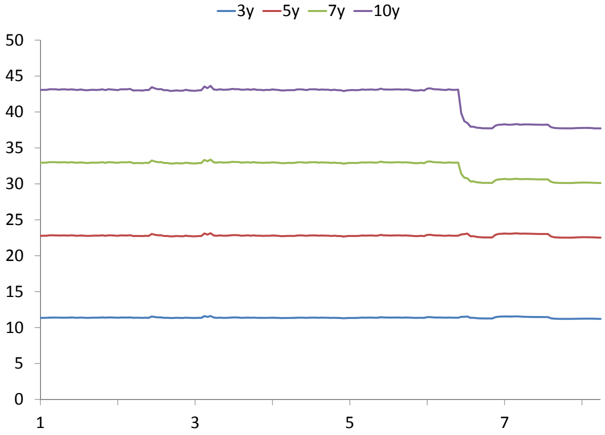
No Default until Maturity

Time	0	0.5	1	...	$T - 0.5$	T
Fee Leg	0	δS_0	δS_0	...	δS_0	δS_0
Protection Leg	0	0	0	...	0	0

Default before Maturity

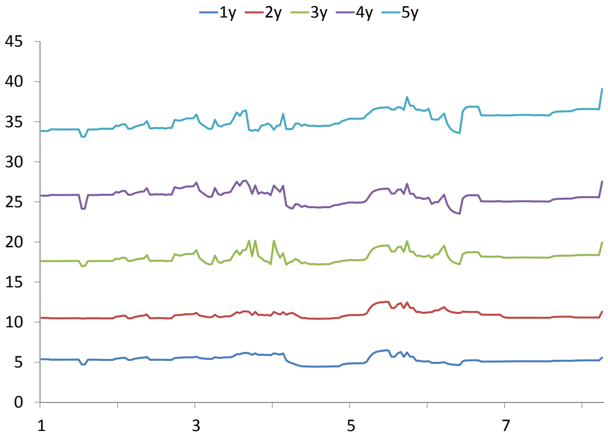
Time	0	0.5	1	...	t_{j-1}	τ	...	T
Fee Leg	0	δS_0	δS_0	...	δS_0	$(\tau - t_{j-1})S_0$	0	0
Protection Leg	0	0	0	...	0	ℓ	0	0

CDS Spreads: Google



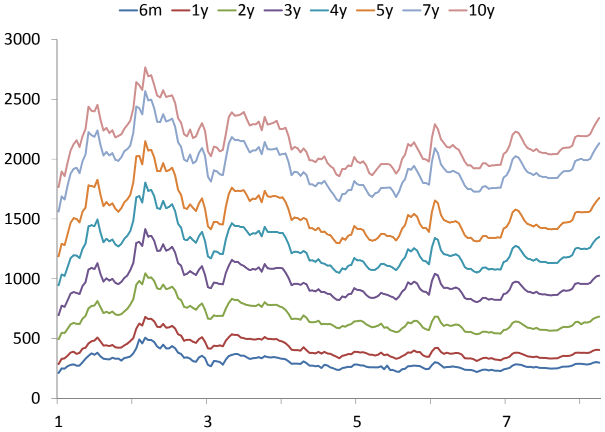
Google spreads for different maturities (daily, 01/01/16-09/14/16)

CDS Spreads: Microsoft



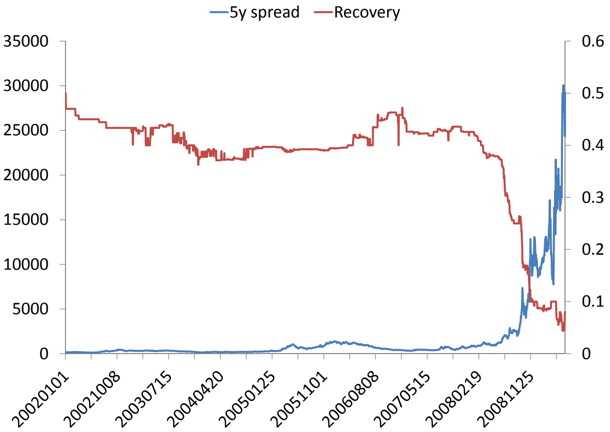
Microsoft spreads for different maturities (daily, 01/01/16-09/14/16)

CDS Spreads: Ford



Ford spreads for different maturities (daily, 01/01/16-09/14/16)

CDS Spread: GM until Default



5y GM spread and recovery (daily, 01/01/02-05/29/09)

CDS: Fee Leg

- The fee payment at time t_j is given by

$$S_0 \, \delta \, \mathbf{1}_{\{\tau > t_j\}},$$

  where S_0 denotes the time-0 CDS spread with maturity T
- If a default happens between t_{j-1} and t_j, then an accrued fee payment is due at default.
- We **disregard** accrued fees since contribution to PV negligible
- We **assume** that r and τ are **independent**
- The time-0 present value of the fee leg is then

$$\hat{V}_0^{fee} = \sum_{j=1}^{n} \mathrm{E}\left[e^{-\int_0^{t_j} r_s \, ds} S_0 \delta \mathbf{1}_{\{\tau > t_j\}}\right] = S_0 \delta \sum_{j=1}^{n} \underbrace{p(0, t_j) Q(\tau > t_j)}_{= p^d(0, t_j)}$$

Interpretation of the Value of the Fee Leg

The value of the fee leg is equivalent to the value of a portfolio of defaultable zeros with notionals $S_0\delta$ and maturities t_j.

CDS: Fee Leg

We also need the value of the fee leg per one unit of fee payments

Value of Normalized Fee Leg (Discrete Fee Payments)

The value of the fee leg per one unit of fee payments is

$$V_0^{fee} = \delta \sum_{j=1}^{n} p(0, t_j) Q(\tau > t_j) = \delta \sum_{j=1}^{n} p^d(0, t_j)$$

Sometimes it is assumed that the fee is paid continuously. In this case the value of the normalized fee leg is given by

Value of Normalized Fee Leg (Continuous Payment)

The value of the fee leg per one unit of fee payments is

$$V_0^{fee} = \mathrm{E}\left[\int_0^T e^{-\int_0^s r_u\,du} \mathbf{1}_{\{\tau > s\}}\,ds\right] = \int_0^T p(0, s) Q(\tau > s)\,ds$$

This simplification has the advantage that accrued fees are avoided.

CDS: Protection Leg and CDS Spread

In any case, the value of the protection leg is given by

Value of the Protection Leg

$$V_0^{prot} = \ell \, \mathrm{E}\left[e^{-\int_0^\tau r_u \, du} \mathbf{1}_{\{\tau \leq T\}}\right] = \ell \int_0^T p(0,s) \mathrm{E}\left[\lambda_s e^{-\int_0^s \lambda_u \, du}\right] ds$$

where ℓ denotes the loss which is assumed to be constant.

Recall that

$$\mathrm{E}\left[e^{-\int_0^\tau r_s \, ds} \mathbf{1}_{\{\tau \leq T\}}\right] = p(0,T) Q(\tau \leq T) - \int_0^T \partial_s \{p(0,s)\} Q(\tau \leq s) \, ds$$

Therefore, the value of the protection leg can also be expressed as

$$V_0^{prot} = \ell \left(p(0,T) Q(\tau \leq T) - \int_0^T \partial_s \{p(0,s)\} Q(\tau \leq s) \, ds \right),$$

i.e. in terms of the default probability.

CDS Spread

The CDS spread S_0 is computed such that the initial value of the CDS is zero, i.e.

$$S_0 V_0^{fee} = \hat{V}_0^{fee} \overset{!}{=} V_0^{prot}$$

Therefore, we obtain

Fair CDS Spread

$$S_0 = \frac{V_0^{prot}}{V_0^{fee}}.$$

CDS: Additional Remarks

- We assume that the protection seller cannot default.
- This is a reasonable assumption if there is collateralization.
- Otherwise, we have to take this counterparty risk into account.

CDS Valuation in Affine Models

- Assume that r and λ are independent.
- Then the fee leg is given by (continuous payments)

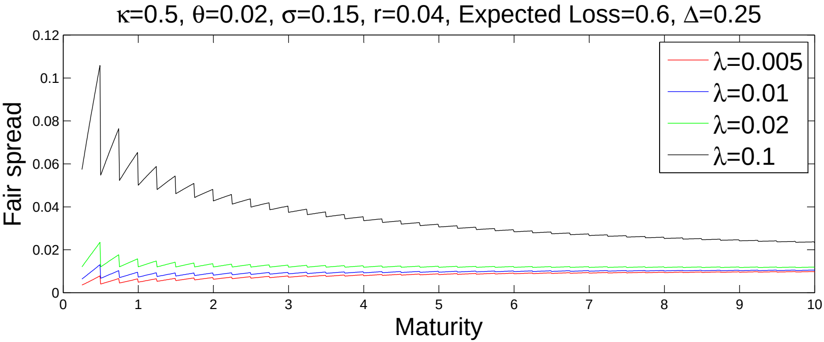
$$
\begin{aligned}
V_0^{fee} &= \int_0^T p(0,s)\mathrm{E}\left[e^{-\int_0^s \lambda_u\,du}\right] ds \\
&= \int_0^T p(0,s)e^{A(s)-B(s)\lambda_0}\,ds
\end{aligned}
$$

- The protection leg becomes

$$
\begin{aligned}
V_0^{prot} &= \ell\int_0^T p(0,s)\mathrm{E}\left[\lambda_s\,e^{-\int_0^s \lambda_u\,du}\right] ds \\
&= \ell\int_0^T p(0,s)\big(C(s)+H(s)\lambda_0\big)e^{A(s)-B(s)\lambda_0}\,ds
\end{aligned}
$$

- The corresponding functions A, B, C, and H depend on the affine model and are given above.
- These formulas are fully explicit up to a numerical integration.

CDS Spreads with CIR Intensity



CDS Spreads with CIR intensity and quarterly fee payments.

Dependence and Accrued Fees

- Typically, it is sufficient to make the independence assumption when pricing CDS contracts.
- Additionally, the contribution of accrued fees is small.
- The following slides summarize the general case where
  - r and λ can be dependent,
  - accrued fees are included.
- The money market account is denoted by

$$B(t, T) = e^{\int_t^T r_s \, ds}$$

Dependence and Accrued Fees: Fee Leg

- The fee payment at time t_j is given by

$$S_0\,\delta\,\mathbf{1}_{\{\tau>t_j\}},$$

where S_0 denotes the CDS spread with maturity T which is contracted today.

- If a default happens between t_{j-1} and t_j, then an accrued fee payment is due at default:

$$S_0\,\delta_d^j\,\mathbf{1}_{\{t_{j-1}\le\tau\le t_j\}},$$

where $\delta_d^j \equiv \tau - t_{j-1}$ is the length of the last interval.

Value of Normalized Fee Leg (Discrete Fee Payments)

$$V_0^{fee} = \sum_{j=1}^{n}\left(\delta \mathrm{E}\left[\frac{\mathbf{1}_{\{\tau>t_j\}}}{B(0,t_j)}\right] + \mathrm{E}\left[\frac{\delta_d^j\,\mathbf{1}_{\{t_{j-1}\le\tau\le t_j\}}}{B(0,\tau)}\right]\right)$$

Dependence and Accrued Fees: Fee Leg

This can be rewritten

$$
\begin{aligned}
V_0^{fee} &= \sum_{j=1}^{n} \left(\delta \mathrm{E}\left[\frac{\mathbf{1}_{\{\tau > t_j\}}}{B(0,t_j)} \right] + \mathrm{E}\left[\frac{\delta_d^j \, \mathbf{1}_{\{t_{j-1} \leq \tau \leq t_j\}}}{B(0,\tau)} \right] \right) \\
&= \delta \sum_{j=1}^{n} \mathrm{E}\left[\frac{\mathbf{1}_{\{\tau > t_j\}}}{B(0,t_j)} \right] + \sum_{j=1}^{n} \mathrm{E}\left[\frac{(\tau - t_{j-1}) \, \mathbf{1}_{\{t_{j-1} \leq \tau \leq t_j\}}}{B(0,\tau)} \right],
\end{aligned}
$$

Sometimes it is assumed that the fee is paid continuously. In this case the value of the fee leg is given by

Value of Normalized Fee Leg (Continuous Fee Payments)

$$
V_0^{fee} = \mathrm{E}\left[\int_0^T \frac{\mathbf{1}_{\{\tau > s\}}}{B(0,s)} \, ds \right] = \int_0^T \mathrm{E}\left[\frac{\mathbf{1}_{\{\tau > s\}}}{B(0,s)} \right] \, ds.
$$

This simplification has the advantage that accrued fees are avoided.

Dependence and Accrued Fees: Protection Leg

In any case, the value of the protection leg is given by

Value of the Protection Leg

$$V_0^{prot} = \ell \, \mathrm{E}\left[\frac{\mathbf{1}_{\{0<\tau\leq T\}}}{B(0,\tau)}\right].$$

where ℓ denotes the loss which is assumed to be constant.

Of course, we obtain again

CDS Spread

$$S_0 = \frac{V_0^{prot}}{V_0^{fee}}.$$

CDS Valuation: Mark-to-Market

- Initially, the value of a CDS is zero, i.e.

$$V_0^{prot} - S_0 V_0^{fee} = 0.$$

- Over time market conditions change, i.e. in general $S_t \neq S_0$. Therefore,

$$V_t^{prot} - S_0 V_t^{fee} \neq 0.$$

- Since $V_t^{prot} = S_t V_t^{fee}$, we get the following result:

Mark-to-market of CDS

The time-t value of a CDS long position initiated at time 0 reads

$$S_t V_t^{fee} - S_0 V_t^{fee} = V_t^{fee}\big(S_t - S_0\big).$$

CDS: What Is It Exactly?

- Assume that there are **two identical floating rate notes** (FRN).
- Only difference: One is default-free, the other is defaultable.
- The **default-free** FRN has an annual coupon rate of L_t that is paid semiannually.
- The **defaultable** FRN has an annual coupon rate of $L_t + S$ that is paid semiannually.
- One can think of L as the 6-month LIBOR rate.
- The spread S is constant.
- **Default** can **only** occur **at a coupon payment date**.
- The contract is immediately settled by **physical delivery** of the defaultable FRN.

CDS: What Is It Exactly?

Consider the following strategy

- Long position in default-free FRN
- Short position in defaultable FRN

No Default

Time	0.5	...	$T - 0.5$	T
Long FRN	δL_t	...	δL_t	$1 + \delta L_t$
Short FRN^d	$-\delta(L_t + S)$	...	$-\delta(L_t + S)$	$-1 - \delta(L_t + S)$
Net position	$-\delta S$	...	$-\delta S$	$-\delta S$

Default

Time	0.5	1	...	$\tau - 0.5$	τ
Long FRN	δL_t	δL_t	...	δL_t	1
Short FRN^d	$-\delta(L_t + S)$	$-\delta(L_t + S)$	...	$-\delta(L_t + S)$	$-R$
Net position	$-\delta S$	$-\delta S$	...	$-\delta S$	ℓ

This is equivalent to a CDS!

CDS vs. Asset Swap

- An asset swap is a **combination** of a **defaultable coupon bond** (the asset) and an **interest rate swap** (IRS).
- The IRS swaps the the coupon of the bond into "LIBOR + spread"
- The asset swap spread is chosen such that value of the whole package is the par value of the defaultable bond.
- At **initiation** seller (A) receives par value from buyer (B) who receives the payment stream of
  - defaultable coupon bond (issued by third party),
  - interest rate swap.
- At **coupon dates**
  - B receives the coupon
  - B pays to A: coupon
  - A pays to B: LIBOR + spread

CDS vs. Asset Swap

- Therefore, an asset swap is similar to a **synthetic defaultable floater**.
- Since the spread of a CDS can be interpreted as spread on such a floater (see above), an asset swap is similar to a CDS.
- However, the **interest rate swap** is **not affected by credit events**, i.e. is not automatically canceled at default.
- This is one reason why asset swap spreads can be different from CDS spreads.
- Other reasons include liquidity, margin requirements etc.

CDS Basis

Basis $\equiv$ CDS spread minus asset swap spread

Big Bang and CDS Quotes

- Until 2009 most CDS were quoted and traded via the fair spread.
- Since 2009 CDS contracts usually have a running spread (**coupon**) of 100bps or 500bps and an upfront payment, which can be positive or negative.
- Usually, the fair spread ('**conventional spread**') is also provided.
- Since the protection leg is not affected by the different conventions, one has to make sure that for both conventions the fee legs have the same value (disregarding accrued payments):

$$S_0 \sum_{j=1}^{n} \delta p^d(0, t_j) = \hat{V}_0^{fee} = U_0 + C_0 \sum_{j=1}^{n} \delta p^d(0, t_j),$$

where S_0 is the conventional spread, U_0 is the upfront payment, and C_0 is the coupon.

Markit Quotes from 09/14/2012

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Free CDS Pricing Report

Markit Group has developed this free "Last Quote for the most Liquid Credit Default Swaps" pricing report to address a public interest in CDS prices. This Last Quote report is based on the most recent price quote any active dealer in the CDS market provided to its institutional customers before 4:00PM Eastern. In total, 1000 CDS (based on DTCC's Top 1000), as well as several Markit CDS Indices, are contained in this pricing report. The report is intended to provide the general public an indication of where CDS trades could last be made on each trading day. However, as those prices represent the view of a single dealer and often precede the market close by several hours, they are not appropriate for end of day mark to market purposes.

Data and pricing (5 year tenor only) as of 14Sep2012. The numbers for CDS Contracts and Notional are provided by DTCC on week ending 07Sep2012.

Reference Entity	Region	Industry	# of CDS Contracts Outstanding	# of CDS Contracts (Weekly Δ)	Net Notional CDS ($)	Net Notional CDS (Weekly Δ $)	Fixed Coupon	Price	Daily Change	Price Type
*MARKIT CDX.NA.HY			12,344	360	127,284,170,278	(652,403,495)		101.97	1.50	Price
*MARKIT CDX.NA.IG			26,224	298	377,033,092,849	(794,305,818)		84.88	(6.34)	Spread
*MARKIT ITRAXX ASIA EX-JAPAN IG			6,103	60	9,449,690,000	166,000,000		117.50	(9.00)	Spread
*MARKIT ITRAXX EUROPE			38,772	1,102	456,701,770,806	9,790,020,570		119.50	(6.50)	Spread
3I GROUP PLC	Europe	FINANCIALS	1,338	(2)	843,626,770	12,060,302	100	215.00	N/A	ConvSpread
3M COMPANY	Americas	INDUSTRIALS	941	0	555,683,774	438,284	100	25.50	0	ConvSpread
ABB INTERNATIONAL FINANCE LIMITED	Europe	FINANCIALS	2,221	0	609,231,634	8,400,417	100	40.00	(10.00)	ConvSpread
ABBOTT LABORATORIES	Americas	HEALTHCARE	1,249	(69)	582,963,198	1,434,495	100	43.50	2.00	ConvSpread
ABU DHABI	Europe	GOVERNMENT	1,463	(12)	1,160,776,029	(22,781,170)	100	97.00	(6.00)	ConvSpread
ACCOR	Europe	CONSUMER SERVICES	3,041	2	901,917,821	33,832,009	100	125.00	(7.00)	ConvSpread
ACE LIMITED	Americas	FINANCIALS	2,533	1	1,575,009,742	21,912,689	100	53.00	0.50	ConvSpread
ACOM CO., LTD.	Japan	FINANCIALS	1,773	10	605,822,192	1,323,076	100	190.00	(20.00)	ConvSpread
ADECCO S.A.	Europe	INDUSTRIALS	2,471	0	662,428,108	(5,237,264)	100	130.00	(5.00)	ConvSpread
ADVANCED MICRO DEVICES, INC.	Americas	TECHNOLOGY	1,548	(152)	416,191,111	1,206,299	500	3.25	(1.25)	Upfront
AEGON N.V.	Europe	FINANCIALS	4,160	(1)	1,459,525,784	10,503,097	100	200.00	(15.00)	ConvSpread
AEON CO., LTD.	Japan	CONSUMER SERVICES	554	10	456,432,601	11,334,008	100	100.00	0	ConvSpread
AETNA INC.	Americas	HEALTHCARE	2,145	1	963,024,830	9,298,744	100	60.50	3.00	ConvSpread
AGRIUM INC.	Americas	BASIC MATERIALS	1,548	6	462,007,854	10,762,181	100	86.00	(7.00)	ConvSpread
AIR PRODUCTS AND CHEMICALS, INC.	Americas	BASIC MATERIALS	1,049	1	355,044,523	6,372,505	100	46.50	(3.50)	ConvSpread
AK STEEL CORPORATION	Americas	BASIC MATERIALS	1,543	10	362,089,476	6,464,774	500	13.75	(2.75)	Upfront
AKTIEBOLAGET ELECTROLUX	Europe	CONSUMER GOODS	3,255	(1)	903,898,536	(4,805,824)	100	60.00	(4.00)	ConvSpread
AKTIEBOLAGET SKF	Europe	INDUSTRIALS	467	0	163,280,340	2,139,933	100	60.00	0	ConvSpread
AKTIEBOLAGET VOLVO	Europe	INDUSTRIALS	3,868	(6)	1,316,088,819	13,904,153	100	152.00	(10.00)	ConvSpread
AKZO NOBEL N.V.	Europe	BASIC MATERIALS	3,587	16	956,713,266	2,559,969	100	95.00	(4.50)	ConvSpread
ALCATEL LUCENT	Europe	TECHNOLOGY	3,920	17	697,828,703	13,180,791	500	27.50	(2.00)	Upfront
ALCATEL-LUCENT USA INC.	Americas	TECHNOLOGY	1,279	(130)	330,220,435	(2,577,923)	500	23.50	(0.25)	Upfront
ALCOA INC.	Americas	BASIC MATERIALS	3,908	5	1,540,375,042	(36,743,424)	100	279.50	(5.50)	ConvSpread
ALL NIPPON AIRWAYS CO., LTD.	Japan	INDUSTRIALS	671	2	325,053,588	339,779	100	275.00	0	ConvSpread

Markit Quotes from 09/24/2015

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Welcome, Holger Kraft | Support

Markets

Source

Credit Default Swaps

- Snapshot
- News & Commentary
- Credit Events
- Settlement Prices
- Index Trade Volume
- Index Volatility
- Education & Documentation

Markit Eurex CDS Settlement Prices | Markit ICE Settlement Prices | Markit JSCC Settlement Prices | Markit LCH Settlement Prices

Single Names | Indices

As of 24-Sep

North America

All single names are Senior Unsecured Debt, USD, No Restructuring and 5 Year unless noted.

Markit makes a subset of the Markit ICE daily settlement prices on select liquid 5 year single name CDS and indices available free to the public on this website. For access to the full set of Markit ICE Settlement Prices, visit Markit ICE Settlement Price product page.

Name	Ticker	Coupon	Conventional Spread (bps)	Upfront (pts)	1 Day Change
1st Data Corp No Restructuring (ISDA 2014)	FDC	500	345.22	-6.8782	+7.72 (+2.29%) ↑ bps
21st Century Fox America Inc No Restructuring (ISDA 2014)	STCENT	100	58.16	-2.0991	+0.34 (+0.59%) ↑ bps
AB Electrolux EUR, Modified Modified Restructuring (ISDA 2014)	ELTLX	25	61.86	1.8958	+1.32 (+2.18%) ↑ bps
AB Electrolux EUR, Modified Modified Restructuring (ISDA 2014)	ELTLX	100	60.70	-2.0223	+1.31 (+2.21%) ↑ bps
AB Volvo EUR, Modified Modified Restructuring (ISDA 2014)	VLVY	25	140.13	5.7233	+9.29 (+7.10%) ↑ bps
AB Volvo EUR, Modified Modified Restructuring (ISDA 2014)	VLVY	100	138.35	1.9082	+9.24 (+7.16%) ↑ bps

Calibration to Data: Constant Intensity

- The simplest way to calibrate a single CDS quote is to assume a constant default intensity.
- Then for a given loss rate, $\ell = 0.5$ say, we can use the relation

$$S_t = \ell\lambda.$$

- For instance, if the CDS spread is 100bp, then

$$\lambda = \frac{S_t}{\ell} = 2\% = 0.02$$

- If for a given entity there are two or more spread quotes (e.g. 5y and 10y spread) at a particular point in time, then a perfect fit is not possible any more.

Calibration to Data: Deterministic Intensity

- If at time t you observe spread quotes for different maturities, then a perfect fit can still be achieved.
- The idea is to assume the intensity to be **piecewise constant**.
- For a given company assume that the 5y CDS spread is 100bp and the 10y CDS spread is 110bp.
- Assume that the intensity is given by

$$\lambda_t = \begin{cases} \lambda_5, & t \in [0,5], \\ \lambda_{10}, & t \in (5,10], \end{cases}$$

- As above the intensity for the first five years is

$$\lambda_5 = \frac{S_0(5)}{\ell} = 2\% = 0.02$$

Calibration to Data: Piecewise Constant Intensity

- The **fee leg** for the 10y CDS is given by (continuous)

$$V_0^{fee} = \int_0^5 p(0,s)e^{-\lambda_5 s}\,ds + \int_5^{10} p(0,s)e^{-\lambda_5 5-\lambda_{10}(s-5)}\,ds$$

- The **protection leg** is given by

$$V_0^{prot} = \ell\lambda_5 \int_0^5 p(0,s)e^{-\lambda_5 s}\,ds + \ell\lambda_{10} \int_5^{10} p(0,s)e^{-\lambda_5 5-\lambda_{10}(s-5)}\,ds$$

- Therefore, one can write the 10y spread as

$$S_0(10) = \ell(w_1\lambda_5 + w_2\lambda_{10}),$$

where w_1 and w_2 are weights with $w_1 + w_2 = 1$.

Calibration to Data: Piecewise Constant Intensity

The weights are given by

$$\begin{aligned} w_1 &= \frac{\int_0^5 p(0,s)e^{-\lambda_5 s}\,ds}{V_0^{fee}}, \\ w_2 &= 1 - w_1 \\ &= \frac{e^{(\lambda_{10}-\lambda_5)5}\int_5^{10} p(0,s)e^{-\lambda_{10} s}\,ds}{V_0^{fee}} \end{aligned}$$

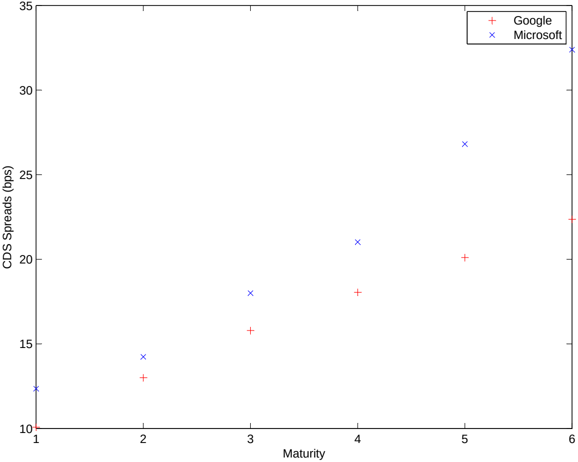
where

$$V_0^{fee} = \int_0^5 p(0,s)e^{-\lambda_5 s}\,ds + e^{(\lambda_{10}-\lambda_5)5}\int_5^{10} p(0,s)e^{-\lambda_{10} s}\,ds$$

Calibration to Data: Piecewise Constant Intensity

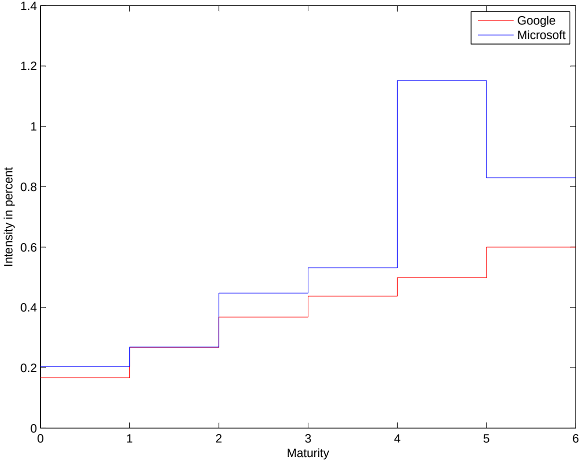
- One has to be careful since the weights w_1 and w_2 are λ_{10} dependent.
- Assume that interest rates are constant, $r = 0.02$.
- Then we obtain in our example $\lambda_{10} = 0.02447$.
- **Warning:** Piecewise calibration is not always possible (e.g. too steep CDS curve).
- Way out: Reformulation as a minimization problem.
- In any case, the results can be used to
  - construct an implied term structure of defaultable zero-coupon bonds,
  - mark-to-market CDS positions.

CDS Term Structure: Google and Microsoft



Term structure of CDS spreads (03/15/10)

Calibration: Google and Microsoft



Calibration of piecewise constant intensity (03/15/10)

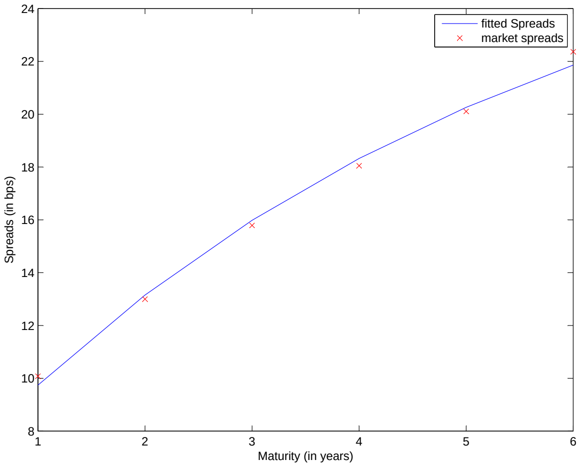
Calibration to Data: Stochastic Intensity

- Instead a piecewise constant model, one can use a stochastic intensity model such as

$$d\lambda_t = \kappa(\theta - \lambda_t)dt + \sigma\sqrt{\lambda_t}dW_t.$$

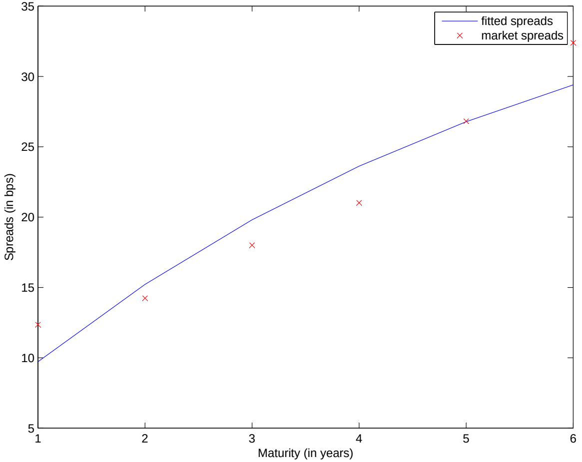
- One has to calibrate four parameters: λ_0, κ, θ, and σ.
- This means that one needs at least four observations (e.g. 1y, 3y, 5y, 7y spread).
- If one has more observations, then a perfect fit is usually not feasible.
- Way out: Minimization of the pricing error.

Calibration: Google



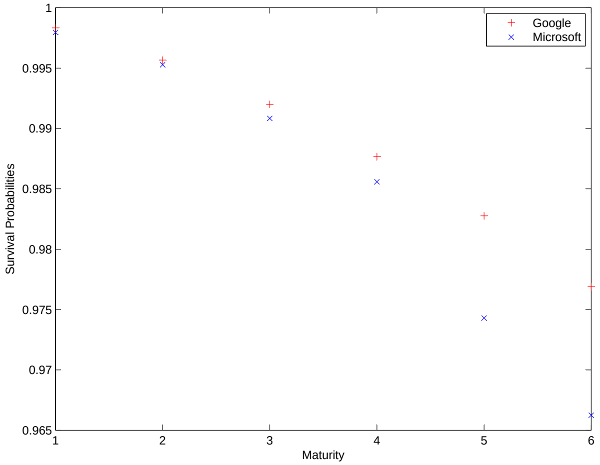
Calibration of CIR intensity (03/15/10):
$\lambda_0 = 0.0009$, $\kappa = 0.1964$, $\theta = 0.0084$, $\sigma = 0.2005$.

Calibration: Microsoft



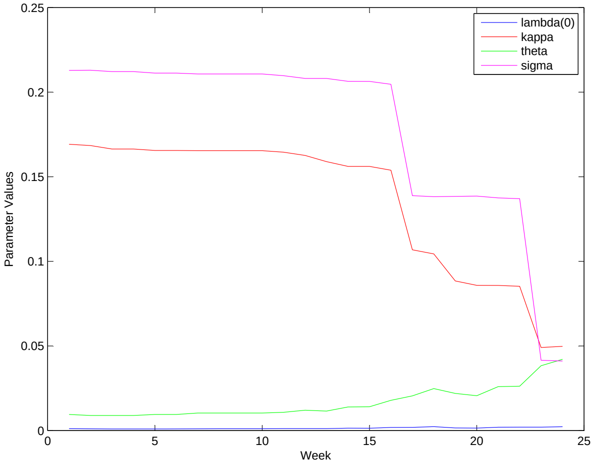
Calibration of CIR intensity (03/15/10):
$\lambda_0 = 0.0005$, $\kappa = 0.1988$, $\theta = 0.0123$, $\sigma = 0.1998$.

Implied Survival Probabilities: Google and Microsoft



Calibrated risk-neutral survival probabilities for a CIR model (03/15/10)

Calibrated CIR Model: Google



Calibrated parameters of CIR model over time (weekly, 02/01/10-07/12/10). Matlab did a bad job at the end!

Agenda: Single-Name Credit

Forward CDS

- So far we have only discussed **spot CDS** contracts where protection starts at initiation of the contract.
- On the contrary, a **forward CDS** is issued today (time t), but the protection starts at a future date $U > t$.
- Therefore, the fee payments also start at time U.
- If default occurs before time U, then the forward CDS is knocked out.
- We denote the maturities of spot or forward contracts by T.

Notation

- A forward CDS has a fee leg and a protection leg.
- Three points in time are relevant ($t \leq U < T$):
    1. t: Initiation of the contract
    2. U: Start protection and fee payments
    3. T: End protection and fee payments
- $V_t^{fee}(U, T)$ denotes the value of the fee leg.
- $V_t^{prot}(U, T)$ denotes the value of the protection leg.
- $S_t(U, T)$ denotes the forward CDS spread.

Relation Spot and Forward CDS

$$V_t^{fee}(T) = V_t^{fee}(t, T), V_t^{prot}(T) = V_t^{prot}(t, T), S_t(T) = S_t(t, T).$$

Forward CDS: Fee Leg

If there is a liquid market for spot CDS contracts, we can create forward CDS contracts via buy-and-hold strategies.

Forward Fee Leg (No Default, 1bp)

Time	0.5	1	...	$U-0.5$	U	$U+0.5$	...	T
$-V_t^{fee}(U)$	$-\delta$	$-\delta$	...	$-\delta$	$-\delta$	0	...	0
$V_t^{fee}(T)$	δ	δ	...	δ	δ	δ	...	δ
Net position	0	0	...	0	0	δ	...	δ

If a default occurs before T, then the net position is zero from that point on.

Value of Forward Fee Leg

The value of the forward fee leg reads

$$S_t(U,T)V_t^{fee}(U,T) = S_t(U,T)\big(V_t^{fee}(T) - V_t^{fee}(U)\big).$$

Forward CDS: Protection Leg

Consider long/short positions in spot CDS with maturities T, U.

If no default occurs until T, then there are no protection payments.

Protection Leg (Default before U)

Time	τ	...	U	$U+0.5$	...	T
$-V_t^{prot}(U)$	$-\ell$	0	0	0	0	0
$V_t^{prot}(T)$	ℓ	0	0	0	0	0
Net position	0	0	0	0	0	0

Protection Leg (Default between U and T)

Time	...	U	τ	...	T
$-V_t^{prot}(U)$	0	0	0	0	0
$V_t^{prot}(T)$	0	0	ℓ	0	0
Net position	0	0	ℓ	0	0

This is the protection leg of a forward CDS!

Forward CDS: Protection Leg

We have just shown the following.

Value of Forward Protection Leg

The value of the forward protection leg reads

$$V_t^{prot}(U,T) = V_t^{prot}(T) - V_t^{prot}(U).$$

The spot spreads $S_t(U)$, $S_t(T)$ are chosen such that the fee and protection legs have the same value, i.e.

$$\begin{aligned} V_t^{prot}(U) &= S_t(U)V_t^{fee}(U), \\ V_t^{prot}(T) &= S_t(T)V_t^{fee}(T). \end{aligned}$$

The same is true for the forward spread $S_t(U,T)$, i.e.

$$V_t^{prot}(U,T) = S_t(U,T)V_t^{fee}(U,T).$$

Forward CDS

From the last slide

$$V_t^{prot}(U, T) = S_t(U, T)V_t^{fee}(U, T).$$

For the left-hand side we get:

$$V_t^{prot}(U, T) = V_t^{prot}(T) - V_t^{prot}(U) = S_t(T)V_t^{fee}(T) - S_t(U)V_t^{fee}(U).$$

For the right-hand side we get:

$$S_t(U, T)V_t^{fee}(U, T) = S_t(U, T)\big(V_t^{fee}(T) - V_t^{fee}(U)\big).$$

We can thus solve for the $S_t(U, T)$.

Forwards CDS Spread

The fair forward spread is given by

$$S_t(U, T) = \frac{S_t(T)V_t^{fee}(T) - S_t(U)V_t^{fee}(U)}{V_t^{fee}(T) - V_t^{fee}(U)}.$$

Forward CDS: Remarks

$$S_t(U,T) = \frac{S_t(T)V_t^{fee}(T) - S_t(U)V_t^{fee}(U)}{V_t^{fee}(T) - V_t^{fee}(U)}.$$

- Notice that the forward spread can be represented without referring to the protection legs.
- This is reasonable since these legs do not involve any spreads.
- Applying the above formula, we can back out implied forward spreads if spot spreads are available.

Forward CDS Valuation

- Initially, the value of a forward CDS is zero, i.e.

$$V_0^{prot}(U,T) - S_0(U,T)V_0^{fee}(U,T) = 0$$

- Over time market conditions change, i.e. in general

$$S_U(T) \neq S_0(U,T).$$

- Since $V_U^{prot}(T) = S_U(T)V_U^{fee}(T)$, the time-$U$ value of the forward CDS long position initiated at time 0 reads

$$\begin{aligned}
&V_U^{prot}(U,T) - S_0(U,T)V_U^{fee}(U,T) \\
&= V_U^{prot}(T) - S_0(U,T)V_U^{fee}(T) \\
&= V_U^{fee}(T)\big(S_U(T) - S_0(U,T)\big)
\end{aligned}$$

CDS Option

- A **long position** in a **forward CDS contract** is the obligation to enter into a spot CDS contract as the protection buyer at time U.
- Its value at maturity U of the forward is (see last slide)

$$V_U^{fee}(T)\big(S_U(T) - S_0(U,T)\big).$$

- A CDS option (swaption) gives the holder the right to enter into a spot CDS contract at time U.
- The payoff of a **payer swaption** at U equals

$$V_U^{fee}(T)\big[S_U(T) - K\big]^+,$$

  where K denotes the strike spread.
- The word "payer" refers to the fact that the holder can enter into a CDS contract as the **protection buyer** who **makes the fee payments**.
- Knock-out provision: Contract is canceled if default before U.

CDS Option

- A **short position** in a **forward CDS contract** is the obligation to enter into a spot CDS contract as the protection seller at time U.
- Its value at maturity U of the forward is

$$V_U^{fee}(T)\big(S_0(U,T) - S_U(T)\big).$$

- Again this value can be positive or negative.
- The payoff of a **receiver swaption** at U equals

$$V_U^{fee}(T)\big[K - S_U(T)\big]^+,$$

  where K denotes the strike spread.
- The word "receiver" refers to the fact that the holder can enter into a CDS contract as the **protection seller** who **receives the fee payments**.

Market Model for CDS Options

- In LIBOR-models, it is assumed that forward LIBOR rates are assumed to have the dynamics

$$dL_t(U,T) = L_t(U,T)\mathrm{vol}_t(U,T)dW_t,$$

where $\mathrm{vol}_t(U,T)$ is the volatility structure.

- Consequently, one gets Black-type formulas for caplets if the volatility structure is assumed to be constant.
- To price CDS options, we can also assume a **Brownian setting**,

$$dS_t(U,T) = S_t(U,T)\mathrm{vol}_t(U,T)dW_t,$$

where $\mathrm{vol}_t(U,T)$ is the volatility structure of the CDS options.

- In general, the volatilities are stochastic.

Pricing CDS Options: Deterministic Volatility

Black Formula

Under the assumption that the volatility structure $v = \mathrm{vol}_t(U, T)$ is constant, the time-t pre-default value of a payer swaption with strike K is

$$V_t^{fee}(U, T)\Big(S_t(U, T)\mathcal{N}(d_1) - K\mathcal{N}(d_2)\Big),$$

where

$$d_{1/2} = \frac{\ln(S_t(U, T)/K) \pm 0.5v^2(U-t)}{v\sqrt{U-t}}.$$

If the volatility structure is assumed to be stochastic, then one obtains Heston-like formulas.

Pricing CDS Options

- The arguments that we used are very similar to the ones used to price Caps/Floors or Swaptions.
- **Warning**: Specifying a volatility structure $\mathrm{vol}_t(U, T)$ is not necessarily consistent with intensity models discussed above.
- However, this problem also occurs in fixed-income models (LIBOR-market models vs. Swap-market models).

Constant Maturity CDS

- The spread $S_0(T)$ of a CDS contract is fixed at time zero.
- Over time the spread might vary a lot such that $S_t(T) \neq S_0(T)$.
- Therefore, a CDS contract has to mark-to-market every day and it can get a non-negligible value over time.
- Is there a way to reduce the volatility of the value?
- If the spread would be floating, then the volatility goes down.

Constant Maturity CDS

In a constant maturity CDS, the relevant spread is the spread of a CDS contract with a fixed maturity.

CDS vs. “Floating-rate CDS” vs. Constant Maturity CDS

No Default until Maturity

Time	0	0.5	1	...	$T-0.5$	T
CDS	0	$\delta S_0(T)$	$\delta S_0(T)$	...	$\delta S_0(T)$	$\delta S_0(T)$
Fee Leg FRCDS	0	$\delta S_0(0.5)$	$\delta S_{0.5}(1)$	...	$\delta S_{T-1}(T-0.5)$	$\delta S_{T-0.5}(T)$
Fee Leg CMCDS	0	$\delta S_0(c)$	$\delta S_{0.5}(0.5+c)$	...	$\delta S_{T-1}(T-1+c)$	$\delta S_{T-0.5}(T-0.5+c)$
Protection Leg	0	0	0	...	0	0

Default before Maturity

Time	0	0.5	1	...	t_{j-1}	τ
CDS	0	$\delta S_0(T)$	$\delta S_0(T)$	...	$\delta S_0(T)$	$\delta_d^j S_0(T)$
Fee Leg FRCDS	0	$\delta S_0(0.5)$	$\delta S_{0.5}(1)$	...	$\delta S_{t_{j-2}}(t_{j-1})$	$\delta_d^j S_{t_{j-1}}(t_j)$
Fee Leg CMCDS	0	$\delta S_0(c)$	$\delta S_{0.5}(0.5+c)$	...	$\delta S_{t_{j-2}}(t_{j-2}+c)$	$\delta_d^j S_{t_{j-1}}(t_{j-1}+c)$
Protection Leg	0	0	0	...	0	ℓ

- A floating-rate CDS (FRCDS) would have a value of zero at the payments dates, but such a contract is not traded.
- In contrast, the value of a constant maturity CDS (CMCDS) is in general non-zero at payment dates.
- We need a model to price such a CMCDS.

Constant Maturity CDS: Valuation of the Legs

The protection leg is identical to an ordinary CDS, i.e.

$$V_t^{prot}(T) = \ell \, \mathrm{E}_t^Q \left[\frac{\mathbf{1}_{\{t_0 < \tau \leq T\}}}{B(t,\tau)} \right].$$

As shown above, the fee leg differs. We have to include the (stochastic) spread:

$$\begin{aligned} \hat{V}_t^{fee}(T) &= \sum_{j=1}^{n} \Bigg(\delta \mathrm{E}_t^Q \left[\frac{\mathbf{1}_{\{\tau > t_j\}} S_{t_{j-1}}(t_{j-1}+c)}{B(t,t_j)} \right] \\ &\quad + \mathrm{E}_t^Q \left[\frac{\delta_d^j \, \mathbf{1}_{\{t_{j-1} \leq \tau \leq t_j\}} S_{t_{j-1}}(t_{j-1}+c)}{B(t,\tau)} \right] \Bigg) \end{aligned}$$

Participation Rate

The participation rate $P_t(T)$ is chosen such that the initial value of the CMCDS is zero, i.e. $V_t^{prot}(T) = P_t(T)\hat{V}_t^{fee}(T)$.