Credit Markets

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Agenda: Single-Name Credit

Reduced-form vs. Firm Value Models

- The models considered so far are so-called **reduced-form models**.
- **Default** happens as a **sudden surprise** ("jump of a Poisson process").
- These models are well-suited to calibrate CDS prices etc.
- Certain **information** is however **disregarded** (stock and option prices).
- For instance, a default is sometimes preceded by a period of a declining stock price.
- **Firm value models** (syn. structural models) explicitly model the **link between equity and debt**.

Firm Value Model

Q: What is the **economic intuition** behind a default?

Firm value models shed some light on the mechanism driving a default.

We consider a firm with a simple **capital structure**:

- Debt: Zero with face value F and maturity T
- Equity: Stock
- Firm Value $V =$ Value Equity $+$ Value Debt

Firm Value Model

Upon maturity T of the zero, two scenarios can occur:

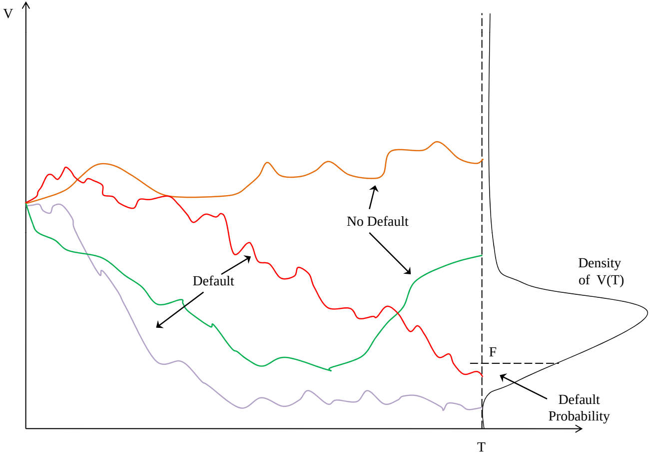
	Stock Holders	Bond Holders	
$V(T) \geq F$	$V(T) - F$	F	Zero redeemed
$V(T) < F$	0	$V(T)$	Zero defaults

Equity and Debt in a Firm Value Model

$$\textbf{Equity} \quad : \quad \underbrace{\max\{V(T) - F; 0\}}_{\text{“Call on firm value”}}$$

$$\textbf{Debt} \quad : \quad F - \underbrace{\max\{F - V(T); 0\}}_{\text{“Put on firm value”}}$$

Firm Value Model: Merton's Model



Firm Value Model: Lognormal Firm Value

In a Black-Scholes world, one can easily calculate the value of equity and debt since the risk-neutral firm value dynamics read

$$dV_t = V_t[rdt + \sigma dW_t].$$

Merton (1974)

$$\begin{aligned} E_t &= V_t N(d_1) - Fe^{-r(T-t)} N(d_2) \\ D_t &= V_t N(-d_1) + Fe^{-r(T-t)} N(d_2) \end{aligned}$$

where

$$\begin{aligned} d_1 &= \frac{\ln\left(\frac{V_t}{F}\right) + (r + 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}}, \\ d_2 &= d_1 - \sigma\sqrt{T-t}. \end{aligned}$$

Default Probability in Merton's Model

Q: When do we have a default in this model?
A:

- Default can only happen at maturity (no premature default).
- Default if firm value smaller than bond's face value.

Q: What is the probability that $V(T) < F$?

Default Probability

$$\mathrm{PD}^{Q} = Q(V(T) < F) = 1 - N(d_2) = N(-d_2).$$

KMV calls d_2 the distance to default.

Default Probability in Merton's Model

Unfortunately, d_2 depends on the **firm value** and the **volatility** of the firm value that **cannot be observed** directly! We thus assume that the **stocks are traded** securities.

$\Longrightarrow$ E and σ_E are observable

$\Longrightarrow$ V and σ can be calculated from

$$\begin{aligned} E &= VN(d_1) - Fe^{-rT}N(d_2) \\ \sigma_E &= \frac{\partial E}{\partial V}\frac{V}{E}\sigma = N(d_1)\frac{V}{E}\sigma \end{aligned}$$

Two equations with two unknowns!

Default and Recovery in Merton's Model

- Formally, the default stopping time is given by

$$\tau_M = \begin{cases} T & \text{if} \quad V_T < F, \\ \infty & \text{if} \quad V_T \geq F. \end{cases}$$

- If a default occurs ($V_T < F$), then the bondholders receive V_T.
- Therefore, the loss rate equals

$$\ell = \frac{F - V_T}{F}.$$

- Although Merton's model is simple, the recovery payment is thus **stochastic**!

Firm Value Models: Some Extensions

- Premature default (first-passage models): Black and Cox (1976)
- More complex capital structure (senior and junior debt): Black and Cox (1976)
- Stochastic interest rates: Briys and de Varenne (1997)
- Jump-diffusion model: Zhou (2001)
- Hybrid model: Longstaff and Schwartz (1995)
- Endogenous default: Leland (1994)
- Incomplete information: Duffie and Lando (2001)

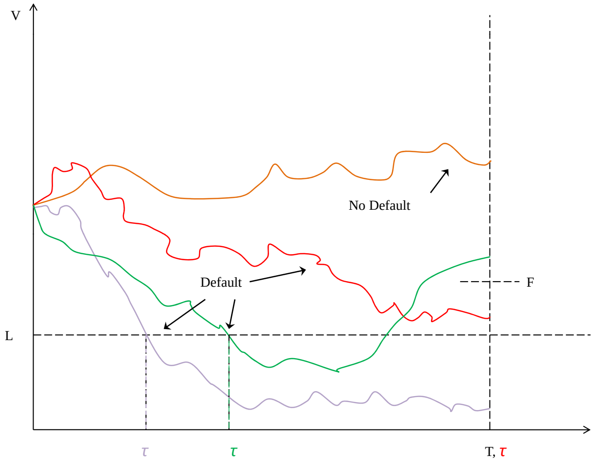
First-Passage Models

- Black and Cox (1976) consider a firm with the same capital structure as in Merton (1974).
- However, the zero bonds have **safety covenants**.
- This contractual provision gives the bondholders the right to trigger default when the firm value reaches a **lower boundary**

$$L_t = k \cdot e^{-\kappa(T-t)}$$

  with $k, \kappa > 0$ and $k \leq F$.
- If **default** occurs during the life time of the bond, the bondholders immediately take over the firm.
- **Otherwise**, the terminal value of the bond is identical to the value in **Merton's model**.

First-Passage Models



First-Passage Models

- There are two reasons for default
  1. Premature default: Firm value falls below barrier
  2. Default at maturity: $V_T < F$
- Therefore, the **default stopping time** is given by

$$\tau = \min\{\tau_H, \tau_M\},$$

  where τ_M is the default stopping time in Merton's model and

$$\tau_H \equiv \inf\{t \geq 0 : V_t \leq L_t\}$$

  is the first-hitting time of the firm value.
- τ_H models premature default, whereas τ_M default at maturity.

First-Passage Models: Debt

At maturity, the payoff of the zero-coupon bond is given by

$$D_T = p^d(T,T) = \Big(F - \max\{F - V_T; 0\}\Big)\mathbf{1}_{\{\tau_H > T\}} + V_T \mathbf{1}_{\{\tau_H \leq T\}}$$

This can be rewritten as

$$\begin{aligned} p^d(T,T) &= \Big(F - \max\{F - V_T; 0\}\Big)\mathbf{1}_{\{\tau_H > T\}} + V_T \mathbf{1}_{\{\tau_H \leq T\}} \\ &= F - \max\{F - V_T; 0\} \\ &\quad + \Big(-F + \max\{F - V_T; 0\} + V_T\Big)\mathbf{1}_{\{\tau_H \leq T\}} \\ &= \underbrace{F - \max\{F - V_T; 0\}}_{\text{Merton}} + \underbrace{\max\{V_T - F; 0\}\mathbf{1}_{\{\tau_H \leq T\}}}_{\text{Covenant}} \end{aligned}$$

- Due to the safety covenant, the bond price is **higher than in Merton's model**.
- Formally, the covenant is a down-and-in call.

First-Passage Models: Equity

The value of equity at maturity is equal to

- zero if a premature default has occurred,
- a call on firm value if no premature default has occurred.

Therefore, the equity value at maturity T is

$$E_T = \max\{V_T - F; 0\}\mathbf{1}_{\{\tau_H > T\}},$$

which is a down-and-out call on firm value. This also follows from

$$\begin{aligned} E_T &= V_T - D_T \\ &= V_T - \Big(F - \max\{F - V_T; 0\}\Big)\mathbf{1}_{\{\tau_H > T\}} - V_T\mathbf{1}_{\{\tau_H \leq T\}} \\ &= \Big(V_T - F + \max\{F - V_T; 0\}\Big)\mathbf{1}_{\{\tau_H > T\}} \\ &= \max\{V_T - F; 0\}\mathbf{1}_{\{\tau_H > T\}} \end{aligned}$$

Due to the safety covenant, the equity value is **lower** than in Merton's model.

First-Passage Models: Lognormal Model

- For simplicity, assume a constant default barrier, i.e. $L_t = k$.
- The challenge is to calculate

$$\begin{aligned} E_0 &= \mathrm{E}^Q\Big[\max\{V_T - F; 0\}\mathbf{1}_{\{\tau_H > T\}}\Big]e^{-rT} \\ &= \mathrm{E}^Q\Big[\max\{V_T - F; 0\}\mathbf{1}_{\{m_T^V > k\}}\Big]e^{-rT}, \end{aligned}$$

  where $m_t^V = \min_{s \le t} V_s$ is the running minimum of the firm value.
- If V is a geometric Brownian motion, then there is an explicit formula for the joint density of (V_T, m_T^V).
- Therefore, there is an explicit formula for the equity value.

First-Passage Models: Lognormal Model

One can show the following ($\longrightarrow$ reflection principle).

Default Probability

In the Black-Cox model the default probability reads

$$
\begin{aligned}
&Q_t(\tau \leq T) \\
&\quad = \mathcal{N}\left(\frac{\ln(F/V_t) - (r - 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}}\right) \\
&\quad + \left(\frac{L_t}{V_t}\right)^{\frac{2(r-\kappa)}{\sigma^2}-1} \mathcal{N}\left(\frac{\ln(L_t^2/(FV_t)) + (r - 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}}\right)
\end{aligned}
$$

with default barrier $L_t = ke^{-\kappa(T-t)}$.

See, e.g., Harrison (1985).

First-Passage Models: Lognormal Model

Debt Value

$$D_t = \underbrace{Fe^{-r(T-t)} - \left[Fe^{-r(T-t)}\mathcal{N}(-z_1) - V_t\mathcal{N}(-z_3)\right]}_{\text{Merton}}$$
$$+ \underbrace{y_t^{2\theta-2}\left[V_t y_t^2 \mathcal{N}(z_4) - Fe^{-r(T-t)}\mathcal{N}(z_2)\right]}_{\text{Covenant}},$$

where

$$z_{1/3} = \frac{\ln(\frac{V_t}{F}) + (r \mp 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}},$$
$$z_{2/4} = \frac{\ln(\frac{V_t}{F}) + 2\ln(y_t) + (r \mp 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}},$$

$y_t = ke^{-\kappa(T-t)}/V_t$, and $\theta = (r - \kappa + 0.5\sigma^2)/\sigma^2$.

First-Passage Models: Lognormal Model

- The value of **equity** is **not necessarily increasing in volatility**.
- This is due to the default barrier.
- Large volatility increases the probability to default if the firm value is close to the barrier.
- This property **complicates** the **calibration** of the model.

Firm Value Models: Calibration to Time Series

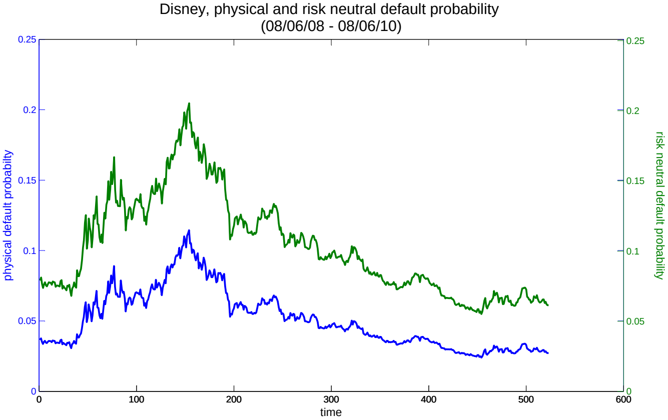
- The equity price can be expressed as a function of six input parameters

$$E_t = f(V_t, F, T, r, \sigma, L).$$

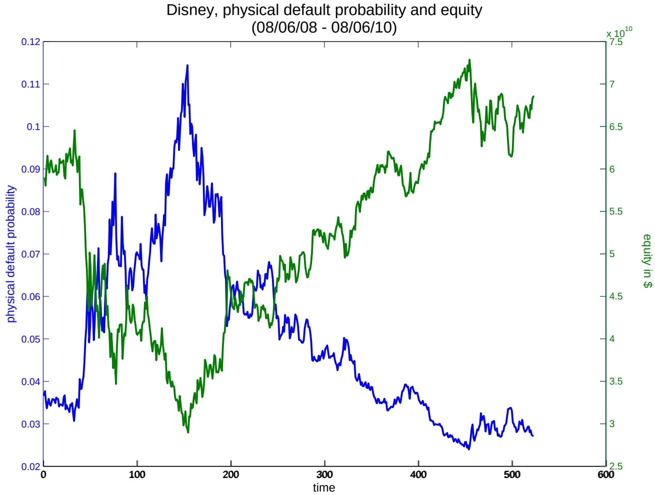
- r can be estimated from Treasury yields.
- Let STD denote the value of short term debt (less than 1y) and LTD denote the value of long term debt.
- Then a rule of thumb is to choose

$$\begin{aligned} F &= STD + 0.5LTD, \\ T &= \frac{0.5STD + 15LTD}{STD + LTD}, \\ L &= F = const. \end{aligned}$$

Merton Model



Calibrated parameters: $\mu = 0.0957$, $\sigma = 0.3449$. Default probabilities over 8.5y horizon.



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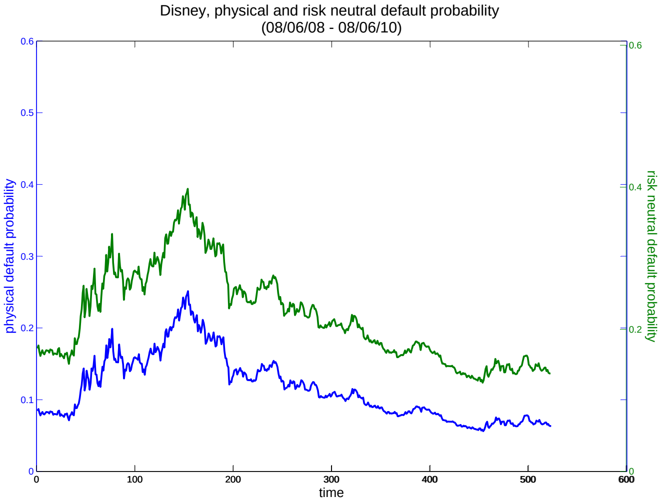
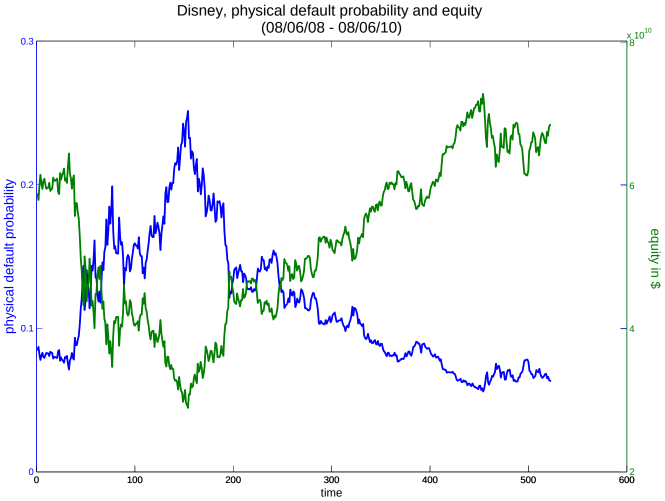
First Passage Model



Calibrated parameters: $\mu = 0.1085$, $\sigma = 0.3573$. Default probabilities over 8.5y horizon

First Passage Model



Calibrated parameters: $\mu = 0.1085$, $\sigma = 0.3573$. Default probabilities over 8.5y horizon.