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常用导函数证明.

1) $(x^{\alpha})' = \alpha \cdot x^{\alpha-1}$

2) $(\sin x)' = \cos x$

3) $(a^x)' = a^x \ln a$

证: 只能利用已有的导数的定义去求证

$$f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$$

1) $$(x^{\alpha})' = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^{\alpha} - (x)^{\alpha}}{\Delta x}$$

注意此种提取公因式的方法

提取公因式 $$= \lim_{\Delta x \to 0} \frac{x^{\alpha}\left(\left(1+\frac{\Delta x}{x}\right)^{\alpha} - 1\right)}{\Delta x}$$

$(1+\Delta)^{\alpha} - 1 \sim \alpha\Delta$ $$\lim_{\Delta x \to 0} \frac{x^{\alpha} \cdot \alpha\left(\frac{\Delta x}{x}\right)}{\Delta x}$$

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$$= \alpha \cdot x^{\alpha-1}$$

2) $(\sin x)' = \lim\limits_{\Delta x \to 0} \dfrac{\sin(x+\Delta x) - \sin x}{\Delta x}$

$\xlongequal{\text{和差化积}} \lim\limits_{\Delta x \to 0} \dfrac{2\cos\left(\frac{x+\Delta x+x}{2}\right)\sin\left(\frac{x+\Delta x-x}{2}\right)}{\Delta x}$

$= \lim\limits_{\Delta x \to 0} \dfrac{2\cos(x+\frac{\Delta x}{2}) \cdot \sin\frac{\Delta x}{2}}{\Delta x}$

$= \lim\limits_{\Delta x \to 0} \dfrac{\cos(x+\frac{\Delta x}{2}) \cdot \sin\frac{\Delta x}{2}}{\Delta x/2}$

$\xlongequal{\frac{\sin\Delta}{\Delta}\text{重要极限}} \lim\limits_{\Delta x \to 0} \cos(x+\frac{\Delta x}{2}) \cdot \lim\limits_{\Delta x \to 0} \dfrac{\sin\frac{\Delta x}{2}}{\Delta x/2}$

$= \cos(x+0) \cdot 1 = \cos x$

3) $(a^x)' = \lim\limits_{\Delta x \to 0} \dfrac{a^{x+\Delta x} - a^x}{\Delta x}$

$= \lim\limits_{\Delta x \to 0} \dfrac{a^x(a^{\Delta x} - 1)}{\Delta x}$

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等价无穷小

$a^{\Delta}-1=\Delta\ln a$ $\lim_{\Delta x\to 0}\frac{a^{x}(\Delta x\ln a)}{\Delta x}$

由X是相对固定的 $a^{x}\ln a.$

是求x的导卷