

A general form for ODE predator prey modelling in \mathbb{R}^N (i.e. N species) is given by the autonomous system:

$$\dot{\mathbf{x}} = F(\mathbf{x}), \quad (1)$$

where $\mathbf{x}, \dot{\mathbf{x}} \in \mathbb{R}^N$ and $F : \mathbb{R}^N \rightarrow \mathbb{R}^N$ such that:

$$F(\mathbf{x}) = \begin{pmatrix} f_1(\mathbf{x}) \\ f_2(\mathbf{x}) \\ \vdots \\ f_N(\mathbf{x}) \end{pmatrix} \quad (2)$$

We may write the elements of F in terms of an intrinsic growth term and an interaction term:

$$f_i(\mathbf{x}) = g_i(x_i) + \sum_{j=1}^N a_{ij} (x_j h_j(x_i) + x_i h_i(x_j)), \quad (3)$$

where g_i is the intrinsic growth function; $a_{ij} \in \mathbb{R}$ is the interaction coefficient; and $h_k(x_l)$ is the functional response of species k when it predaes on species l . The terms g_i, a_{ij} and $h_k(x_l)$ take the following conditional values:

$$g_i(x_i) = \begin{cases} r_i x_i \left(1 - \frac{x_i}{K_i}\right) & \text{if } i \text{ is basal} \\ -r_i x_i & \text{if } i \text{ is non-basal} \end{cases} \quad (4)$$

and

$$a_{ij} = \begin{cases} > 0 & \text{if } i \text{ eats } j \\ < 0 & \text{if } j \text{ eats } i \\ = 0 & \text{if no interaction} \end{cases} \quad (5)$$

and

$$h_k x_l = \begin{cases} 0 & \text{if } k \text{ is basal} \\ x_l & \text{if } k \text{ is type I predator} \\ \frac{x_l}{x_l + S_k} & \text{if } k \text{ is type II predator} \end{cases}, \quad (6)$$

where $r_i \in \mathbb{R}^+$ is the intrinsic growth rate; K_i is the carrying capacity for species i ; and S_k is the predator saturation constant of species k .

I think that this is general enough? From this I can derive the rescaled equations for the 2,3,4 species systems that I simulated.