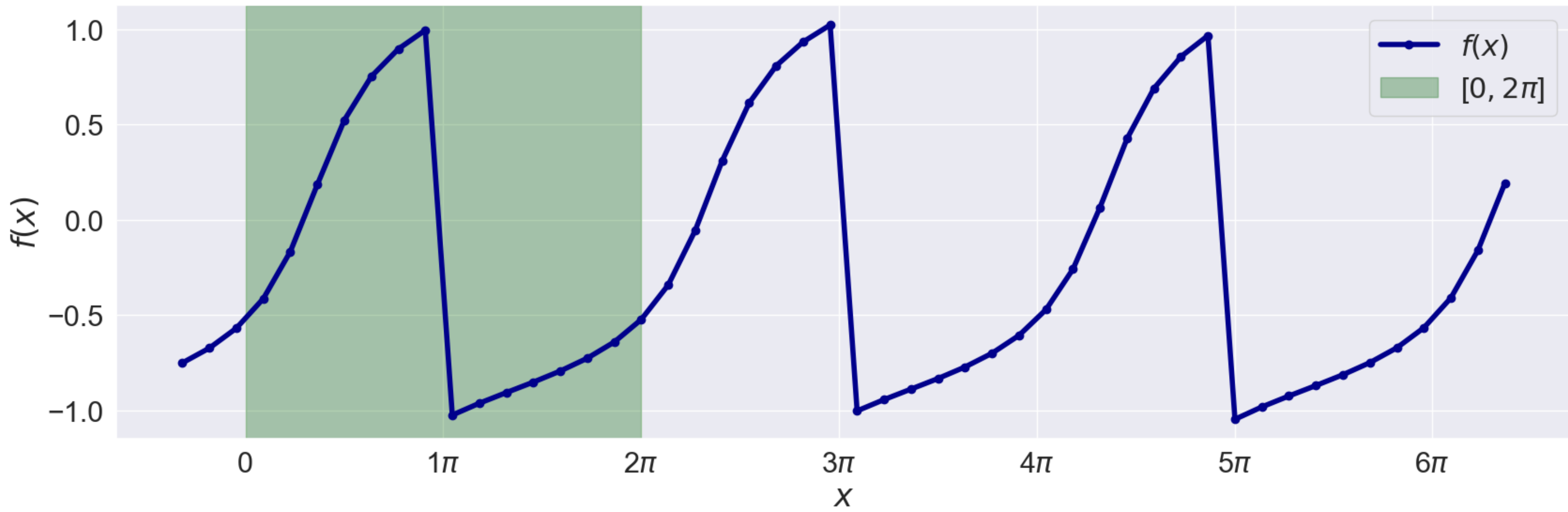
FTC SGA, by Magomedov Rustam

To solve the problem, let us assess the $f(x)$. Let us first draw the functions' plot. Given the $\arctan(\tan(x))$ function, we expect the function to be discontinuous

In [2]:
```
# importing libraries
import matplotlib.pyplot as plt
from matplotlib.ticker import FuncFormatter, MultipleLocator
import seaborn as sns
import numpy as np

# settings for plots
sns.set()
sns.set_context("notebook", font_scale=2.0, rc={"lines.linewidth": 4.0})
sns.set_palette('cubehelix')

%matplotlib inline

# function obtained by differentiation
x = np.linspace(-1, 20)
y = (-2/np.sqrt(3)) * (np.arctan(1-2*np.tan(x/2))/np.sqrt(3))

# init plot
fig,ax = plt.subplots()
fig.set_size_inches(20, 6)

# plot function
ax.plot(x, y, 'o-', label='$f(x)$', color='darkblue')

# labels, integrated interval
ax.set_xlabel('$x$')
ax.set_ylabel('$f(x)$')
ax.axvspan(0, 2*np.pi, color='darkgreen', alpha=.3, label='[$0, 2\pi$]')

# formatting to display pi ticks on x axis
ax.xaxis.set_major_formatter(FuncFormatter(
   lambda val,pos: '{:.0g}$\pi$'.format(val/np.pi) if val !=0 else '0'
))
ax.xaxis.set_major_locator(MultipleLocator(base=np.pi))

# legend
ax.legend()
plt.show()
```



Let us interpret the graph. Since we need to integrate the function on the interval $[0, 2\pi]$, let us check the textbook condition that must be hold for Fundamental Theorem of Calculus to be applied. It can be inferred from our lectures and then confirmed in our textbooks by *Stewart (2016, p.398)*, and *Friedman (1970, p.130)*, that `to apply FTC, it must hold true that` $f(x)$ `is continuous on` $[a, b]$. In our case, we are dealing with a discontinuity point of the 2-nd kind, since we have a function of type $\arctan(\tan(x))$. The stated function will only be the antiderivative in the interval $[0 : \pi)$ or $(\pi : 2\pi]$, excluding the point $x = \pi$. Put simply, it can be concluded from the graphical representation of $f(x)$ above, that the function is indeed periodic, but has a discontinuity on the interval $[0, 2\pi]$. Hence, we can classify this as the improper integral of the 2-nd kind, and cannot apply FTC to obtain the integral value.

To sum up, the function's continuity condition does not hold.
$\therefore$ it is impossible to obtain the integral value using FTC.