Q1

Let $\vec{a} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \vec{b} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}, \vec{p} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}, \vec{q} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

$R_2 \mapsto R_2 \therefore$ the matrix of the linear operator f is $M = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 2 & -1 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix}$

We consider it a matrix in a canonical basis C. To find the basis AQ transformed by the liear operator f given by vectors $\{\overrightarrow{a, q}\}$, let us apply the change of basis equation.

$T_{C \to AQ}{}^{-1} \cdot M_C \cdot T_{C \to AQ} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & -0.4 \end{bmatrix} \cdot \begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -2 & 11 \end{bmatrix}$

Sanity check using *numpy* below

In [1]:
```
import numpy as np

t = np.array([[2,-1], [-1,-2]])
t_inv = np.linalg.inv(t)
m = np.array([[3,4], [6,8]])

print(t_inv @ m @ t)
```

```
[[ 0.  0.]
 [-2. 11.]]
```