Q4

Let us reduce the given matrix to the row reduced echelon form to find its rank.

$$\begin{pmatrix} a & 1 & 2 \\ 5 & 5 & -2 \\ 3 & 3 & 6 \\ 1 & a & 2 \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{a} & \frac{2}{a} \\ 5 & 5 & -2 \\ 3 & 3 & 6 \\ 1 & a & 2 \end{pmatrix}^{I=I\cdot 1/a} = \begin{pmatrix} 1 & \frac{1}{a} & \frac{2}{a} \\ 0 & 5-\frac{5}{a} & -2-\frac{10}{a} \\ 3 & 3 & 6 \\ 1 & a & 2 \end{pmatrix}^{II=II-5\cdot I} =$$

$$\begin{pmatrix} 1 & \frac{1}{a} & \frac{2}{a} \\ 5 & 5 & -2 \\ 3 & 3 & 6 \\ 1 & a & 2 \end{pmatrix}^{I=I\cdot 1/a} = \ldots = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

The code below also confirms the obtained row echelon reduced form.

Since all rows are linearly independent $\implies$ matrix rank = 3 $\implies C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Substituting a for 7, we obtain $B = \begin{pmatrix} 7 & 1 & 2 \\ 5 & 5 & -2 \\ 3 & 3 & 6 \\ 1 & 7 & 2 \end{pmatrix}$

Since matrix C is an identity matrix, we obtain $B * C = B$

$$\therefore B = \begin{pmatrix} 7 & 1 & 2 \\ 5 & 5 & -2 \\ 3 & 3 & 6 \\ 1 & 7 & 2 \end{pmatrix}$$

In [2]:

```
import sympy as sp
import numpy as np

matrix = np.array([[7,1,2], [5,5,-2], [3,3,6], [1,7,2]])
# checking the obtaind RREF

sp.Matrix(matrix).rref()[0]
```

Out[2]:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

In [ ]: