Q2

Let us apply the naive approach first by using the Rouche-Capelli theorem; let us prove the inconsistence by checking the rank of the coefficient matrix and the augmented matrix. When ranks differ, the system is proved to be inconsistent.

Transforming the SLAE into the augmented matrix, we obtain $\begin{pmatrix} 1 & 2 & 3 & 8 \\ 1 & 0 & 3 & -2 \\ 3 & 2 & -2 & 0 \\ 1 & -1 & 2 & 16 \end{pmatrix}$.

Given that there persists linear independence, the rank of the augmented matrix = $rk = 4$

The coefficient matrix transformed to the row reduced echelon form has

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & 3 \\ 3 & 2 & -2 \\ 1 & -1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -2 & 0 \\ 3 & 2 & -2 \\ 1 & -1 & 2 \end{pmatrix}^{II=II-1\cdot I} = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -2 & 0 \\ 0 & -4 & -11 \\ 1 & -1 & 2 \end{pmatrix}^{III=III-3\cdot I} =$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & -2 & 0 \\ 0 & -4 & -11 \\ 0 & -3 & -1 \end{pmatrix}^{IV=IV-I} = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -2 & 0 \\ 0 & -4 & -11 \\ 0 & -3 & -1 \end{pmatrix}^{IV=IV-I} = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & -4 & -11 \\ 0 & -3 & -1 \end{pmatrix}^{II=II-IV}$$

$$= \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & -11 \\ 0 & -3 & -1 \end{pmatrix}^{III=III-4\cdot II} =$$

$$= \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & -11 \\ 0 & 0 & -1 \end{pmatrix}^{IV=IV-3\cdot II} = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{pmatrix}^{III=III-12\cdot IV} = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{pmatrix}^{I=I-2\cdot II}$$

$$= \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{pmatrix}^{I=I-2\cdot III} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}^{IV=IV-(-1\cdot I)}$$

$\implies$ the coefficient matrix has a rank of $rk = 3$.

Matrix rank differs for the augmented and coefficient matrices $\therefore$ the system is inconsistent. We can also prove our claim empirically using *numpy*.

I present the least squares solution below using Python for a more precise answer. The formula for least squares $A^T @ B = A^T @ A @ X$. We obtain that approximate values of x, y, z are:

$x : 2.8357810413885183$
$y : -2.2056074766355147$
$z : 1.704939919893191$

In [1]:

```
# consistency check
import numpy as np
# augmented matrix
aug = np.array([[1,2,3,8], [1,0,3,-2], [3,2,-2,0], [1,-1,2,16]])

# coefficient matrix
coef = np.array([[1,2,3], [1,0,3], [3,2,-2], [1,-1,2]])

# check ranks
if np.linalg.matrix_rank(aug) == np.linalg.matrix_rank(coef):
    print('The system is consistent')
elif np.linalg.matrix_rank(aug) != np.linalg.matrix_rank(coef):
    print('The system is inconsistent!\n')

# find least squares solution of the matrix
y = np.array([[8], [-2], [0], [16]])

x, y, z = np.linalg.lstsq(coef, y, rcond=None)[0]
print(f"Least squares solution:\tx = {x[0]}, y = {y[0]}, z = {z[0]}'
```

```
The system is inconsistent!

Least squares solution: x = 2.8357810413885183, y = -2.20560747663
55147, z = 1.704939919893191
```