Use definition of variance and properties of expected value to prove that for any discrete random variable X, $VarX = \mathbb{E}(X^2) - (\mathbb{E}X)^2$

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1. Let us start with the variance formula: $Var(X) = \mathbb{E}((X - \mathbb{E}X)^2)$
2. Now, we can expand the square and obtain: $\mathbb{E}(X - \mathbb{E}X)^2 = \mathbb{E}(X^2 - 2X\mathbb{E}X + \mathbb{E}X^2)$.
3. Using the properties of the expected value, we get: $\mathbb{E}(X^2 - 2X\mathbb{E}X + \mathbb{E}X^2) = \mathbb{E}(X^2) - 2\mathbb{E}(X)\mathbb{E}(X) + \mathbb{E}(\mathbb{E}(X^2)) = \mathbb{E}(X^2) - 2\mathbb{E}(X)^2 + \mathbb{E}(X^2)$.
4. Simplifying, we get: $\mathbb{E}(X^2) - 2\mathbb{E}(X)^2 + \mathbb{E}(X^2) = \mathbb{E}(X^2) - \mathbb{E}(X)^2$

$$\implies Var(X) = \mathbb{E}(X^2) - \mathbb{E}(X)^2$$

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Overall solution:
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$$Var(X) = \mathbb{E}((X - \mathbb{E}X)^2) = \mathbb{E}(X^2 - 2X\mathbb{E}X + \mathbb{E}X^2) = \mathbb{E}(X^2) - 2\mathbb{E}(X)\mathbb{E}(X) + \mathbb{E}(\mathbb{E}(X^2)) = \mathbb{E}(X^2) - 2\mathbb{E}(X)^2 + \mathbb{E}(X^2) = \mathbb{E}(X^2) - \mathbb{E}(X)^2 \blacksquare$$