`1.1 We need to prove that` $\sum_{j=1}^{k} r_j \log p_j - \sum_{j=1}^{k} r_j \log r_j \leq 0$, which, by properties of the logarithm, can be re-written as:

$\sum_{j=1}^{k} r_j \log \frac{p_j}{r_j} \leq 0 \implies$ we can use Jensen's inequality to prove that the left-hand side of the inequality is less than or equal to $0$

$\implies \sum_{j=1}^{k} r_j \log \frac{p_j}{r_j} \leq \log \sum_{j=1}^{k} \frac{r_j \cdot p_j}{r_j}$, where the right side can be simplified to the logarithm of the sum of all probabilities, which must be equal to $1$.

$\implies \log(1) = 0$

$\therefore \sum_{j=1}^{k} r_j \log \frac{p_j}{r_j} \leq 0$ ∎

`1.2 Prove that to obtain maximum log-likelihood (and therefore maximum likelihood) for fixed` $(f_1, \ldots, f_k)$ we have to put $p_i = r_i, i = 1, \ldots, k$.

Given the proof above, it can be deduced that the maximal likelihood is 0, which is satisfied only when $\frac{p_j}{r_j} = 1$.

$\implies \max(\sum_{j=1}^{k} r_j \log \frac{p_j}{r_j}) = 0 \iff \frac{p_j}{r_j} = 1 \therefore p_j = r_j$ ∎