```
Let X and Y be two independent normally distributed random variables with expected value 0 and variance 1. Find their joint PDF. Plot its level curves.
```

Independence of X and Y implies that the joint PDF is the mat. product of their distinctive PDFs $\implies pdf(X,Y) = pdf(X) \cdot pdf(Y)$

The PDF of a normally distributed random variable is given by

$$pdf(X) = pdf(Y)\frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}}$$

$$\implies pdf(X,Y) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}} \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{y^2}{2}}$$

$$\implies pdf(X,Y) = \frac{1}{2\pi} \cdot e^{-\frac{x^2+y^2}{2}} \quad \blacksquare$$

The level curves of the joint PDF are defined by

$$\frac{1}{2\pi} \cdot e^{-\frac{x^2+y^2}{2}} = c$$

$$\implies e^{-\frac{x^2+y^2}{2}} = c \cdot 2\pi \quad \blacksquare$$

The visuals below prove that the mathematical proof is correct.

In [8]:

```python
import numpy as np
import scipy.stats
import matplotlib.pyplot as plt

# plotting func
def plot(x,y, plot_type, title):
    fig = plt.figure(figsize=(8,6))
    if plot_type == 'surface':
        ax = fig.add_subplot(111, projection='3d')
        ax.plot_surface(x, y, Z, cmap='viridis')
        ax.set_zlabel('Z')
    elif plot_type == 'contonur':
        ax = fig.add_subplot(111)
        ax.contour(x, y, Z)
    ax.set_xlabel('X')
    ax.set_ylabel('Y')
    ax.set_title(title)
    plt.show()
```

In [9]:

```python
# Define X and Y
x,y  = np.linspace(-5, 5, 1000), np.linspace(-7, 7, 1000)

# Create the meshgrid, find joint PDF based on X and Y with EX = 0 and VarX = 1
X, Y = np.meshgrid(x, y)
Z = scipy.stats.norm.pdf(X, 0, 1) * scipy.stats.norm.pdf(Y, 0, 1)

# Plot the joint PDF, plot the level curves
plot(X, Y, 'surface', 'Joint PDF of the normal distribution')
plot(X, Y, 'contonur', 'Level curves of the normal distribution')
```