*Consider Bernoulli random variable, i.e. random variable that takes value $1$ with probability $p$ and value $0$ with probability $1-p$. Find its variance. Then find variance of binomial random variable with probability of success (i.e. head) $p$ and number of trials (i.e. tosses) $n$*

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If $X$ is a Bernoulli random variable with probability of success $p \implies X$ takes value $1$ with probability $p$ and value $0$ with probability $1-p$.

$\implies \mathbb{E}(X) = 1 \cdot p + 0 \cdot (1-p) = p$
$\implies \mathbb{E}(X^2) = 1^2 \cdot p + 0^2 \cdot (1-p) = p$

Using the variane formula, we obtain $Var(X) = \mathbb{E}(X^2) - \mathbb{E}(X)^2 = p - p^2 = p(1-p) \therefore$ the variance of a Bernoulli random variable is $p \cdot (1-p)$

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If $X_1, X_2, \ldots, X_n$ are independent Bernoulli random variables with probability of success $p$ and $X_1 + X_2 + \cdots + X_n$ is a binomial random variable with probability of success $p$ and number of trials $n$

$\implies \mathbb{E}(X_1 + X_2 + \cdots + X_n) = \sum_{i=1}^{n} \mathbb{E}(X_i) = np$

Given that events are independent, the variance can be calculated as:

$Var(X) = Var(X_1, X_2, \ldots, X_n) = Var(X_1) + Var(X_2) + \ldots + Var(X_n) = \sum_{i=1}^{n} \mathbb{Var}(X_i) = np(1-p) + np(1-p) + \ldots + np(1-p)$

$\implies Var(X) = np \cdot (1-p)$