Let (X) be uniform random variable on a segment ([0, 2]). Consider random variable (Y=X^2).

1.1 Find CDF and PDF of Y. 1.2 Is PDF a bounded function?

The CDF(X) can be defined as:
$$F_X(x) = P(X \le x) = \begin{cases} 0 & x < 0 \\ \frac{x}{2} & x \in [0,2] \\ 1 & x > 2 \end{cases}$$

$$\implies$$
 the CDF of Y is given by $F_Y(y) = P(Y \le y) = P(X^2 \le y) = P(X \le \sqrt{y}) = \frac{\sqrt{y}}{2}$

Given that X is a uniform distribution on the segment [0,2], Y can take values **only** on the segment $[0,4] \implies$ to obtain PDF we need to find the derivative of the CDF.

$$\implies$$
 the PDF of Y is given by $f_Y(y) = \frac{dFY(y)}{dy} = \frac{d\sqrt{y}}{2dy} = \frac{1}{4\sqrt{y}}$

 \therefore the PDF of Y is an unbounded function.