Consider arbitrary sample $x = (x_1, \dots, x_n)$. Let us find such value a that $\sum_{i=1}^{n}(x_i - a)^2$ is minimized. Prove that a is sample average $\bar{x}$ of x.

Proof:

$$\frac{\partial}{\partial a}\sum_{i=1}^{n}(x_i - a)^2 = \sum_{i=1}^{n} 2(x_i - a)\cdot(-1) = -2\sum_{i=1}^{n}(x_i - a) = 0$$

Simplifying, we obtain

$$\sum_{i=1}^{n}(x_i - a) = 0$$

$$\sum_{i=1}^{n} x_i - \sum_{i=1}^{n} a = 0$$

$$\sum_{i=1}^{n} x_i - n \cdot a = 0$$

$$\sum_{i=1}^{n} x_i = n \cdot a$$

$$a = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$a = \bar{x}$$

$\blacksquare$