Exploring Approaches to Solving a Linear Partial Differential Equation Using Quantum-Inspired Algorithms

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1 Overview

The purpose of this project is to explore the different approaches of setting up quantum-inspired algorithms by comparing the spectral split-step[1], and linear combination of Hamiltonian simulation (LCHS) methods[2], to the classical algorithms like finite differences method which is a standard technique to solving linear PDEs. These quantum-inspired algorithms use a matrix product state (MPS) encoding of a quantum register[1] to solve the differential equations. By analyzing the results given by the classical versus our two distinct quantum approaches we hope to gain insight for how we can choose and refine our initial approach and the impact the efficiency and efficacy of solving our second-order linear partial differential equations, and by also looking at the numerical stability, help us better understand how we can maximize the number of qubits while also keeping the error to a minimum.

The standard approach to simulations involving randomness and uncertainty is the Monte-Carlo method, the issue is the time cost of running this algorithm can get very high depending on the parameters being varied for your chosen model. The optimization of these algorithms is extremely important when it comes to these types of probabilistic algorithms as we want to minimize these time costs as much as possible with maximizing the accuracy of our output.

2 Methods

- Examine how our approach differs depending on the PDE chosen, with there being a few that have already been solved and can be used to compare our results to already solved PDEs such as, the Fokker-Planck equation, the acoustic equation or the non-conservative heat equation.
- Use Python code to simulate the quantum computing space as well as the Trotter-Suzuki technique for implementing the Hamiltonian in quantum computing[1].
- Conduct error analysis on our quantum-inspired algorithmic approaches to see the benefits, drawbacks and efficiency in comparison to the classical algorithm.
- Compare the results of the spectral split-step method, and linear combination of Hamiltonian simulation method to the finite differences technique to see what differences other than efficiency, efficacy, and cost that may arise when applying these methods to general PDEs in comparison to our classical approach.

3 Proposed Timeline

November 18th: Finalize thesis goals and begin formulating the way in which we will approach second-order, linear, PDEs in non-conservative and conservative systems.

January 23rd: Complete literature review rough draft

January 30th: Code for simulating quantum space, including, the finite differences,the LCHS, and the spectral split-step methods completed.

February 1st: Rough draft of project report submitted to supervisor.

February 10th: Collect data and begin error analysis of different approaches.

March 13th: Pre-rough draft of thesis submitted to supervisor.

4 References

References

[1] Juan José García-Ripoll. "Quantum-inspired algorithms for multivariate analysis: from interpolation to partial differential equations". In: *Quantum* 5 (Apr. 2021), p. 431. DOI: 10.22331/q-2021-04-15-431. arXiv: 1909.06619 [quant-ph].

[2] Yuki Sato et al. "Quantum algorithm for partial differential equations of non-conservative systems with spatially varying parameters". In: *arXiv e-prints*, arXiv:2407.05019 (July 2024), arXiv:2407.05019. DOI: 10.48550/arXiv.2407.05019. arXiv: 2407.05019 [quant-ph].