1. Let $\phi : \mathbf{X} \to \mathbf{Y}$ be an affine morphism. Show that

1. ϕ is injective if and only if $\mathrm{lin}(\phi)$ is injective.
2. ϕ is surjective if and only if $\mathrm{lin}(\phi)$ is surjective.
3. ϕ is bijective if and only if $\mathrm{lin}(\phi)$ is bijective.

2. Consider $\mathbf{v}(2,1,1) \in \mathbb{R}^3$ and $Q(2,2,2) \in \mathbf{A}^3(\mathbb{R})$.

1. Give the matrix form for the parallel projection on the plane $\pi : z = 0$ along the line $Q + \langle \mathbf{v} \rangle$.
2. Give the matrix form for the parallel reflection in the plane $\pi : z = 0$ along the line $Q + \langle \mathbf{v} \rangle$.

3. Write down the vector forms and matrix forms for parallel projections and reflections in $\mathbf{A}^3(\mathbf{K})$.

4. In $\mathbf{A}^2(\mathbf{K})$, for the lines/hyperplanes

$$\pi : ax + by + c = 0, \quad \ell : \frac{x - x_0}{v_1} = \frac{y - y_0}{v_2}$$

with $\pi \nparallel \ell$, deduce the matrix forms of $\mathrm{Pr}_{\pi,\ell}$ and $\mathrm{Ref}_{\pi,\ell}$.

5. Let H be a hyperplane and let $\mathbf{v}$ be a vector. Use the deduced compact matrix forms to show that

1. $\mathrm{Pr}_{H,\mathbf{v}} \circ \mathrm{Pr}_{H,\mathbf{v}} = \mathrm{Pr}_{H,\mathbf{v}}$ and
2. $\mathrm{Ref}_{H,\mathbf{v}} \circ \mathrm{Ref}_{H,\mathbf{v}} = \mathrm{Id}$.

6. Give Cartesian equations for the line passing through the point $M(1,0,7)$, parallel to the plane $\pi : 3x - y + 2z - 15 = 0$ and intersecting the line

$$\ell : \frac{x-1}{4} = \frac{y-3}{2} = \frac{z}{1}.$$

7. Give Cartesian equations for the projection of the line

$$\ell : 2x - y + z - 1 = 0 \cap x + y - z + 1 = 0$$

on the plane $\pi : x + 2y - z = 0$ parallel to the direction of $\overrightarrow{u}(1,1,-2)$. Write down Cartesian equations of the line obtained by reflecting ℓ in the plane π parallel to the direction of $\overrightarrow{u}$.

8. Consider the Euclidean space $\mathbb{E}^3$. Show that the orthogonal reflection $\mathrm{Ref}_\pi^\perp(x)$ in the plane $\pi : \langle n, x \rangle = p$ is given by

$$\mathrm{Ref}_\pi(x) = Ax + b$$

where $A = \left(I - 2\frac{nn^t}{\|n\|^2}\right)$ and $b = \frac{2p}{\|n\|^2} n$.

9. Give the matrix form for the orthogonal reflections in the planes

$$\pi_1 : 3x - 4z = -1 \quad \text{and} \quad \pi_2 : 10x - 2y + 3z = 4 \quad \text{respectively.}$$

10. Let $\mathbf{X}$ be an affine space. Show that the set T of all translations is a subgroup of $\mathrm{AGL}(\mathbf{X})$. Show that T is a normal subgroup of $\mathrm{AGL}(\mathbf{X})$.

11. Show that $\mathrm{AGL}(\mathbb{R}^n)$ is a subgroup of $\mathrm{GL}(\mathbb{R}^{n+1})$.