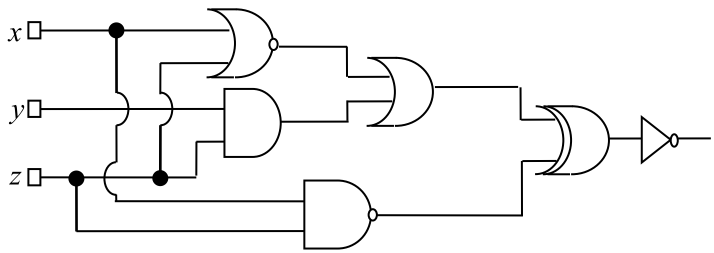
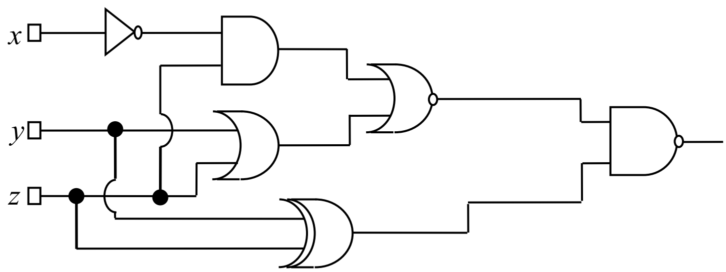
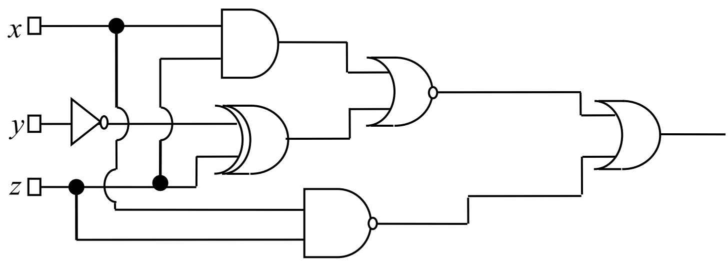
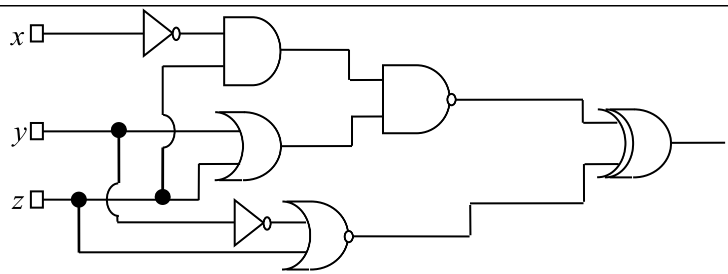
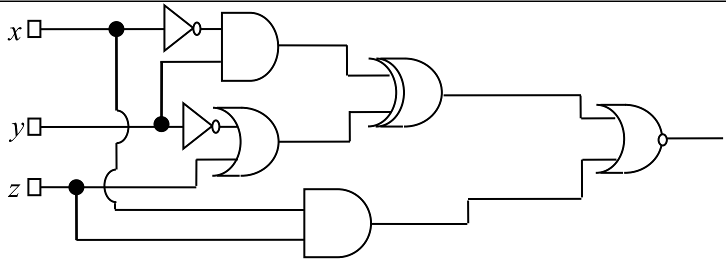
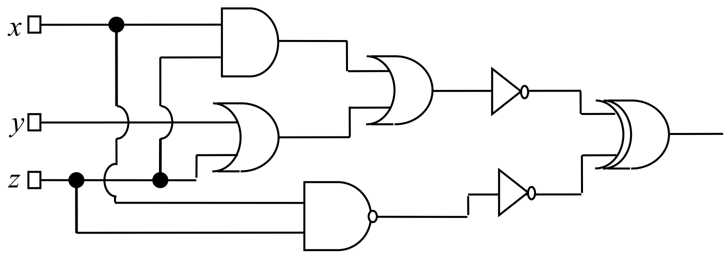
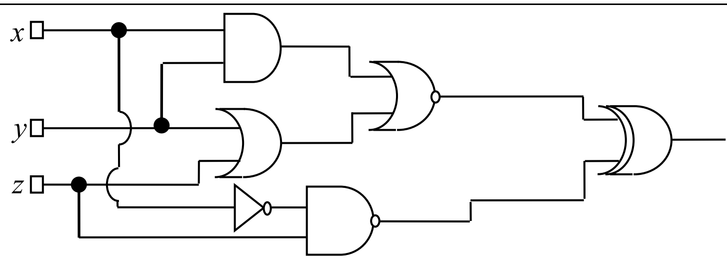
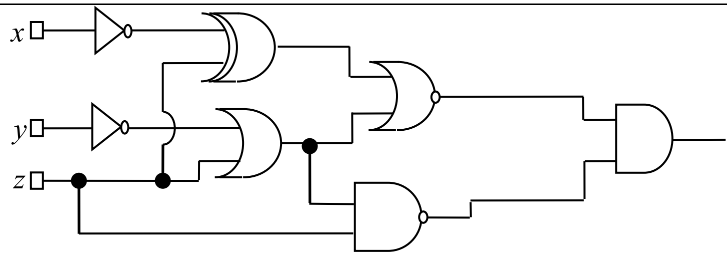
Exercises – LOGIC CIRCUITS

Exercise 1.

Write the corresponding Boolean function associated to the following logic circuit, then simplify it and draw a simplified equivalent circuit using only basic gates:

1.	
2.	
3.	
4.	

5.	
6.	
7.	
8.	

Exercise 2.

For each of the following Boolean functions draw the corresponding logic circuit using derived gates, simplify the function and draw the logic circuits associated to all simplified forms of the initial function using only basic gates.

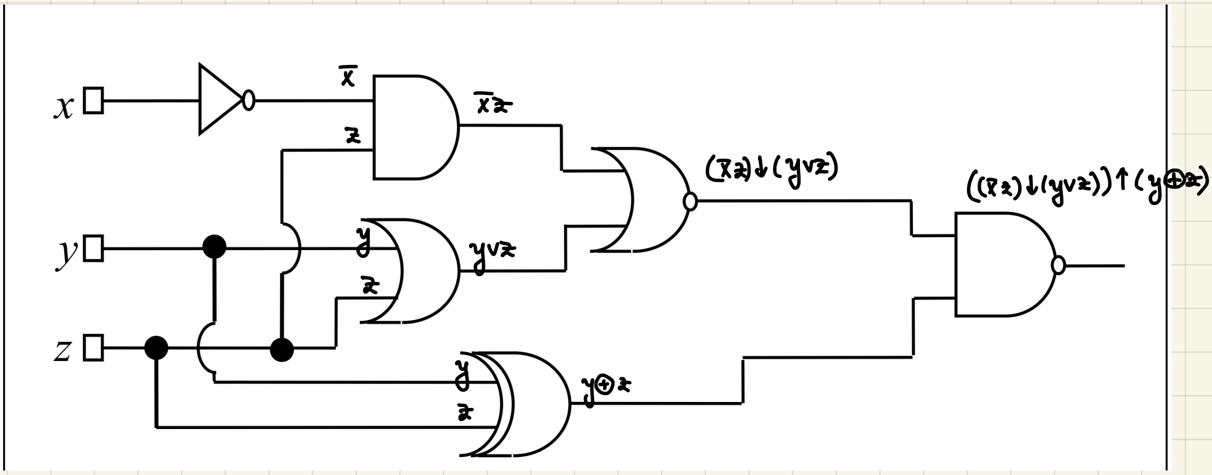
1. $f_1(x, y, z) = x(y \oplus z) \vee y(x \oplus z) \vee x(\overline{y} \downarrow \overline{z}) \vee (x \downarrow y)\overline{z}$;
2. $f_2(x, y, z) = x(y \uparrow z) \vee \overline{x}(\overline{y} \oplus z) \vee y(\overline{x} \oplus \overline{z})$;
3. $f_3(x, y, z) = x(\overline{y} \oplus z) \vee y(\overline{x} \oplus z) \vee \overline{x}(\overline{y} \downarrow z) \vee (\overline{x} \downarrow y)z$;
4. $f_4(x, y, z) = \overline{x}(y \uparrow \overline{z}) \vee x(\overline{y} \oplus z) \vee \overline{y}(\overline{x} \oplus z)$;

5. $f_5(x, y, z) = \overline{x}(y \oplus \overline{z}) \vee \overline{y}(x \oplus z) \vee \overline{x}(y \downarrow z) \vee (\overline{x} \downarrow y)\overline{z}$;
6. $f_6(x, y, z) = x(\overline{y} \uparrow \overline{z}) \vee \overline{x}(y \oplus z) \vee \overline{y}(\overline{x} \oplus z)$;
7. $f_7(x, y, z) = x(y \oplus \overline{z}) \vee y(\overline{x} \oplus z) \vee x(y \downarrow z) \vee (x \downarrow y)\overline{z}$;
8. $f_8(x, y, z) = x(\overline{y} \uparrow z) \vee \overline{x}(\overline{y} \oplus z) \vee y(x \oplus \overline{z})$.

Exercise 3.
Draw a logic circuit having 3 input wires and containing all basic and derived gates. Write the corresponding Boolean function, simplify it and then draw a simplified circuit equivalent to the initial one.

Exercise 4.
Write a Boolean function of 4 variables given by its table of values, simplify it and draw the logic circuits corresponding to all its simplified forms.

2.



$(\overline{x}z\downarrow(y\vee z))\uparrow(y\oplus z)$

$\Leftrightarrow \overline{(\overline{x}z\downarrow(y\vee z))}\vee(y\otimes z)$

$\Leftrightarrow (\overline{x}z\vee z\vee y)\vee((\overline{y}\vee z)\wedge(y\vee\overline{z}))$

$\Leftrightarrow z\vee y\vee(\cancel{\overline{y}y}\vee\overline{y}\overline{z}\vee zy\vee\cancel{z\overline{z}})$

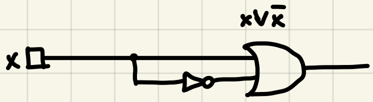
$\Leftrightarrow z\vee y\vee\overline{y}\overline{z}\vee zy$

$\Leftrightarrow z\vee y\vee\overline{y}\overline{z}$

$\Leftrightarrow z\vee(\underbrace{(y\vee\overline{y})}_{1}\wedge(y\vee\overline{z})$

$\Leftrightarrow z\vee y\vee\overline{z}$

$\Leftrightarrow 1$



3. $f_3(x, y, z) = x(\overline{y}\oplus z)\vee y(\overline{x}\oplus z)\vee\overline{x}(\overline{y}\downarrow z)\vee(\overline{x}\downarrow y)z$;

replace $\oplus$, $\downarrow$

$\Leftrightarrow x(\overline{y}\overline{z}\vee yz)\vee y(\overline{x}\overline{z}\vee xz)\vee\overline{x}(y\overline{z})\vee(x\overline{y})z$

distrib.

$\Leftrightarrow x\overline{y}\overline{z}\vee xyz\vee\overline{x}y\overline{z}\vee\cancel{xyz}\vee\cancel{\overline{x}y\overline{z}}\vee x\overline{y}z$

idem.

$\Leftrightarrow x\overline{y}\overline{z}\vee xyz\vee\overline{x}y\overline{z}\vee x\overline{y}z$

$\Leftrightarrow y(xz\vee\overline{x}\overline{z})\vee\overline{y}(x\overline{z}\vee xz)$

$\Leftrightarrow y(xz\vee\overline{x}\overline{z})\vee x\overline{y}(z\vee\overline{z})$

$\Leftrightarrow y(xz\vee\overline{x}\overline{z})\vee x\overline{y}$

$\Leftrightarrow xyz\vee\overline{x}y\overline{z}\vee x\overline{y}$

$\Leftrightarrow x(yz\vee\overline{y})\vee\overline{x}y\overline{z}$

$\Leftrightarrow x(\underbrace{(y\vee\overline{y})}_{1}\wedge(\overline{y}\vee z))\vee\overline{x}y\overline{z}$

$\Leftrightarrow x(\overline{y}\vee z)\vee\overline{x}y\overline{z}$

$\Leftrightarrow x\overline{y}\vee xz\vee\overline{x}y\overline{z}$