Calculus on $\mathbb{R}$ Lect. dr. Anca Grad

Excercise 1 Fill in the following table, with the help of the notions from the lecture:

Nr.	A	LB A	UB A	inf A	sup A	min A	max A
1	$(-\infty,-1]\cup(2,+\infty)$	$\emptyset$	$\emptyset$	$-\infty$	$+\infty$	$-\infty$	$+\infty$
2	$(-1,9]\cup[10,20)$	$(-\infty,-1]$	$[20,+\infty)$	-1	20	$\nexists$	$\nexists$
3	$\Big((-1,9]\cup[10,20)\Big)\cap\mathbb{N}$	$(-\infty,1]$	$[20,+\infty)$	1	20	1	$\nexists$
4	$\{1,2,3\}$	$(-\infty,1]$	$[3,+\infty)$	1	3	1	3
5	$\mathbb{N}$	$(-\infty,1]$	$\emptyset$	1	$+\infty$	1	$+\infty$
6	$\mathbb{R}\setminus\{1,2,3\}$	$(-\infty,1]$	$[3,+\infty)$	1	3	$\nexists$	$\nexists$
7	$\mathbb{R}\setminus\mathbb{N}$	$\emptyset$	$\emptyset$	$-\infty$	$+\infty$	$\nexists$	$\nexists$
8	$\mathbb{Z}$	$\emptyset$	$\emptyset$	$-\infty$	$+\infty$	$\nexists$	$\nexists$
9	$\mathbb{R}\setminus\mathbb{Z}$	$\emptyset$	$\emptyset$	$-\infty$	$+\infty$	$\nexists$	$\nexists$
10	$\mathbb{Q}$	$\emptyset$	$\emptyset$	$-\infty$	$+\infty$	$\nexists$	$\nexists$
11	$\mathbb{R}\setminus\mathbb{Q}$	$\emptyset$	$\emptyset$	$-\infty$	$+\infty$	$\nexists$	$\nexists$
12	$\mathbb{R}$	$\emptyset$	$\emptyset$	$-\infty$	$+\infty$	$\nexists$	$\nexists$

Exercise 2:

Determine the same requirements as for Exercise 1, this time, for the sets:

$$A = \bigcup_{n\in\mathbb{N}\setminus\{1\}} \left(-1+\frac{1}{n}, 1-\frac{1}{n}\right), \quad B = \bigcup_{n\in\mathbb{N}} \left[-1+\frac{1}{n}, 1-\frac{1}{n}\right]$$

$$C = \bigcap_{n\in\mathbb{N}\setminus\{1\}} \left(-1+\frac{1}{n}, 1-\frac{1}{n}\right) \quad D = \bigcap_{n\in\mathbb{N}} \left[-1+\frac{1}{n}, 1-\frac{1}{n}\right]$$

$$E = \bigcup_{n\in\mathbb{N}} \left[-1-\frac{1}{n}, 1+\frac{1}{n}\right] \quad D = \bigcap_{n\in\mathbb{N}} \left(-1-\frac{1}{n}, 1+\frac{1}{n}\right)$$

Exercise 3:

Fill in the following table, by using ✓when the set if a neighbourhood of -1 and ✗ when it is not :

$(-1,2]$	$(-2,1)$	$[-1,1]$	$\mathbb{R}\setminus\{1\}$	Z	$\mathbb{R}\setminus(-1,0)$	$\mathbb{Q}$
✗	✓	✗	✓	✗	✗	✗

Argumentaţi (demonstraţi) fiecare afimaţie folosind rezultatele teoretice de la curs.

A set $U \in \mathbb{R}$ is a neighborhood of -1 if $U \supset (-1-\varepsilon, -1+\varepsilon)$ for some $\varepsilon > 0$.

If $a < -1 < b$, then the closed interval $[a, b]$ is a neighborhood of -1, since it contains the interval $(x-\varepsilon, x+\varepsilon)$ for sufficiently small $\varepsilon > 0$.

Exercise 2:

Determine the same requirements as for Exercise 1, this time, for the sets:

$$A = \bigcup_{n\in\mathbb{N}\setminus\{1\}} \left(-1+\frac{1}{n}, 1-\frac{1}{n}\right), \quad B = \bigcup_{n\in\mathbb{N}} \left[-1+\frac{1}{n}, 1-\frac{1}{n}\right]$$

$$C = \bigcap_{n\in\mathbb{N}\setminus\{1\}} \left(-1+\frac{1}{n}, 1-\frac{1}{n}\right) \quad D = \bigcap_{n\in\mathbb{N}} \left[-1+\frac{1}{n}, 1-\frac{1}{n}\right]$$

$$E = \bigcup_{n\in\mathbb{N}} \left[-1-\frac{1}{n}, 1+\frac{1}{n}\right] \quad F = \bigcap_{n\in\mathbb{N}} \left(-1-\frac{1}{n}, 1+\frac{1}{n}\right)$$

$$A = \bigcup_{m\in\mathbb{N}\setminus\{1\}} \left(-1+\frac{1}{m}, 1-\frac{1}{m}\right) = \underbrace{\left(-1+\frac{1}{m}, 1-\frac{1}{m}\right)}_{\substack{m\to\infty \\ m\in\mathbb{N}}} = (-1, 1)$$

$LB(A) = (-\infty, -1]$ $\quad \inf A = -1 \notin A \Rightarrow \nexists \min A$

$UP(A) = [1, +\infty)$ $\quad \sup A = 1 \notin A \Rightarrow \nexists \max A$

$$B = \bigcup_{m\in\mathbb{N}} \left[-1+\frac{1}{m}, 1-\frac{1}{m}\right] = \underbrace{[-1+1, 1-1] \cup \dots \cup \left[-1+\frac{1}{m}, 1-\frac{1}{m}\right]}_{\substack{m\to\infty \\ m\in\mathbb{N}}} =$$

$$= \{0\} \cup \dots \cup \left[-1+\frac{1}{m}, 1-\frac{1}{m}\right] =$$

$$= \left[-1+\frac{1}{m}, 1-\frac{1}{m}\right] = [-1, 1]$$

$LB(B) = (-\infty, -1]$ $\quad \inf B = -1 \in B \Rightarrow \exists \min B = -1$

$UP(B) = [1, +\infty)$ $\quad \sup B = 1 \in B \Rightarrow \exists \max B = 1$

$$C = \bigcap_{m\in\mathbb{N}\setminus\{1\}} \left(-1+\frac{1}{m}, 1-\frac{1}{m}\right) = \left(\overset{2}{-\frac{1}{1}} + \frac{1}{2}, \overset{2}{1} - \frac{1}{2}\right) = \left(\frac{-2+1}{2}, \frac{1}{2}\right) =$$

$$= \left(-\frac{1}{2}, \frac{1}{2}\right)$$

$LB(C) = \left(-\infty, -\frac{1}{2}\right]$ $\quad \inf C = -\frac{1}{2} \notin C \Rightarrow \nexists \min C$

$UP(C) = \left[\frac{1}{2}, +\infty\right)$ $\quad \sup C = \frac{1}{2} \notin C \Rightarrow \nexists \max C$

$$D = \bigcap_{m \in \mathbb{N}} \left[-1 + \frac{1}{m}, 1 - \frac{1}{m}\right] = \left[-1+1, 1-1\right] = \{0\}$$

$LB(D) = (-\infty, 0]$ $\qquad$ $\inf D = 0 \in D \Rightarrow \exists \min D = 0$

$UP(D) = [0, +\infty)$ $\qquad$ $\sup D = 0 \in D \Rightarrow \exists \max D = 0$

$$E = \bigcup_{m \in \mathbb{N}} \left[-1 - \frac{1}{m}, 1 + \frac{1}{m}\right] = \left[-1 - \frac{1}{1}, 1 + \frac{1}{1}\right] \cup \dots \cup \left[-1 - \frac{1}{m}, 1 + \frac{1}{m}\right] = [-2, 2] \cup \dots \cup \left[-\frac{m+1}{m}, \frac{m+1}{m}\right] =$$

$$= \left[-\frac{m+1}{m}, \frac{m+1}{m}\right] = [-1, 1]$$

$LB(E) = (-\infty, -1]$ $\qquad$ $\inf E = -1 \in E \Rightarrow \exists \min E = -1$

$UP(E) = [1, +\infty)$ $\qquad$ $\sup E = 1 \in E \Rightarrow \exists \max E = 1$

$$F = \bigcap_{m \in \mathbb{N}} \left(-1 - \frac{1}{m}, 1 + \frac{1}{m}\right) = \left(-1 - \frac{1}{1}, 1 + \frac{1}{1}\right) \cap \dots \cap \left(-1 - \frac{1}{m}, 1 + \frac{1}{m}\right) = (-2, 2) \cap \dots \cap \left(-\frac{m+1}{m}, \frac{m+1}{m}\right) =$$

$$= (-2, 2)$$

$LB(F) = (-\infty, -2]$ $\qquad$ $\inf F = -2 \notin F \Rightarrow \nexists \min F$

$UP(F) = [2, +\infty)$ $\qquad$ $\sup F = 2 \notin F \Rightarrow \nexists \max F$