Babeș-Bolyai University Cluj-Napoca
Faculty of Mathematics and Computer Science

**Final Exam in Calculus (2)**
**Groups 811, 812, 813, 814 – July 9, 2020**

$\boxed{1.}$ **(2 points)** Consider the set $A := \{(x,y) \in \mathbb{R}^2 \mid x > y > 0\}$ and the function

$$f : A \to \mathbb{R}, \qquad f(x,y) := \left(x^2 - y^2\right) \arctan \frac{x-y}{x+y}\,.$$

Prove that

$$x\,\frac{\partial f}{\partial x}(x,y) + y\,\frac{\partial f}{\partial y}(x,y) = 2f(x,y), \quad \forall\,(x,y) \in A.$$

$\boxed{2.}$ **(2 points)** Calculate $\displaystyle\iiint_A \arctan\sqrt{x^2+y^2+z^2}\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$, where

$$A := \{(x,y,z) \in \mathbb{R}^3 \ \mid\ x^2+y^2+z^2 \le 1\}.$$

$\boxed{3.}$ **(1.5 points)** Determine $\alpha \in \mathbb{R}$ such that the function $f : \mathbb{R}^2 \to \mathbb{R}$, defined by

$$f(x,y) := \begin{cases} \dfrac{\ln\left(1+x^2+y^2\right)}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ \alpha & \text{if } (x,y) = (0,0), \end{cases}$$

is continuous at $(0,0)$. For that value of $\alpha$ study the differentiability of $f$ at $(0,0)$.

$\boxed{4.}$ **(2 points)** Determine the volume (i.e. the Jordan measure) of the solid body bounded below by the cone $z = \sqrt{3(x^2+y^2)}$ and above by the sphere $x^2+y^2+z^2 = 2z$.

$\boxed{5.}$ **(1.5 points)** Calculate $\displaystyle\iint_A (x^2+y^2)\,\mathrm{d}x\mathrm{d}y$, where $A$ is the set of all points lying inside the circle $x^2+y^2 = 4x$ and outside the circle $(x-3)^2+y^2 = 1$.

**All problems are mandatory. One point is awarded ex officio.**
**The solutions will be sent to the e-mail address tiberiutrif@gmail.com.**