Exercises Boolean functions

Exercise 1.
For the following Boolean functions of 3 variables, given by their truth tables, write the corresponding *disjunctive canonical form* (*DCF*) and *conjunctive canonical form* (*CCF*).
Using Karnaugh diagrams simplify both *DCF* and *CCF*.

x	y	z	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
0	0	0	0	1	1	1	0	1	0	1
0	0	1	1	1	1	0	1	0	0	0
0	1	0	0	0	1	0	1	1	1	1
0	1	1	1	0	0	1	0	1	0	0
1	0	0	1	0	1	0	0	0	1	0
1	0	1	0	1	0	0	1	0	1	1
1	1	0	0	0	0	1	1	1	0	1
1	1	1	1	1	0	1	0	0	1	0

Exercise 2.
Simplify the following Boolean functions of 4 variables using Karnaugh diagrams.

1. $f_1(x_1,x_2,x_3,x_4)= x_1x_2\bar{x}_3x_4 \vee x_1x_2x_3\bar{x}_4 \vee x_1x_2\bar{x}_3\bar{x}_4 \vee x_1\bar{x}_2x_3\bar{x}_4 \vee \bar{x}_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1\bar{x}_2x_3\bar{x}_4 \vee x_1\bar{x}_2x_3x_4 \vee \bar{x}_1\bar{x}_2\bar{x}_3x_4 \vee \bar{x}_1\bar{x}_2x_3x_4$;
2. $f_2(x_1,x_2,x_3,x_4)= x_1x_2x_3x_4 \vee x_1x_2x_3\bar{x}_4 \vee \bar{x}_1x_2x_3\bar{x}_4 \vee x_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1\bar{x}_2x_3\bar{x}_4 \vee x_1\bar{x}_2\bar{x}_3x_4 \vee \bar{x}_1\bar{x}_2\bar{x}_3x_4 \vee \bar{x}_1\bar{x}_2x_3x_4$;
3. $f_3(x_1,x_2,x_3,x_4)= \bar{x}_1x_2x_3x_4 \vee \bar{x}_1x_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1x_2x_3\bar{x}_4 \vee x_1\bar{x}_2x_3\bar{x}_4 \vee x_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee x_1\bar{x}_2x_3x_4 \vee x_1\bar{x}_2\bar{x}_3x_4 \vee \bar{x}_1\bar{x}_2\bar{x}_3x_4$;
4. $f_4(x_1,x_2,x_3,x_4)= \bar{x}_1x_2\bar{x}_3x_4 \vee x_1x_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1x_2\bar{x}_3\bar{x}_4 \vee x_1\bar{x}_2x_3\bar{x}_4 \vee x_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1\bar{x}_2x_3\bar{x}_4 \vee x_1\bar{x}_2x_3x_4 \vee x_1\bar{x}_2\bar{x}_3x_4 \vee \bar{x}_1\bar{x}_2x_3x_4$;
5. $f_5(x_1,x_2,x_3,x_4)= x_1x_2x_3x_4 \vee x_1x_2\bar{x}_3x_4 \vee x_1x_2x_3\bar{x}_4 \vee \bar{x}_1x_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1x_2x_3\bar{x}_4 \vee x_1\bar{x}_2x_3\bar{x}_4 \vee x_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee x_1x_2\bar{x}_3\bar{x}_4 \vee x_1\bar{x}_2\bar{x}_3x_4$;
6. $f_6(x_1,x_2,x_3,x_4)= x_1x_2x_3x_4 \vee \bar{x}_1x_2\bar{x}_3x_4 \vee \bar{x}_1x_2x_3x_4 \vee x_1x_2x_3\bar{x}_4 \vee \bar{x}_1x_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1x_2x_3\bar{x}_4 \vee x_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee x_1\bar{x}_2x_3x_4 \vee x_1\bar{x}_2\bar{x}_3x_4$;
7. $f_7(x_1,x_2,x_3,x_4)= x_1x_2x_3x_4 \vee \bar{x}_1x_2\bar{x}_3x_4 \vee \bar{x}_1x_2x_3x_4 \vee x_1x_2\bar{x}_3\bar{x}_4 \vee x_1\bar{x}_2x_3\bar{x}_4 \vee x_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee x_1\bar{x}_2x_3x_4 \vee \bar{x}_1\bar{x}_2\bar{x}_3x_4 \vee \bar{x}_1\bar{x}_2x_3x_4$;
8. $f_8(x_1,x_2,x_3,x_4)= x_1x_2x_3x_4 \vee x_1x_2\bar{x}_3x_4 \vee \bar{x}_1x_2\bar{x}_3x_4 \vee \bar{x}_1x_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1x_2x_3\bar{x}_4 \vee x_1\bar{x}_2x_3\bar{x}_4 \vee x_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee x_1\bar{x}_2x_3x_4 \vee x_1\bar{x}_2\bar{x}_3x_4$.

Exercise 3.
Using Veitch diagrams, simplify the following Boolean functions:

1. $f_1(x_1,x_2,x_3) = \bar{x}_1(x_2 \downarrow x_3) \vee \bar{x}_1\bar{x}_2x_3 \vee \overline{(x_1 \vee (x_2 \uparrow x_3))} \vee x_1x_2\bar{x}_3$
2. $f_2(x_1,x_2,x_3) = \bar{x}_1\bar{x}_2\bar{x}_3 \vee x_1(\bar{x}_2 \downarrow x_3) \vee \overline{(x_1 \vee (x_2 \uparrow \bar{x}_3))} \vee x_1\bar{x}_2x_3$
3. $f_3(x_1,x_2,x_3) = \bar{x}_1(x_2 \downarrow x_3) \vee x_1\bar{x}_2x_3 \vee \overline{(x_1 \vee (x_2 \uparrow x_3))} \vee x_1x_2x_3$
4. $f_4(x_1,x_2,x_3) = \bar{x}_1\bar{x}_2\bar{x}_3 \vee x_1(\bar{x}_2 \downarrow x_3) \vee \overline{(x_1 \vee (\bar{x}_2 \uparrow x_3))} \vee x_1\bar{x}_2x_3$
5. $f_5(x_1,x_2,x_3) = x_1(x_2 \downarrow x_3) \vee \bar{x}_1\bar{x}_2x_3 \vee \overline{(\bar{x}_1 \vee (\bar{x}_2 \uparrow x_3))} \vee \bar{x}_1x_2\bar{x}_3$
6. $f_6(x_1,x_2,x_3) = x_1\bar{x}_2x_3 \vee \bar{x}_1(\bar{x}_2 \downarrow x_3) \vee \overline{(\bar{x}_1 \vee (x_2 \uparrow x_3))} \vee x_1\bar{x}_2\bar{x}_3$
7. $f_7(x_1,x_2,x_3) = \bar{x}_1(x_2 \downarrow x_3) \vee x_1\bar{x}_2x_3 \vee \overline{(x_1 \vee (x_2 \uparrow x_3))} \vee x_1\bar{x}_2\bar{x}_3$
8. $f_8(x_1,x_2,x_3) = \bar{x}_1\bar{x}_2x_3 \vee x_1(x_2 \downarrow x_3) \vee \overline{(x_1 \vee (x_2 \uparrow x_3))} \vee \bar{x}_1x_2\bar{x}_3$

Exercise 4.
Simplify the following Boolean functions of 4 variables using Veitch diagrams.

1. $f_1(x_1,x_2,x_3,x_4)= x_1\bar{x}_4 \vee x_1x_2\bar{x}_3x_4 \vee \bar{x}_1x_2x_4 \vee \bar{x}_1x_3 \vee x_3\bar{x}_4$;
2. $f_2(x_1,x_2,x_3,x_4)= x_1x_2 \vee x_1\bar{x}_2\bar{x}_3x_4 \vee \bar{x}_1\bar{x}_2x_4 \vee \bar{x}_1x_3 \vee x_2x_3$;
3. $f_3(x_1,x_2,x_3,x_4)= x_1x_4 \vee x_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1\bar{x}_2\bar{x}_4 \vee \bar{x}_1\bar{x}_3 \vee x_3x_4$;
4. $f_4(x_1,x_2,x_3,x_4)= x_1\bar{x}_2 \vee x_1x_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1x_2\bar{x}_4 \vee \bar{x}_1x_3 \vee \bar{x}_2x_3$;
5. $f_5(x_1,x_2,x_3,x_4)= x_3\bar{x}_4 \vee x_1x_2x_3x_4 \vee \bar{x}_3x_2x_4 \vee \bar{x}_1\bar{x}_3 \vee \bar{x}_1\bar{x}_4$;
6. $f_6(x_1,x_2,x_3,x_4)= \bar{x}_1\bar{x}_4 \vee \bar{x}_1x_2x_3x_4 \vee x_1x_2x_4 \vee x_1\bar{x}_3 \vee \bar{x}_3\bar{x}_4$;
7. $f_7(x_1,x_2,x_3,x_4)= \bar{x}_3\bar{x}_4 \vee \bar{x}_1x_2\bar{x}_3x_4 \vee x_2x_3x_4 \vee x_1x_3 \vee x_1\bar{x}_4$;
8. $f_8(x_1,x_2,x_3,x_4)= x_3x_4 \vee x_1\bar{x}_2x_3\bar{x}_4 \vee \bar{x}_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1\bar{x}_3 \vee \bar{x}_1x_4$

Exercise 5.

Using Quine's method, simplify the following Boolean functions given by their values 0.

1. $f_1(0,1,0) = f_1(0,1,1) = f_1(1,0,1) = 0$;
2. $f_2(0,0,0) = f_2(0,0,1) = f_2(1,1,1) = 0$;
3. $f_3(0,0,1) = f_3(0,1,0) = f_3(1,1,0) = 0$;
4. $f_4(0,0,0) = f_4(0,1,1) = f_4(1,0,0) = 0$;
5. $f_5(0,0,0) = f_5(1,1,0) = f_5(1,1,1) = 0$;
6. $f_6(0,1,0) = f_6(1,0,0) = f_6(1,0,1) = 0$;
7. $f_7(0,1,1) = f_7(1,0,0) = f_7(1,1,1) = 0$;
8. $f_8(0,0,1) = f_8(1,0,1) = f_8(1,1,0) = 0$.

Exercise 6.

Using Quine's method, simplify the following Boolean functions given in *DCF* (disjunction of minterms):

1. $f_1(x_1,x_2,x_3) = m_0 \vee m_3 \vee m_4 \vee m_5 \vee m_6 \vee m_7$;
2. $f_2(x_1,x_2,x_3) = m_1 \vee m_2 \vee m_4 \vee m_5 \vee m_6 \vee m_7$;
3. $f_3(x_1,x_2,x_3) = m_1 \vee m_2 \vee m_3 \vee m_4 \vee m_5 \vee m_7$;
4. $f_4(x_1,x_2,x_3) = m_0 \vee m_1 \vee m_2 \vee m_3 \vee m_5 \vee m_6$;
5. $f_5(x_1,x_2,x_3) = m_0 \vee m_1 \vee m_2 \vee m_4 \vee m_6 \vee m_7$;
6. $f_6(x_1,x_2,x_3) = m_0 \vee m_1 \vee m_3 \vee m_5 \vee m_6 \vee m_7$;
7. $f_7(x_1,x_2,x_3) = m_0 \vee m_1 \vee m_2 \vee m_3 \vee m_4 \vee m_7$;
8. $f_8(x_1,x_2,x_3) = m_0 \vee m_2 \vee m_3 \vee m_4 \vee m_5 \vee m_6$.

Exercise 7.

Simplify the following Boolean functions of 4 variables given by their values 1, using Quine's method.

1. $f_1(1,1,1,1) = f_1(1,1,0,1) = f_1(0,1,1,1) = f_1(1,1,0,0) = f_1(0,1,0,0) = f_1(0,0,0,0) = f_1(0,0,0,1) = f_1(0,0,1,1) = 1$;
2. $f_2(1,1,0,1) = f_2(0,1,0,1) = f_2(0,1,0,0) = f_2(0,0,0,0) = f_2(0,0,1,0) = f_2(1,0,1,1) = f_2(1,0,0,1) = f_2(0,0,1,1) = 1$;
3. $f_3(0,1,0,1) = f_3(0,1,0,0) = f_3(0,1,1,0) = f_3(1,0,1,0) = f_3(1,0,0,0) = f_3(0,0,1,0) = f_3(1,0,0,1) = f_3(0,0,0,1) = 1$;
4. $f_4(0,1,0,1) = f_4(0,1,1,1) = f_4(1,1,1,0) = f_4(1,1,0,0) = f_4(0,1,1,0) = f_4(1,0,0,0) = f_4(0,0,0,0) = f_4(0,0,0,1) = 1$;
5. $f_5(1,1,1,1) = f_5(0,1,0,1) = f_5(0,1,1,1) = f_5(1,1,1,0) = f_5(1,1,0,0) = f_5(1,0,0,0) = f_5(1,0,0,1) = f_5(0,0,0,1) = 1$;
6. $f_6(1,1,0,1) = f_6(0,1,0,1) = f_6(0,1,1,1) = f_6(1,1,1,0) = f_6(0,1,1,0) = f_6(1,0,1,0) = f_6(1,0,1,1) = f_6(1,0,0,1) = 1$;
7. $f_7(1,1,1,1) = f_7(1,1,0,1) = f_7(0,1,0,1) = f_7(0,1,0,0) = f_7(0,1,1,0) = f_7(0,0,1,0) = f_7(1,0,1,1) = f_7(0,0,1,1) = 1$;
8. $f_8(1,1,1,1) = f_8(1,1,1,0) = f_8(1,1,0,0) = f_8(1,0,0,0) = f_8(0,0,0,0) = f_8(0,0,1,0) = f_8(1,0,1,1) = f_8(0,0,1,1) = 1$.

Exercise 8

Using Moisil's method simplify the following Boolean functions of 3 variables:

1. $f_1(x_1,x_2,x_3) = m_0 \vee m_1 \vee m_2 \vee m_5 \vee m_6$;
2. $f_2(x_1,x_2,x_3) = m_0 \vee m_1 \vee m_3 \vee m_4 \vee m_7$;
3. $f_3(x_1,x_2,x_3) = m_1 \vee m_2 \vee m_3 \vee m_5 \vee m_6$;
4. $f_4(x_1,x_2,x_3) = m_0 \vee m_3 \vee m_4 \vee m_5 \vee m_7$;
5. $f_5(x_1,x_2,x_3) = m_1 \vee m_2 \vee m_5 \vee m_6 \vee m_7$;
6. $f_6(x_1,x_2,x_3) = m_0 \vee m_2 \vee m_3 \vee m_4 \vee m_7$;
7. $f_7(x_1,x_2,x_3) = m_1 \vee m_2 \vee m_4 \vee m_5 \vee m_6$;
8. $f_8(x_1,x_2,x_3) = m_0 \vee m_3 \vee m_4 \vee m_6 \vee m_7$.

Exercise 1.
For the following Boolean functions of 3 variables, given by their truth tables, write the corresponding *disjunctive canonical form* (*DCF*) and *conjunctive canonical form* (*CCF*).
Using Karnaugh diagrams simplify both *DCF* and *CCF*.

x	y	z	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
0	0	0	0	1	1	1	0	1	0	1
0	0	1	1	1	1	0	1	0	0	0
0	1	0	0	0	1	0	1	1	1	1
0	1	1	1	0	0	1	0	1	0	0
1	0	0	1	0	1	0	0	0	1	0
1	0	1	0	1	0	0	1	0	1	1
1	1	0	0	0	0	1	1	1	0	1
1	1	1	1	1	0	1	0	0	1	0

$DCF(f_8) = m_0 \vee m_2 \vee m_5 \vee m_6$

$x_1 \backslash x_2x_3$	00	01	11	10
0	m_0			m_2
1		m_5		m_6

$max1 = m_2 \vee m_6 = y\bar{z}$

$max2 = m_2 \vee m_0 = \bar{x}\bar{z}$

$max3 = m_5 = x\bar{y}z$

$M(f) = \{max1, max2, max3\}$

$C(f) = \{max1, max2, max3\}$

$M(f) = C(f) \Rightarrow 1^{st}$ case

$f_8^{DS}(x,y,z) = max1 \vee max2 \vee max3$

$= y\bar{z} \vee \bar{x}\bar{z} \vee x\bar{y}z$

$m_0 = x^0 y^0 z^0 = \bar{x}\bar{y}\bar{z}$

$m_2 = x^0 y^1 z^0 = \bar{x}y\bar{z}$

$m_5 = x^1 y^0 z^1 = x\bar{y}z$

$m_6 = x^1 y^1 z^0 = xy\bar{z}$

0,2,5,6

$CCF(f_8) = M_1 \wedge M_3 \wedge M_4 \wedge M_7$

$x_1 \backslash x_2x_3$	00	01	11	10
0		M_1	M_3	
1	M_4		M_7	

$maxd_1 = M_1 \wedge M_3 = x \vee \bar{z}$

$maxd_2 = M_3 \wedge M_7 = \bar{y} \vee \bar{z}$

$maxd_3 = M_4 = \bar{x} \vee y \vee z$

$Md(f) = \{maxd_1, maxd_2, maxd_2\}$

$Cd(f) = \{maxd_1, maxd_2, maxd_2\}$

$Md(f) = Cd(f) \Rightarrow 1^{st}$ case

$f_8^{CS}(x,y,z) = (x \vee \bar{z}) \wedge (\bar{y} \vee \bar{z}) \wedge (\bar{x} \vee y \vee z)$

$M_1 = x^{\bar{0}} \vee y^{\bar{0}} \vee z^{\bar{1}} = x \vee y \vee \bar{z}$

$M_3 = x^{\bar{0}} \vee y^{\bar{1}} \vee z^{\bar{1}} = x \vee \bar{y} \vee \bar{z}$

$M_4 = x^{\bar{1}} \vee y^{\bar{0}} \vee z^{\bar{0}} = \bar{x} \vee y \vee z$

$M_7 = x^{\bar{1}} \vee y^{\bar{1}} \vee z^{\bar{1}} = \bar{x} \vee \bar{y} \vee \bar{z}$

Exercise 2.
Simplify the following Boolean functions of 4 variables using Karnaugh diagrams.

2. $f_2(x_1,x_2,x_3,x_4) = x_1x_2x_3x_4 \vee x_1x_2x_3\bar{x}_4 \vee \bar{x}_1x_2x_3\bar{x}_4 \vee x_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1\bar{x}_2x_3\bar{x}_4 \vee x_1\bar{x}_2\bar{x}_3x_4 \vee \bar{x}_1\bar{x}_2\bar{x}_3x_4 \vee \bar{x}_1\bar{x}_2x_3x_4$;

$= m_{15} \vee m_{14} \vee m_6 \vee m_8 \vee m_0 \vee m_2 \vee m_9 \vee m_1 \vee m_3$

$x_1x_2 \backslash x_3x_4$	00	01	11	10
00	m_0	m_1	m_3	m_2
01				m_6
11			m_{15}	m_{14}
10	m_8	m_9		

$max_1 = m_0 \vee m_1 \vee m_3 \vee m_2 = \bar{x}_1\bar{x}_2$

$max_2 = m_0 \vee m_1 \vee m_8 \vee m_9 = \bar{x}_2\bar{x}_3$

$max_3 = m_2 \vee m_6 = \bar{x}_1x_3\bar{x}_4$

$max_4 = m_6 \vee m_{14} = x_2x_3\bar{x}_4$

$max_5 = m_{15} \vee m_{14} = x_1x_2x_3$

$M(f) = \{max_1 \ldots max_5\}$

$C(f) = \{max1, max2, max5\}$

$f_1^S(x_1,x_2,x_3,x_4) = \bar{x}_1\bar{x}_2 \vee \bar{x}_2\bar{x}_3 \vee x_1x_2x_3 \vee \bar{x}_1x_3\bar{x}_4$

$f_1^S(x_1,x_2,x_3,x_4) = \bar{x}_1\bar{x}_2 \vee \bar{x}_2\bar{x}_3 \vee x_1x_2x_3 \vee x_2x_3\bar{x}_4$

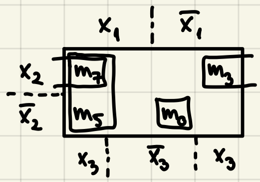
Exercise 3.
Using Veitch diagrams, simplify the following Boolean functions:

3. $f_3(x_1,x_2,x_3)=\bar{x}_1(x_2 \downarrow x_3) \vee x_1\bar{x}_2x_3 \vee \overline{(x_1 \vee (x_2 \uparrow x_3))} \vee x_1x_2x_3$

$= \bar{x}_1(\bar{x}_2 \wedge \bar{x}_3) \vee x_1\bar{x}_2x_3 \vee \overline{(x_1 \vee (\bar{x}_2 \vee \bar{x}_3))} \vee x_1x_2x_3$

$= (\bar{x}_1 \wedge \bar{x}_2 \wedge \bar{x}_3) \vee x_1\bar{x}_2x_3 \vee (\bar{x}_1 \wedge x_2 \wedge x_3) \vee x_1x_2x_3$

$= m_0 \vee m_5 \vee m_3 \vee m_7$



$max1 = m_7 \vee m_5 = x_1x_3$

$max2 = m_3 \vee m_7 = x_2x_3$

$max3 = m_0 = \bar{x}_1\bar{x}_2\bar{x}_3$

$M(f) = \{max1, max2, max3\} = C(f) \Rightarrow f^S(x_1,x_2,x_3) = x_1x_2 \vee x_2x_3 \vee \bar{x}_1\bar{x}_2\bar{x}_3$

Exercise 4.
Simplify the following Boolean functions of 4 variables using Veitch diagrams.

3. $f_3(x_1,x_2,x_3,x_4) = x_1x_4 \vee \underbrace{x_1\bar{x}_2\bar{x}_3\bar{x}_4}_{m_8} \vee \bar{x}_1\bar{x}_2\bar{x}_4 \vee \bar{x}_1\bar{x}_3 \vee x_3x_4;$

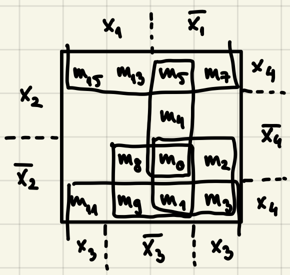
x_1x_4: $x_1x_2x_3x_4$ (15), $x_1x_2\bar{x}_3x_4$ (13), $x_1\bar{x}_2x_3x_4$ (11), $x_1\bar{x}_2\bar{x}_3x_4$ (9)

$\bar{x}_1\bar{x}_2\bar{x}_4$: $\bar{x}_1\bar{x}_2\bar{x}_3\bar{x}_4$ (0), $\bar{x}_1\bar{x}_2x_3\bar{x}_4$ (2)

$\bar{x}_1\bar{x}_3$: $\bar{x}_1x_2\bar{x}_3x_4$ (5), $\bar{x}_1x_2\bar{x}_3\bar{x}_4$ (4), $\bar{x}_1\bar{x}_2\bar{x}_3x_4$ (1), $\bar{x}_1\bar{x}_2\bar{x}_3\bar{x}_4$ (0)

x_3x_4: $x_1x_2x_3x_4$ (15), $x_1\bar{x}_2x_3x_4$ (11), $\bar{x}_1x_2x_3x_4$ (7), $\bar{x}_1\bar{x}_2x_3x_4$ (3)

$f(x_1,x_2,x_3,x_4) = m_0 \vee m_1 \vee m_2 \vee m_3 \vee m_4 \vee m_5 \vee m_7 \vee m_8 \vee m_9 \vee m_{11} \vee m_{13} \vee m_{15}$



$max_1 = m_1 \vee m_3 \vee m_5 \vee m_7 \vee m_9 \vee m_{11} \vee m_{13} \vee m_{15} = x_4$

$max_2 = m_0 \vee m_1 \vee m_8 \vee m_9 = \bar{x}_2\bar{x}_3$

$max_3 = m_0 \vee m_1 \vee m_2 \vee m_3 = \bar{x}_1\bar{x}_2$

$max_4 = m_0 \vee m_1 \vee m_4 \vee m_5 = \bar{x}_1\bar{x}_3$

$M(f) = \{max_1 \dots max_4\} = C(f)$

$f^S(x_1,x_2,x_3,x_4) = x_4 \vee \bar{x}_1\bar{x}_3 \vee \bar{x}_2\bar{x}_3 \vee \bar{x}_1\bar{x}_2$

Exercise 5.

Using Quine's method, simplify the following Boolean functions given by their values 0.

2. $f_2(0,0,0) = f_2(0,0,1) = f_2(1,1,1) = 0$;

$$f_2(x_1, x_2, x_3) = \overset{1}{m_2} \vee \overset{2}{m_3} \vee \overset{1}{m_4} \vee \overset{2}{m_5} \vee \overset{2}{m_6}$$

		x_1	x_2	x_3	
I	✓	0	1	0	m_2
	✓	1	0	0	m_4
II	✓	0	1	1	m_3
	✓	1	0	1	m_5
	✓	1	1	0	m_6
I+II		0	1	–	$m_2 \vee m_3 = max_1 = \bar{x}_1 x_2$
		–	1	0	$m_2 \vee m_6 = max_2 = x_2 \bar{x}_3$
		1	0	–	$m_4 \vee m_5 = max_3 = x_1 \bar{x}_2$
		1	–	0	$m_4 \vee m_6 = max_4 = x_1 \bar{x}_3$

min \ max	max_1	max_2	max_3	max_4
m_2	*	*		
m_4			*	*
m_3	*			
m_5			*	
m_6		*		*

$$\left.\begin{array}{l} M(f) = \{max_1, \dots max_4\} \\ \qquad\quad \neq \\ C(f) = \{max_1, max_3\} \end{array}\right\} \Rightarrow g \vee max_2,\ g \vee max_4$$

$$f_1^S = \bar{x}_1 x_2 \vee x_1 \bar{x}_2 \vee x_2 \bar{x}_3$$
$$f_2^S = \bar{x}_1 x_2 \vee x_1 \bar{x}_2 \vee x_1 \bar{x}_3$$

2. $f_2(1,1,0,1) = f_2(0,1,0,1) = f_2(0,1,0,0) = f_2(0,0,0,0) = f_2(0,0,1,0) = f_2(1,0,1,1) = f_2(1,0,0,1) = f_2(0,0,1,1) = 1$;

$$f(x_1, x_2, x_3, x_4) = \overset{2}{m_{13}} \vee \overset{2}{m_5} \vee \overset{1}{m_4} \vee \overset{0}{m_0} \vee \overset{1}{m_2} \vee \overset{3}{m_{11}} \vee \overset{2}{m_9} \vee \overset{2}{m_3}$$

		x_1	x_2	x_3	x_4	
I	✓	0	0	0	0	m_0
II	✓	0	0	1	0	m_2
	✓	0	1	0	0	m_4
III	✓	0	0	1	1	m_3
	✓	0	1	0	1	m_5
	✓	1	0	0	1	m_9
IV	✓	1	0	1	1	m_{11}
	✓	1	1	0	1	m_{13}
V = I+II		0	0	–	0	$m_0 \vee m_2 = max_1 = \bar{x}_1 \bar{x}_2 \bar{x}_4$
		0	–	0	0	$m_0 \vee m_4 = max_2 = \bar{x}_1 \bar{x}_3 \bar{x}_4$
VI = II+III		0	0	1	–	$m_2 \vee m_3 = max_3 = \bar{x}_1 \bar{x}_2 x_3$
		0	1	0	–	$m_4 \vee m_5 = max_4 = \bar{x}_1 x_2 \bar{x}_3$
VII = III+IV		–	0	1	1	$m_3 \vee m_{11} = max_5 = \bar{x}_2 x_3 x_4$
		–	1	0	1	$m_5 \vee m_{13} = max_6 = x_2 \bar{x}_3 x_4$
		1	0	–	1	$m_9 \vee m_{11} = max_7 = x_1 \bar{x}_2 x_4$
		1	–	0	1	$m_9 \vee m_{13} = max_8 = x_1 \bar{x}_3 x_4$

	max_1	max_2	max_3	max_4	max_5	max_6	max_7	max_8
m_0	*	*						
m_2	*		*					
m_4		*		*				
m_3			*		*			
m_5				*		*		
m_9							*	*
m_{11}					*		*	
m_{13}						*		*

$$\left.\begin{array}{l} M(f) = \{max_1, \dots max_n\} \\ C(f) = \emptyset \end{array}\right\} \Rightarrow \text{case } 3$$

$$f_1^S(x_1, x_2, x_3, x_4) = max_1 \vee max_4 \vee max_5 \vee max_8 = \bar{x}_1 \bar{x}_2 \bar{x}_4 \vee \bar{x}_1 x_2 \bar{x}_3 \vee \bar{x}_2 x_3 x_4 \vee x_1 \bar{x}_3 x_4$$
$$f_2^S(x_1, x_2, x_3, x_4) = max_2 \vee max_3 \vee max_6 \vee max_7 = \bar{x}_1 \bar{x}_3 \bar{x}_4 \vee \bar{x}_1 \bar{x}_2 x_3 \vee x_2 \bar{x}_3 x_4 \vee x_1 \bar{x}_2 x_4$$

Exercise 8
Using Moisil's method simplify the following Boolean functions of 3 variables:

2. $f_2(x_1,x_2,x_3) = m_0 \vee m_1 \vee m_3 \vee m_4 \vee m_7$;

		x_1	x_2	x_3	
I	✓	0	0	0	m_0
II	✓	0	0	1	m_1
	✓	1	0	0	m_4
III	✓	0	1	1	m_3
IV	✓	1	1	1	m_7
I+II		0	0	-	$m_0 \vee m_1 = max_1 = \overline{x_1}\,\overline{x_2}$
		-	0	0	$m_0 \vee m_4 = max_2 = \overline{x_2}\,\overline{x_3}$
II+III		0	-	1	$m_1 \vee m_3 = max_3 = \overline{x_1} x_3$
III+IV		-	1	1	$m_3 \vee m_7 = max_4 = x_2 x_3$

"m_0 is covered by max_1 or max_2": $p_1 \vee p_2$
"m_1 is covered by max_1 or max_3": $p_1 \vee p_3$
"m_3 is covered by max_3 or max_4": $p_3 \vee p_4$
"m_4 is covered by max_2": p_2
"m_7 is covered by max_4": p_4

$$(p_1 \vee p_2) \wedge (p_1 \vee p_3) \wedge (p_3 \vee p_4) \wedge p_2 \wedge p_4 \equiv T$$
$$\Leftrightarrow p_2 \wedge p_4 \wedge (p_1 \vee p_3) \equiv T$$
$$\Leftrightarrow p_2 \wedge ((p_1 \wedge p_4) \vee (p_3 \wedge p_4)) \equiv T$$
$$\Leftrightarrow (p_1 \wedge p_2 \wedge p_4) \vee (p_2 \wedge p_3 \wedge p_4) \equiv T$$

- $p_1 \wedge p_2 \wedge p_4 = f_1^S(x_1, x_2, x_3, x_4) = max_1 \vee max_2 \vee max_4 = \overline{x_1}\,\overline{x_2} \vee \overline{x_2}\,\overline{x_3} \vee x_2 x_3$
- $p_2 \wedge p_3 \wedge p_4 = f_2^S(x_1, x_2, x_3, x_5) = max_2 \vee max_3 \vee max_4 = \overline{x_2}\,\overline{x_3} \vee \overline{x_1} x_3 \vee x_2 x_3$

$$f(x_1,x_2,x_3,x_4) = \overset{1}{m_4} \vee \overset{2}{m_6} \vee \overset{3}{m_7} \vee \overset{1}{m_8} \vee \overset{2}{m_9} \vee \overset{2}{m_{10}} \vee \overset{3}{m_{11}} \vee \overset{2}{m_{12}}$$

		x_1	x_2	x_3	x_4	
I	✓	0	1	0	0	m_4
	✓	1	0	0	0	m_8
II	✓	0	1	1	0	m_6
	✓	1	0	0	1	m_9
	✓	1	0	1	0	m_{10}
	✓	1	1	0	0	m_{12}
III	✓	0	1	1	1	m_7
	✓	1	0	1	1	m_{11}

IV = I + II		0	1	–	0	$m_6 \vee m_4 = max_1 = \bar{x}_1 x_2 \bar{x}_4$
		–	1	0	0	$m_4 \vee m_{12} = max_2 = x_2 \bar{x}_3 \bar{x}_4$
	✓	1	0	0	–	$m_8 \vee m_9$
	✓	1	0	–	0	$m_8 \vee m_{10}$
		1	–	0	0	$m_8 \vee m_{12} = max_3 = x_1 \bar{x}_3 \bar{x}_4$
V = II + III		0	1	1	–	$m_6 \vee m_7 = max_4 = \bar{x}_1 x_2 x_3$
	✓	1	0	–	1	$m_9 \vee m_{11}$
	✓	1	0	1	–	$m_{10} \vee m_{11}$
		1	0	–	–	$m_8 \vee m_9 \vee m_{10} \vee m_{11} = max_5 = x_1 \bar{x}_2$

max. monoms / minterms	max_1	max_2	max_3	max_4	max_5
m_4	*	*			
m_8			*		*
m_6	*			*	
m_9					*
m_{10}					*
m_{12}		*	*		
m_7				*	
m_{11}					*

$M(f) = \{max_1 \dots max_5\}$

$C(f) = \{max_4, max_5\}$

We choose the maximal monom with most unshaded '*' = max_2

$f^S = max_4 \vee max_5 \vee max_2 = \bar{x}_1 x_2 x_3 \vee x_1 \bar{x}_2 \vee x_2 \bar{x}_3 \bar{x}_4$