Antiderivatives of a function

$\emptyset \neq D \subseteq \mathbb{R}$ $f: D \to \mathbb{R}$ $J \subseteq D$ is an interval	f has antiderivatives on J if $\exists\, F: J \to \mathbb{R}$ s.t. $\to F$ is differential on J $\to F'(x) = f(x)\,,\ \forall x \in J$ F is called an antiderivative of f on J

The set of all antiderivatives of f on J:

$\int f(x)\,dx = F(x) + \mathcal{C} \longrightarrow$ the set of all constants

$\hookrightarrow$ an antiderivative

Theorem:

Each two different antiderivatives of f are different through a constant.

$J \subseteq \mathbb{R}$ an interval $f: J \to \mathbb{R}$ a function $F_1, F_2: J \to \mathbb{R}$ two antiderivatives of f	$\Longrightarrow$	$\exists\, c \in \mathbb{R}$ s.t. $\forall x \in J \quad F_2(x) = F_1(x) + c$

Proof:

F_1, F_2 differentiable $\Rightarrow F_2 - F_1$ differentiable $\Rightarrow (F_2 - F_1)'(x) = 0 \quad f(x) - f(x) = 0$

$\Downarrow \qquad \forall x \in [a, b]$

$F_2 - F_1 = 0$

Theorem:

(concerning the antiderivative when the function is continuous)

$J \subseteq \mathbb{R}$ an interval $x_0 \in J$ $f: J \to \mathbb{R}$ LRI f continuous at x_0	$\Longrightarrow$	$\forall a \in J$, the function $F: J \to \mathbb{R}$, $F(x) = \int_a^x f(t)\,dt$, $\forall x \in J$ is $\to$ differential at x_0 $\to F'(x_0) = f(x_0)$

Theorem:

$(\exists)$

$J \subseteq \mathbb{R}$ an interval $f: J \to \mathbb{R}$ continuous $a \in J$	$\Longrightarrow$	The function $F: J \to \mathbb{R}$, $F(x) = \int_a^x f(t)\,dt$, $\forall x \in J$ is an antiderivative of f on J and $F(a) = 0$.

Theorem:

$J \subseteq \mathbb{R}$ an interval $f: J \to \mathbb{R}$ continuous $a \in J$	$\Longrightarrow$	The function $F: J \to \mathbb{R}$, is an antiderivative of f on J s.t $F(a) = 0$ then $F(x) = \int_a^x f(t)\,dt$, $\forall x \in J$.

Leibniz-Newton theorem:

$f: [a, b] \to \mathbb{R}$ a function f RI on $[a, b]$ f has antiderivatives on $[a, b]$	$\Longrightarrow$	$\forall F: [a, b] \to \mathbb{R}$ an antiderivative of f $\int_a^b f(x)\,dx = F(b) - F(a)$.