iulian.simion@ubbcluj.ro

1. Establish which of the following triples of points in $\mathbf{A}^3(\mathbb{C})$ are collinear:

1. $\{(2,1,-3),(1,-1,2),(3/2,0,-1/2)\}$
2. $\{(\mathbf{i},0,0),(1+\mathbf{i},2\mathbf{i},1),(1,2,-\mathbf{i})\}$

2. In each of the following, find the value (if it exists) of the real parameter m for which the triple of points is collinear in $\mathbf{A}^3(\mathbb{R})$

1. $\{(2,-1,2),(1,1,1),(4,-m+1,4)\}$
2. $\{(3,0,0),(0,1,1),(m,m,m)\}$

3. After checking, for each of the following, that the points are not collinear, find parametric and Cartesian equations for the planes determined by the points

1. $\{(2,\sqrt{2},1),(1,1,\sqrt{2}),(0,0,1)\}$
2. $\{(5,-1,0),(1,1,\sqrt{5}),(-3,1,\pi/2)\}$

4. In each of the following, find a Cartesian equation of the plane in $\mathbf{A}^3(\mathbb{C})$ passing through Q and parallel to the plane π

1. $Q=(-1,2,2)$, $\pi: x+2y+3z+1=0$
2. $Q=(\mathbf{i},\mathbf{i},\mathbf{i})$, $\pi: 2x-y=0$

5. For each of the following, determine whether or not the three planes belong to the same pencil

1. $x-y+z=0$, $-x+3y-5z+2=0$, $y-2z+1=0$
2. $2x-3y+3=0$, $x-y+6=0$, $x-3z=-1$

6. For each of the following, find parametric and Cartesian equations for the line in $\mathbf{A}^3(\mathbb{R})$ passing through the point Q and parallel to the vector $\mathbf{v}$.

1. $Q=(1,1,0)$, $\mathbf{v}=(2,-1,\sqrt{2})$
2. $Q=(-2,2,-2)$, $\mathbf{v}=(1,1,0)$

7. Find parametric equations for each of the following lines in $\mathbf{A}^3(\mathbb{C})$

1. $x-\mathbf{i}y=0$, $2y+z+1=0$
2. $3x+z-1=0$, $y+z-5=0$

8. For each of the following, find parametric equations for the line in $\mathbf{A}^3(\mathbb{C})$ passing through the point Q and parallel to the line ℓ.

1. $Q(1,1,0)$, $\ell: x-\mathbf{i}y=0, z+1=0$
2. $Q(2,1,-5)$, $\ell: y=2, x=\mathbf{i}z+7$

9. In each of the following, find a Cartesian equation of the plane in $\mathbf{A}^3(\mathbb{R})$ passing through Q and parallel to the lines ℓ and ℓ':

1. $Q(1,-1,-2)$, $\ell : x-y=1, x+z=5$, $\ell' : x=1, z=2$
2. $Q(0,1,3)$, $\ell : x+y=-5, x-y+2z=0$, $\ell : 2x-2y=1, x-y+2z=1$

10. In each of the following, determine whether the lines ℓ and ℓ' are skew or coplanar. If they are coplanar, find whether they are incident or parallel, and then, after checking that they are distinct, find a Cartesian equation for the plane containing them,

1. $\ell : x=1+t, y=-t, z=2+2t$, $\ell' : x=2-t, y=-1+3t, z=t$
2. $\ell : 2x+y+1=0, y-z=2$, $\ell' : x=2-t, y=3+2t, z=1$

11. In each of the following, find the relative positions of the line ℓ and the plane π in $\mathbf{A}^3(\mathbb{R})$, and, if they are incident, determine the point of intersection.

1. $\ell : x=1+t, y=2-2t, z=1-4t$, $\pi : 2x-y+z-1=0$
2. $\ell : x=2-t, y=1+2t, z=-1+3t$, $\pi : 2x+2y-z+1=0$

12. In each of the following, find a Cartesian equation for the plane in $\mathbf{A}^3(\mathbb{R})$ containing the point Q and the line ℓ.

1. $Q=(3,3,1)$, $\ell : x=2+3t, y=5+t, z=1+7t$
2. $Q=(2,1,0)$, $\ell : x-y+1=0, 3x+5z-7=0$

13. In each of the following, find Cartesian equations for the line ℓ in $\mathbf{A}^3(\mathbb{R})$ passing through Q, contained in the plane π and intersecting the line ℓ'

1. $Q=(1,1,0)$, $\pi : 2x-y+z-1=0$, $\ell' : x=2-t, y=2+t, z=t$
2. $Q=(-1,-1,-1)$, $\pi : x+y+z+3=0$, $\ell' : x-2z+4=0, 2y-z=0$

14. In each of the following, find Cartesian equations for the line ℓ in $\mathbf{A}^3(\mathbb{R})$ passing through Q and coplanar to the lines ℓ' and ℓ''. Furthermore, establish whether ℓ meets or is parallel to ℓ' and ℓ''

1. $Q=(1,1,2)$, $\ell' : 3x-5y+z=-1, 2x-3z=-9$, $\ell'' : x+5y=3, 2x+2y-7z=-7$
2. $Q=(2,0,-2)$, $\ell' : -x+3y=2, x+y+z=-1$, $\ell'' : x=2-t, y=3+5t, z=-t$

15. In each of the following, find the value of the real parameter k for which the lines ℓ and ℓ' are coplanar. Find a Cartesian equation for the plane that contains them, and find the point of intersection whenever they meet

1. $\ell : x=k+t, y=1+2t, z=-1+kt$, $\ell' : x=2-2t, y=3+3t, z=1-t$
2. $\ell : x=3-t, y=1+2t, z=k+t$, $\ell' : x=1+t, y=1+2t, z=1+3t$

16. Find a Cartesian equation for the plane π in $\mathbf{A}^3(\mathbb{R})$ which contains the line of intersection of the two planes

$$x+y=3 \quad \text{and} \quad 2y+3z=4$$

and is parallel to the vector $\mathbf{v}=(3,-1,2)$.