ANALYTIC GEOMETRY, PROBLEM SET 11

Mostly distances in 3D.

1. Find the distance from the point $P(1, 2, -1)$ to the line $d : x = y = z$.

2. Find the distance from $P(3, 1, -1)$ to the plane $\pi : 22x + 4y - 20z - 45 = 0$.

3. Find the distance between the planes
$\pi_1 : 2x - 3y + 4z - 7 = 0$ and $\pi_2 : 4x - 6y + 8z - 3 = 0$.

4. Find the distance between the lines $d_1 : \frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{1}$ and $d_2 : \frac{x+1}{3} = \frac{y}{4} = \frac{z-1}{3}$.

5. Find the distance between the lines $d_1 : x = 1 - 2t$, $y = 3t$, $z = -2t + t$, where $t \in \mathbb{R}$ and $d_2 : x = 7 + 4s$, $y = 5 - 6s$, $z = 4 - 2s$, where $s \in \mathbb{R}$.

6. Show that the line
$d : \frac{x+1}{1} = \frac{y-3}{2} = \frac{z}{-1}$ and the plane $\pi : 2x - 2y - 2z + 3 = 0$ are parallel and find the distance between them.

7. Given the point $P(6, -5, 5)$ and the plane $\pi : 2x - 3y + z - 4 = 0$, find the coordinates of the symmetric P' of the point P with respect to the plane π.

8. Consider the point $P(4, 3, 10)$ the line $d : \frac{x-1}{2} = \frac{y-2}{4} = \frac{z-3}{5}$. Find the coordinates of the symmetric point P' of P with respect to the line d.

11.[From the previous set.] Determine the equations of the planes which pass through the points $P(0, 2, 0)$ and $Q(-1, 0, 0)$ and which form an angle of $60°$ with the Oz axis.

12. Find the geometric locus of the lines passing through a given point and having a constant distance to a given line.

The setup of the next problem is in the Euclidean plane $\mathcal{E}_2$.

13. In each of the following situations, find the equation of the circle:
a) of diameter $[AB]$, where $A(1, 2)$ and $B(-3, -1)$; b) of center $I(2, -3)$ and radius $R = 7$;
c) of center $I(-1, 2)$ and which passes through $A(2, 6)$;
d) centered at the origin and tangent to $d : 3x - 4y + 20 = 0$; e) passing through $A(3, 1)$ and $B(-1, 3)$ and having the center on the line $d : 3x - y - 2 = 0$;
f) determined by $A(1, 1)$, $B(1, -1)$ and $C(2, 0)$; g) tangent to both $d_1 : 2x + y - 5 = 0$ and $d_2 : 2x + y + 15 = 0$, if the tangency point with d_1 is $M(3, 1)$.

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