ANALYTIC GEOMETRY, PROBLEM SET 1

Rectangular (cartesian) coordinates

1. Give the coordinates of the vertices of the rectangular parallelepiped whose faces are the coordinate planes and the planes $x = 1$, $y = 3$ and $z = 6$.

2. Describe the locus of points $P(x, y, z) \in \mathcal{E}_3$ if their Cartesian coordinates satisfy:
a) $xyz = 0$; b) $x^2 + y^2 + z^2 = 0$; c) $(x+1)^2 + (y-2)^2 + (z+3)^2 = 0$; d) $(x-2)(z-8) = 0$;
e) $z^2 - 25 = 0$.

3. Find $x \in \mathbb{R}$ if:
a) $P_1(x, 2, 3)$, $P_2(2, 1, 1)$ and $P_1P_2 = \sqrt{21}$; b) $Q_1(x, x, 1)$, $Q_2(0, 3, 5)$ and $Q_1Q_2 = 5$.

4. Show that the given points are collinear:
a) $P_1(1, 2, 0)$, $P_2(-2, -2, -3)$, $P_3(7, 10, 6)$; b) $Q_1(2, 3, 2)$, $Q_2(1, 4, 4)$, $Q_3(5, 0, -4)$.

5. The coordinates of the midpoint of the segment $[P_1P_2]$, determined by $P_1(x_1, y_1, z_1)$ and $P_2(2, 3, 6)$ are $(-1, -4, 8)$. Find the coordinates of P_1.

6. Let P_3 be the midpoint of the segment joining the points $P_1(-3, 4, 1)$ and $P_2(-5, 8, 3)$. Find the coordinates of the midpoint of the segment: a) joining P_1 and P_3; b) joining P_3 and P_2.

7. Compute the area of the triangle whose vertices have coordinates $P_1 = (-1, 0, 1)$, $P_2 = (0, 2, 2)$ and $P_3 = (0, -1, 2)$.

8*. Let $\mathcal{R}$ be the region consisting of the set of points in the coordinate plane that satisfy both $|8 - x| + y \leq 10$ and $3y - x \geq 15$. When $\mathcal{R}$ is revolved around the line whose equation is $3y - x = 15$, the volume of the resulting solid is $\frac{m\pi}{n\sqrt{p}}$, where m, n, and p are positive integers, m and n are relatively prime, and p is not divisible by the square of any prime. Find $m + n + p$.

Polar coordinates

9. Graph the points P, whose polar coordinates are given by: a) $(2, \pi)$; b) $(3, \pi/3)$; c) $(4, 3\pi/2)$; d) $(5, \pi/6)$.

10. Find the polar coordinates of the points whose rectangular (Cartesian) coordinates are given by: a) $(-3, -3)$; b) $(0, -5)$; c) $(\sqrt{3}, -1)$; d) $(\sqrt{2}, \sqrt{6})$.

11. Describe, in each case, the geometric locus of the set represented in the plane by the following equation given in polar coordinates:
a) $r = r_0$, where $r_0 > 0$ is fixed; b) $\theta = \theta_0$, where $\theta_0 \in [0, 2\pi)$ is fixed.

12. If the points A, B have the polar coordinates (r_A, θ_A) and (r_B, θ_B) respectively, show that
$$|AB| = \sqrt{r_A^2 + r_B^2 - 2r_Ar_B\cos(\theta_A - \theta_B)}.$$

Date: September 27, 2021. The starred problem is not examinable material.