ANALYTIC GEOMETRY, PROBLEM SET 8

Line bundles, angle between two lines (in 2D) and the equation of the plane (3D)

1. Given the bundle of lines of equations $(1-t)x+(2-t)y+t-3=0, t \in \mathbb{R}$ and $x+y-1=0$, find:

(1) the coordinates of the vertex of the bundle;
(2) the equation of the line in the bundle which cuts Ox and Oy in M, respectively N, such that $OM^2 \cdot ON^2 = 4(OM^2+ON^2)$;

2. Let B be the bundle of vertex $M_0(5,0)$. An arbitrary line from B intersects the lines $d_1 : y-2=0$ and $d_2 : y-3=0$ in M_1 respectively M_2. Prove that the line passing through M_1 and parallel to OM_2 passes through a fixed point.

3. Determine the angle between the lines:

(1) $y=2x+1$ and $y=-x+2$;
(2) $y=3x-4$ and $x=3+t$, $y=-1-2t$ for $t \in \mathbb{R}$.
(3) $y=\frac{2}{5}x+1$ and $4x+3y-12=0$.

4. Determine the equation of the line which passes through $A(3,1)$ and makes an angle of $45°$ with the line $2x+3y-1=0$.

5. Consider the triangle given by the points $A(1,-2)$, $B(5,4)$ and $C(-2,0)$. Find the equations of the internal, respectively external bisectors corresponding to the vertex A of this triangle.

6. The points of intersection of the lines $d_1 : x+2y-1=0$, $d_2 : 5x+4y-17=0$ and $d_3 : x-4y+11=0$ determine a triangle. Find the equations of the altitudes of these triangles without determining the coordinates of the vertices of the triangle!

7. Let $P_1(x_1,y_1,z_1)$ and $P_2(x_2,y_2,z_2)$ be distinct points in space. Prove that the equation of the plane containing P_1 and P_2 that is parallel to a vector $\overline{a}=(l,m,n)$ is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l & m & n \end{vmatrix} = 0.$$

8. Find the equation of the plane passing through $P(7,-5,1)$ which determines on the positive half-axes three segments of the same length.

9. Find the equation for each of the following planes:

(a) containing $P(2,1,-1)$ and perpendicular to the vector $\overline{n}=(1,-2,3)$;

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(b) determined by $O(0,0,0)$, $P_1(3,-1,2)$ and $P_2(4,-2,-1)$;
(c) containing $P(3,4,-5)$ and parallel to both $\overline{a_1}(1,-2,4)$ and $\overline{a_2}(2,1,1)$;
(d) containing the points $P_1(2,-1,-3)$ and $P_2(3,1,2)$ and parallel to the vector $\overline{a}(3,-1,-4)$.

10. Find the equation of the plane containing the perpendicular lines through $P(-2,3,5)$ on the planes $\pi_1 : 4x + y - 3z + 13 = 0$ and $\pi_2 : x - 2y + z - 11 = 0$.