ANALYTIC GEOMETRY, PROBLEM SET 10

Mostly angles in 3D.

1. Show that the line $d = \begin{cases} x = 0 \\ y = t \\ z = t \end{cases}$ is contained inside the plane $6x + 4y - 4z = 0$.

2. Determine whether the line given by $x = 3 + 8t$, $y = 4 + 5t$, and $z = -3 - t$, $t \in \mathbb{R}$ is parallel to the plane $x - 3y + 5z - 12 = 0$.

3. Prove that the lines $d_1 : \begin{cases} x = 1 + 4t \\ y = 5 - 4t \\ z = -1 + 5t \end{cases}, t \in \mathbb{R}$ and $d_2 : \begin{cases} x = 2 + 8t \\ y = 4 - 3t \\ z = 5 + t \end{cases}, t \in \mathbb{R}$ are skew.

4. Find the parametric equations of the line passing through $(5, 0, -2)$ and parallel to the planes $x - 4y + 2z = 0$ and $2x + 3y - z + 1 = 0$.

5. Find the equation of the plane containing the point $P(2, 0, 3)$ and the line $d : \begin{cases} x = -1 + t \\ y = t \\ z = -4 + 2t \end{cases}$.

6. Let $M_1(2, 1, -1)$ and $M_2(-3, 0, 2)$ be two points. Find:

a) the equation of the bundle of planes passing through M_1 and M_2;
b) the plane π from the bundle, which is orthogonal on xOy;
c) the plane ρ from the bundle, which is orthogonal on π.

7. Find the angle determined by d_1 and d_2, when: a) $d_1 : x = 4-t, y = 3+2t, z = -2t, t \in \mathbb{R}$ and $d_2 : x = 5 + 2s, y = 1 + 3s, z = 5 - 6s, s \in \mathbb{R}$.
b) $d_1 : \frac{x-1}{2} = \frac{y+5}{7} = \frac{z-1}{-1}$ and $d_2 : \frac{x+3}{-2} = \frac{y-9}{1} = \frac{z}{4}$.

8. Find the angle determined by the planes $\pi_1 : x-\sqrt{2}y+z-1 = 0$ and $\pi_2 : x+\sqrt{2}y-z+3 = 0$.

9. Find the coordinates of the orthogonal projection of the point P(2, 1, 1) on the plane $\pi : x + y + 3z + 5 = 0$.

10. Determine the orthogonal projection of the point $A(1, 3, 5)$ on the line which is given as the intersection of the planes $2x + y + z - 1 = 0$ and $3x + y + 2z - 3 = 0$.

Date: December 6, 2021.

11. Determine the equations of the planes which pass through the points $P(0,2,0)$ and $Q(-1,0,0)$ and which form an angle of $60°$ with the Oz axis.

12. Find the equations of the projection of the line $d : \begin{cases} 2x - y + z - 1 = 0 \\ x + y - z + 1 = 0 \end{cases}$ on the plane $\pi : x + 2y - z = 0$.

13. Find the angle determined by the lines $d_1 : \begin{cases} x + 2y + z - 1 = 0 \\ x - 2y + z + 1 = 0 \end{cases}$ and $d_2 : \begin{cases} x - y - z - 1 = 0 \\ x - y + 2x + 1 = 0 \end{cases}$.

14. Find the angle determined by the planes $\pi_1 : x+3y+2z+1 = 0$ and $\pi_2 : 3x+2y-z-6 = 0$.

15. Find the angle determined by the plane xOy and the line M_1M_2, where $M_1(1,2,3)$ and $M_2(-2,1,4)$.