1. Show that the congruency of matrices is an equivalence relation in $\mathrm{Mat}_{n\times n}(\mathbf{K})$.

2. Which of the following are bilinear forms on $\mathbb{R}^n$?

1. $\langle \mathbf{x}, \mathbf{y}\rangle = \sum_{i=1}^n x_i |y_i|$
2. $\langle \mathbf{x}, \mathbf{y}\rangle = |\sum_{i=1}^n x_i y_i|$
3. $\langle \mathbf{x}, \mathbf{y}\rangle = (\sum_{i=1}^n x_i)(\sum_{i=1}^n y_i)$
4. $\langle \mathbf{x}, \mathbf{y}\rangle = \sqrt{\sum_{i=1}^n x_i^2 y_i^2}$
5. $\langle \mathbf{x}, \mathbf{y}\rangle = \sum_{i=1}^n (x_i + y_i)^2 - \sum_{i=1}^n x_i^2 - \sum_{i=1}^n y_i^2$

3. In each of the following, determine the polar bilinear form associated to the given quadratic form $q : \mathbb{R}^2 \to \mathbb{R}$.

1. $q(x,y) = 3x^2 - 8xy - 3y^2$
2. $q(x,y) = 4x^2 - 9xy + 5y^2$
3. $q(x,y) = 6xy$

4. Determine the matrix and the rank of each of the quadratic forms from the previous exercise.

5. In each of the following, determine the polar bilinear form associated to the given quadratic form $q : \mathbb{R}^3 \to \mathbb{R}$.

1. $q(x,y,z) = xz + xy + yz$
2. $q(x,y,z) = 2xy + y^2 - 2xz$
3. $q(x,y,z) = -x^2 - 4xy + 3y^2 + 2z^2$

6. Determine the matrix and the rank of each of the quadratic forms from the previous exercise.

7. Let b be a symmetric bilinear form on a vector space $\mathbf{V}$ and let S be a non-empty subset of $\mathbf{V}$. Show that $S^\perp$ is a vector subspace of $\mathbf{V}$.

8. In each of the following cases find a basis with respect to which the given quadratic form on $\mathbb{R}^3$ is in normal form and calculate the signatures:

1. $4x^2 - 5y^2 + 12z^2$
2. $-x^2 + 9z^2$
3. $-x^2 - y^2 + z^2$
4. $y^2 + 16z^2$

9. Diagonalize each of the quadratic forms of exercise 3, determining the change of coordinates required, and the signatures of the forms.

10. For each of the forms of the preceding exercise, express the matrix B of the diagonalized form as $B = M^t AM$, where A is the matrix of the given form.

11. Diagonalize each of the quadratic forms of exercise 5, determining the change of coordinates required, and the signatures of the forms.

12. For each of the forms of the preceding exercise, express the matrix B of the diagonalized form as $B = M^t AM$, where A is the matrix of the given form.