Exam simulation

- The generic exam sheet will be as follows.

> ## No. 1
>
> 1. **(3 points)** Theoretical subject.
> 2. **(3 points)** Exercise from Group Theory.
> 3. **(3 points)** Exercise from Ring Theory.
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> **Plus 1 point for free and bonus points.**

- **You are invited to participate in an exam simulation on the Microsoft Teams platform. It will take place on Monday, May 24, 2021, starting at 14:00.**

- For the exam simulation, choose one theoretical subject [**1**] and only one exercise (either from Group Theory [**2**] or from Ring Theory [**3**]) from the following sample list.

1. Normal subgroup. Factor group.

1. Polynomial rings.

2. Let $n \in \mathbb{N}^*$ and $U_n = \{z \in \mathbb{C} \mid z^n = 1\}$. Show that U_n is a stable subset of the group $(\mathbb{C}^*, \cdot)$ and $(U_n, \cdot)$ is an abelian group.

2. Determine the subgroups of the group $(\mathbb{Z}_2 \times \mathbb{Z}_2, +)$, and then draw the Hasse diagram of its subgroup lattice.

2. Let $f : \mathbb{R} \to \mathbb{C}^*$ be defined by $f(x) = \cos x + \mathrm{i} \sin x$. Show that f is a group homomorphism between $(\mathbb{R}, +)$ and $(\mathbb{C}^*, \cdot)$.

2. Determine the order of each element and the cyclic subgroups of the quaternion group $(Q, \cdot)$.

2. Let $n \in \mathbb{N}, n \geq 2$. Show the group isomorphism $(\mathrm{GL}_n(\mathbb{R})/\mathrm{SL}_n(\mathbb{R}), \cdot) \simeq (\mathbb{R}^*, \cdot)$ using the first isomorphism theorem.

3. Let $n \in \mathbb{N}$, $n \geq 2$ and $\widehat{0} \neq \widehat{a} \in \mathbb{Z}_n$. Show that $\widehat{a}$ is invertible in the ring $(\mathbb{Z}_n, +, \cdot) \Leftrightarrow \gcd(a, n) = 1$.

3. Show that the ring $(M_2(\mathbb{R}), +, \cdot)$ has the center $Z(M_2(\mathbb{R}), +, \cdot) = \{a \cdot I_2 \mid a \in \mathbb{R}\}$.

3. Let $\mathcal{M} = \left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \middle| \, a, b \in \mathbb{R} \right\}$. Show that $\mathcal{M}$ is a subring of the ring $(M_2(\mathbb{R}), +, \cdot)$.

3. Let $\mathcal{M} = \left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \middle| \, a, b \in \mathbb{R} \right\}$. It is known that $(\mathbb{C}, +, \cdot)$ and $(\mathcal{M}, +, \cdot)$ are fields. Show that they are isomorphic.

3. Determine the ideals and the factor rings of the ring $(\mathbb{Z}_{12}, +, \cdot)$.

- Prepare them in advance, and present them in about 5-10 minutes during our (public or private) audio-video meeting.