SEMINAR 8

1) Let V be a K-vector space, $S \leq_K V$ and $x, y \in V$. We denote $\langle S, x\rangle = \langle S \cup \{x\}\rangle$. Show that if $x \in V \setminus S$ and $x \in \langle S, y\rangle$ then $y \in \langle S, x\rangle$.

2) Let V be a K-vector space and $\alpha, \beta, \gamma \in K$, $x, y, z \in V$ such that $\alpha\gamma \neq 0$ and $\alpha x + \beta y + \gamma z = 0$. Show that $\langle x, y\rangle = \langle y, z\rangle$.

3) Is the real vector space $\mathbb{R}_3[X] = \{f \in \mathbb{R}[X] \mid \deg f \leq 3\}$ generated by the set

$$\{f_1 = 3X + 2,\ f_2 = 4X^2 - X + 1,\ f_3 = X^3 - X^2 + 3\}\,?$$

Why?

4) Let V, V' be K-vector spaces, $f : V \to V'$ a linear map, $A \leq_K V$ and $A' \leq_K V'$. Show that:

a) $f(A) = \{f(a) \in V' \mid a \in A\} \leq_K V'$;

b) $\overset{-1}{f}(A') = \{x \in V \mid f(x) \in A'\} \leq_K V$.

5) In the $\mathbb{R}$-vector space $\mathbb{R}^{\mathbb{R}} = \{f \mid f : \mathbb{R} \to \mathbb{R}\}$ we consider

$$\mathbb{R}^{\mathbb{R}}_o = \{f : \mathbb{R} \to \mathbb{R} \mid f \text{ is odd}\},\ \mathbb{R}^{\mathbb{R}}_e = \{f : \mathbb{R} \to \mathbb{R} \mid f \text{ is even}\}.$$

Show that $\mathbb{R}^{\mathbb{R}}_o$ şi $\mathbb{R}^{\mathbb{R}}_e$ are subspaces of $\mathbb{R}^{\mathbb{R}}$ and $\mathbb{R}^{\mathbb{R}} = \mathbb{R}^{\mathbb{R}}_o \oplus \mathbb{R}^{\mathbb{R}}_e$.

6) Show that the property of being a direct summand is transitive.