Propositional Logic

***Exercises- Propositional Logic***

## Exercise 1

Check the following properties for $\downarrow$ ('*nor*'), $\uparrow$ ('*nand*') and $\oplus$ ('*xor*') connectives using the truth table method.

1. associativity of ' $\uparrow$ ' connective: $p \uparrow (q \uparrow r) \equiv (p \uparrow q) \uparrow r$;
2. associativity of ' $\downarrow$ ' connective: $p \downarrow (q \downarrow r) \equiv (p \downarrow q) \downarrow r$;
3. associativity of ' $\oplus$ ' connective: $p \oplus (q \oplus r) \equiv (p \oplus q) \oplus r$;
4. distribution of ' $\uparrow$ ' connective over ' $\downarrow$ ' connective:
   $p \uparrow (q \downarrow r) \equiv (p \uparrow q) \downarrow (p \uparrow r)$;
5. distribution of ' $\downarrow$ ' connective over ' $\uparrow$ ' connective:
   $p \downarrow (q \uparrow r) \equiv (p \downarrow q) \uparrow (p \downarrow r)$;
6. De Morgan's laws for ' $\downarrow$ ' and ' $\uparrow$ ':
   $\neg(p \downarrow q) \equiv \neg p \uparrow \neg q$ and $\neg(p \uparrow q) \equiv \neg p \downarrow \neg q$;
7. $p \uparrow (q \vee r) \equiv (p \uparrow q) \wedge (p \uparrow r)$ and $p \downarrow (q \wedge r) \equiv (p \downarrow q) \vee (p \downarrow r)$.
8. $p \downarrow (q \uparrow p) \equiv F$ and $p \uparrow (q \downarrow p) \equiv T$;

## Exercise 2

Using the truth table method decide what kind of formula (consistent, inconsistent, tautology, contingent) is $U_j, j \in \{1,2,\ldots,8\}$. Write all the models and anti-models of $U_j, j \in \{1,2,\ldots,8\}$.

1. $U_1 = q \vee \neg p \vee r \rightarrow \neg(p \vee r)$;
2. $U_2 = \neg p \vee \neg(q \wedge r) \rightarrow q \wedge \neg p$;
3. $U_3 = \neg p \wedge (\neg q \vee r) \rightarrow q \wedge \neg p \vee r$;
4. $U_4 = \neg(\neg p \wedge q) \vee r \rightarrow p \wedge (\neg q \vee r)$;
5. $U_5 = \neg p \vee q \wedge r \rightarrow q \wedge \neg p$;
6. $U_6 = \neg p \vee (\neg q \wedge \neg r) \rightarrow q \wedge \neg p \wedge r$;
7. $U_7 = p \rightarrow (p \wedge r) \vee q$;
8. $U_8 = (p \vee q) \wedge \neg r \rightarrow p \wedge q \wedge r$.

## Exercise 3

Using the truth table method, check whether the following logical consequences hold:

1. $p \rightarrow q \models (p \rightarrow r) \rightarrow (p \rightarrow q \wedge r)$;
2. $p \rightarrow q \models (q \rightarrow r) \rightarrow (p \rightarrow r)$;
3. $p \rightarrow (q \rightarrow r) \models (p \rightarrow q) \rightarrow (p \rightarrow r)$;
4. $p \rightarrow r \models (q \rightarrow r) \rightarrow ((p \vee q) \rightarrow r)$;
5. $p \rightarrow q \models (\neg p \rightarrow q) \rightarrow q$;
6. $p \rightarrow q \models (q \rightarrow r) \rightarrow (p \rightarrow q \wedge r)$;
7. $p \rightarrow q \models (q \rightarrow r) \rightarrow (p \rightarrow q \vee r)$;
8. $r \rightarrow (q \rightarrow p) \models (r \rightarrow q) \rightarrow (r \rightarrow p)$.

**Exercise 4.**

Prove that the following formulas are tautologies using the truth table method.

1. the left-distribution of ' $\rightarrow$ ' over ' $\wedge$ ': $(p \rightarrow (q \wedge r)) \rightarrow ((p \rightarrow q) \wedge (p \rightarrow r))$;
2. the permutation of the premises law: $(p \rightarrow (q \rightarrow r)) \rightarrow (q \rightarrow (p \rightarrow r))$;
3. the reunion of the premises law: $(p \rightarrow (q \rightarrow r)) \rightarrow (p \wedge q \rightarrow r)$;
4. the separation of the premises law: $(p \wedge q \rightarrow r) \rightarrow (p \rightarrow (q \rightarrow r))$;
5. the left-distribution of ' $\vee$ ' over ' $\leftrightarrow$ ': $(p \vee (q \leftrightarrow r)) \rightarrow ((p \vee q) \leftrightarrow (p \vee r))$;
6. the left-distribution of ' $\vee$ ' over ' $\rightarrow$ ': $p \vee (q \rightarrow r) \rightarrow ((p \vee q) \rightarrow (p \vee r))$;
7. the left-distribution of ' $\rightarrow$ ' over ' $\leftrightarrow$ ': $(p \rightarrow (q \leftrightarrow r)) \rightarrow ((p \rightarrow q) \leftrightarrow (p \rightarrow r))$;
8. the left-distribution of ' $\rightarrow$ ' over ' $\vee$ ': $(p \rightarrow (q \vee r)) \rightarrow ((p \rightarrow q) \vee (p \rightarrow r))$.

**Exercise 5**

Transform the formulas $U_j, j \in \{1,2,...,8\}$ into their equivalent conjunctive and disjunctive normal forms. Using one of these forms prove that $U_j, j \in \{1,2,...,8\}$ are valid formulas in propositional logic.

1. $U_1 = (p \rightarrow \neg q) \wedge (q \vee r) \rightarrow (p \rightarrow r)$;
2. $U_2 = (p \rightarrow q) \wedge (p \wedge q \rightarrow r) \rightarrow (p \rightarrow r)$;
3. $U_3 = (p \wedge q \rightarrow r) \rightarrow (p \rightarrow (q \rightarrow r))$;
4. $U_4 = (p \rightarrow (q \vee r)) \rightarrow ((p \rightarrow q) \vee (p \rightarrow r))$;
5. $U_5 = (p \rightarrow q) \wedge (p \wedge q \rightarrow r) \rightarrow (p \rightarrow r)$;
6. $U_6 = (p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$;
7. $U_7 = (p \rightarrow (q \wedge r)) \rightarrow ((p \rightarrow q) \wedge (p \rightarrow r))$;
8. $U_8 = p \vee (q \rightarrow r) \rightarrow ((p \vee q) \rightarrow (p \vee r))$.

**Exercise 6**

Using the appropriate normal form write all the models of the following formulas:

1. $U_1 = (p \vee q \rightarrow r) \rightarrow (p \rightarrow r) \wedge q$;
2. $U_2 = \neg(\neg p \vee q) \vee r \rightarrow \neg p \wedge \neg(q \wedge r)$;
3. $U_3 = (p \wedge q \rightarrow r) \rightarrow (p \rightarrow r) \wedge q$;
4. $U_4 = (p \vee q) \wedge \neg r \rightarrow p \wedge q \wedge r$;
5. $U_5 = p \vee \neg(q \wedge \neg r) \rightarrow p \wedge q \wedge \neg r$;
6. $U_6 = (p \vee q \rightarrow r) \rightarrow (q \rightarrow r) \wedge p$;
7. $U_7 = (q \vee r \rightarrow p) \rightarrow (p \rightarrow r) \wedge q$;
8. $U_8 = (q \wedge r \rightarrow p) \rightarrow (p \rightarrow r) \wedge q$.

**Exercise 7**

Using the appropriate normal form, prove that the following formulas are inconsistent:

1. $U_1 = (p \rightarrow (q \rightarrow r)) \wedge \neg((p \rightarrow q) \rightarrow (p \rightarrow r))$;
2. $U_2 = (\neg p \vee q) \wedge \neg(\neg q \rightarrow \neg p)$;

3. $U_3 = (p \to q) \wedge (p \wedge q \to r) \wedge (p \wedge \neg r)$;
4. $U_4 = (p \to (q \vee r)) \wedge (\neg(p \to q) \wedge \neg(p \to r))$;
5. $U_5 = p \wedge (q \to r) \wedge ((p \wedge q) \wedge \neg(p \wedge r))$;
6. $U_6 = (p \to (q \to r)) \wedge (p \wedge q \wedge \neg r)$;
7. $U_7 = (p \to (q \to r)) \wedge \neg(q \to (p \to r))$;
8. $U_8 = (p \wedge q \to r) \wedge \neg(p \to (q \to r))$.

**Exercise 8**

Write all the anti-models of the following formulas using CNF.

1. $U_1 = (q \wedge r \to p) \to (p \to r) \wedge q$;
2. $U_2 = (q \vee r \to p) \to (p \to r) \wedge q$;
3. $U_3 = (p \vee q \to r) \to (q \to r) \wedge p$;
4. $U_4 = p \vee \neg(q \wedge \neg r) \to p \wedge q \wedge \neg r$;
5. $U_5 = p \wedge \neg(q \wedge \neg r) \to p \vee q \wedge \neg r$;
6. $U_6 = (p \wedge q \to r) \to (p \to r) \wedge q$;
7. $U_7 = \neg(\neg p \vee q) \vee r \to \neg p \wedge \neg(q \wedge r)$;
8. $U_8 = (p \vee q \to r) \to (p \to r) \wedge q$.

**Exercise 9**

Using the definition of deduction, prove the following deductions:

? 1. $p \to q, r \to t, p \vee r, \neg q \vdash t$; 2. $p \to r, p \vee r \to q, r \vdash q$;
3. $q \to p, t \to r, q \vee t, \neg p \vdash r$; 4. $p \vee (q \to r), p \vee q, \neg p \vdash r$;
5. $\neg p \vee \neg q \vee r, q, p \vdash r$; 6. $p \to \neg q \vee r, p \wedge q, p \vdash r$;
7. $r \vee (q \to p), r \vee q, \neg r \vdash p$; 8. $\cancel{p \to q, q \to r, r \to t, p \vdash t}$.

**Exercise 10**

Prove the following theorems using the theorem of deduction and its reverse.

1. $\vdash p \vee (q \to r) \to ((p \vee q) \to (p \vee r))$;
2. $\vdash (p \to (\neg r \to q)) \to (r \vee \neg p \vee q)$;
3. $\vdash (p \to (q \to r)) \to (p \wedge q \to r)$;
4. $\vdash (p \wedge q \to r) \to (p \to (q \to r))$;
5. $\vdash (p \to (q \to r)) \to (q \to (p \to r))$;
6. $\vdash (p \to (q \to r)) \to ((p \to q) \to (p \to r))$;
7. $\vdash (p \to q) \wedge (p \wedge q \to r) \to (p \to r)$;
8. $\vdash (p \to q) \to ((p \to r) \to (p \to q \wedge r))$.

**Exercise 11**
Using the theorem of deduction and its reverse prove that:

1. $\vdash (p \to (q \vee r)) \to ((p \to q) \vee (p \to r))$;
2. $\vdash (p \to q) \to ((\neg r \vee p) \to (r \to q))$;
3. $\vdash p \vee (q \to r) \to ((p \vee q) \to (p \vee r))$;
4. $\vdash (p \to r) \to ((q \to r) \to (p \vee q \to r))$;
5. $\vdash (p \to q) \to ((r \to t) \to (p \wedge r \to q \wedge t))$;
6. $\vdash (p \to r) \to ((p \wedge r \to q) \to (p \to q))$;
7. $\vdash (\neg q \vee p) \to ((s \to q) \to (s \to p))$;
8. $\vdash (p \to (q \to r)) \to (p \to (\neg r \to \neg q))$.

**Exercise 12**
H1: It is not sunny this afternoon and it is colder than yesterday.
H2: We will go swimming only if it is sunny.
H3: If we do not go swimming, then we will take a canoe trip.
H4: If we take a canoe trip, then we will be home by sunset.
C: We will be home by sunset.

Is C deducible from the set of hypotheses {H1,H2,H3,H4}?
If yes, build its deduction.

**Exercise 1**

Check the following properties for $\downarrow$ ('*nor*'), $\uparrow$ ('*nand*') and $\oplus$ ('*xor*') connectives using the truth table method.

$p \downarrow (q \downarrow r) \equiv (p \downarrow q) \downarrow r$ $\qquad$ $\boxed{q \downarrow r = \neg(q \vee r)}$

equivalence → identical truth table

| $p$ | $q$ | $r$ | $q \downarrow r$ | $p \downarrow (q \downarrow r)$ | $p \downarrow q$ | $(p \downarrow q) \downarrow r$ |
|---|---|---|---|---|---|---|
| T | T | T | F | F | F | F |
| T | T | F | F | F | F | T |
| T | F | T | F | F | F | F |
| T | F | F | T | F | F | T |
| F | T | T | F | T | F | F |
| F | T | F | F | T | F | T |
| F | F | T | F | T | T | F |
| F | F | F | T | F | T | F |

⇒ property doesn't hold

**Exercise 2**

Using the truth table method decide what kind of formula (consistent, inconsistent, tautology, contingent) is $U_j, j \in \{1,2,\ldots,8\}$. Write all the models and anti-models of $U_j, j \in \{1,2,\ldots,8\}$.

$U_2 = \neg p \vee \neg(q \wedge r) \rightarrow q \wedge \neg p$

| $p$ | $q$ | $r$ | $\neg p$ | $q \wedge r$ | $\neg(q \wedge r)$ | $\neg p \vee \neg(q \wedge r)$ | $q \wedge \neg p$ | $U_2$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |

- consistent (at least one model) ✓
- inconsistent (no models)
- tautology (all models)
- contingent (at least one of each) ✓
- models (interpretations which are true)
- anti-models (interpretations which are false)

MODELS ( $i : \{p, q, r\} \rightarrow \{T, F\}$ s.t. $U_2(p, q, r) = T$ )

$i_1(p) = F$ $\qquad$ $i(p) = F$ $\qquad$ $i_3(p) = T$

$i_1(q) = T$ $\qquad$ $i(q) = T$ $\qquad$ $i_3(q) = T$

$i_1(r) = F$ $\qquad$ $i(r) = T$ $\qquad$ $i_3(r) = T$

ANTI MODELS ( $i : \{p, q, r\} \rightarrow \{T, F\}$ s.t. $U_2(p, q, r) = F$ )

**Exercise 3**
Using the truth table method, check whether the following logical consequences hold:

$$p \to q \vDash (q \to r) \to (p \to r)$$

| $p$ | $q$ | $r$ | $p \to q$ | $q \to r$ | $p \to r$ | $(q \to r) \to (p \to r)$ |
|---|---|---|---|---|---|---|
| F | F | F | T | T | T | T |
| F | F | T | T | T | T | T |
| F | T | F | T | F | T | T |
| F | T | T | T | T | T | T |
| T | F | F | F | T | F | F |
| T | F | T | F | T | T | T |
| T | T | F | T | F | F | T |
| T | T | T | T | T | T | T |

The *logical consequence* notion is a generalization of the *tautology* notion:

**Definition**: The formula $V$ is a *logical consequence* of the formula $U$, notation: $U \vDash V$, if $\forall i : F_P \to \{T, F\}$ such that $i(U)$=T, we have $i(V)$=T.

**Definition:** The formulas $U$ and $V$ are *logically equivalent*, notation: $U \equiv V$, if they have identical truth tables.

$A \vDash B$ when $A = T$, $B = T$

## Exercise 4.

Prove that the following formulas are tautologies using the truth table method.

$$(p \to (q \wedge r)) \to ((p \to q) \wedge (p \to r))$$

| $p$ | $q$ | $r$ | $q \wedge r$ | $p \to (q \wedge r)$ | $p \to q$ | $p \to r$ | $(p \to q) \wedge (p \to r)$ | $U$ |
|---|---|---|---|---|---|---|---|---|
| F | F | F | F | T | T | T | T | T |
| F | F | T | F | T | T | T | T | T |
| F | T | F | F | T | T | T | T | T |
| F | T | T | T | T | T | T | T | T |
| T | F | F | F | F | F | F | F | T |
| T | F | T | F | F | F | T | F | T |
| T | T | F | F | F | T | F | F | T |
| T | T | T | T | T | T | T | T | T |

$\Rightarrow$ TAUTOLOGY

**Exercise 5**

Transform the formulas $U_j, j \in \{1,2,\ldots,8\}$ into their equivalent conjunctive and disjunctive normal forms. Using one of these forms prove that $U_j, j \in \{1,2,\ldots,8\}$ are valid formulas in propositional logic.

$$U_2 = (p \to q) \wedge (p \wedge q \to r) \to (p \to r)$$

$$\Leftrightarrow (\bar{p} \vee q) \wedge (\overline{p \wedge q} \vee r) \to (\bar{p} \vee r)$$

$$\Leftrightarrow (\bar{p} \vee q) \wedge (\bar{p} \vee \bar{q} \vee r) \to \bar{p} \vee r$$

$$\Leftrightarrow \overline{(\bar{p} \vee q) \wedge (\bar{p} \vee \bar{q} \vee r)} \vee \bar{p} \vee r$$

$$\Leftrightarrow \overline{(\bar{p} \vee q)} \vee \overline{(\bar{p} \vee \bar{q} \vee r)} \vee \bar{p} \vee r$$

$$\Leftrightarrow (p \wedge \bar{q}) \vee (p \wedge q \wedge \bar{r}) \vee (\bar{p} \vee r)$$

$$\Leftrightarrow (p \wedge \bar{q}) \vee (\bar{p} \vee r) \vee (p \wedge q \wedge \bar{r})$$

$$\Leftrightarrow (\overset{T}{p \vee \bar{p}} \vee r) \wedge (\bar{q} \vee \bar{p} \vee r) \vee (p \wedge q \wedge \bar{r})$$

$$\Leftrightarrow (\bar{q} \vee \overbrace{\bar{p} \vee r \vee p}^{T}) \wedge (\overbrace{\bar{q} \vee \bar{p} \vee r \vee q}^{T}) \wedge (\bar{q} \vee \bar{p} \vee \overbrace{r \vee \bar{r}}^{T})$$

$$\Leftrightarrow T \Rightarrow \text{TAUTOLOGY}$$

**Exercise 6**
Using the appropriate normal form write all the models of the following formulas:

$$U_2 = \overline{(\bar p \vee q)} \vee r \to \bar p \wedge \overline{(q \wedge r)}$$

$$\Leftrightarrow (p \wedge \bar q) \vee r \to \bar p \wedge (\bar q \vee \bar r)$$

$$\Leftrightarrow \overline{(p \wedge \bar q) \vee r} \ \vee (\bar p \wedge \bar q) \vee (\bar p \wedge \bar r)$$

$$\Leftrightarrow (\bar p \vee q) \wedge \bar r \ \vee \ (\bar p \wedge \bar q) \vee (\bar p \wedge \bar r)$$

$$\Leftrightarrow \underset{①}{(\bar p \wedge \bar r)} \vee \underset{②}{(q \wedge \bar r)} \vee \underset{③}{(\bar p \wedge \bar q)} \vee \cancel{(\bar p \wedge \bar r)}$$

① $= T \Rightarrow p = F, r = F, q \in \{F, T\}$

② $= T \Rightarrow q = T, r = F, p \in \{F, T\}$

③ $= T \Rightarrow p = F, q = F, r \in \{F, T\}$

**Exercise 7**
Using the appropriate normal form, prove that the following formulas are inconsistent:

$$(\bar p \vee q) \wedge \overline{(\bar q \to \bar p)}$$

$$\Leftrightarrow (\bar p \vee q) \wedge \overline{(q \vee \bar p)}$$

$$\Leftrightarrow (\bar p \vee q) \wedge (\bar q \wedge p)$$

$$\Leftrightarrow (\overbrace{\bar p \wedge \bar q \wedge p}^{F}) \vee (\overbrace{q \wedge \bar q}^{F} \wedge p) \Leftrightarrow F \Leftrightarrow \text{INCONSISTENT}$$

**Exercise 8**
Write all the anti-models of the following formulas using CNF.

$$U_2 = (q \vee r \to p) \to (p \to r) \wedge q$$

$$\Leftrightarrow (\bar q \wedge \bar r) \vee p \to (\bar p \vee r) \wedge q$$

$$\Leftrightarrow (q \vee \bar r) \wedge \bar p \ \vee (\bar p \wedge q) \vee (r \wedge q)$$

$$\Leftrightarrow (q \wedge \bar p) \vee (\bar r \wedge \bar p) \vee \cancel{(\bar p \wedge q)} \vee (r \wedge q)$$

$$\Leftrightarrow (q \vee \bar r) \wedge (q \vee \bar p) \wedge (\bar p \vee \bar r) \wedge \bar p \ \vee (r \wedge q)$$

$$\Leftrightarrow \underset{①}{(q \vee r)} \wedge \underset{②}{(q \vee \bar p)} \wedge \underset{③}{(\bar p \vee r)} \wedge \cancel{(\bar p \vee r)} \vee \cancel{(\bar p \wedge q)}$$

① $= F \Rightarrow q = F, r = F, p \in \{T, F\}$

② $= F \Rightarrow q = F, p = T, r \in \{T, F\}$

③ $= F \Rightarrow p = T, r = F, q \in \{T, F\}$

## Exercise 9

Using the definition of deduction, prove the following deductions:

$$\boxed{p \to r,\ p \vee r \to q,\ r \vdash q}$$

$f_1 : p \to r$

$f_2 : p \vee r \to q$

$f_3 : r$

$f_3 \vdash_{\text{addition}} p \vee r$

$f_4 : p \vee r$

$f_4, f_2 \vdash_{mp} q$

$f_5 : q$

| ADDITION |
|---|
| $U \vdash U \vee V$ |

| MODUS PONEMS |
|---|
| $U,\ U \to V \vdash V$ |

## Exercise 10

Prove the following theorems using the theorem of deduction and its reverse.

$$\vdash (p \to (\bar{r} \to q)) \to (r \vee \bar{p} \vee q)$$

STEP 1

if $\vdash (p \to (\bar{r} \to q)) \to (r \vee \bar{p} \vee q)$

then $(p \to (\bar{r} \to q)) \vdash (r \vee \bar{p} \vee q)$

if $(p \to (\bar{r} \to q)) \vdash \overline{(r \vee \bar{p})} \to q$

then $(p \to (\bar{r} \to q)),\ \bar{r} \wedge p \vdash q$

(apply the reverse of th. of deduction)

STEP 2

$f_1 : p \to (\bar{r} \to q)$

$f_2 : \bar{r} \wedge p$

$f_2 \vdash_{\text{simpl.}} \bar{r}$

$f_2 \vdash_{\text{simpl.}} p$

$f_3 : \bar{r}$

$f_4 : p$

$f_1, f_4 \vdash_{mp} \bar{r} \to q$

$f_5 : \bar{r} \to q$

$f_3, f_5 \vdash_{mp} q$

$f_6 : q$

! We can deduce two theorems depending on the order in which we apply theorem of deduction !

**Exercise 11**

Using the theorem of deduction and its reverse prove that:

$$\vdash (p \to q) \to ((\bar{r} \vee p) \to (r \to q))$$

STEP 1: APPLY TH. OF DEDUCTION (REVERSE)

if $\vdash (p \to q) \to ((\bar{r} \vee p) \to (r \to q))$
then if $p \to q \vdash ((\bar{r} \vee p) \to (r \to q))$
then if $p \to q, \bar{r} \vee p \vdash r \to q$
then $p \to q, \bar{r} \vee p, r \vdash q$

STEP 2: PROVE THIS DEDUCTION

$f_1: p \to q$

$f_2: \bar{r} \vee p \equiv r \to p$

$f_3: r$

$f_2, f_3 \vdash_{mp} p$

$f_4: p$

$f_1, f_4: q$

$f_5: q$

STEP 3: To the deduction $p \to q, \bar{r} \vee q, r \vdash q$, we apply 3 times th. of deduction

There are $3! = 6$ possibilities to move the premises to the right-hand side of $\vdash$ and we can prove 6 theorems.

Possibilities:

1. $p \to q, \bar{r} \vee p, r$
2. $p \to q, r, \bar{r} \vee p$
3. $\bar{r} \vee p, r, p \to q$
4. $\bar{r} \vee p, p \to q, r$
5. $r, \bar{r} \vee p, p \to q$
6. $r, p \to q, \bar{r} \vee p$

(move in this order)

Possibility 1:

if $p \to q, \bar{r} \vee p, r \vdash q$
then if $p \to q, \bar{r} \vee p \vdash r \to q$
then if $p \to q \vdash (\bar{r} \vee p) \to (r \to q)$
then $\vdash (p \to q) \to ((\bar{r} \vee p) \to (r \to q))$

⋮

**Exercise 12**

H1: It is not sunny this afternoon and it is colder than yesterday.
H2: We will go swimming only if it is sunny.
H3: If we do not go swimming, then we will take a canoe trip.
H4: If we take a canoe trip, then we will be home by sunset.
C: We will be home by sunset.

Is C deducible from the set of hypotheses {H1,H2,H3,H4}?
If yes, build its deduction.

$\overline{sun} \wedge col,\ sun \leftrightarrow swim,\ \overline{swim} \to canoe,\ canoe \to home \vdash home$

$H_1$: $\overline{sun} \wedge col$
$H_2$: $sun \leftrightarrow swim \equiv (sun \to swim) \wedge (swim \to sun)$
$H_3$: $\overline{swim} \to canoe$
$H_4$: $canoe \to home$
$H_5$: $\overline{sun}$ ($H_1 \vdash_{simp}$)
$H_6$: $swim \to sun$ ($H_2 \vdash_{simp}$)
$H_7$: $\overline{sun} \to \overline{swim}$ ($H_6 \vdash_{mt}$)
$H_8$: $\overline{swim}$ ($H_5, H_7 \vdash_{mp}$)
$H_9$: $canoe$ ($H_8, H_3 \vdash_{mp}$)
$H_{10}$: $home$ ($H_9, H_4 \vdash_{mp}$)

$(H_1, H_2, \dots, H_{10})$ is the deduction of our conclusion from premises