1. Find a Cartesian equation of the line ℓ in $\mathbf{A}^2(\mathbb{R})$ containing the points $P = S \cap S'$ and $Q = T \cap T'$ where

$$S : x + 5y - 8 = 0, \quad S' : 3x + 6 = 0, \quad T : 5x - \frac{1}{2}y = 1, \quad T' : x - y = 5.$$

2. Deterimine an equation for the line in $\mathbf{A}^2(\mathbb{C})$ parallel to $\mathbf{v}$ and passing through $S \cap T$ in each of the following cases:

1. $\mathbf{v} = (2,4)$, $S : 3x - 2y - 7 = 0$, $T : 2x + 3y = 0$,
2. $\mathbf{v} = (-5\sqrt{2}, 7)$, $S : x - y = 0$, $T : x + y = 1$.

3. Let ABC be a triangle in some affine space X. Consider the points C' and B' on the sides AB and AC of the triangle ABC such that

$$\overrightarrow{AC'} = \lambda \overrightarrow{BC'} \quad \text{and} \quad \overrightarrow{AB'} = \mu \overrightarrow{CB'}.$$

The lines BB' and CC' meet in the point M. For a fixed but arbitrary point $O \in X$ show that

$$\overrightarrow{OM} = \frac{\overrightarrow{OA} - \lambda \overrightarrow{OB} - \mu \overrightarrow{OC}}{1 - \lambda - \mu}.$$

4. Consider the triangle ABC in $\mathbb{E}^n$ with side lengths a, b, c. Let G be its centroid, H the orthocenter and I the incenter. For a fixed but arbitrary point $O \in X$, show that

1. $\overrightarrow{OG} = \frac{\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}}{3}$,
2. $\overrightarrow{OI} = \frac{a\overrightarrow{OA} + b\overrightarrow{OB} + c\overrightarrow{OC}}{a+b+c}$,
3. $\overrightarrow{OH} = \frac{(\tan\hat{A})\overrightarrow{OA} + (\tan\hat{B})\overrightarrow{OB} + (\tan\hat{C})\overrightarrow{OC}}{\tan\hat{A} + \tan\hat{B} + \tan\hat{C}}$.

5. In some affine space, consider the angle BOB' and the points $A \in [OB]$, $A' \in [OB']$. Show that

$$\overrightarrow{OM} = m\frac{1-n}{1-mn}\overrightarrow{OA} + n\frac{1-m}{1-mn}\overrightarrow{OA'}$$
$$\overrightarrow{ON} = m\frac{n-1}{n-m}\overrightarrow{OA} + n\frac{m-1}{m-n}\overrightarrow{OA'}$$

where $M = AB' \cap A'B$ and $N = AA' \cap BB'$ and where $\overrightarrow{OB} = m\overrightarrow{OA}$ and $\overrightarrow{OB'} = n\overrightarrow{OA'}$.

6. Show that the midpoints of the diagonals of a complete quadrilateral are collinear.

7. Prove the following generalziation of Thales' theorem.

In an affine space $\mathbf{A}$ over $\mathbf{K}$ let H, H', H'' be three distinct parallel hyperplanes, and ℓ_1 and ℓ_2 be lines which are not parallel to H, H', H''. Let $P_i = \ell_i \cap H$, $P_i' = \ell_i \cap H'$, $P_i'' = \ell_i \cap H''$ (for $i = 1, 2$), and let k_1, k_2 be the scalars such that

$$\overrightarrow{P_iP_i''} = k_i \overrightarrow{P_iP_i'} \quad i = 1, 2.$$

Then $k_1 = k_2$.