

Final Exam in Calculus (2)
Group 812 – June 23, 2020

1. (2 points) Let $f : \mathbb{R} \times (0, \infty) \times (0, \infty) \rightarrow \mathbb{R}$ be defined by

$$f(x, y, z) = x^3 y^2 + 2yz + \ln(xy).$$

Determine $\nabla f(x, y, z)$ for an arbitrary point $(x, y, z) \in \mathbb{R} \times (0, \infty) \times (0, \infty)$. Compute the particular value $\nabla f(0, 1, 2)$.

2. (2 points) Evaluate

$$\iiint_A (x^2 + y^2)^2 \, dx dy dz$$

where

$$A = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 \leq 1, 0 \leq z \leq 1, y \geq 0\}.$$

3. (1.5 points) Consider the function $f : \mathbb{R}^3 \rightarrow \mathbb{R}$, defined by

$$f(x, y, z) = 2xy^2 - 4xy + x^2 + z^2 - 2z.$$

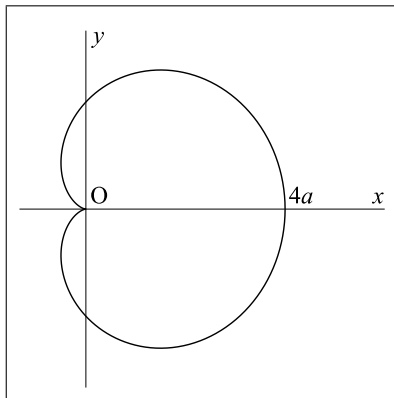
Study its local extrema points.

4. (2 points) Calculate $\iint_A (x^2 + y^2) \, dx dy$, where $A := \{(x, y) \in \mathbb{R}^2 \mid 2y \leq x^2 + y^2 \leq 4y\}$.

5. (1.5 points) Let $a > 0$ and let C be the set defined by

$$C := \{(x, y) \mid x^2 + y^2 - 2a(x + \sqrt{x^2 + y^2}) = 0\}.$$

The points of C are located on a plane curve, called *cardioid* (see the figure below). Find the greatest distance from a point on the cardioid to the Ox axis.



All problems are mandatory. One point is awarded ex officio.
The solutions will be sent to the e-mail address tiberiutrif@gmail.com.