SEMINAR 7

1) Show that the Abelian group $(\mathbb{R}_+^*, \cdot)$ is an $\mathbb{R}$-vector space with the external operation $*$ defined by

$$\alpha * x = x^\alpha,\ \alpha \in \mathbb{R},\ x \in \mathbb{R}_+^*\,.$$

2) Let V be a K-vector space an let M be a set. Show that V^M is a K-vector space with the pointwise operations on V^M, i.e.

$$(f+g)(x) = f(x) + g(x),\ (\alpha f)(x) = \alpha f(x),\ \forall f, g \in V^M,\ \forall \alpha \in K.$$

3) Can one organize a finite set M as a vector space over an infinite field K?

4) Let $p \in \mathbb{N}$ be a prime. Can one organize the Abelian group $(\mathbb{Z}, +)$ as a vector space over the field $(\mathbb{Z}_p, +, \cdot)$?

5) Which of the following subsets is a subspace in the space mentioned nearby:

a) $A = \{(x,y) \in \mathbb{R}^2 \mid ax + by = 0\}$, $(a, b \in \mathbb{R}$ are given) in ${}_{\mathbb{R}}\mathbb{R}^2$;

b) $D = [-1, 1]$ in ${}_{\mathbb{R}}\mathbb{R}$;

b') $D' = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1\}$ in ${}_{\mathbb{R}}\mathbb{R}^2$;

b") $D'' = \{(x_1, \dots, x_n) \in \mathbb{R}^n \mid x_1^2 + \dots + x_n^2 \leq 1\}$ in ${}_{\mathbb{R}}\mathbb{R}^n$;

c) $P_n(\mathbb{R}) = \{f \in \mathbb{R}[X] \mid \mathrm{grad} f \leq n\}$ in ${}_{\mathbb{R}}\mathbb{R}[X]$ ($n \in \mathbb{N}$ is given);

d) $B = \{f \in \mathbb{R}[X] \mid \mathrm{grad} f = n\}$ in ${}_{\mathbb{R}}\mathbb{R}[X]$ ($n \in \mathbb{N}$ is given)?

6) Let V be a K-vector space, $A \leq_K V$ and $C_V A = V \setminus A$.

i) Is $C_V A$ a subspace in ${}_K V$?

ii) What about $C_V A \cup \{0\}$?