1. Show that similarity of matrices is an equivalence relation in $\mathrm{Mat}_n(\mathbf{K})$.

2. Show that the eigenvalue associated to an eigenvector is uniquely determined.

3. Show that if $\mathbf{v}_1, \mathbf{v}_2 \in \mathbf{V}$ are eigenvectors with the same eigenvalue λ, then for every $c_1, c_2 \in \mathbf{K}$ the vector $c_1\mathbf{v}_1 + c_2\mathbf{v}_2$, if it is non-zero, is also an eigenvector with eigenvalue λ.

4. Give the characteristic polynomial, the eigenvectors and eigenvalues for the identity matrix Id_n and the zero matrix 0_n.

5. Give the eigenvalues of $\mathrm{lin}(\mathrm{Pr}_{H,\mathbf{v}})$, $\mathrm{lin}(\mathrm{Pr}_{\ell,\mathbf{W}})$, $\mathrm{lin}(\mathrm{Ref}_{H,\mathbf{v}})$ and $\mathrm{lin}(\mathrm{Ref}_{\ell,\mathbf{W}})$. What can you say about the eigenvectors?

6. Give the characteristic polynomial, the eigenvectors and eigenvalues for the matrices

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & & & & & \vdots \\ 0 & 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & 1 & \dots & 0 & 0 \\ \vdots & & & & & \vdots \\ 0 & 0 & 0 & \dots & 1 & 1 \\ 0 & 0 & 0 & \dots & 0 & 1 \end{bmatrix}$$

Are these matrices diagonalizable?

7. Consider the matrix

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Show that A doesn't have eigenvectors when considered in $\mathrm{Mat}_{n\times n}(\mathbb{R})$. Show that A is diagonalizable when considered in $\mathrm{Mat}_{n\times n}(\mathbb{C})$ and find the eigenvectors of A.

8. Find the eigenvalues and eigenvectors of the following matrices in $\mathrm{Mat}_{2\times 2}(\mathbb{R})$:

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

9. Let $\phi : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear operator

$$\phi(x, y, z) = (x + y - z, y + z, 2x).$$

Find the matrix $[\phi]_{\boldsymbol{b}}$ where

$$\boldsymbol{b} = \{(1,1,0), (-1,0,1), (1,1,1)\}.$$

10. Calculate the eigenvalues and their algebraic and geometric multiplicities for the following matrices in $\mathrm{Mat}_{3\times 3}(\mathbb{R})$, and deduce whether or not they are diagonalizable:

$$\begin{bmatrix} -6 & 2 & -5 \\ -4 & 4 & -2 \\ 10 & -3 & 8 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & -15 \\ 0 & 2 & 8 \end{bmatrix}$$

11. Show that if $A = (a_{ij}) \in \mathrm{Mat}_{2\times 2}(\mathbf{K})$ then

$$P_A(T) = T^2 - \mathrm{tr}(A)T + \det(A)$$

where $\mathrm{tr}(A) = a_{11} + a_{22}$ is the *trace* of A.