SEMINAR 12

1) In the $\mathbb{Q}$-vector space $\mathbb{Q}^3$ we consider the vectors

$$a = (-2, 1, 3),\ b = (3, -2, -1),\ c = (1, -1, 2),\ d = (-5, 3, 4),\ e = (-9, 5, 10).$$

Does the following equality $\langle a, b\rangle = \langle c, d, e\rangle$ hold?

2) In the $\mathbb{R}$-vector space $\mathbb{R}^4$ one considers the subspaces:

a) $S = \langle u_1, u_2\rangle$, with $u_1 = (1, 1, 0, 0)$, $u_2 = (1, 0, 1, 1)$,
$T = \langle v_1, v_2\rangle$, with $v_1 = (0, 0, 1, 1)$, $v_2 = (0, 1, 1, 0)$;

b) $S = \langle u_1, u_2, u_3\rangle$, with $u_1 = (1, 2, -1, -2)$, $u_2 = (3, 1, 1, 1)$, $u_3 = (-1, 0, 1, -1)$,
$T = \langle v_1, v_2\rangle$, with $v_1 = (-1, 2, -7, -3)$, $v_2 = (2, 5, -6, -5)$;

c) $S = \langle u_1, u_2\rangle$, with $u_1 = (1, 2, 1, 0)$, $u_2 = (-1, 1, 1, 1)$,
$T = \langle v_1, v_2\rangle$, with $v_1 = (2, -1, 0, 1)$, $v_2 = (1, -1, 3, 7)$;

d) $S = \langle u_1, u_2, u_3\rangle$, with $u_1 = (1, 2, 1, -2)$, $u_2 = (2, 3, 1, 0)$, $u_3 = (1, 2, 2, -3)$,
$T = \langle v_1, v_2, v_3\rangle$, with $v_1 = (1, 1, 1, 1)$, $v_2 = (1, 0, 1, -1)$, $v_3 = (1, 3, 0, -3)$.

Find a basis and the dimension for each of the subspaces S, T, $S + T$ and $S \cap T$.