Babeş-Bolyai University Cluj-Napoca
Faculty of Mathematics and Computer Science

Final Exam in Calculus (2)
Group 814 – June 24, 2020

$\boxed{1.}$ **(2 points)** Let $f : (0, \infty) \times (0, \infty) \times (0, \infty) \to \mathbb{R}$ be defined by

$$f(x, y, z) = xy + \cos(3y) + \ln(xyz).$$

Determine $\nabla f(x, y, z)$ for an arbitrary point $(x, y, z) \in (0, \infty)\times(0, \infty)\times(0, \infty)$. Compute the particular value $\nabla f\left(1, \frac{\pi}{6}, 1\right)$.

$\boxed{2.}$ **(2 points)** Evaluate

$$\iiint_A e^{\left(x^2+y^2+z^2\right)^{\frac{3}{2}}} \,\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

where

$$A = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \le 1, z \ge 0\}.$$

$\boxed{3.}$ **(1.5 points)** Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$, defined by

$$f(x, y) = 4x^2 + 9y^2 + 8x - 36y + 24.$$

Study its local extrema points.

$\boxed{4.}$ **(2 points)** Determine $\alpha \in \mathbb{R}$ such that the function $f : \mathbb{R}^2 \to \mathbb{R}$, defined by

$$f(x, y) := \begin{cases} \dfrac{\ln(1 + x^2y^2)}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ \alpha & \text{if } (x, y) = (0, 0), \end{cases}$$

is continuous at $(0, 0)$. For that value of α study the differentiability of f at $(0, 0)$.

$\boxed{5.}$ **(1.5 points)** Calculate $\iint_A (x^2 + y^2)\,\mathrm{d}x\mathrm{d}y$, where $A = \{(x, y) \in \mathbb{R}^2 \mid x \ge |y|,\ x^2 + y^2 \le 2x\}$.

All problems are mandatory. One point is awarded ex officio.
The solutions will be sent to the e-mail address tiberiutrif@gmail.com.