1. Show that translations are isometries.

2. Show that an isometry is bijective.

3. Determine the matrix form of a rotation with angle 45° having the same center of rotation as the rotation

$$f(\mathbf{x}) = \frac{1}{\sqrt{13}}\begin{bmatrix} 2 & -3 \\ 3 & 2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ -2 \end{bmatrix}.$$

4. Determine the cosine of the angle of the rotation f given in the previous exercise and find the inverse rotation, f^{-1}.

5. Let T be the isometry obtained by applying a rotation of angle $-\frac{\pi}{3}$ around the origin after a transation with vector $(-2,5)$. Determine the inverse transformation, T^{-1}.

6. Find the eigenvectors for each of the following symmetric matrices:

$$A = \begin{bmatrix} 73 & 36 \\ 36 & 52 \end{bmatrix}, \quad B = \begin{bmatrix} -94 & 180 \\ 180 & 263 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 128 & 240 \\ 240 & 450 \end{bmatrix}.$$

7. Determine the sum-of-angles formulas for sine and cosine using rotation matrices.

8. Verify that the matrices

$$A = \frac{1}{3}\begin{bmatrix} -1 & 2 & -2 \\ -2 & -2 & -1 \\ -2 & 1 & 2 \end{bmatrix} \quad \text{and} \quad B = \frac{1}{11}\begin{bmatrix} -9 & -2 & 6 \\ 6 & -6 & 7 \\ 2 & 9 & 6 \end{bmatrix}$$

belong to $SO(3)$. Moreover, determine the axis of rotation and the rotation angle.

9. Show that $O(n)$ is a subgroup of $\mathrm{AGL}(\mathbb{R}^n)$. Show that $SO(n)$ is a normal subgroup of $O(n)$.

10. Show that the Gram-Schmidt process produces an orthonormal basis.

11. In an orthonormal basis, consider the vectors $\mathbf{v}_1(0,1,0)$, $\mathbf{v}_2(2,1,0)$ and $\mathbf{v}_3(-1,0,0)$. Use the Gram-Schmidt process to find an orthonormal basis containing $\mathbf{v}_1$.

12. Prove that in a Euclidean vector space $(\mathbf{V}, \langle \_, \_ \rangle)$ the following identities hold, for any $\mathbf{v}, \mathbf{w} \in \mathbf{V}$.

1. $||\mathbf{v}+\mathbf{w}||^2 + ||\mathbf{v}-\mathbf{w}||^2 = 2||\mathbf{v}||^2 + 2||\mathbf{w}||^2$,
2. $||\mathbf{v}+\mathbf{w}||^2 - ||\mathbf{v}-\mathbf{w}||^2 = 4\langle \mathbf{v}, \mathbf{w} \rangle$.

13. Consider two points $P(a,b)$ and $Q(c,d)$. Show that a rotation around P with angle θ followed by a rotation around Q with angle $-\theta$ is a translation and determine the corresponding translation vector.

14. Show that in $\mathbb{E}^2$ orthogonal reflections in lines are isometries. Show that in $\mathbb{E}^n$ orthogonal reflections in hyperplanes are isometies.