Babeș-Bolyai University Cluj-Napoca
Faculty of Mathematics and Computer Science

Final Exam in Calculus (2)
Group 811 – June 24, 2020

$\boxed{\textbf{1.}}$ **(2 points)** Let $f : \mathbb{R}^3 \to \mathbb{R}$ be defined by

$$f(x, y, z) = e^{x+y+z} + xy^2 + \sin z.$$

Determine $\nabla f(x, y, z)$ for an arbitrary point $(x, y, z) \in \mathbb{R}^3$. Compute the particular value $\nabla f(0, 1, 0)$.

$\boxed{\textbf{2.}}$ **(2 points)** Evaluate

$$\iiint_A e^{(x^2+y^2+z^2)^{\frac{3}{2}}}\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

where

$$A = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 1, z \geq 0\}.$$

$\boxed{\textbf{3.}}$ **(1.5 points)** Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$, defined by

$$f(x, y) = x^3 + 2xy - 6x - 4y^2.$$

Study its local extrema points.

$\boxed{\textbf{4.}}$ **(2 points)** Determine $\alpha \in \mathbb{R}$ such that the function $f : \mathbb{R}^2 \to \mathbb{R}$, defined by

$$f(x, y) := \begin{cases} \dfrac{\sin(xy)}{\sqrt{x^2 + y^2}} & \text{if } (x, y) \neq (0, 0) \\ \alpha & \text{if } (x, y) = (0, 0), \end{cases}$$

is continuous at $(0, 0)$. For that value of α study the differentiability of f at $(0, 0)$.

$\boxed{\textbf{5.}}$ **(1.5 points)** Calculate $\displaystyle\iint_A (x^2 + y^2)\,\mathrm{d}x\mathrm{d}y$, where $A = \{(x, y) \in \mathbb{R}^2 \mid y \geq |x|,\ x^2 + y^2 \leq 2y\}$.

All problems are mandatory. One point is awarded ex officio.
The solutions will be sent to the e-mail address tiberiutrif@gmail.com.