$(x,y)$ randomly selected = midpoint of a chord
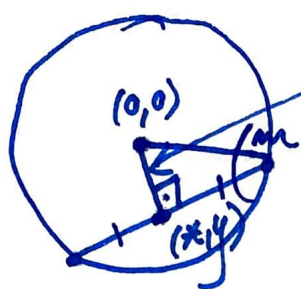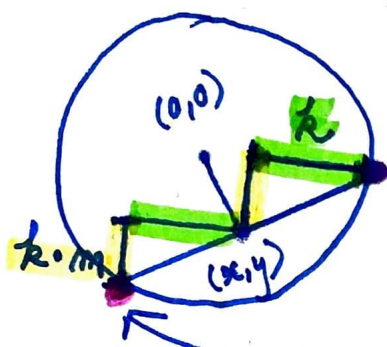


bisector (perpendicular)

$$slope = \frac{-x}{y} = m$$



$\Rightarrow$ $(x-k, \; y-k*m) =$ chord_x

$(x+k, \; y+k*m) =$ chord_y

$$\cancel{r^2 = d((x+k))}$$

$$r^2 = d((x+k, y+k\cdot m), (x-k, y-k\cdot m))^2$$
$$= ((y+km) - (y-km))^2 + ((x+k) - (x-k))^2$$
$$= (2km)^2 + (2k)^2$$
$$= 4k^2 m^2 + 4k^2$$
$$= 4k^2(m^2+1)$$

solve for $k$, since we know $r$:

$$k = \sqrt{\frac{r^2}{4(m^2+1)}} = \sqrt{\frac{8h(R-\frac{h}{2})}{4(m^2+1)}} = \sqrt{\frac{2h(R-\frac{h}{2})}{\frac{x^2}{y^2}+1}}$$