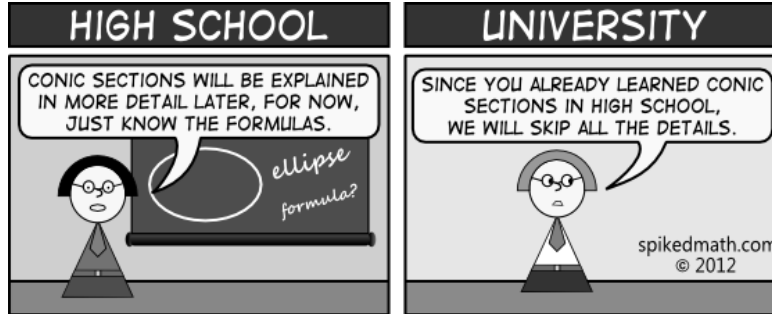


## ANALYTIC GEOMETRY, PROBLEM SET 12



1. Find the equation of the circle:
  - a) passing through  $A(3, 1)$  and  $B(-1, 3)$  and having the center on the line  $d : 3x - y - 2 = 0$ ;
  - b) determined by  $A(1, 1)$ ,  $B(1, -1)$  and  $C(2, 0)$ ;
  - c) tangent to both  $d_1 : 2x + y - 5 = 0$  and  $d_2 : 2x + y + 15 = 0$ , if the tangency point with  $d_1$  is  $M(3, 1)$ .
2.
  - a) Determine the position of the point  $A(1, -2)$  relative to the circle  $C : x^2 + y^2 - 8x - 4y - 5 = 0$ ;
  - b) Find the intersection between the line  $d : 7x - y + 12 = 0$  and the circle  $C : (x - 2)^2 + (y - 1)^2 - 25 = 0$ ;
  - c) Determine the position of the line  $d : 2x - y - 3 = 0$  relative to the circle  $C : x^2 + y^2 - 3x + 2y - 3 = 0$ .
3. Find the equation of
  - a) the tangent line to  $C : x^2 + y^2 - 5 = 0$  at the point  $A(-1, 2)$ ;
  - b) the tangent lines to  $C : x^2 + y^2 + 10x - 2y + 6 = 0$ , parallel to  $d : 2x + y - 7 = 0$ ;
  - c) the tangent lines to  $C : x^2 + y^2 - 2x + 4y = 0$ , orthogonal on  $d : x - 2y + 9 = 0$ .
4. Find the foci of the ellipse  $\mathcal{E} : 9x^2 + 25y^2 - 225 = 0$ .
5. Find the intersection points between the line  $d_1 : x + 2y - 7 = 0$  and the ellipse given by the equation  $\mathcal{E} : x^2 + 3y^2 - 25 = 0$ .
6. Find the position of the line  $d : 2x + y - 10 = 0$  relative to the ellipse  $\mathcal{E} : \frac{x^2}{9} + \frac{y^2}{4} - 1 = 0$ .
7. Find the equation of a line which is orthogonal on  $d_1 : 2x - 2y - 13 = 0$  and tangent to the ellipse  $\mathcal{E} : x^2 + 4y^2 - 20 = 0$ .
8. Consider the ellipse  $x^2 + 4y^2 = 25$ . Find the chords on the ellipse which have the point  $A(7/2, 7/4)$  as their midpoint.
9. Consider the ellipse  $\frac{x^2}{4} + y^2 = 1$  with  $F_1, F_2$  as foci. Find the points  $M$ , situated on the ellipse for which the angle  $\angle F_1 M F_2$  is right.

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**10.** Consider the ellipse  $\frac{x^2}{4} + y^2 = 1$  with  $F_1, F_2$  as foci. Find the point  $M$  on the ellipse for which  $\angle F_1 M F_2$  is maximal.

**11.** Consider the ellipse  $\frac{x^2}{25} + \frac{y^2}{9} = 1$ . Determine the geometric locus of the midpoints of the chords on the ellipse which are parallel to the line  $x + 2y = 1$ .