BABEŞ-BOLYAI UNIVERSITY
Faculty of Mathematics and Computer Science

Algorithms and Programming

Lecture 9 – Search and sorting algorithms

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Course content

Programming in the large

- Introduction in the software development process
- Procedural programming
- Modular programming
- Abstract data types
- Software development principles
- Testing and debugging

Programming in the small

- Recursion
- Complexity of algorithms
- **Search and sorting algorithms**
- Backtracking and other problem solving methods
- Recap

Last time

- Recursion
    - Basic concept
    - Mechanism
    - Recursive functions

- Computational complexity
    - Analyzing the efficiency of a program
    - Run time complexity
    - Classes of complexity

Today

- Search
  - Objective and problem specification
  - Types
    - Sequential seach
    - Binary search
- Sort
  - Objective and problem specification
  - Types
    - Selection sort
    - Insert sort
    - Bubble sort
    - Quick sort

Search methods: Objective

- For a set of data stored in memory as a list of elements (el1, el2, ..., eln)
  - The list may contain elements in any order
  - The list contains elements ordered by some criteria
- Look for
  - A certain element
  - Elements that satisfy different criteria
- Return
  - True or False – if the element(s) exist in the list
  - The index of the element found

Search methods: problem specification

- Unorderd list of elements
  - Input data:
    - elem, n, list = ($list_i$) i=0,1, 2,...,n-1 (n – natural number)
  - Results:
    - p, where $0 \leq p \leq n-1$, if elem = list[p] or -1, if elem is not in the list
- Ordered list of elements
  - Input data:
    - elem, n, list = ($list_i$), list[0]<list[1]<...<list[n-1], i=0,1, 2,...,n-1 (n – natural number)
  - Results:
    - p, where $0 \leq p \leq n-1$, if elem = list[p] or -1, if elem is not in the list

Search methods: implementation

- **Sequential search**
  - Basic idea: the elements of the list are examined one by one (the list can be ordered or not)
  - Versions: simple and improved
- **Binary search**
  - Basic idea: the problem is divided in two similar but smaller subproblems (the list has to be ordered)

- Python
  - Functions index and find

Sequential search: implementation
Unordered list

```python
def searchSeq(el, l):
    '''
    Descr: search for an element in a list
    Data: an element and a list
    Res: the position of element in list or -1 if the elemnt is not in the list
    '''
    pos = -1
    for i in range(0, len(l)):
        if (el == l[i]):
            pos = i
    return pos

def test_searchSeq():
    assert searchSeq(2, [3,2,4]) == 1
    assert searchSeq(2, [3,5,7,2]) == 3
    assert searchSeq(2, [2,5,4]) == 0
    assert searchSeq(2, [3,7,4]) == -1
    assert searchSeq(2, [3,2,4,2,7]) == 3

test_searchSeq()
```

Case	T(n)
Best case	$\sum_{i=1}^{n} 1 = \mathrm{n}$
Worst case	$\sum_{i=1}^{n} 1 = \mathrm{n}$
Average case	$\sum_{i=1}^{n} 1 = \mathrm{n}$

O(n)

Sequential search: implementation
Unordered list – Improved version

```python
def searchSeq_v2(el, l):
    '''
    Descr: search for an element in a list
    Data: an element and a list
    Res: the position of element in list or
    -1 if the elemnt is not in the list
    '''
    i = 0
    while ((i < len(l)) and (l[i] != el)):
        i = i + 1
    if (i < len(l)):
        return i
    else:
        return -1

def test_searchSeq_v2():
    assert searchSeq_v2(2, [3,2,4]) == 1
    assert searchSeq_v2(2, [3,5,7,2]) == 3
    assert searchSeq_v2(2, [2,5,4]) == 0
    assert searchSeq_v2(2, [3,7,4]) == -1
    assert searchSeq_v2(2, [3,2,4,2,7]) == 1
test_searchSeq_v2()
```

Case	**T(n)**
Best case	1
Worst case	$\sum_{i=1}^{n} 1 = \mathrm{n}$
Average case	(0+1+2+...+n-1)/n

Sequential search: implementation *Ordered list*

```python
def searchSeqOrder(el, l):
    '''
    Descr: search for an element in a list
    Data: an element and a list of ordered elements
    Res: the position of element in list or the position where the element can be inserted
    '''
    if (len(l) == 0): #l==[]
        return 0
    pos = -1
    for i in range(len(l) - 1, -1, -1):
        if (el <= l[i]):
            pos = i
    if (pos == -1):
        return len(l)
    return pos

def test_searchSeqOrder():
    assert searchSeqOrder(2, [2,3,4]) == 0
    assert searchSeqOrder(4, [2,3,4,5]) == 2
    assert searchSeqOrder(2, [1,3,5,7]) == 1
    assert searchSeqOrder(9, [1,2,3]) == 3

test_searchSeqOrder()
```

Case	T(n)
Best case	$\sum_{i=1}^{n} 1 = \mathrm{n}$
Worst case	$\sum_{i=1}^{n} 1 = \mathrm{n}$
Average case	$\sum_{i=1}^{n} 1 = \mathrm{n}$

Sequential search: implementation
Ordered list – Improved version

```
def searchSeqOrder_v2(el, l):
    '''
    Descr: search for an element in a list
    Data: an element and a list of ordered elements
    Res: the position of element in list or
    the position where the element can be inserted
    '''
    if (len(l) == 0): #l==[]
        return 0
    if (el <= l[0]):
        return 0
    if (el > l[len(l)-1]):
        return len(l)
    i = 0
    while ((i < len(l)) and (l[i] < el)):
        i = i + 1
    return i
def test_searchSeqOrder_v2():
    assert searchSeqOrder_v2(2, [2,3,4]) == 0
    assert searchSeqOrder_v2(4, [2,3,4,5]) == 2
    assert searchSeqOrder_v2(2, [1,3,5,7]) == 1
    assert searchSeqOrder_v2(9, [1,2,3]) == 3
test_searchSeqOrder_v2()
```

Case	**T(n)**
Best case	1
Worst case	$\sum_{i=1}^{n} 1 = n$
Average case	(0+1+2+...+n-1)/n

Binary search: implementation
Ordered list – recursive version

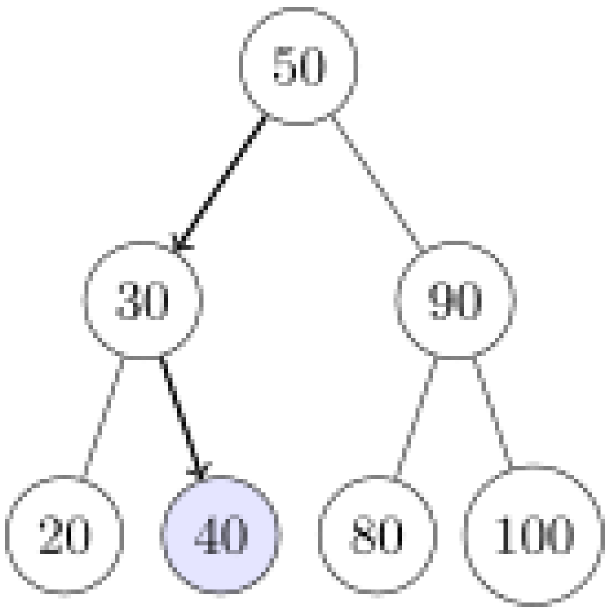
```
def binarySearch(el, l, start, end):
    if (start > end):
        return -1
    middle = (start + end) // 2
    if (el < l[middle]):
        return binarySearch(el, l, start, middle)
    elif (el > l[middle]):
        return binarySearch(el, l, middle + 1, end)
    else: #el == l[middle]
        return middle

def binarySearchRec(el, l):
    #Descr: search for an element in a list
    #Data: an element and a list
    #Res: the position of element in list or
    # -1 if the element is not in the list
    if (len(l) == 0):
        return -1
    elif (el < l[0]) or (el > l[len(l) -1]):
        return -1
    else:
        return binarySearch(el, l, 0, len(l)-1)
```

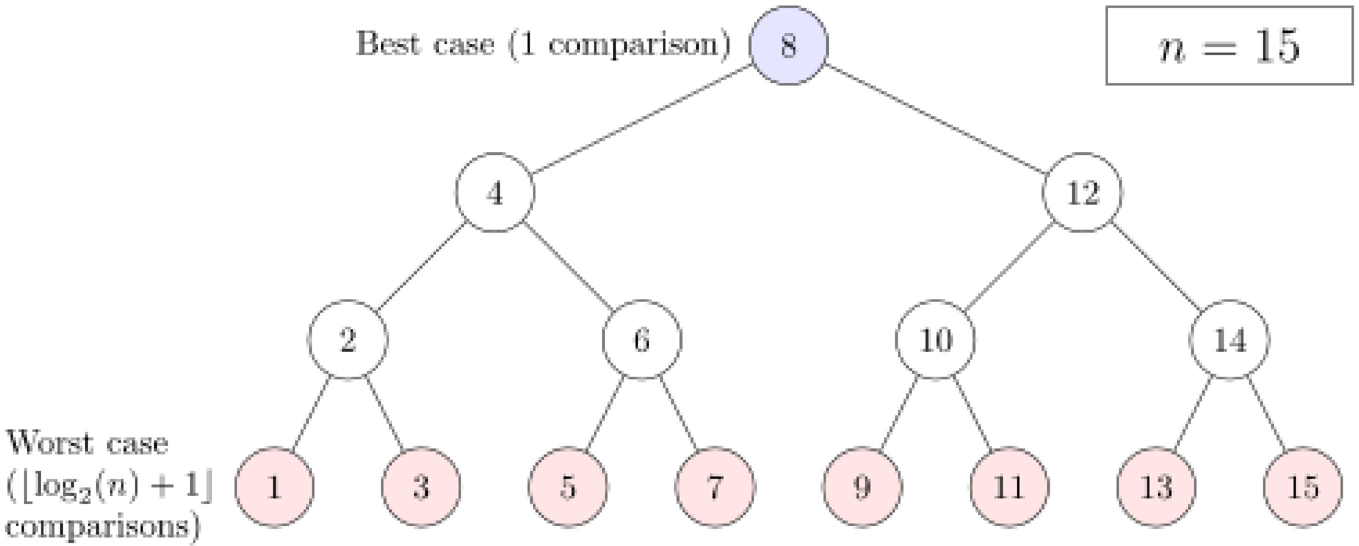
```
def test_binarySearchRec():
    assert binarySearchRec(4, [3,4,5]) == 1
    assert binarySearchRec(2, [-3,0,1,2]) == 3
    assert binarySearchRec(2, [2,5,6]) == 0
    assert binarySearchRec(2, [3,7,9]) == -1
    assert binarySearchRec(2, [1,2,2,5,6]) == 2

test_binarySearchRec()
```

Case	T(n)
Best case	1
Worst case	$\log_2 n$
Average case	$\log_2 n$

Binary search: example

- List is [20, 30, 40, 50, 80, 90, 100]
- Search for element 40





https://en.wikipedia.org/wiki/Binary_search_algorithm

Binary search: implementation
Ordered list – iterative version

```python
def binarySearchIter(el, l):
    '''
    Descr: search for an element in a list
    Data: an element and a list
    Res: the position of element in list or
    -1 if the elemnt is not in the list
    '''
    if (len(l) == 0):
        return -1
    elif (el < l[0]) or (el > l[len(l) -1]):
        return -1
    else:
        start = 0
        end = len(l) - 1
        while (start <= end):
            middle = (start + end) // 2
            if (el < l[middle]):
                end = middle
            elif (el > l[middle]):
                start = middle + 1
            else:
                return middle
```

```python
def test_binarySearchIter():
    assert binarySearchIter(4, [3,4,5]) == 1
    assert binarySearchIter(2, [-3,0,1,2]) == 3
    assert binarySearchIter(2, [2,5,6]) == 0
    assert binarySearchIter(2, [3,7,9]) == -1
    assert binarySearchIter(2, [1,2,2,5,6]) == 2
test_binarySearchIter()
```

$$T(n) = \begin{cases} 1, if\ n = 1 \\ T(n/2) + 1, otherwise \end{cases}$$

$$if\ n = 2^k \rightarrow T(2^k) = T(2^{k-1}) + 1$$

$$T(2^k) = T(2^{k-1}) + 1$$

$$T(2^{k-1}) = T(2^{k-2}) + 1$$

$$\dots$$

$$T(2^1) = T(2^0) + 1$$

$$T(2^k) = k + 1 \qquad k = \log_2 n \rightarrow T(n) = \log_2 n + 1$$

Search methods: Python functions

- list.index(element)
  - Returns the index of the element in the list
  - If the element does not exist in the list, throws an exception
- list.count(element)
  - Returns the number of times the element appears in the list (if it exists)
  - Returns 0 if the element is not in the list

```python
def test_index():
    l = [7,2,13,4,1]
    assert l.index(2) == 1
    assert l.index(1) == 4
    try:
        l.index(3)
        assert False
    except ValueError as ex:
        print("elem not found")
        assert True

test_index()
```

```python
def test_count():
    l = [7,2,13,4,1]
    assert l.count(2) == 1
    assert l.count(1) == 1
    assert l.count(3) == 0

test_count()
```

Sorting methods

- Objective
  - Rearrange the elements of a container such that they are in a certain relation of order

- Problem specification
  - Input data:
    - n, list = ($list_i$) i=0,1, 2,...,n-1 (n – natural number)
  - Results:
    - n, list' = ($list_i$') i=0,1, 2,...,n-1, orderRelation($list_i$', $list_{i+1}$')=True for any i=0,1,...,n-2

Sorting methods: taxonomy

- Place where the elements are stored
  - **Internal sort** – data to be sorted is available in the internal memory
  - **External sort** – data available from files (external memory)
- The order relation
  - **Ascending sort**
  - **Descending sort**
- Keeping the initial order of the elements
  - **Stable sort –** keep the initial order of equal elements
  - **Instable sort** – the initial order of equal elements is not kept
- Space complexity
  - **In-place sort** – the additional space (to that needed for the container) is small
  - **Not-in-place / Out-of-place sort** – large additional space
- Mechanism
  - **Selection sort**
  - **Insert sort**
  - **Bubble sort**
  - **Quick sort**

Selection Sort

- Basic idea
  - Determine the smallest element from the collection and place it in first position (swap the smallest element with the first one)
  - Repeat the first step for all elements different from the smallest element
- Algorithm
- Complexity
  - Time

    $T(n) = \sum_{i=0}^{n-2}\left(\sum_{j=i}^{n-1} 1 + 3\right) \approx n(n-1)/2$ -> ***$O(n^2)$***
  - Space

    S(n)=n+1+1

```python
def selectionSort(l):
    '''
    descr: sorts the leemnts of a list
    data: a list of elements
    res: the ordered list
    '''
    for i in range(0, len(l) - 1):
        min_pos = i
        for j in range(i + 1, len(l)):
            if (l[j] < l[min_pos]):
                min_pos = j
        if (i < min_pos):
            aux = l[i]
            l[i] = l[min_pos]
            l[min_pos] = aux
    return l
```

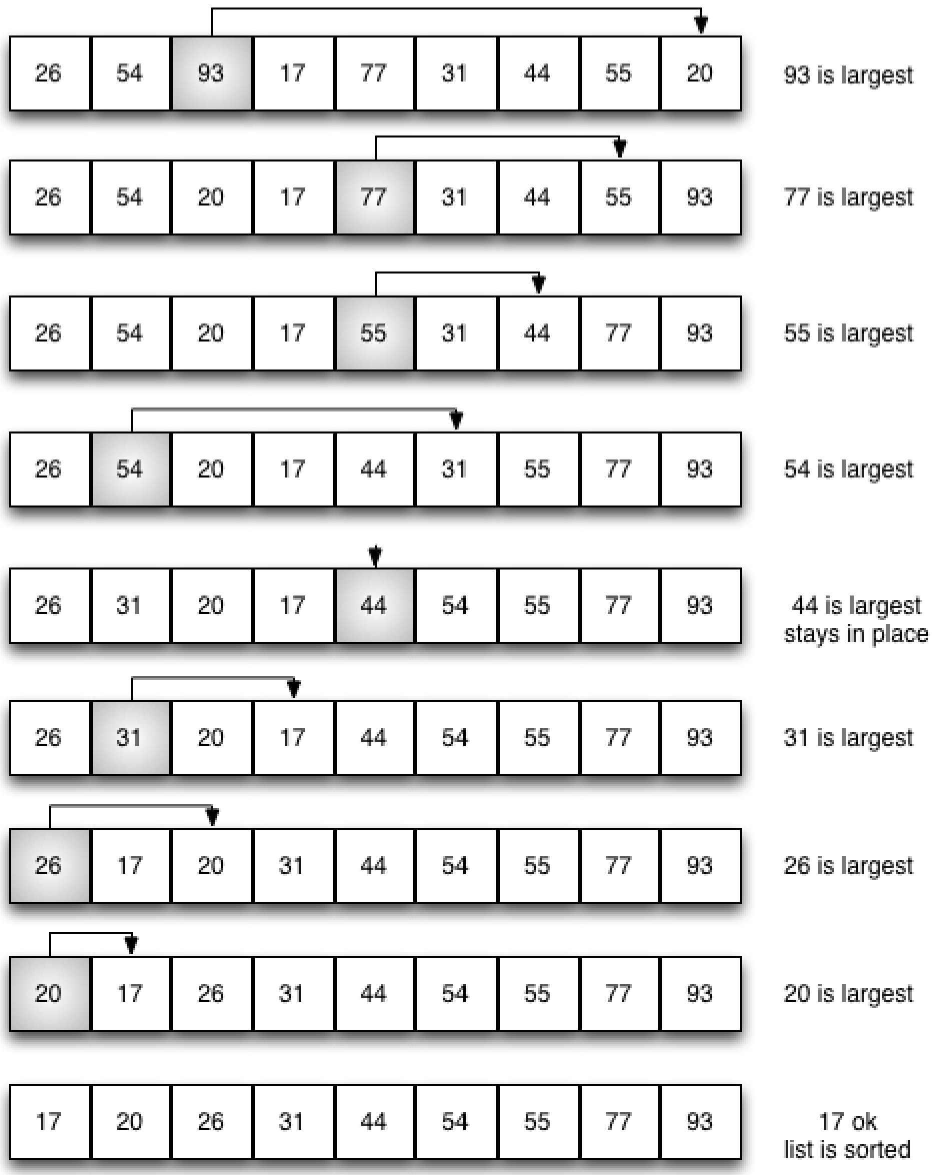
```python
l[i], l[min_pos] = l[min_pos], l[i]
```

```python
def test_selectionSort():
    assert selectionSort([1,2,3]) == [1,2,3]
    assert selectionSort([3,2,1]) == [1,2,3]
    assert selectionSort([1,2,1]) == [1,1,2]

test_selectionSort()
```

Selection Sort: Example

- Each step, select the *largest* item and place it in the proper position

Alternative: each step, select the *smallest* item and place it in the proper position



http://interactivepython.org/

Insertion Sort

- Basic idea
  - Traverse the elements of the container and insert each element at the correct position in the sub-container with the elements already sorted
  - At the end of the algorithm, the sub-container will have all the initial elements sorted
- Algorithm
- Complexity
  - Time

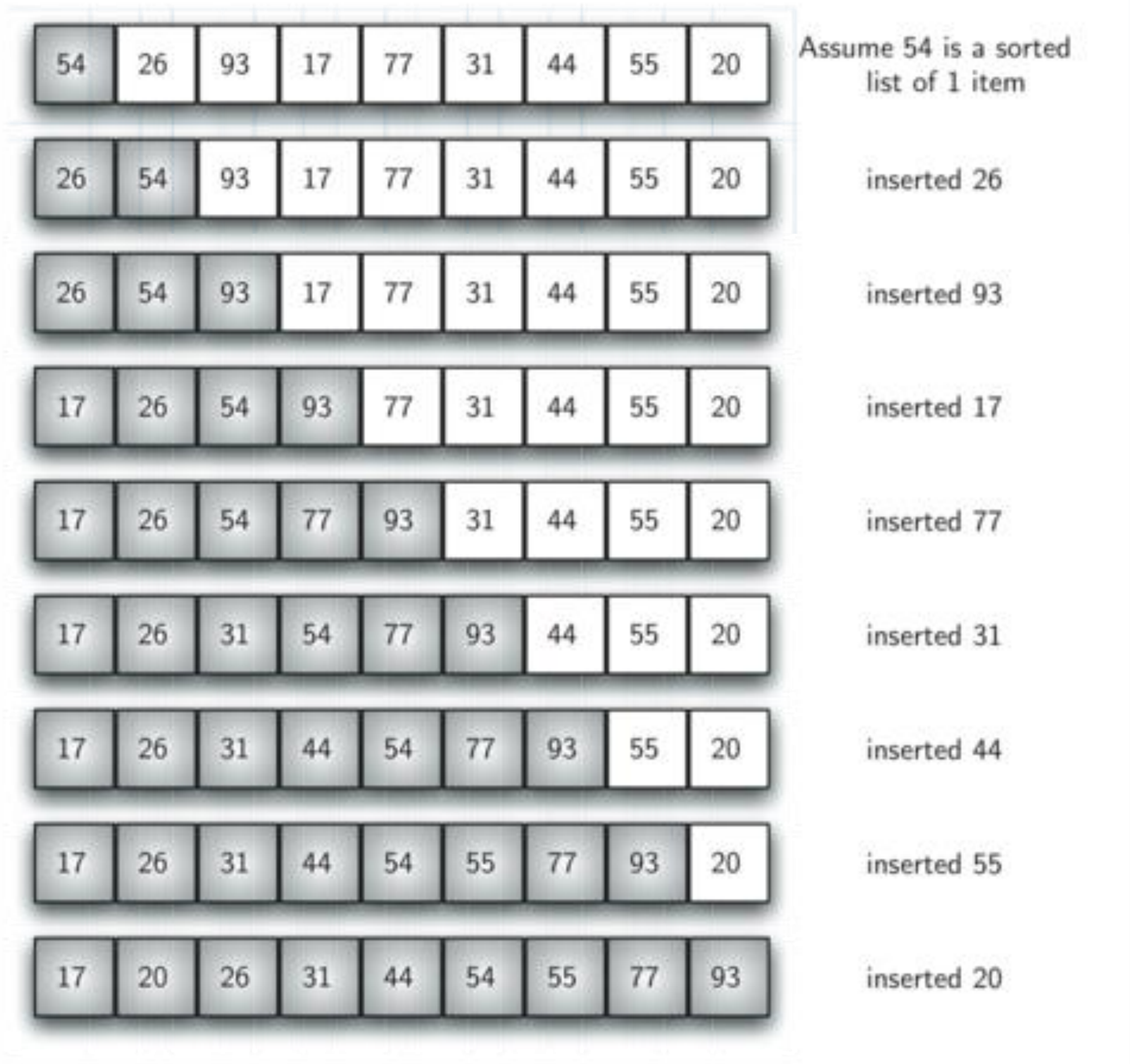
    $T(n) = \sum_{i=1}^{n-1}\left(1 + \sum_{j=0}^{i-1} 2 + 1\right) \approx n^2 + n - 2$ -> ***O(n²)***
  - Space

    S(n)=n+1+1+1

```python
def insertSort(l):
    '''
    descr: sorts the elemnts of a list
    data: a list of elements
    res: the ordered list
    '''
    for i in range(1, len(l)):
        noOfAlreadySort = i - 1
        crtElem = l[i]
        # insert crtElem in the right position
        # (<= noOfAlreadySort)
        j = noOfAlreadySort
        while ((j >= 0) and (crtElem < l[j])):
            l[j + 1] = l[j]
            j = j - 1
        l[j + 1] = crtElem
    return l

def test_insertSort():
    assert insertSort([1,2,3]) == [1,2,3]
    assert insertSort([3,2,1]) == [1,2,3]
    assert insertSort([1,2,1]) == [1,1,2]

test_insertSort()
```

Insertion Sort: Example

- Maintain sorted sublists
- Insert each new item in the sorted sublist at the proper position



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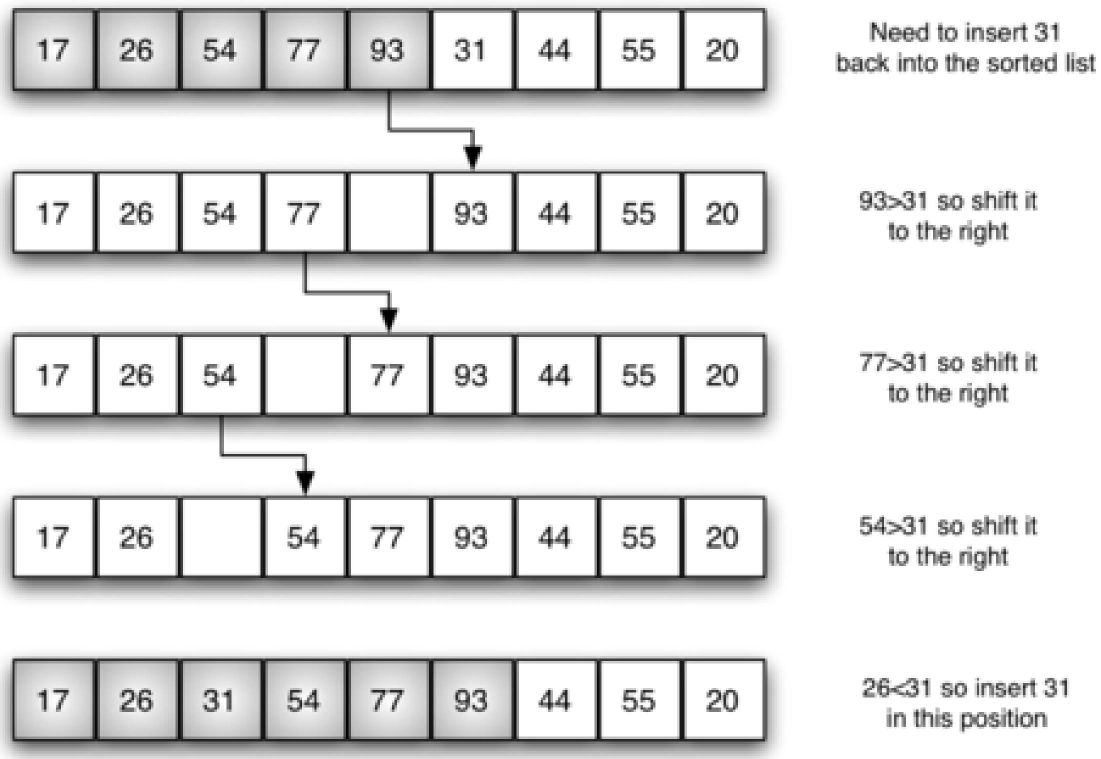
Insertion Sort: Example



http://interactivepython.org/

Bubble Sort

- Basic idea
  - Compare any 2 consecutive elements
    - If they are not in correct order, swap them
  - Until any 2 consecutive elements are in the correct order
- Algorithm
- Complexity
  - Time

    $T(n)$ -> ***O(n²)***
  - Space

    S(n)=n+1+1+1

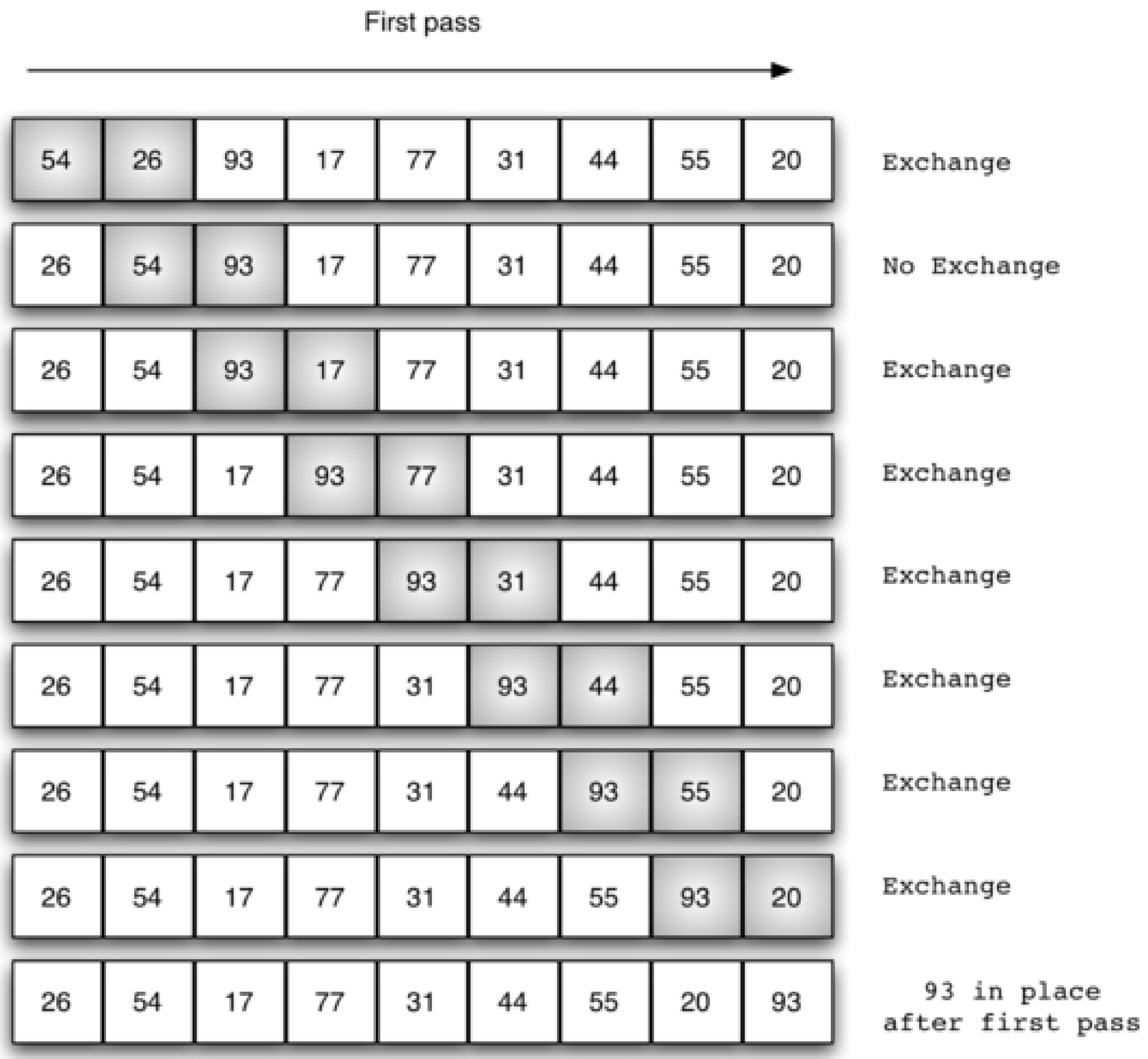
```python
def bubbleSort(l):
    '''
    descr: sorts the elemnts of a list
    data: a list of elements
    res: the ordered list
    '''
    isSort = False
    while (not isSort):
        isSort = True
        for i in range(0, len(l) - 1):
            if (l[i] > l[i + 1]):
                aux = l[i]
                l[i] = l[i + 1]
                l[i + 1] = aux
                isSort = False
    return l

def test_bubbleSort():
    assert bubbleSort([1,2,3]) == [1,2,3]
    assert bubbleSort([3,2,1]) == [1,2,3]
    assert bubbleSort([1,2,1]) == [1,1,2]

test_bubbleSort()
```

Bubble Sort: Example

- Exchange adjacent items that are not in correct order



http://interactivepython.org/

Quick Sort

- Basic idea
  - Divide and conquer technique
  1. **Divide:** divide the container in 2 parts such that any element in the first sub-container ≤ any element in the second sub-container
  2. **Conquer:** sort the two sub-containers (recursively)
- Algorithm

```
def test_quickSort():
    assert quickSort([1,2,3]) == [1,2,3]
    assert quickSort([3,2,1]) == [1,2,3]
    assert quickSort([1,2,1]) == [1,1,2]

test_quickSort()
```

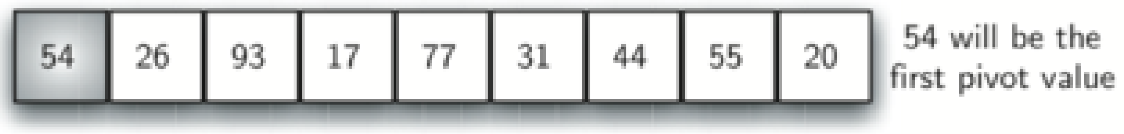
```
def partition(l, start, end):
    pivot = l[start]
    i = start
    j = end
    while (i != j):
        while ((pivot <= l[j]) and (i < j)):
            j = j - 1
        l[i] = l[j]
        while ((l[i] <= pivot) and (i < j)):
            i = i + 1
        l[j] = l[i]
        l[i] = pivot
    return i

def quickSortRec(l, start, end):
    pivotPos = partition(l, start, end)
    if (start < pivotPos - 1):
        quickSortRec(l, start, pivotPos - 1)
    if (pivotPos + 1 < end):
        quickSortRec(l, pivotPos + 1, end)

def quickSort(l):
    '''
    descr: sorts the elemnts of a list
    data: a list of elements
    res: the ordered list
    '''
    quickSortRec(l, 0, len(l) - 1)
    return l
```

Quick Sort: Example

- First pivot value – first item



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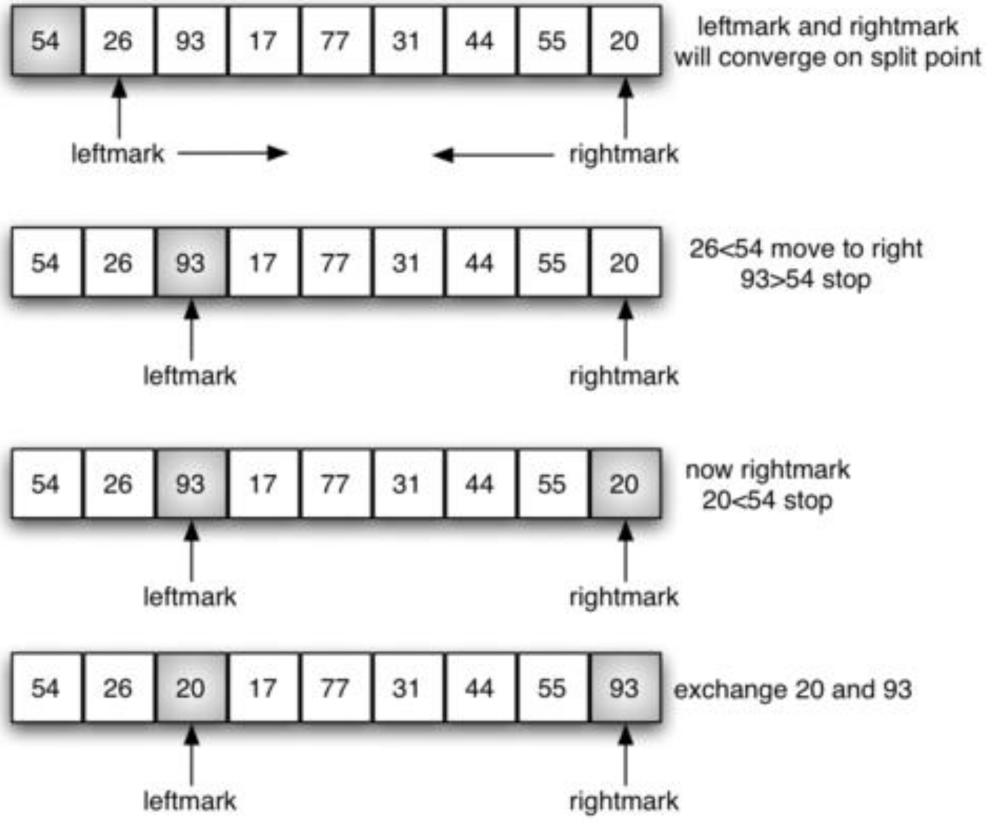
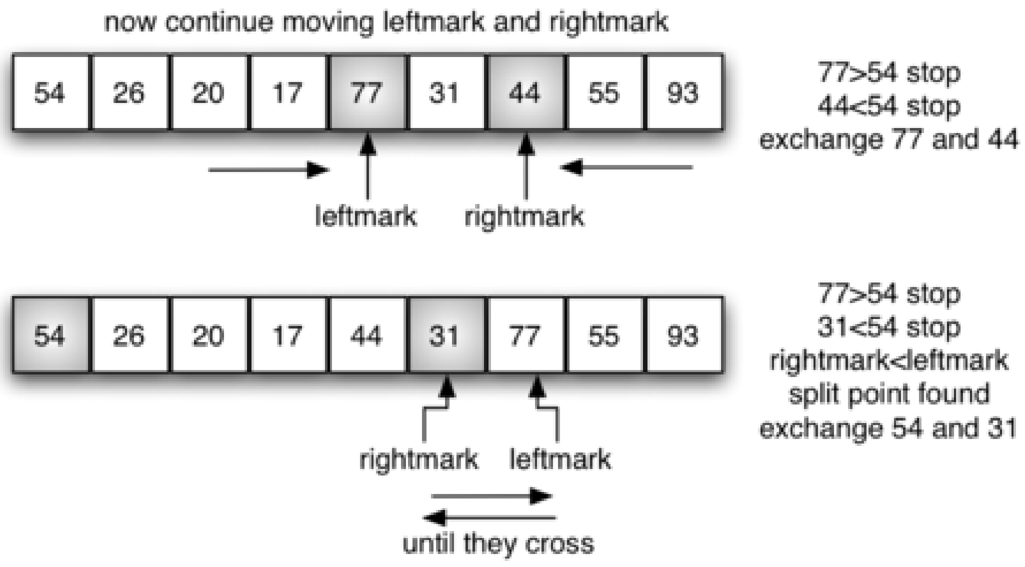
Quick Sort: Example

- Partition process

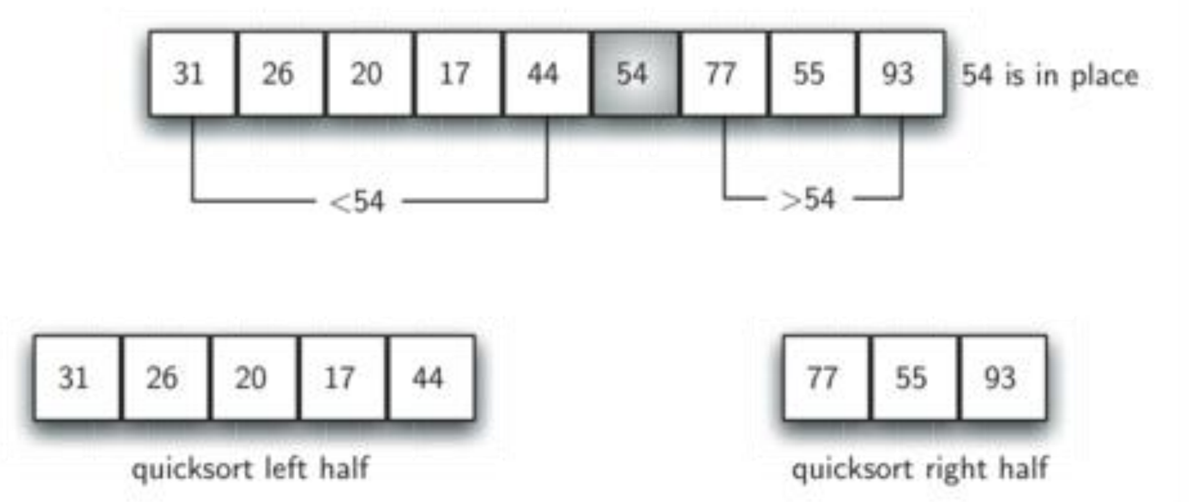




http://interactivepython.org/

Quick Sort: Example

- Recursive call for the left half and right half



http://interactivepython.org/

Quick Sort: complexity

- The run time of quick-sort depends on the distribution of splits
  - The partitioning function requires linear time
  - Best case is when the partitioning function splits the array evenly

Best case	$T(n)=2*T(n/2)+n \rightarrow O(n\, log_2\, n)$
Worst case	T(n)=T(1)+T(n-1)+n=T(n-1)+n+1 T(n-1)=T(1)+T(n-2)+(n-1)=T(n-2)+n … T(2)=T(1)+T(1)+2=4 _______________ T(n)=(n+1)(n+2)/2-(1+2+3)=>$O(n^2)$
Average case	L(n)=2*U(n/2)+n U(n)=L(n-1)+n L(n)=2*(L(n/2-1)+n/2)+n=2L(n/2-1_+2n=>$O(n\, log_2\, n)$

- Space complexity
  - Average: S(n) = $\log_2 n$
  - Worst: S(n) = n^2

Sorting in Python

- `list.sort()`
- `sorted(lista)`

```
>>> t = (5, 1, 17, 12)
>>> sorted(t)
[1, 5, 12, 17]
>>> t
(5, 1, 17, 12)
```

```
>>> a = [2, 1, 5, 7, 9]
>>> a
[2, 1, 5, 7, 9]
>>> sorted(a)
[1, 2, 5, 7, 9]
>>> a
[2, 1, 5, 7, 9]
>>> sorted(a, reverse=True)
[9, 7, 5, 2, 1]
>>> a
[2, 1, 5, 7, 9]
>>> a.sort()
>>> a
[1, 2, 5, 7, 9]
>>> a.sort(reverse=True)
>>> a
[9, 7, 5, 2, 1]
```

Sorting in Python: list of lists / tuples

- Use the key argument in sorted/sort

```
>>> my_list = [[2,1,3], [1,5,7], [7,2,1]]
>>> def getKey(item):
        return item[1]

>>> sorted(my_list, key=getKey)
[[2, 1, 3], [7, 2, 1], [1, 5, 7]]
>>> my_list
[[2, 1, 3], [1, 5, 7], [7, 2, 1]]
>>> sorted([(1,1,1), (15,0,16), (25,5,0)], key=getKey)
[(15, 0, 16), (1, 1, 1), (25, 5, 0)]
```

key argument &
lambda expressions

```
>>> sorted(my_list, key=lambda x: x[1])
[[2, 1, 3], [7, 2, 1], [1, 5, 7]]
>>> sorted(my_list, key=lambda x: x[2])
[[7, 2, 1], [2, 1, 3], [1, 5, 7]]
>>> sorted(my_list, key=lambda x: x[0])
[[1, 5, 7], [2, 1, 3], [7, 2, 1]]
```

Sorting in Python: list of custom objects

```python
class Student:
    def __init__(self, name, grade):
        self.__name = name
        self.__grade = grade

    def getName(self):
        return self.__name

    def getGrade(self):
        return self.__grade
```

```
>>> st_list = [Student("Sara", 8), Student("Erin", 10), Student("Emma", 9)]
>>> def getKey(s):
        return s.getGrade()

>>> sorted(st_list, key=getKey)
[<__main__.Student object at 0x02E5BFD0>, <__main__.Student object at 0x02E73090
>, <__main__.Student object at 0x02E5B3B0>]
>>> st_list
[<__main__.Student object at 0x02E5BFD0>, <__main__.Student object at 0x02E5B3B0
>, <__main__.Student object at 0x02E73090>]
```

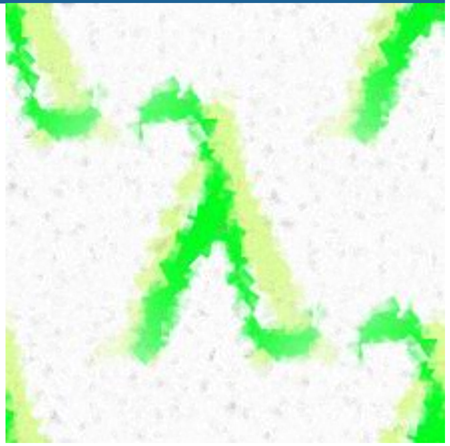
Sorting in Python: list of custom objects

```python
class Student:
    def __init__(self, name, grade):
        self.__name = name
        self.__grade = grade

    def getName(self):
        return self.__name

    def getGrade(self):
        return self.__grade

    def __repr__(self):
        return self.__name + " - " + str(self.__grade)
```

```
>>> sorted(st_list, key=lambda x: x.getGrade())
[Sara - 8, Emma - 9, Erin - 10]
>>> sorted(st_list, key=lambda x: x.getName())
[Emma - 9, Erin - 10, Sara - 8]
>>>
>>> sorted(st_list, key=lambda Student: Student.getName())
[Emma - 9, Erin - 10, Sara - 8]
```

```
>>> st_list = [Student("Sara", 8), Student("Erin", 10), Student("Emma", 9)]
>>> def getKey(s):
        return s.getGrade()

>>> sorted(st_list, key=getKey)
[Sara - 8, Emma - 9, Erin - 10]
>>> st_list
[Sara - 8, Erin - 10, Emma - 9]
```

Lambda expressions



- Small anonymous functions
- Defined and used in the same place

✓ *Syntactically restricted to a single expression*

✓ *Can reference variables from the containing scope (just like nested functions)*

✓ *They are syntactic sugar for a function definition*

Lambda expressions

- Syntax

```
lambda arg1, arg2, ...argN : expression using arguments
```

- Lambda is an expression

(def - a function with name, statement)

- Body – is simply an expression

(not a block of stataments,

no return statement)

```
>>> def f(x):
        return x**3

>>> f(2)
8
>>> g = lambda x: x**3
>>> g(2)
8
>>> (lambda x: x**3) (2)
8
```

Map and Lambda expressions

- Function map
  - r=map(function, sequence)

```
>>> m = map(f, [1,2,3])
>>> list(m)
[1, 8, 27]
>>> list(map(lambda x: x**3, [1,2,3]))
[1, 8, 27]
>>> list(map(lambda x,y: x**y, [1,2,3], [2, 3, 4]))
[1, 8, 81]
```

Filter and lambda expressions

- Function filter
  - `filter(function, sequence)`

```
>>> my_list = [1, 4, 5, 8, 9, 10]
>>> filter(lambda x: x%2 == 0, my_list)
<filter object at 0x02E4CFB0>
>>> list(filter(lambda x: x%2 == 0, my_list))
[4, 8, 10]
>>>
>>> fibonacci = [0,1,1,2,3,5,8,13,21,34,55]
>>> odd_numbers = list(filter(lambda x: x % 2, fibonacci))
>>> odd_numbers
[1, 1, 3, 5, 13, 21, 55]
>>>
>>> names = ["Zara", "Erin", "Carla", "Ana", "Nico"]
>>> filtered_names = list(filter(lambda x: x[-1] == "a", names))
>>> filtered_names
['Zara', 'Carla', 'Ana']
```

Sort and Lambda expressions

```
class Person:
    def __init__(self, n, a):
        self.name = n
        self.age = a

    def getName(self):
        return self.name

    def getAge(self):
        return self.age

    def __repr__(self):
        return self.name + "-" + str(self.age)
```

```
def sort_python():
    l1 = [4,2,3,1]
    l1.sort()
    print(l1)

    l1s = sorted(l1)
    print(l1s)

    p1 = Person("nnnn", 20)
    p2 = Person("eeee", 21)
    p3 = Person("ttt", 10)
    l2 = [p1, p2, p3]
    l2s = sorted(l2, key=lambda Person:Person.getName())
    print(l2s)
    l2s = sorted(l2, key=lambda Person:Person.getAge())
    print(l2s)

sort_python()
```

Recap today

- Search
  - Sequential seach
  - Binary search

- Sort
  - Selection sort
  - Insert sort
  - Bubble sort
  - Quick sort

Next time

- Algorithms
  - Backtracking
  - Divide and conquer

Reading materials and useful links

1. The Python Programming Language - https://www.python.org/
2. The Python Standard Library - https://docs.python.org/3/library/index.html
3. The Python Tutorial - https://docs.python.org/3/tutorial/
4. M. Frentiu, H.F. Pop, Fundamentals of Programming, Cluj University Press, 2006.
5. MIT OpenCourseWare, Introduction to Computer Science and Programming in Python, https://ocw.mit.edu, 2016.
6. K. Beck, Test Driven Development: By Example. Addison-Wesley Longman, 2002. http://en.wikipedia.org/wiki/Test-driven_development
7. M. Fowler, Refactoring. Improving the Design of Existing Code, Addison-Wesley, 1999. http://refactoring.com/catalog/index.html