ANALYTIC GEOMETRY, PROBLEM SET 9

The line in 3D. Relative positions of lines and planes

1. Find the equation of the plane containing the points $P_1(2,-1,-3)$, $P_2(3,1,2)$ and parallel to the vector $\overline{a}(3,-1,-4)$.

2. Find the equation of the plane containing the perpendicular lines through $P(-2,3,5)$ on the planes $\pi_1 : 4x+y-3z+13=0$ and $\pi_2 : x-2y+z-11=0$.

3. Find the equation of the plane passing through the points A, B and C, where:
(a) $A(-2,1,1)$, $B(0,2,3)$ and $C(1,0,-1)$; (b) $A(3,2,1)$, $B(2,1,-1)$ and $C(-1,3,2)$.

4. Show that the points $A(1,0,-1)$, $B(0,2,3)$, $C(-2,1,1)$ and $D(4,2,3)$ are coplanar.

5. Let d_1 and d_2 be two lines in $\mathcal{E}_3$, given by $d_1 : \frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-5}{6}$ and $d_2 : \frac{x-1}{1} = \frac{y+1}{1} = \frac{z-5}{-3}$.
(a) Find the parametric equations of d_1 and d_2;
(b) Prove that they intersect and find the coordinates of their intersection point;
(c) Find the equation of the plane determined by d_1 and d_2.

6. Given the lines $d_1 : x=1+t, y=1+2t, z=3+t, t\in\mathbb{R}$ and $d_2 : x=3+s, y=2s, z=-2+s, s\in\mathbb{R}$, show that $d_1 \parallel d_2$ and find the equation of the plane determined by d_1 and d_2.

7. Find the parametric equations of the line $\begin{cases} -2x+3y+7z+2=0 \\ x+2y-3z+5=0 \end{cases}$.

8. Find the parametric equations of the line passing through $P_1(5,-2,1)$ and $P_2(2,4,2)$. Find the equations of the line passing through $P(6,4,-2)$ and parallel to the line $d : \frac{x}{2} = \frac{y-1}{-3} = \frac{z-5}{6}$.

9. Given the points $A(1,2,-7)$, $B(2,2,-7)$ and $C(3,4,5)$, find the equation(s) of the internal bisector passing through the vertex A in the triangle ABC.

10. Find the equations of the line passing through the origin and parallel to the line given by the parametric equations: $x=t$, $y=-1+t$ and $z=2$.

11. Given the lines $d_1 : x=4-2t, y=1+2t, z=9+3t$ and $d_2 : \frac{x-1}{2} = \frac{y+2}{3} = \frac{z-4}{2}$, find the intersection points between the two lines and the coordinate planes.

12. Let d_1 and d_2 be the lines given by $d_1 : x=3+t, y=-2+t, z=9+t, t\in\mathbb{R}$ and $d_2 : x=1-2s, y=5+s, z=-2-5s, s\in\mathbb{R}$.

a) Prove they are coplanar.

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b) Find the equation of the line passing through the point $P(4,1,6)$ and orthogonal on the plane determined by d_1 and d_2.

13. Prove that the intersection lines of the planes $\pi_1 : 2x-y+3z-5=0$, $\pi_2 : 3x+y+2z-1=0$ and $\pi_3 : 4x+3y+z+2=0$ are parallel.

14. Verify that the lines $d_1 : \frac{x-3}{1} = \frac{y-8}{3} = \frac{z-3}{4}$ and $d_2 : \frac{x-4}{1} = \frac{y-9}{2} = \frac{z-9}{5}$ are coplanar and find the equation of the plane determined by the two lines.

15. Determine whether the line given by $x = 3+8t$, $y = 4+5t$, and $z = -3-t$, $t \in \mathbb{R}$ is parallel to the plane $x - 3y + 5z - 12 = 0$.