# ANALYTIC GEOMETRY, PROBLEM SET 2

## More polar coordinates

**1.** Let $ABCDEF$ be a regular hexagon with size length $l$. Find the polar coordinates of its vertices in each of the following cases:

a) The center of the hexagon $O$ is chosen as *the pole* and the half-line $[OA$ is set as *the polar axis*.

b) The vertex $A$ is chosen as *the pole* and the half-line $[AB$ is set as *the polar axis*.

**2.** Find the polar equation corresponding to the given Cartesian equation: a) $y = 5$; b) $x + 1 = 0$; c) $y = 7x$; d) $3x + 8y + 6 = 0$; e) $y^2 = -4x + 4$; f) $x^2 - 12y - 36 = 0$; g) $x^2 + y^2 = 36$; h) $x^2 - y^2 = 25$. Briefly give a geometric interpretation for the solutions to these equations.

**3.** Find the polar coordinates of the point $P \in \mathcal{E}_2$, whose rectangular (Cartesian) coordinates are $(1 + \cos\alpha, \sin\alpha)$, where $\alpha \in (0, 2\pi)$ is fixed.

## Cylindrical and spherical (everything is in 3D here)

**Warmp-up 1.** In the cylindrical coordinate system, what do the following equations represent in $\mathcal{E}_3$?

a) $r = r_0$, where $r_0 \in \mathbb{R}_{\geq 0}$ is fixed; b) $\theta = \theta_0$, where $\theta_0 \in [0, 2\pi)$ is fixed;
c) $z = z_0$, where $z_0 \in \mathbb{R}$ is fixed.

**Warm-up 2.** In the spherical coordinate system, what do the following equations represent in $\mathcal{E}_3$?

a) $\rho = \rho_0$, where $\rho_0 \in \mathbb{R}_{\geq 0}$ is fixed; b) $\theta = \theta_0$, where $\theta_0 \in [0, 2\pi)$ is fixed;
c) $\varphi = \varphi_0$, where $\varphi_0 \in [0, \pi]$ is fixed.

**4.** Let $P_1(r_1, \theta_1, z_1)$ and $P_2(r_2, \theta_2, z_2)$ be points in $\mathcal{E}_3$ expressed using their cylindrical coordinates. Find the distance $P_1P_2$, as an expression of $r_i, \theta_i, z_i$, where $i \in \{1, 2\}$.

**5.** Let $P_1(r_1, \theta_1, \varphi_1)$ and $P_2(r_2, \theta_2, \varphi_2)$ be points in $\mathcal{E}_3$, expressed using their spherical coordinates. Find the distance $P_1P_2$, as an expression of $r_i, \theta_i, \varphi_i$, where $i \in \{1, 2\}$.

**6.** Determine, in cylindrical coordinates, the equation of the surface whose equation in rectangular coordinates is $z = x^2 + y^2 - 2x + y$.

**7.** Find the equation, in rectangular coordinates, of the surface whose equation in cylindrical coordinates is $r = 4\cos(\theta)$. Explain what the equation describes geometrically.

**8.** (Non-examinable) Three spheres are pairwise exterior tangent; a plane is tangent to these spheres at points $A$, $B$ and $C$. Find the radii of the spheres in terms of $a$, $b$, $c$, representing the lengths of the sides of triangle $ABC$.

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**Time permitting...**

**9.** Let $M$ and $N$ be the midpoints of two opposite sides of a quadrilateral $ABCD$ and let $P$ be the midpoint of $[MN]$. Prove that $\overline{PA} + \overline{PB} + \overline{PC} + \overline{PD} = \overline{0}$.

**10.** In the plane determined by the triangle $ABC$, let us consider the points $M$, $N$, $P$, $Q$ such that $\overrightarrow{AM} = \frac{2}{3}\overrightarrow{AB}$, $2\overrightarrow{NA} + \overrightarrow{NC} = \overrightarrow{0}$, $\overrightarrow{AP} = \frac{2}{5}\overrightarrow{AB}$ and $3\overrightarrow{QA} + 2\overrightarrow{QB} + \overrightarrow{QC} = \overrightarrow{0}$.

(1) Find $\alpha \in \mathbb{R}$ such that $\overrightarrow{QN} = \alpha \cdot \overrightarrow{QM}$.
(2) Find $\beta \in \mathbb{R}$ such that $\overrightarrow{CQ} = \beta \cdot \overrightarrow{QP}$.
(3) Find the value of the ratio $\frac{QA}{QR}$, where $AQ \cap BC = \{R\}$.