

SEMINAR 5 + 6

1) Using Gauss elimination, solve the following systems:

$$a) \begin{cases} x_1 + x_2 + 2x_3 = -1 \\ 2x_1 - x_2 + 2x_3 = -4 \text{ (in } \mathbb{R}^3); \\ 4x_1 + x_2 + 4x_3 = -2 \end{cases} \quad b) \begin{cases} 3x_1 + 4x_2 + x_3 + 2x_4 = 3 \\ 6x_1 + 8x_2 + 2x_3 + 5x_4 = 7 \text{ (in } \mathbb{R}^4); \\ 9x_1 + 12x_2 + 3x_3 + 10x_4 = 13 \end{cases}$$

$$c) \begin{cases} x_1 + x_2 - 3x_3 = -1 \\ 2x_1 + x_2 - 2x_3 = 1 \\ x_1 + x_2 + x_3 = 3 \\ x_1 + 2x_2 - 3x_3 = 1 \end{cases} \quad (\text{in } \mathbb{R}^3).$$

2) Discuss on the real parameter α the consistency of the following systems, then solve them:

$$a) \begin{cases} 5x_1 - 3x_2 + 2x_3 + 4x_4 = 3 \\ 4x_1 - 2x_2 + 3x_3 + 7x_4 = 1 \\ 8x_1 - 6x_2 - x_3 - 5x_4 = 9 \\ 7x_1 - 3x_2 + 7x_3 + 17x_4 = \alpha \end{cases}, \quad b) \begin{cases} 2x_1 - x_2 + 3x_3 + 4x_4 = 5 \\ 4x_1 - 2x_2 + 5x_3 + 6x_4 = 7 \\ 6x_1 - 3x_2 + 7x_3 + 8x_4 = 9 \\ \alpha x_1 - 4x_2 + 9x_3 + 10x_4 = 11 \end{cases};$$

$$c) \begin{cases} \alpha x_1 + x_2 + x_3 = 1 \\ x_1 + \alpha x_2 + x_3 = 1 \\ x_1 + x_2 + \alpha x_3 = 1 \end{cases}.$$

3) Solve the following matrix equations:

$$a) \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} X = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}; \quad b) \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} X = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix};$$

$$c) \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} X = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; \quad d) X \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \end{pmatrix};$$

$$e) \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} X = \begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \end{pmatrix}.$$

4) Are these matrices invertible? If yes, find their inverses:

$$a) \begin{pmatrix} 1 & 1 & 2 \\ 2 & -1 & 1 \\ 4 & 1 & 4 \end{pmatrix}; \quad b) \begin{pmatrix} 3 & 4 & 2 \\ 6 & 8 & 5 \\ 9 & 12 & 10 \end{pmatrix}; \quad c) \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}.$$