ANALYTIC GEOMETRY, PROBLEM SET 11

Mostly distances in 3D.

1. Find the distance from the point $P(1,2,-1)$ to the line $d: x=y=z$.

2. Find the distance from $P(3,1,-1)$ to the plane $\pi: 22x+4y-20z-45=0$.

3. Find the distance between the planes
$\pi_1: 2x-3y+4z-7=0$ and $\pi_2: 4x-6y+8z-3=0$.

4. Find the distance between the lines $d_1: \frac{x-1}{2}=\frac{y+1}{3}=\frac{z}{1}$ and $d_2: \frac{x+1}{3}=\frac{y}{4}=\frac{z-1}{3}$.

5. Find the distance between the lines $d_1: x=1-2t$, $y=3t$, $z=-2t+t$, where $t\in\mathbb{R}$ and $d_2: x=7+4s$, $y=5-6s$, $z=4-2s$, where $s\in\mathbb{R}$.

6. Show that the line
$d: \frac{x+1}{1}=\frac{y-3}{2}=\frac{z}{-1}$ and the plane $\pi: 2x-2y-2z+3=0$ are parallel and find the distance between them.

7. Given the point $P(6,-5,5)$ and the plane $\pi: 2x-3y+z-4=0$, find the coordinates of the symmetric P' of the point P with respect to the plane π.

8. Consider the point $P(4,3,10)$ the line $d: \frac{x-1}{2}=\frac{y-2}{4}=\frac{z-3}{5}$. Find the coordinates of the symmetric point P' of P with respect to the line d.

11.[From the previous set.] Determine the equations of the planes which pass through the points $P(0,2,0)$ and $Q(-1,0,0)$ and which form an angle of $60°$ with the Oz axis.

12. Find the geometric locus of the lines passing through a given point and having a constant distance to a given line.

The setup of the next problem is in the Euclidean plane $\mathcal{E}_2$.

13. In each of the following situations, find the equation of the circle:
a) of diameter $[AB]$, where $A(1,2)$ and $B(-3,-1)$; b) of center $I(2,-3)$ and radius $R=7$;
c) of center $I(-1,2)$ and which passes through $A(2,6)$;
d) centered at the origin and tangent to $d: 3x-4y+20=0$; e) passing through $A(3,1)$ and $B(-1,3)$ and having the center on the line $d: 3x-y-2=0$;
f) determined by $A(1,1)$, $B(1,-1)$ and $C(2,0)$; g) tangent to both $d_1: 2x+y-5=0$ and $d_2: 2x+y+15=0$, if the tangency point with d_1 is $M(3,1)$.

Date: December 12, 2021.

1. Find the distance from the point $P(1, 2, -1)$ to the line $d : x = y = z$.

from the lecture:

$$d(P_0, d) = \frac{|\overline{v} \times \overline{P_1 P_0}|}{|\overline{v}|}. \qquad (15)$$

where $\overline{v}$ is a direction vector for d, in our case $\overline{v} = \overline{v}(1,1,1)$

$P_0 = P$

P_1 is a point on d, for example, in our case, $P_1 = P_1(0,0,0)$

so $\overrightarrow{P_1P} = (1, 2, -1)$

$$\Rightarrow \overline{v} \times \overrightarrow{P_1P} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 1 & 2 & -1 \end{vmatrix} = i \underbrace{\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix}}_{-3} - j \underbrace{\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}}_{-2} + k \underbrace{\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}}_{1}$$

$$= -3i + 2j + k = (-3, 2, 1)$$

$$\Rightarrow d(P, d) = \frac{|\overline{v} \times \overrightarrow{P_1P}|}{|\overline{v}|} = \frac{\sqrt{9+4+1}}{\sqrt{3}} = \sqrt{\frac{14}{3}}$$

2. Find the distance from $P(3,1,-1)$ to the plane $\pi : 22x + 4y - 20z - 45 = 0$.

from the lecture

Theorem

The distance from the point $P_0(x_0, y_0, z_0)$ to the plane $\pi : ax + by + cz + d = 0$ is given by

$$d(P_0, \pi) = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}. \tag{14}$$

$$d(P,\pi) = \frac{|\overbrace{22\cdot 3}^{66} + \overbrace{4\cdot 1}^{4} \overbrace{- 20\cdot(-1)}^{+20} \overbrace{- 45}^{-45}|}{\sqrt{22^2 + 4^2 + 20^2}} = \frac{45}{30} = \frac{3}{2}$$

$2^2\cdot 11^2 + 2^2\cdot 2^2 + 2^2\cdot 10^2$

$2\sqrt{121 + 4 + 100}$

$225 = 15^2$

3. Find the distance between the planes
$\pi_1 : 2x - 3y + 4z - 7 = 0$ and $\pi_2 : 4x - 6y + 8z - 3 = 0$.

Remark: if the planes intersect then $d(\pi_1, \pi_2) = 0$

- the equation of π_1 shows that $n_1(2, -3, 4)$ is a normal vector for π_1
- the equation of π_2 shows that $n_2(4, -6, 8)$ — " — for π_2
- $n_2 = 2 \cdot n_1 \Rightarrow \pi_1$ and π_2 have the same normal vectors

$\Rightarrow \pi_1 \parallel \pi_2$

- from the lecture:

any point of π_1 — any point of π_2

$$d(\pi_1, \pi_2) = d(P_1, \pi_2). = d(\pi_1, P_2)$$

if we choose $x = 1$ and $y = -1$ then
$2 \cdot 1 - 3(-1) + 4z - 7 = 0$ so $z = 0$

$\Rightarrow (1, -1, 0) \in \pi_1$ choose this to be P_1

$$d(P_1, \pi_2) = \frac{|4 \cdot 1 - 6 \cdot (-1) + 8 \cdot 0 - 3|}{\sqrt{4^2 + 6^2 + 8^2}} = \frac{7}{2\sqrt{29}} = d(\pi_1, \pi_2)$$

$$\sqrt{4^2 + 6^2 + 8^2} = 2\sqrt{2^2 + 3^2 + 4^2}$$

4. Find the distance between the lines $d_1 : \frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{1}$ and $d_2 : \frac{x+1}{3} = \frac{y}{4} = \frac{z-1}{3}$.

from the lecture

Theorem

The distance between two skew lines d_1 and d_2 is given by

$$d(d_1, d_2) = \frac{|A(x_1 - x_2) + B(y_1 - y_2) + C(z_1 - z_2)|}{\sqrt{A^2 + B^2 + C^2}}. \quad (16)$$

where

- $n(A, B, C) = \underset{\substack{\uparrow \\ \text{direction vector of } d_1}}{v_1} \times \underset{\substack{\uparrow \\ \text{direction vector of } d_2}}{v_2} = \begin{vmatrix} i & j & k \\ 2 & 3 & 1 \\ 3 & 4 & 3 \end{vmatrix} = i \underbrace{\begin{vmatrix} 3 & 1 \\ 4 & 3 \end{vmatrix}}_{5} - j \underbrace{\begin{vmatrix} 2 & 1 \\ 3 & 3 \end{vmatrix}}_{3} + k \underbrace{\begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix}}_{-1}$

$= 5i - 3j - k$

$= (5, -3, -1) = (A, B, C)$

- (x_1, y_1, z_1) is a point on d_1 for example $(1, -1, 0)$
- (x_2, y_2, z_2) — " — d_2 — " — $(-1, 0, 1)$

$\Rightarrow (x_1 - x_2, y_1 - y_2, z_1 - z_2) = (2, -1, -1)$

$$\Rightarrow d(d_1, d_2) = \frac{|5 \cdot 2 - 3 \cdot (-1) - 1 \cdot (-1)|}{\sqrt{25 + 9 + 1}} = \frac{14}{\underset{5 \cdot 7}{\sqrt{35}}} = \frac{2\sqrt{7}}{\sqrt{5}}$$

5. Find the distance between the lines $d_1 : x = 1 - 2t,\ y = 3t,\ z = -2t + t$, where $t \in \mathbb{R}$ and $d_2 : x = 7 + 4s,\ y = 5 - 6s,\ z = 4 - 2s$, where $s \in \mathbb{R}$.

$$d_1 : \begin{cases} x = 1 - 2t \\ y = 3t \\ z = -t \end{cases} \qquad d_2 : \begin{cases} x = 7 + 4s \\ y = 5 - 6s \\ z = 4 - 2s \end{cases}$$

$$P_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad v_1 \begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix} \qquad P_2 \begin{pmatrix} 7 \\ 5 \\ 4 \end{pmatrix} \quad v_2 \begin{pmatrix} 4 \\ -6 \\ -2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}$$

$$n(A,B,C) = v_1 \times \tilde{v}_2 = \begin{vmatrix} i & j & k \\ -2 & 3 & -1 \\ 2 & -3 & -1 \end{vmatrix} = i \underbrace{\begin{vmatrix} 3 & -1 \\ -3 & -1 \end{vmatrix}}_{-6} - j \underbrace{\begin{vmatrix} -2 & -1 \\ 2 & -1 \end{vmatrix}}_{4} + k \underbrace{\begin{vmatrix} -2 & 3 \\ 2 & -3 \end{vmatrix}}_{0}$$

$$= -6i - 4j = (-6, -4, 0) = (A, B, C)$$

$$(x_1 - x_2, y_1 - y_2, z_1 - z_2) = \overrightarrow{P_2P_1} = (-6, -5, -4)$$

$$\Rightarrow d(d_1, d_2) = \frac{|-6(-6) - 4(-5) - 4 \cdot 0|}{\sqrt{36 + 16}} = \frac{56}{\sqrt{42}}$$

$$d_1 : \begin{cases} x = 1 - 2t \\ y = 3t \\ z = -2 + t \end{cases} \qquad d_2 : \begin{cases} x = 7 + 4s \\ y = 5 - 6s \\ z = 4 - 2s \end{cases}$$

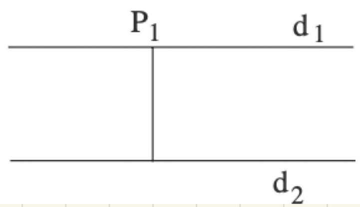
$$P_1 \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} \hookleftarrow \ \hookrightarrow v_1 \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} \qquad P_2 \begin{pmatrix} 7 \\ 5 \\ 4 \end{pmatrix} \hookleftarrow \ \hookrightarrow v_2 \begin{pmatrix} 4 \\ -3 \\ -2 \end{pmatrix} = -2 \cdot v_1$$

$$\Downarrow$$

$$d_1 \parallel d_2$$

from the lecture:

$d(d_1, d_2) = d(P_1, d_2)$.



$$\Rightarrow d(d_1, d_2) = d(P_1, d_2)$$

$$= \frac{|\overrightarrow{P_1P_2} \times v_2|}{|v_2|} = \dots$$

6. Show that the line
$d : \frac{x+1}{1} = \frac{y-3}{2} = \frac{z}{-1}$ and the plane $\pi : 2x - 2y - 2z + 3 = 0$ are parallel and find the distance between them.

has direction vector $v(1,2,-1)$

has normal vector $n(2,-2,-2)$

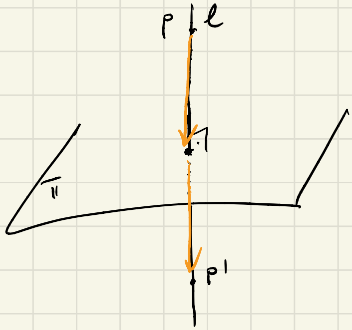
$n \cdot v = 1 \cdot 2 + 2 \cdot (-2) - 1(-2) = 0 \Rightarrow n \perp v \Rightarrow d \parallel \pi$

(you can also show that $d \parallel \pi$ by showing that $d \cap \pi = \emptyset$)

$d(d, \pi) = d(P, \pi) = \ldots\ldots$

point on d
for example $\begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix}$

7. Given the point $P(6,-5,5)$ and the plane $\pi : 2x-3y+z-4=0$, find the coordinates of the symmetric P' of the point P with respect to the plane π.



π has $n(2,-3,1)$ as normal vector

so, the line $\ell \perp \pi$ and containing P is

$$\ell : \begin{cases} x = 6+2t \\ y = -5-3t \\ z = 5+t \end{cases}$$

$\ell \cap \pi$: $2(6+2t)-3(-5-3t)+(5+t)-4=0$

$12+4t+15+9t+5+t-4=0$

$28+14t=0 \implies t=-2$

$\implies \ell \cap \pi = P_0(2,1,3)$

$\implies \overrightarrow{PP_0} = (-4,6,-2)$

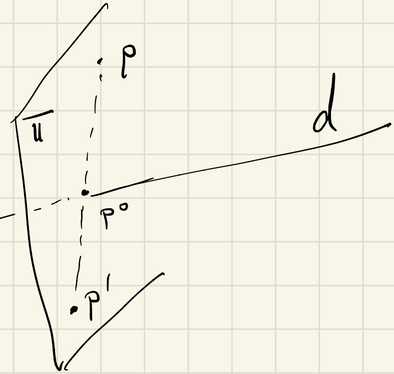
$\implies P' = P + 2\overrightarrow{PP_0} = \begin{pmatrix} 6 \\ -5 \\ 5 \end{pmatrix} + 2\begin{pmatrix} -4 \\ 6 \\ -2 \end{pmatrix} = \begin{pmatrix} -2 \\ 7 \\ 1 \end{pmatrix}$

8. Consider the point $P(4,3,10)$ the line $d : \frac{x-1}{2} = \frac{y-2}{4} = \frac{z-3}{5}$. Find the coordinates of the symmetric point P' of P with respect to the line d.

d has direction vector $v(2,4,5)$

so the plane $\Pi \parallel d$ and containing P

is



$\Pi: 2(x-4)+4(y-3)+5(z-10)=0$

$$d: \begin{cases} x = 1+2t \\ y = 2+4t \\ z = 3+5t \end{cases}$$

$P_0 = d \cap \Pi : \quad 2(1+2t-4) + 4(2+4t-3) + 5(3+5t-10) = 0$

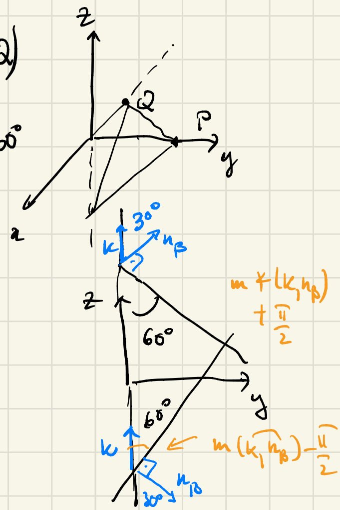
$$-6+4t \qquad -4+16t \qquad -35+25t$$

$$-45+45t = 0 \Rightarrow t = 1$$

$$\Rightarrow P_0 = (3,6,8) = d \cap \Pi$$

P_0 is the midpoint of $PP' \Rightarrow P' = \ldots$

11.[From the previous set.] Determine the equations of the planes which pass through the points $P(0,2,0)$ and $Q(-1,0,0)$ and which form an angle of $60°$ with the Oz axis.



- from all the planes containing P and Q (the line PQ) we want to select those which form an angle of 60° with Oz.

- PQ : $\dfrac{x-0}{-1-0} = \dfrac{y-2}{0-2} = \dfrac{z-0}{0}$

$$-2x = -y+2 \qquad\qquad \underbrace{z=0}_{\pi_2}$$

$$\underbrace{2x-y+2=0}_{\pi_1}$$

$PQ = \pi_1 \cap \pi_2$

- the bundle of planes passing through the line PQ is

$$\pi_{\alpha\beta} : \alpha(x-y+2) + \beta z = 0$$

a reduced bundle is $\pi_\beta : x - y + 2 + \beta z = 0$

↳ has normal vector $n_\beta(1,-1,\beta)$

- a direction vector for Oz is $k(0,0,1)$

$$60° = m(\sphericalangle(Oz, \pi_\beta)) = m(\sphericalangle(k, n_\beta)) \pm 90°$$

so $m(\sphericalangle(k, n_\beta)) = 150°$ or $30°$

$$\underbrace{\frac{\pi}{2} + \frac{\pi}{3}}_{\frac{5\pi}{6}} \qquad\qquad \frac{\pi}{6}$$

so $\cos \measuredangle(k, n_\beta) = -\frac{\sqrt{3}}{2}$ or $\frac{\sqrt{3}}{2}$

$$\frac{k \cdot n_\beta}{\|k\| \cdot \|n_\beta\|} = \frac{\beta}{1 \cdot \sqrt{1+1+\beta^2}} = \pm \frac{\sqrt{3}}{2}$$

So $\frac{\beta^2}{2+\beta^2} = \frac{3}{4}$ $\iff 4\beta^2 = 6 + 3\beta^2$

$\iff \beta^2 = 6$

$\iff \beta = \sqrt{6}$ or $-\sqrt{6}$

we found two planes

$\pi_{-\sqrt{6}} : x - y - \sqrt{6}z + 2 = 0$

and

$\pi_{\sqrt{6}} : x - y + \sqrt{6}z + 2 = 0$

12. Find the geometric locus of the lines passing through a given point and having a constant distance to a given line.

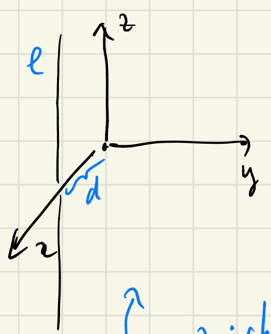
we have
- a fixed point P and
- a fixed line ℓ

$\underline{\text{I}}$ if $P \in \ell$ then any line ℓ' containing P intersects ℓ (in P)

so $d(\ell, \ell') = 0$

So, in this case the geometric locus is the set of all lines passing through P

$\underline{\text{II}}$ if $P \notin \ell$: in order to investigate this case we fix a coordinate system in order to translate the requirement into equations. We can choose the coordinate system as we want (the best choices are the ones which simplify the calculations)



- choose the origin to be the given poin P
- choose the Oz axis parallel to ℓ
- choose the Ox axis such that it intersects ℓ

then

$$\ell : \begin{cases} x = d \\ y = 0 \\ z = t \end{cases} \quad t \in \mathbb{R} \qquad \text{and } \ell' : \begin{cases} x = t v_x \\ y = t v_y \\ z = t v_z \end{cases}$$

(a line passing through P)

- if $\ell \parallel \ell'$ then $d(\ell,\ell') = d$
- assume that $\ell \nparallel \ell'$

$$d(\ell,\ell') = \frac{\left| \begin{vmatrix} d-0 & 0-0 & 0-0 \\ 0 & 0 & 1 \\ v_x & v_y & v_z \end{vmatrix} \right|}{\sqrt{\begin{vmatrix} 0 & 0 \\ v_x & v_y \end{vmatrix}^2 + \begin{vmatrix} 0 & 1 \\ v_x & v_z \end{vmatrix}^2 + \begin{vmatrix} 0 & 1 \\ v_y & v_z \end{vmatrix}^2}} = \frac{|d\cdot(-v_y)|}{\sqrt{v_x^2 + v_y^2}}$$

$$d(\ell,\ell') = \frac{|d\cdot v_y|}{\sqrt{v_x^2+v_y^2}}$$

- $v = (v_x, v_y, v_z)$ is a direction vector for ℓ' and we may rescale v as we like

  choose v such that $v_x^2 + v_y^2 = 1$, i.e replace v by $\frac{v}{\sqrt{v_x^2+v_y^2}}$

- then $d(\ell,\ell') = |d\cdot v_y| = c$

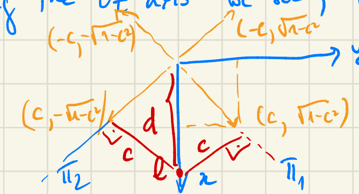
  Case 1 $\quad d\cdot v_y = c \Rightarrow v_y = \frac{c}{d} \Rightarrow v_x = \pm\sqrt{1-\frac{c^2}{d^2}} = \pm\frac{\sqrt{1-c^2}}{d^2}$

  case 2 $\quad d\cdot v_y = -c \Rightarrow v_y = -\frac{c}{d} \Rightarrow v_x = \pm\sqrt{1+\frac{c^2}{d^2}} = \pm\frac{\sqrt{1-c^2}}{d^2}$

$$\Rightarrow \ell' : \begin{cases} x = t\cdot(\pm\sqrt{1-c^2}) \\ y = t\cdot(\pm c) \\ z = t\cdot v_z \end{cases} \quad \text{a direction vector is } \begin{pmatrix} \pm\sqrt{1-c} \\ \pm c \\ v_z \end{pmatrix} \xrightarrow{\text{rescaling this by } d}$$

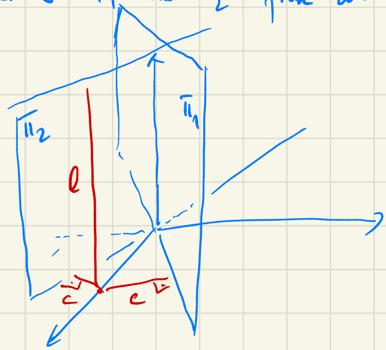
So we have no restriction on v_z, but if the constant c is given, the we care restrictions on v_x and v_y

viewing along the Oz axis we see, for a given distance c



so, the union of all possible lines ℓ' (in this case) is

the union of two planes Π_1 and Π_2 from which we remove Oz

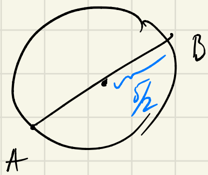


13. In each of the following situations, find the equation of the circle:
a) of diameter $[AB]$, where $A(1,2)$ and $B(-3,-1)$; b) of center $I(2,-3)$ and radius $R=7$;
c) of center $I(-1,2)$ and which passes through $A(2,6)$;
d) centered at the origin and tangent to $d: 3x-4y+20=0$; e) passing through $A(3,1)$ and $B(-1,3)$ and having the center on the line $d: 3x-y-2=0$;
f) determined by $A(1,1)$, $B(1,-1)$ and $C(2,0)$; g) tangent to both $d_1: 2x+y-5=0$ and $d_2: 2x+y+15=0$, if the tangency point with d_1 is $M(3,1)$.

a) $A(1,2)$, $B(-3,-1)$

$[A,B]$ diameter $\Rightarrow$ midpoint of $[AB]$ is center of $\mathcal{C}$ $= \left(-1, \frac{1}{2}\right)$

and $\frac{|AB|}{2}$ is the radius $= \frac{\sqrt{4^2+3^2}}{2} = \frac{5}{2}$

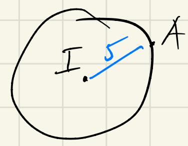


$\Rightarrow \mathcal{C}: \underbrace{(x-(-1))^2 + \left(y-\frac{1}{2}\right)^2}_{\text{distance to the center squared}} = \frac{25}{4}$

distance to the center squared = radius squared.

b) $\mathcal{C}: (x-2)^2 + (y-(-3))^2 = 7^2$

c) radius is $d(I,A) = \sqrt{3^2+4^2} = 5$



so $\mathcal{C}: (x-(-1))^2 + (y-2)^2 = 25$

d) $\mathcal{C}: x^2+y^2 = r^2$ but what is r?

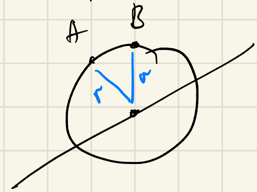
since $d: 3x-4y+20=0$ is tangent to $\mathcal{C}$

$r = d(O,d) = \frac{|20|}{\sqrt{3^2+4^2}} = \frac{20}{25} = \frac{4}{5}$

e.) $A(3,1)$, $B(-1,3) \in \mathcal{C}$

center of $\mathcal{C}$ is on $d: 3x - y - 2 = 0;$

generic picture



a point on d has coordinates

$P(x, 3x-2)$

$$d(P,A)^2 = (3-x)^2 + (3-3x)^2 = 9 - 6x + x^2 + 9 - 18x + 9x^2$$
$$= 18 - 24x + 10x^2$$

$$d(P,B)^2 = (-1-x)^2 + (5-3x)^2 = 1 + 2x + x^2 + 25 - 30x + 9x^2$$
$$= 26 - 28x + 10x^2$$

since $r = d(P,A) = d(P,B)$

$$\Rightarrow 18 - 24x + 10x^2 = 26 - 28x + 10x^2$$

$$-8 = -4x \Rightarrow x = 2$$

$\Rightarrow$ the center has coords $I(2,4)$

$$\Rightarrow r = d(I,A) = \sqrt{1^2 + 3^2} = \sqrt{10}$$

test: $d(I,B) = \sqrt{3^2 + 1^2} = \sqrt{10}$

so $\mathcal{C}: (x-2)^2 + (y-4)^2 = 10$

f) $\mathcal{C} \ni A(1,1)$, $B(1,-1)$, $C(2,0)$

$$\mathcal{C} : (x-a)^2 + (y-b)^2 = r^2$$

$$A \in \mathcal{C} \Rightarrow (1-a)^2 + (1-b)^2 = r^2$$

$$1 - 2a + a^2 + 1 - 2b + b^2 = r^2 \qquad (1)$$

$$B \in \mathcal{C} \Rightarrow (1-a)^2 + (1+b)^2 = r^2$$

$$1 - 2a + a^2 + 1 + 2b + b^2 = r^2 \qquad (2)$$

$$C \in \mathcal{C} \Rightarrow (2-a)^2 + b^2 = r^2$$

$$4 - 4a + a^2 + b^2 = r^2 \qquad (3)$$

so, a, b, r have to satisfy

$$(*) \begin{cases} 1 - 2a + a^2 + 1 - 2b + b^2 = r^2 & (1) \\ 1 - 2a + a^2 + 1 + 2b + b^2 = r^2 & (2) \\ 4 - 4a + a^2 + b^2 = r^2 & (3) \end{cases}$$

"$(1) - (2)$": $\quad -4b = 0 \Rightarrow \boxed{b = 0}$

$$\Rightarrow (*) \overset{b=0}{\Leftrightarrow} \begin{cases} 2 - 2a + a^2 = r^2 \\ 4 - 4a + a^2 = r^2 \end{cases}$$

$$-2 + 2a = 0 \Rightarrow \boxed{a = 1} \Rightarrow r^2 = 1$$

so $\mathcal{C} : (x-1)^2 + y^2 = 1.$

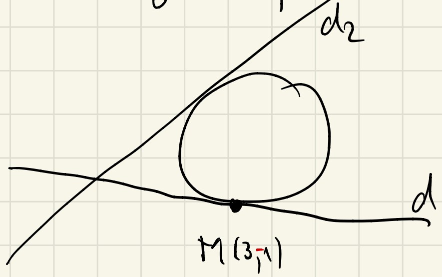
~~f) determined by $A(1,1)$, $B(1,-1)$ and $C(2,0)$;~~ g) tangent to both $d_1 : 2x + y - 5 = 0$ and $d_2 : 2x + y + 15 = 0$, if the tangency point with d_1 is $M(3,-1)$.

$M \in d_1$

$\mathcal{C} : (x-a)^2 + (y-b)^2 = r^2$

$I(a,b)$ is the center of C

generic picture



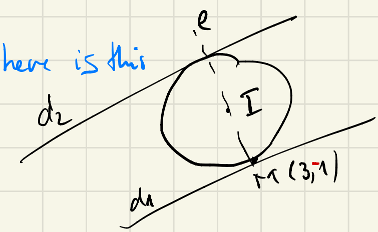
$$d(I,d_1) = d(I,d_2)$$

$$\frac{|2a+b+15|}{\sqrt{4+1}} = \frac{|2a+b-5|}{\sqrt{4+1}} \quad \ldots\ldots$$

I started calculating and only noticed now that the normal vectors of the two lines are the same

so in fact $d_1 \parallel d_2$

so in fact the picture here is this



a line $l \perp d_1$, $l \ni M$ is

$$l : \begin{cases} x = 3 + 2t \\ y = -1 + t \end{cases}$$

and $l \cap d_2$: $2(3+2t) + (-1+t) + 15 = 0$

$6 + 4t$

$5t + 20 = 0 \Rightarrow t = -4$

$\Rightarrow l \cap d_2 = N(-5,-5)$

$\Rightarrow$ center I of $\mathcal{C}$ is mid point of MN, i.e. $I(-1,-3)$

and radius is $d(I,M) = d(I,N) = \sqrt{20}$

$\Rightarrow \mathcal{C} : (x+1)^2 + (y+3)^2 = 20$