

Three different heuristics have been looked at: **move_diff**, **closeness_score** and **centrality**. **move_diff** is the legal move difference between the player and the opponent. It attempts to select moves that result in most moves remaining for the player and fewest for the opponent. **closeness_score** is the measure of closeness to the opponent on the board. It was developed through an observation that, on average, moves that keep the player close to the opponent have a better result than those that allow to diverge far from the opponent. **centrality** is a measure how close the player is to the center of the board. It is only assessed at the beginning of the game since that's when players should aim to occupy the central part of the grid to increase the number of move options later down the game. All three heuristics can be described as 'easy' - they are quick to assess, do not require extensive computation. The flip point is that they achieve results comparable to AB_Improved but not significantly better. When 50 games were played with each of these heuristics alone, the following results were obtained:

Match #	Opponent	AB_Improved		Centrality		Closeness		Move_diff	
		Won	Lost	Won	Lost	Won	Lost	Won	Lost
1	Random	43	7	44	6	44	6	47	3
2	MM_Open	25	25	31	19	26	24	32	18
3	MM_Center	34	16	35	15	37	13	37	13
4	MM_Improved	29	21	20	30	28	22	28	22
5	AB_Open	27	23	22	28	25	25	28	22
6	AB_Center	20	30	19	31	19	31	24	26
7	AB_Improved	24	26	26	24	27	23	25	25
Win Rate:		57.7%		56.3%		58.9%		63.1%	

On average **move_diff** achieved slightly better outcomes. This is to be expected since it assesses the mobility of the players after each potential move - this measure is relevant in all stages of the game and particularly important in later stages when there are fewer move options and moves that would trap the player or benefit the opponent should be avoided. **closeness** heuristic just assesses the proximity of the player to the opponent, it has no other logic behind it. It does not assess if the move is damaging to a player in any other way (e.g. would trap the player if executed) so it is expected to have worse performance outcome than move_diff. Finally, **centrality** encourages the players to occupy central space of the board at the beginning of the game again without any other mobility considerations. This heuristic is expected to do worst since it only provides positional guidance at the start of the game.

A combination of these heuristics was tested in attempt to further improve the player performance. The first custom score combines the scores for **move_diff**, **closeness_score** and **centrality**. The second custom score is written to chase the opponent – it is a combination of **move_diff** and **closeness_score**. **move_diff** this time has higher weight given to opponent moves (i.e. prioritize moves that give opponent very few move options over moves that give the player many move options). Finally, the third custom score only combines the **move_diff** and the **centrality** score (the latter only applied at the beginning of the game). The results of 50 games are shown below:

Match #	Opponent	AB_Improved		AB_Custom		AB_Custom_2		AB_Custom_3	
		Won	Lost	Won	Lost	Won	Lost	Won	Lost
1	Random	47	3	45	5	46	4	42	8
2	MM_Open	27	23	34	16	33	17	29	21
3	MM_Center	39	11	37	13	37	13	35	15
4	MM_Improved	27	23	24	26	23	27	23	27
5	AB_Open	24	26	25	25	29	21	22	28
6	AB_Center	20	30	25	25	21	29	24	26
7	AB_Improved	27	23	25	25	30	20	25	25
Win Rate:		60.3%		61.4%		62.6%		57.1%	

Again, the results are comparable to AB_Improved with marginally better performance for the first and second custom scores. It is of interest that a combination of heuristics does not seem to give a significantly higher performance than a single heuristic. The takeaway from this should be that it might not be worth to spend too much time in developing complex heuristic for assessing values high up in a search tree but instead use a simple one for fast assessment that would enable the search to proceed further down the search tree. In this particular case, **move_diff** seems like the best choice based on (i) the experimental results, (ii) the fact that it is the only heuristic that assesses the mobility of the players which is arguably the most important factor for good player performance in the game, (iii) the fact that this heuristic remains relevant to player performance through all stages of the game and (iv) the fact that, out of three heuristics investigated, it is the only one that assesses potential outcomes of the move (i.e. how many moves will I and my opponent still have?) as opposed to extracting positional information about the move only (i.e. is the move close to the center of the board? Is the move close to the opponent?).