1) $L = \{ w c w^R \quad w \in (a, b)^* \}$

eg string:- {aca, bcb, abcba, abacaba, aacaa, bbcbb, ...

Γ = set of all the stack alphabet.

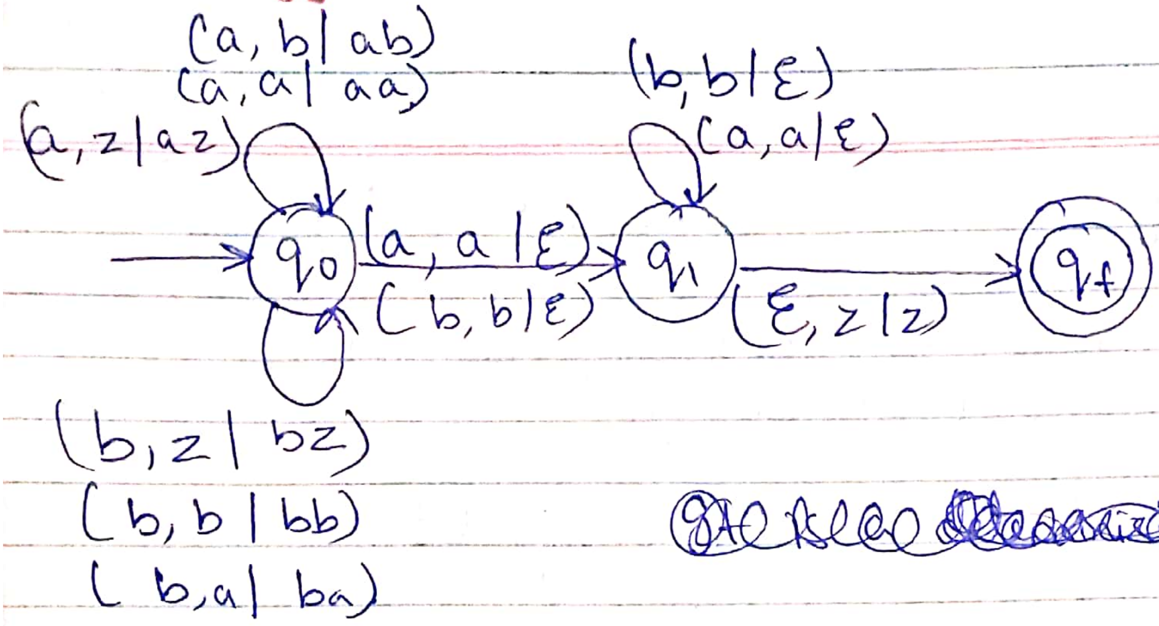
$F = \{a, b, z\}$

Z = stack start symbol.
a = input alphabet.
b = input alphabet.

To construct PDA
q_0 = initial state
q_f = final state
Z = stack start symbol.
ε = indicates pop operation.

Stack transition function.

$\delta(q_0, a, z) \vdash (q_0, az)$
$\delta(q_0, a, a) \vdash (q_0, aa)$
$\delta(q_0, b, z) \vdash (q_0, bz)$
$\delta(q_0, b, b) \vdash (q_0, bb)$
$\delta(q_0, a, b) \vdash (q_0, ab)$
$\delta(q_0, b, a) \vdash (q_0, ba)$
$\delta(q_0, c, a) \vdash (q_1, a)$
$\delta(q_0, c, b) \vdash (q_1, b)$
$\delta(q_1, a, a) \vdash (q_1, \varepsilon)$
$\delta(q_1, b, b) \vdash (q_1, \varepsilon)$
$\delta(q_1, \varepsilon, z) \vdash (q_f, z)$



It is a deterministic PDA.

2) Convert the given grammar to PDA.

$S \to AaBB \mid aAA$
$A \to aBB \mid a$
$B \to bBB \mid A$
$C \to a.$

S1) Convert grammar to GNF.

$B \to A$ is a unit production and not in GNF

$B \longrightarrow A$ $\quad A \to aBB \mid a$

$\therefore B \to bBB \mid aBB \mid a.$

Then, $S \to AaBB \mid aAA$
$A \to aBB \mid a$
$B \to bBB \mid aBB \mid a$
$C \to a$

As, $S \to AaBB$ is not in GNF, substitute
$A \to aBB \mid a$ and $C \to a$.

$S \rightarrow (aBB|a)\ CBB|\ aAA$

$S \rightarrow aBBCBB|\ aCBB\ |aAA.$

$A \rightarrow aBB|a$

$B \rightarrow bBB|aBB|a$

$C \rightarrow a$

The final GNF grammar is

$S \rightarrow aBBCBB\ |\ aCBB|\ aAA$

$A \rightarrow aBB|a$

$B \rightarrow bBB|aBB|a.$

$C \rightarrow a.$

S2) Push S to stack & q_0 is start state and change to q_1

$\delta(q_0, \varepsilon, z_0) = (q_1, Sz_0)$ — 1

S3) For each production $A \rightarrow a\alpha$ introduce $\delta(q_1, a, A) = (q_1, \alpha)$

This can be done as.

Production	Transitions
$S \rightarrow aBBCBB$	$\delta(q_1, a, S) = (q_1, BBCBB)$ — 2
$S \rightarrow aCBB$	$\delta(q_1, a, S) = (q_1, CBB)$ — 3
$S \rightarrow aAA$	$\delta(q_1, a, S) = (q_1, AA)$ — 4
$A \rightarrow aBB$	$\delta(q_1, a, A) = (q_1, BB)$ — 5
$A \rightarrow a$	$\delta(q_1, a, A) = (q_1, \varepsilon)$ — 6
$B \rightarrow bBB$	$\delta(q_1, b, B) = (q_1, BB)$ — 7
$B \rightarrow aBB$	$\delta(q_1, a, B) = (q_1, BB)$ — 8
$B \rightarrow a$	$\delta(q_1, a, B) = (q_1, \varepsilon)$ — 9
$C \rightarrow a$	$\delta(q_1, a, C) = (q_1, \varepsilon)$ — 10

S4) Finally in state q_1, without consuming any input, change the state to q_f.

$$\delta(q_1, \epsilon, z_0) = (q_f, z_0) \quad — (11)$$

The PDA is given by $M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

$Q = \{q_0, q_1, q_f\}$
$\Sigma = \{a, b\}$
$\Gamma = \{S, A, B, C, z_0\}$
q_0 = start ~~state~~ state.
z_0 = initial symbol on stack
$F = \{q_f\} \longrightarrow$ final state.
$\delta \longrightarrow$ transitions.

Obtain a PDA to accept the language $L = \{ w w^R : |w| >= 1$ for $w \in (a+b)^* \}$

equivalent CFG

$S \rightarrow aSa \mid aa$

$S \rightarrow bSb \mid bb$

→ First convert to CNF

i) Eliminate ϵ-productions

ii) Eliminate unit production

iii) Eliminate ~~ab~~ useless symbols

S1)

OV	NV	Productions
ϕ	S	$S \rightarrow aa$
		$S \rightarrow bb$
S	S	$S \rightarrow aSa$
		$S \rightarrow bSb$
S	S	—

S2)

P2	T2	V2
—	—	S
$S \rightarrow aSa \mid aa \mid bSb \mid bb$	a, b	S

resulting grammar. $G = (V_1, T_1, P_1, S)$

$V_1 = \{S\}$

$T_1 = \{0, 1\}$

$P_1 = \{ S \rightarrow aSa \mid bSb \mid aa \mid bb \}$

S ~ start symbol.

CNF.

S) Given productions	Action	Resulting production
S1) $S \rightarrow aSa \mid aa \mid bSb \mid bb$	replace a by D_0 and b by D_1 introduce $D_0 \rightarrow a$, $D_1 \rightarrow b$	$S \rightarrow D_0 S D_0 \mid D_0 D_0 \mid D_1 S D_1 \mid D_1 D_1$, $D_0 \rightarrow a$, $D_1 \rightarrow b$
S2) $S \rightarrow D_0 S D_0 \mid D_1 S D_1$	replace SD_0 by D_2 and SD_1 by D_3. introduce $D_2 \rightarrow SD_0$, $D_3 \rightarrow SD_1$	$S \rightarrow D_0 D_2 \mid D_1 D_3$, $D_2 \rightarrow S D_0$, $D_3 \rightarrow S D_1$

Final grammar in CNF.

$G = (V_1, T_1, P_1, S)$

$V_1 = \{S, D_0, D_1, D_2, D_3\}$

$T = \{a, b\}$

$P = \{ S \rightarrow D_0 D_0 \mid D_1 D_1 \mid D_0 D_2 \mid D_1 D_3$

$D_0 \rightarrow a$

$D_1 \rightarrow b.$

$D_2 \rightarrow S D_0$

$D_3 \rightarrow S D_1 \}$

S = start symbol.

Conversion to GNF.

rename the variables

$S = A_1$

$D_0 = A_2$

$D_1 = A_3$

$D_2 = A_4$

$D_3 = A_5$

So,

$A_1 \rightarrow A_2A_2 \mid A_3A_3 \mid A_2A_4 \mid A_3A_5.$

$A_2 \rightarrow a$

$A_3 \rightarrow b$

$A_4 \rightarrow A_1A_2.$

$A_5 \rightarrow A_1A_3.$

Already in GNF

$A_2 \rightarrow a$

$A_3 \rightarrow b$

$A_1 \Rightarrow aA_2 \mid bA_3 \mid aA_4 \mid bA_5$

$A_4 \rightarrow A_1A_2 \rightarrow aA_2A_2 \mid bA_3A_2 \mid bA_5A_2 \mid aA_4A_2$

$A_5 \rightarrow A_1A_3 \rightarrow aA_2A_3 \mid bA_3A_3 \mid aA_4A_3 \mid bA_5A_3$

Scanned with CamScanner

GNF $G = (V, T, P, S)$

$V = \{A_1, A_2, A_3, A_4, A_5\}$
$T = \{a, b\}$

$P = \{ A_1 \rightarrow aA_2 \mid bA_3 \mid aA_4 \mid bA_5$
$A_2 \rightarrow a$
$A_3 \rightarrow b$
$A_4 \rightarrow aA_2A_2 \mid bA_3A_2 \mid bA_5A_2 \mid aA_4A_2$
$A_5 \rightarrow aA_2A_3 \mid bA_3A_3 \mid aA_4A_3 \mid bA_5A_3 \}$

Conversion to PDA

S1) push A_1 to stack and change state to q_1
$\delta(q_0, \epsilon, z_0) = (q_1, A_1 z_0)$ — 1

S2) for each production $A \rightarrow a\alpha$ introduce
$\delta(q_1, a, A) = (q_1, \alpha)$

Production	Transition
$A_1 \rightarrow aA_2$	$\delta(q_1, a, A_1) = (q_1, A_2)$ — 2
$A_1 \rightarrow bA_3$	$\delta(q_1, b, A_1) = (q_1, A_3)$ — 3
$A_1 \rightarrow aA_4$	$\delta(q_1, a, A_1) = (q_1, A_4)$ — 4
$A_1 \rightarrow bA_5$	$\delta(q_1, b, A_1) = (q_1, A_5)$ — 5
$A_2 \rightarrow a$	$\delta(q_1, a, A_2) = (q_1, \epsilon)$ — 6
$A_3 \rightarrow b$	$\delta(q_1, b, A_3) = (q_1, \epsilon)$ — 7
$A_4 \rightarrow aA_2A_2$	$\delta(q_1, a, A_4) = (q_1, A_2A_2)$ — 8
$A_4 \rightarrow bA_3A_2$	$\delta(q_1, b, A_4) = (q_1, A_3A_2)$ — 9
$A_4 \rightarrow bA_5A_2$	$\delta(q_1, b, A_4) = (q_1, A_5A_2)$ — 10
$A_5 \rightarrow aA_2A_3$	$\delta(q_1, a, A_5) = (q_1, A_2A_3)$ — 11
$A_5 \rightarrow bA_3A_3$	$\delta(q_1, b, A_5) = (q_1, A_3A_3$ — 12

Productions	Transitions.
$A_5 \to a A_4 A_3$	$\delta(q_1, a, A_5) = (q_1, A_4 A_3)$ — 13
$A_4 \to a A_4 A_2$	$\delta(q_1, a, A_4) = (q_1, A_4 A_2)$ — 14
$A_5 \to b A_5 A_3$	$\delta(q_1, b, A_5) = (q_1, A_5 A_3)$ — 15

S3) In state q_1, without consuming any i/p change state to q_f which is an accepting state
i.e, $\delta(q_1, \epsilon, Z_0) = (q_f, Z_0)$ — 14

$M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$Q = \{q_0, q_1, q_f\}$.
$\Sigma = \{a, b\}$
$\Gamma = \{A_1, A_2, A_3, A_4, A_5, Z_0\}$.
$q_0 \in Q \to$ start state
$Z_0 \in \Gamma \to$ initial symbol on stack
$F \in \{q_f\} \to$ final state.

Obtain a CFG that generates the language accepted by PDA

$M = (\{q_0, q_1\}, \{a, b\}, \{A, Z\}, \delta, q_0, Z, \{q_1\})$
with the transitions.

$\delta(q_0, a, Z) = (q_0, AZ)$
$\delta(q_0, b, A) = (q_0, AA)$
$\delta(q_0, a, A) = (q_1, \epsilon)$

Now the tranisition, $\delta(q_0, a, A) = (q_1, \epsilon)$

can be converted production as shown below.

For S of the form.	Resulting production.
$\delta(q_i, a, z) = (q_i, \varepsilon)$	$(q_i\, z\, q_j) \rightarrow a$
$\delta(q_0, a, A) = (q_1, \varepsilon)$	$(q_0\, A\, q_1) \rightarrow a$

Now the transitions

$$\delta(q_0, a, z) = (q_0, Az)$$
$$\delta(q_0, b, A) = (q_0, AA)$$

can be converted into productions using rule.

For S of the form.	Resulting productions
$\delta(q_i, a, z) = (q_j, AB)$	$(q_i\, z\, q_k) \rightarrow a(q_j\, A\, q_l)(q_l\, B\, q_k)$
$\delta(q_0, a, z) = (q_0, Az)$	$(q_0\, z\, q_0) \rightarrow a(q_0\, A\, q_0)(q_0\, z\, q_0) \mid a(q_0\, A\, q_1)(q_1\, z\, q_0)$
	$(q_0\, z\, q_1) \rightarrow a(q_0\, A\, q_0)(q_0\, z\, q_1) \mid a(q_0\, A\, q_1)(q_1\, z\, q_1)$
$\delta(q_0, b, A) = (q_0, AA)$	$(q_0\, z\, q_0) \rightarrow b(q_0\, A\, q_0)(q_0\, A\, q_0) \mid b(q_0\, A\, q_1)(q_1\, A\, q_0)$
	$(q_0\, z\, q_1) \rightarrow b(q_0\, A\, q_0)(q_0\, A\, q_1) \mid b(q_0\, A\, q_1)(q_1\, A\, q_1)$

The start symbol of the grammar will be $q_0 z q_1$