1) $L = \{wcw^R \quad w \in (a,b)^*\}$

eg: string:- {aca, bcb, abcba, abacaba, aacaa, bbcbb, ...

Γ = set of all the stack alphabet.

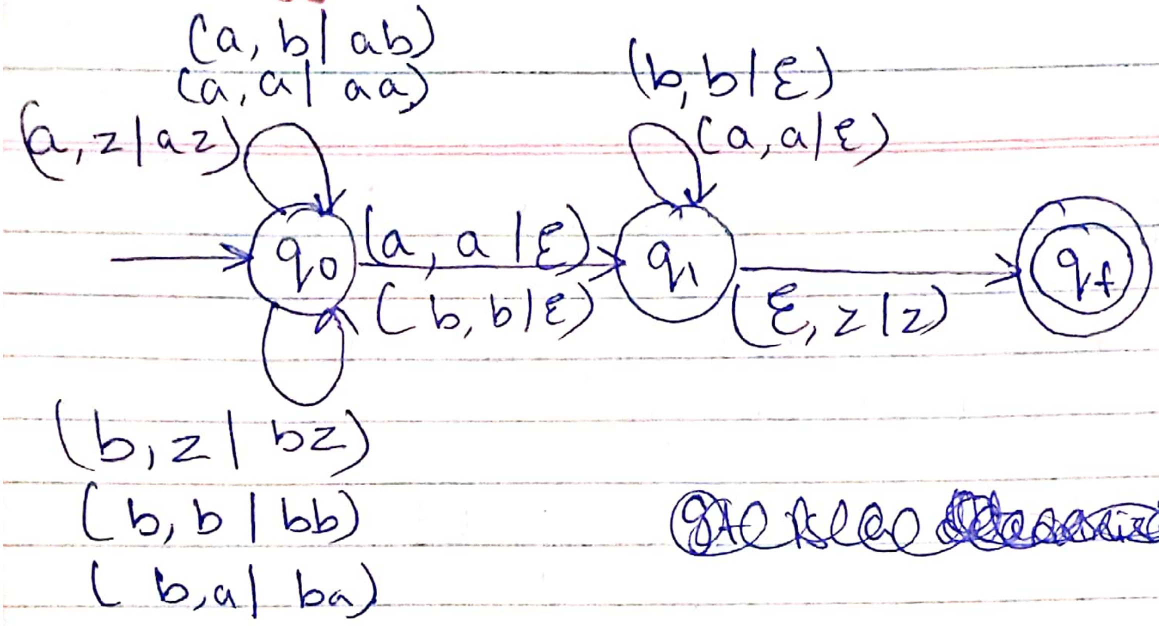
$F = \{a, b, z\}$

Z = stack start symbol.
a = input alphabet.
b = input alphabet.

To construct PDA
q_0 = initial state
q_f = final state
Z = stack start symbol.
ε = indicates pop operation.

Stack transition function.

$\delta(q_0, a, z) \vdash (q_0, az)$
$\delta(q_0, a, a) \vdash (q_0, aa)$
$\delta(q_0, b, z) \vdash (q_0, bz)$
$\delta(q_0, b, b) \vdash (q_0, bb)$
$\delta(q_0, a, b) \vdash (q_0, ab)$
$\delta(q_0, b, a) \vdash (q_0, ba)$
$\delta(q_0, c, a) \vdash (q_1, a)$
$\delta(q_0, c, b) \vdash (q_1, b)$
$\delta(q_1, a, a) \vdash (q_1, \varepsilon)$
$\delta(q_1, b, b) \vdash (q_1, \varepsilon)$
$\delta(q_1, \varepsilon, z) \vdash (q_f, z)$



It is a determinististic PDA.

2) Convert the given grammar to PDA.

$S \rightarrow AaBB \mid aAA$
$A \rightarrow aBB \mid a$
$B \rightarrow bBB \mid A$
$C \rightarrow a.$

s1) Convert grammar to GNF.

$B \rightarrow A$ is a unit production and not in GNF

$B \longrightarrow A$ $A \rightarrow aBB \mid a$

$\therefore B \rightarrow bBB \mid aBB \mid a.$

Then, $S \rightarrow AaBB \mid aAA$
$A \rightarrow aBB \mid a$
$B \rightarrow bBB \mid aBB \mid a$
$C \rightarrow a$

As, $S \rightarrow AaBB$ is not in GNF, substitute
$A \rightarrow aBB \mid a$ and $C \rightarrow a$.

$S \rightarrow (aBB|a)\ CBB|\ aAA$

$S \rightarrow aBBCBB|\ aCBB\ |aAA.$

$A \rightarrow aBB|a$

$B \rightarrow bBB|aBB|a$

$C \rightarrow a$

The final GNF grammar is

$S \rightarrow aBBCBB\ |\ aCBB|\ aAA$

$A \rightarrow aBB|a$

$B \rightarrow bBB|aBB|a.$

$C \rightarrow a.$

S2) Push S to stack & q_0 is start state and change to q_1

$\delta(q_0, \varepsilon, z_0) = (q_1, Sz_0)$ — 1

S3) For each production $A \rightarrow a\alpha$ introduce $\delta(q_1, a, A) = (q_1, \alpha)$

This can be done as.

Production	Transitions
$S \rightarrow aBBCBB$	$\delta(q_1, a, S) = (q_1, BBCBB)$ — 2
$S \rightarrow aCBB$	$\delta(q_1, a, S) = (q_1, CBB)$ — 3
$S \rightarrow aAA$	$\delta(q_1, a, S) = (q_1, AA)$ — 4
$A \rightarrow aBB$	$\delta(q_1, a, A) = (q_1, BB)$ — 5
$A \rightarrow a$	$\delta(q_1, a, A) = (q_1, \varepsilon)$ — 6
$B \rightarrow bBB$	$\delta(q_1, b, B) = (q_1, BB)$ — 7
$B \rightarrow aBB$	$\delta(q_1, a, B) = (q_1, BB)$ — 8
$B \rightarrow a$	$\delta(q_1, a, B) = (q_1, \varepsilon)$ — 9
$C \rightarrow a$	$\delta(q_1, a, C) = (q_1, \varepsilon)$ — 10

S4) Finally in state q_1, without consuming any input, change the state to q_f.

$$\delta(q_1, \varepsilon, z_0) = (q_f, z_0) \quad \text{— } (11)$$

The PDA is given by $M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

$Q = \{q_0, q_1, q_f\}$
$\Sigma = \{a, b\}$
$\Gamma = \{S, A, B, C, Z_0\}$
q_0 = start ~~stats~~ state.
Z_0 = initial symbol on stack
$F = \{q_f\} \longrightarrow$ final state.
$\delta \longrightarrow$ transitions.