I Design a context free grammar.

a) $\{0^n 1^n \mid n \geq 1\}$

$G = (V, T, P, S)$

$V = \{S\}$ $T = \{0, 1\}$

$S = \{S\}$

$P :- S \rightarrow 01 \mid 0S1$

b) $\{a^i b^j c^k \mid i \neq j \text{ or } j \neq k\}$

$G = (V, T, P, S)$

$V = \{S, A, B, C, D, E\}$ $T = \{a, b, c\}$

$S = \{S\}$

$P :- S \rightarrow AB \mid CD$

$A \rightarrow aA \mid \varepsilon$

$D \rightarrow cD \mid \varepsilon$

$B \rightarrow bBc \mid cD \mid E$

$C \rightarrow aCb \mid aA \mid E$

$E \rightarrow bE \mid b$

c) Set of all strings of a's and b's that are not of the form ww (no string repeated)

$G = (V, T, P, S)$

$V = \{S, A, B\}$ $T = \{a, b\}$ $S = \{S\}$

$P \Rightarrow \{ S \rightarrow AB \mid BA \mid A \mid B$

$A \rightarrow aAa \mid aAb \mid bAa \mid bAb \mid a$

$B \rightarrow aBa \mid aBb \mid bBa \mid bBb \mid b \}$

d) The set of all the strings with twice as many 0's as 1's

$G = (V, T, P, S)$

$V = \{S\}$ $T = \{0, 1\}$

$S = \{S\}$

$P :- \{S \to S1S0S0S \mid S0S1S0S \mid S0S0S1S \mid \varepsilon\}$

4) a) Ambiguity → For a given language CFG, if more than one sparse tree exists ⇒ the CFG is ambiguious generated sequences of left and right derivations are different ⇒ ambiguous.
Generated sequences of left and right derivations are same ⇒ unambigious.

For the given language

$S \Rightarrow A1B$ — ①, $A \rightarrow 0A$ ② $| \epsilon$ — ③

$B \Rightarrow 0B$ ④ $| 1B$ ⑤ $| \epsilon$ → ⑥

is ~~ama~~ unambigious because sequence 00101 can be produced by both left and right derivations as shown below.

Left derivation

derivation	Rule	
S	S → A1B	①
A1B	A → 0A	②
0A1B	A → 0A	②
00A1B	A → ε	③
001B	B → 0B	④
0010B	B → 1B	⑤
00101B	B → ε	⑥
00101		

Right derivation.

derivation	Rule	
S	S → A1B	①
A1B	B → 0B	④
A10B	B → 1B	⑤
A101B	B → ε	⑥
A101	A → 0A	②
0A101	A → 0A	②
00A101	A → ε	③
00101		

b) The grammar that is ambigious for this language consider generating 00101 using grammar.

$S \rightarrow A1B \rightarrow$ 1 $0A \rightarrow$ 2 $\epsilon \rightarrow$ 3

$A \rightarrow 0A | \epsilon$ $1B \rightarrow$ 4 $\epsilon \rightarrow$ 5

$B \rightarrow 1B | \epsilon$.

Left Derivation			Right Derivation		
Derivation	Rule		Derivation	Rule	
S	$S \rightarrow A1B$	1	S	$S \rightarrow A1B$	1
A1B	$A \rightarrow 0A$	2	A1B	$B \rightarrow 1B$	4
0A1B	$A \rightarrow 0A$	2	A11B	$B \rightarrow \epsilon$	5
00A1B	$A \rightarrow \epsilon$	3	A11	$A \rightarrow 0A$	2
001B	$B \rightarrow 1B$	4	0A11	$A \rightarrow 0A$	2
0011B	$B \rightarrow \epsilon$		00A11	$A \rightarrow \epsilon$	3
0011			0011		

Hence ambigious, It is not possible to generate 00101.

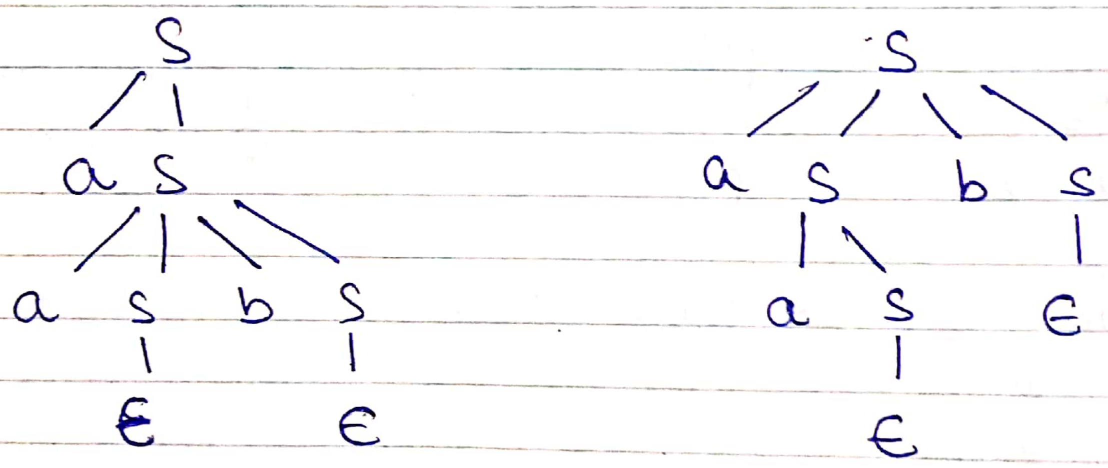
4) Consider the grammar:-

$$S \rightarrow aS \mid aSbS \mid \epsilon$$

This grammar is ambiguous.
Show in particular that the string 'aab' has two:-

a) Parse Trees
b) Left most derivation
c) Right most derivation.

Here are the parse trees:-



Left most derivation 'aab'

① $S \Rightarrow a S$ replaced by $aSbS$
$S \Rightarrow aS \Rightarrow aaSbS$ replaced by ϵ
$S \Rightarrow aS \Rightarrow aabS$ replaced by ϵ
$S \rightarrow aS \Rightarrow aaSbS \Rightarrow aabS \Rightarrow aab$
required ~~strig~~ string : aab

② $S \Rightarrow aSbS$ replace by aS
$S \Rightarrow aaSbS$ replace by ϵ
$S \Rightarrow aabS$ replace by ϵ
$S \Rightarrow aSbS \Rightarrow aaSbS \Rightarrow aabS \Rightarrow aab$

③ Right most derivation 'aab'

i) $S \Rightarrow aS \Rightarrow aaSbS \Rightarrow aaSb \Rightarrow aab.$

replace with aSbS

replace with ϵ

replace with ϵ

ii) $S \Rightarrow aSbS \Rightarrow aSb \Rightarrow aaSb \Rightarrow aab.$

ϵ

aS

ϵ