Consider the following grammar.

$S \rightarrow 0A0 \mid 1B1 \mid BB$

$A \rightarrow C$

$B \rightarrow S \mid A$

$C \rightarrow S \mid \varepsilon$.

a) Eliminate ϵ productions

b) Eliminate any unit production in the resulting grammar

c) Eliminate any useless symbols in the resulting grammar.

d) Put the resulting grammar into Chomsky Normal form.

→ a) Step 1:

	Old variable (OV)	New variable	Production
1.	ϕ	C	$C \rightarrow \varepsilon$
2	C	C, A	$A \rightarrow C$
3.	C, A	C, A, S, B	$S \rightarrow 0A0$ $B \rightarrow A.$
4	C, A, S, B	C, A, S, B	$S \rightarrow 1B1$ $S \rightarrow BB$ $B \rightarrow S$ $C \rightarrow S$
5.	C, A, S, B	C, A, S, B	—

$V_1 = \{S, A, B, C\}$

Step 2:

Productions P	Productions P_1
$S \to OAO$	$S \to OAO \mid OO$
$S \to 1B1$	$S \to 1B1 \mid 11$
$S \to BB$	$S \to BB$
$A \to C$	$A \to C$
$B \to S$	$B \to S$
$B \to A$	$B \to A$
$C \to S$	$C \to S$

Grammar G1 can be written as.

$G_1 = (V_1, T_1, P_1, S)$

$V_1 = \{S, A, B, C\}$

$T_1 = \{0, 1\}$

$P_1 = \{ S \to OAO \mid OO \mid 1B1 \mid 11 \mid BB$
$A \to C$
$B \to S \mid A$
$C \to S$
$\}$

$S \to$ start symbol.

b) Eliminate unit productions

Non-unit productions

$S \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB.$

Unit productions

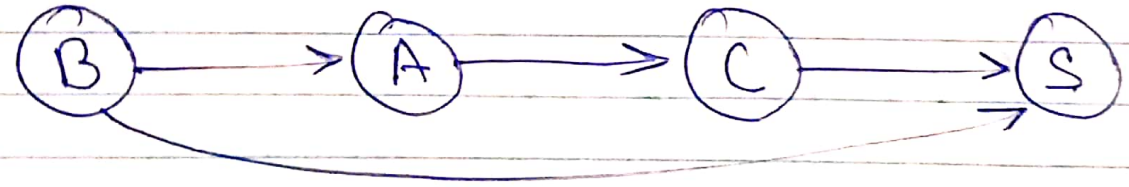
$A \to C$

$B \to S$

$B \to A$

$C \to S$

Dependency graph.



$C \overset{*}{\Rightarrow} S$

$A \overset{*}{\Rightarrow} S$

$A \overset{*}{\Rightarrow} C$

$B \overset{*}{\Rightarrow} S$

$B \overset{*}{\Rightarrow} C$

$B \overset{*}{\Rightarrow} A$

$C \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$

$A \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB.$

$B \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$

(V_1, T_1, P_1, S)

$V_1 = \{S, A, B, C\}$.

$T_1 = \{0, 1\}$.

$P_1 = \{$

$S \rightarrow 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB.$

$A \rightarrow 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$

$B \rightarrow 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB.$

$C \rightarrow 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB.$

$S \rightarrow$ Start symbol.

c) Eliminating useless symbols.

Step 1:

Old variable	new variable	Production.
ϕ	A, B, C, D	$A \rightarrow 00$
		$A \rightarrow 11$
		$B \rightarrow 00$
		$B \rightarrow 11$
		$C \rightarrow 00$
		$C \rightarrow 11.$
		$S \rightarrow 00$
		$S \rightarrow 11$
T		

Old variable	New variable	Production.
A, B, C, S	A, B, C, S	S → OAO | 1B1 | BB A → OAO | 1B1 | BB B → OAO | 1B1 | BB C → OAO | 1B1 | BB.

A, B, C, S A, B, C, S.

Resulting Grammar:-

$G_1 = (V_1, T_1, P_1, S)$.

$V_1 = \{A, B, C, S\}$.

$T_1 = \{0, 1\}$.

$P_1 = \{$

S → OAO | OO | 1B1 | 11 | BB.

A → OAO | OO | 1B1 | 11 | BB.

B → OAO | OO | 1B1 | 11 | BB.

C → OAO | OO | 1B1 | 11 | BB.

$\}$

S → Start Symbols.

Step 2:-

P2	T2	V2
		S
S → 0A0 | 00 | 1B1 | 11 | BB.	0, 1	A, B, S
A → 0A0 | 00 | 1B1 | 11 | BB	0, 1	A, B, S
B → 0A0 | 00 | 1B1 | 11 | BB	0, 1	A, B, S

Resulting Grammar.

$G_2 = (V_2, T_2, P_2, S)$

$V_2 = \{S, A, B\}$.

$T_2 = \{0, 1\}$

$P_2 = \{$

S → 0A0 \| 00 \| 1B1 \| 11 \| BB.
A → 0A0 \| 00 \| 1B1 \| 11 \| BB.
B → 0A0 \| 00 \| 1B1 \| 11 \| BB

$\}$.

S → start symbol.

d) Chomsky Normal Form (CNF.)

Step 1: All production which are in CNF, add to P_1.

S → BB
A → BB
B → BB.

Given productions	Action	Result Production
$S \to 0A0 \mid 00 \mid 1B1 \mid 11$	Replace 0 by B_0 and introduce $B_0 \to 0$, Replace 1 by B_1 and introduce $B_1 \to 1$	$S \to B_0AB_0 \mid B_0B_0 \mid B_1BB_1 \mid B_1B_1$ $B_0 \to 0$ $B_1 \to 1$.
$A \to 0A0 \mid 00 \mid 1B1 \mid 11$	Replace 0 by B_0 and introduce $B_0 \to 0$, replace 1 by B_1 and introduce $B_1 \to 1$	$A \to B_0AB_0 \mid B_0B_0 \mid B_1BB_1 \mid B_1B_1$ $B_0 \to 0$ $B_1 \to 1$.
$B \to 0A0 \mid 00 \mid 1B1 \mid 11$	Replace 0 by B_0 and introduce $B_0 \to 0$, replace 1 by B_1 and introduce $B_1 \to 1$	$B \to B_0AB_0 \mid B_0B_0 \mid B_1BB_1 \mid B_1B_1$ $B_0 \to 0$ $B_1 \to 1$.

Resulting grammar.

$$G_1 = (V_1, T_1, P_1, S)$$

$$V_1 = \{S, A, B, B_0, B_1\}.$$

$$T_1 = \{0, 1\}$$

$P_1 = \{$

$S \rightarrow B_0AB_0 \mid B_0B_0 \mid B_1BB_1 \mid B_1B_1$

$A \rightarrow B_0AB_0 \mid B_0B_0 \mid B_1BB_1 \mid B_1B_1$

$B \rightarrow B_0AB_0 \mid B_0B_0 \mid B_1BB_1 \mid B_1B_1$

$B_0 \rightarrow 0$

$B_1 \rightarrow 1$

$\}$

$S \rightarrow$ Start symbol.

Step 2:-

The production already in CNF

$S \rightarrow BB \mid B_0B_0 \mid B_1B_1$

$A \rightarrow BB \mid B_0B_0 \mid B_1B_1$

$B \rightarrow BB \mid B_0B_0 \mid B_1B_1$

$B_0 \rightarrow 0$

$B_1 \rightarrow 1.$

Not in CNF,

$S \rightarrow B_0AB_0 \mid B_1BB_1$

$A \rightarrow B_0AB_0 \mid B_1BB_1$

$B \rightarrow B_0AB_0 \mid B_1BB_1$

Given Production	Action:	Resulting Production
$S \to B_0 A B_0 \mid B_1 B B_1$	Replace AB_0 with D_1 and introduce $D_1 \to AB_0$. replace $B_0 B_1$ with D_2. and introduce $D_2 \to BB_1$	$S \to B_0 D_1 \mid B_1 D_2$. $D_1 \to AB_0$ $D_2 \to BB_1$
$A \to B_0 A B_0 \mid B_1 B B_1$	,,	$A \to B_0 D_1 \mid B_1 D_2$ $D_1 \to AB_0$ $D_2 \to BB_1$
$B \to B_0 A B_0 \mid B_1 B B_1$	,,	$B \to B_0 D_1 \mid B_1 D_2$ $D_1 \to AB_0$ $D_2 \to BB_1$

Final Grammar,

$G_1 = (V_1, T_1, P_1, S)$.

$V_1 = \{ S, A, B, B_0, B_1, D_1, D_2 \}$

$T_1 = \{ 0, 1 \}$.

$P_1 = \{$

$S \rightarrow BB \mid B_0B_0 \mid B_1B_1 \mid B_0D_1 \mid B_1D_2.$

$A \rightarrow BB \mid B_0B_0 \mid B_1B_1 \mid B_0D_1 \mid B_1D_2.$

$B \rightarrow BB \mid B_0B_0 \mid B_1B_1 \mid B_0D_1 \mid B_1D_2.$

$B_0 \rightarrow 0$

$B_1 \rightarrow 1$

$D_1 \rightarrow AB_0$

$D_2 \rightarrow BB_1$

$\}$.

S → Start Symbol.