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Non-linear optimal control for the hot-steel rolling mill system

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Abstract: Control of hot-steel rolling mills aims at raising the levels of quality of the related industrial production and at minimising the cost of the electric energy consumed by such industrial units. This paper proposes a non-linear optimal control approach for the hot-steel rolling mill system. The non-linear dynamic model of the hot-steel rolling mill undergoes approximate linearisation around a temporary operating point which is recomputed at each iteration of the control method. The linearisation relies on Taylor series expansion and on the calculation of the system's Jacobian matrices. For the approximately linearised model of the hot-steel rolling process, an H-infinity feedback controller is designed. This controller provides the solution of the non-linear optimal control problem for the system under model uncertainty and external perturbations. For the computation of the controller's feedback gain, an algebraic Riccati equation is iteratively solved at each time-step of the control method. The global asymptotic stability properties of the control method are proven through Lyapunov analysis. Finally, to implement state estimation-based control for this system, the H-infinity Kalman filter is proposed as a robust state estimator.

1 Introduction

Control of hot steel rolling mills is of major importance for heavy industries (e.g. the automotive and ship-building industry) or for construction works since it allows to produce metal strips or plates of variable thickness and high strength [1–3]. Hot-steel rolling mills are classified among energy-intensive industries and a major objective is to minimise the electric power consumed by them. The aim of the hot strip finishing mill process is to roll transfer bars, produced in a rough milling process (around 30 mm thick), into steel strips of variable thickness (between 0.8 and 20 mm) [4, 5]. The reduction of the thickness is achieved by moving the metal strips through rolling strands where pressure is exerted [6–8]. Besides, it is possible to apply interstand pressure on the metal strips through their contact with a stiff pole which is called looper and which rotates around a pivot position [9–11]. Actually, by varying the looper's angle with respect to the baseline, the motion of the metal strip is uplifted and variable tension is applied on it [12–14]. As a result, one can control more precisely the finishing of the processed strip. Control of hot steel rolling mills with a variable angle looper is a non-trivial non-linear control problem [15–17]. The difficulties one has to overcome are related with the strongly non-linear multivariable model of the looper's system and the inability to obtain directly measurements of the tension [18–20].

The outputs of the system are taken to be the turn angle of the looper and the tension that is exerted by the looper on the metal strips. The control inputs of the system are the speed (mass flow rate) of the metal strips in the interstand area and the torque that is developed by the actuator which makes the looper rotate. Model-based approaches to the related control problem make use of global or approximate linearisation methods [21–23]. Moreover, in certain cases, model-free control of the rolling mill system has been attempted [24–26]. In this paper, a non-linear optimal control method is developed for the hot-steel roll milling system [27–29]. The non-linear dynamic model of the system undergoes first approximate linearisation around a temporary operating point which is updated at every time-step for the control method [30–32]. This linearisation point consists of the present value of the state vector of the rolling mill and of the last value of the control inputs vector that was applied on it. The linearisation relies on Taylor series expansion and on the computation of the associated Jacobian matrices. For the linearised model of the rolling mill, a stabilising H-infinity feedback controller is designed [33–35].

Actually, the H-infinity controller stands for the solution of the optimal control problem under model uncertainty and external perturbations. The controller represents the solution of a min–max differential game in which the control inputs try to minimise a cost function that comprises quadratic terms of the system's state vector error, whereas the model uncertainty and disturbance terms try to minimise this cost function. To compute the controller's feedback gains, an algebraic Riccati equation has to be solved at each iteration of the control algorithm. The stability properties of the control scheme are proven through Lyapunov analysis. First, it is demonstrated that the control method satisfies the H-infinity tracking performance criterion, which signifies elevated robustness against model uncertainty and external perturbations [36, 37]. Moreover, under moderate conditions, it is also proven that the control loop is globally asymptotically stable. This confirms that the tracking error for all state variables of the system is asymptotically eliminated. Finally, to implement state estimation-based control without the need to measure the entire state vector of the system, the H-infinity Kalman filter is used as a robust state estimator [38, 39].

The topic of control and optimisation of the hot-steel rolling mill process is of interest for several industrial sectors and so far several efforts have been done towards its solution [40–45]. The article offers one of the most effective solutions to the non-linear optimal control problem of the hot-steel rolling mill system. Popular approaches for industrial control such as model predictive control (MPC) or Nonlinear model predictive control (NMPC) may have questionable performance when applied to this control problem. Actually, MPC has been developed for linear dynamical systems and its use in the case of the non-linear model of the hot-steel rolling mill will risk the control loop's destabilisation. Besides, the convergence of NMPC is not assured either. The convergence of the method's iterative search for an optimum depends on initialisation and on specific parameters' selection,

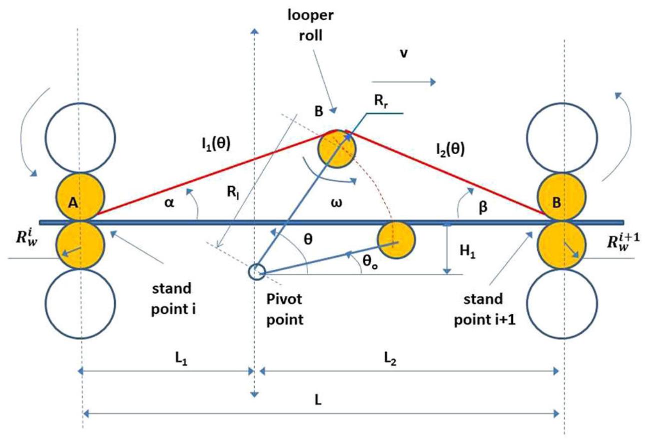


Fig. 1 *Diagram of the hot-steel rolling mill system*

Table 1 Parameters of the hot-steel rolling mill process

Symbol	Parameter's name	Symbol	Parameter's name
θ	looper angle	L	distance between two stands
ω	looper angular speed	L_1	distance of i stand to looper pivot
R_l	looper arm length	L_2	distance of looper pivot to $(i+1)$ stand
h	strip exit thickness	ρ	steel density
H	strip entry thickness	$l_1(\theta)$	distance of i stand to looper roll
w	strip width	$l_2(\theta)$	distance of $(i+1)$ stand to looper roll
R_w^i	work roll radius of i stand	$\xi(t)$	inter-strand strip length deviation from L
R_w^{i+1}	work roll radius of $(i+1)$ stand	R_r	radius of the looper roll
M_L	looper mass	H_1	distance of actual passline to looper pivot
R_G	distance of looper's pivot to center of gravity	g	acceleration of gravity

therefore under NMPC one cannot always guarantee a solution for the non-linear optimal control problem of the hot-steel rolling mill process. Finally, comparing to local models-based optimal control the article's approach exhibits specific advantages: (i) in the local-models based approach linearisation is performed around multiple operating points (equilibria) which are selected off-line and which are not updated in time, whereas in the article's approach there is linearisation only around one single operating which is updated at each iteration of the control algorithm, (ii) in the local-models approach there is need to perform solution of multiple Riccati equations associated with the individual models and this solution is performed offline. On the other side, in the article's approach there is need to solve one single Riccati equation and this solution is repeated at each time-step of the control algorithm. (iii) in local models-based control there is need to find a common solution for the individual Riccati equations, and one cannot assure that such a solution always exists. On the other side, in the article's approach there is need to obtain solution for one single Riccati equation and the existence of such a solution can be assured through suitable selection of the gains and coefficients that appear in it. In conclusion, comparing to local models-based control, the article's control method is computationally more efficient and is subject to less constraining assumptions.

The structure of the article is as follows. In Section 2, the dynamic model of the hot-steel rolling mill system is analysed and its state-space description is obtained. In Section 3, the dynamic model of the rolling mill undergoes approximate linearisation through Taylor series expansion and through the computation of the associated Jacobian matrices. In Section 4, the H-infinity control problem is formulated for the linearised model of the rolling mills and a stabilising feedback control law is proposed for it. In Section 5, the stability properties of the control method are proven through Lyapunov analysis. In Section 6, the H-infinity Kalman filter is introduced as a robust state observer which allows for the implementation of state estimation-based feedback control. In Section 7, the performance of the control method is tested through simulation experiments. Finally, in Section 8 concluding remarks are stated.

2 Dynamic model of the hot-steel rolling mill

The diagram of the hot-steel rolling mill system is given in Fig. 1. The following parameters can be defined [1, 9]: θ is the looper angle, ω is the looper angular speed, R_l is the looper arm length, h is the strip exit thickness, H is the strip entry thickness, w is the strip width, R_w^i is the work roll radius of the ith stand, R_w^{i+1} is the work roll radius of the $(i+1)$th stand, M_L is the looper mass, R_G is the distance between the pivoting point of the looper and the centre of gravity of the looper, R_r is the radius of the looper roll, L is the distance between two stands, L_1 is the distance between the ith stand and the looper pivot, L_2 is the distance between the looper pivot and the $(i+1)$th stand, ρ is the steel density, $l_1(\theta)$ is the distance between the ith stand and the looper roll, $l_2(\theta)$ is the distance between the $i+1$th stand and the looper roll, $\xi(t)$ is the deviation of the inter-strand strip length with respect to L, g is the gravitational constant and H_1 is the distance between the actual passline and the looper pivot. The parameters of the hot-steel rolling mill process are also outlined in Table 1. Besides, the constituents of the system's dynamics are analysed next.

Tension dynamics: The interstand tension dynamics is given by the strip stretch $L'(\theta) - (L + \xi(t))$ and Young's modulus E [1, 9]

$$\sigma(t) = E\left[\frac{L'(\theta) - (L + \xi(t))}{L + \xi(t)}\right] \tag{1}$$

for $L'(\theta)) > L + \xi(t)$, θ is the looper angle, L is the interstrand length, $L + \xi(t)$ is the accumulated material length, $v(t)$ is the mass flow difference of the strip between the stands. Actually $L'(\theta)$ is the sum of the distances between the ith, $(i+1)$th stand and the looper roll. It holds that

$$\begin{aligned} L'(\theta) &= l_1(\theta) + l_2(\theta), \quad \text{where} \\ l_1(\theta) &= \sqrt{(L_1 + R_l\cos(\theta))^2 + (R_l\sin(\theta) + R_r - H_1)^2} \\ l_2(\theta) &= \sqrt{(L_2 - R_l\cos(\theta))^2 + (R_l\sin(\theta) + R_r - H_1)^2} \end{aligned} \tag{2}$$

where $\xi(t)$ is calculated as follows:

$$\begin{aligned} \dot{\xi}(t) &= v(t) + w_\xi(t) \\ v(t) &= v_s^i(t) - V_s^{i+1}(t) \\ v_s^i(t) &= (1 + S_f^i)V_R^i(t) \\ V_s^{i+1}(t) &= (1 - S_b^{i+1})V_R^{i+1}(t) \end{aligned} \tag{3}$$

where $v(t)$ is the difference in the strip speed entering and leaving the interstand space, $v_s^i(t)$ is the strip speed leaving the upstream stand i, $V_s^{i+1}(t)$ is the strip speed exiting the downstream stand $(i+1)$, $V_R^i(t)$ is the linear speed of the ith stand, $V_R^{i+1}(t)$ is the linear speed of the $(i+1)$th stand, S_f^i is the coefficient of forward slip which occurs between the rolls at the ith stand and the strip, S_b^{i+1} is the coefficient of backward slip which occurs between the rolls at the ith stand and the strip. The definition of disturbance terms is (i) $\xi(t)$ is the inter-strand strip length deviation from L, (ii) ω_ξ is the looper's angular speed deviation.

About the denominator of the relation that describes the strip's tension, that is (1) one can consider that the strip's elongation $\xi(t)$ is significantly smaller than L, consequently it holds that

$L + \xi(t) \simeq L$. By differentiating (1) with respect to time, one obtains [1, 9]

$$\dot{\sigma}(t) = \frac{E}{L}\left[\frac{\mathrm{d}}{\mathrm{d}t}L'(\theta) - \dot{\xi}(\theta)\right] \Rightarrow$$
$$\dot{\sigma}(t) = \frac{E}{L}\{R_l[\sin(\theta+\beta) - \sin(\theta-\alpha)]\dot{\theta}(t) - (v(t) + w_g(t))\} \tag{4}$$

where α is the upstream strip angle and β is the downstream strip angle. These angles are geometrically evaluated as [1, 9]

$$\alpha = \tan^{-1}\left(\frac{[R_l \sin(\theta) - H_1 + R_r]}{[L_1 + R_l \cos(\theta)]}\right) \tag{5}$$

$$\beta = \tan^{-1}\left(\frac{[R_l \sin(\theta) - H_1 + R_r]}{[L_2 - R_l \cos(\theta)]}\right) \tag{6}$$

Looper dynamics: The application of Newton's law to the motion of the looper gives [1, 9]

$$J\ddot{\theta}(t) = T_u(t) - T_{\mathrm{Load}}(\theta) + w_\omega(t) \tag{7}$$

where J is the moment of inertia of the looper with respect to the pivoting point, $T_u(t)$ is the actuator torque on the looper, $T_{\mathrm{Load}}(\theta)$ is the load torque of the looper and $w_\omega(t)$ is the unmodelled dynamics (disturbance torque). It holds that [1, 9]

$$T_{\mathrm{Load}}(\theta) = T_\sigma(\theta) + T_s(\theta) + T_L(\theta) \tag{8}$$

where

$$\begin{aligned} T_\sigma(\theta) &= \sigma h \cdot w \cdot R_l[\sin(\theta+\beta) - \sin(\theta-\alpha)] \\ T_L(\theta) &= g M_L R_G \cos(\theta) \\ T_s(\theta) &\simeq 0.5g \cdot \rho \cdot L \cdot h \cdot w \cdot R_l \cdot \cos(\theta) \end{aligned} \tag{9}$$

The above relation means that the load torque of the looper $T_{\mathrm{Load}}(\theta)$ is the sum of the load $T_\sigma(\theta)$ caused by the strip tension, the torque $T_s(\theta)$ caused by the strip weight and the torque $T_L(\theta)$ caused by the looper weight.

State-space model: The cumulative dynamics of the hot steel rolling mill is given by (4) and (7). Actually one has [1, 9]

$$\dot{\theta}(t) = \omega(t) \tag{10}$$

$$\dot{\omega}(t) = \frac{1}{J}[T_u(t) - F_1(\theta) - \sigma A F_3(\theta) + w_\omega(t)] \tag{11}$$

$$\dot{\sigma}(t) = \frac{E}{L}[F_3(\theta)\omega(t) - (v(t) + w_\xi(t))] \tag{12}$$

where $A = h \cdot w$ and

$$\begin{aligned} F_1(\theta) &= 0.5g \cdot \rho L \cdot h \cdot w \cdot R_l \cos(\theta) + g M_L R_G \cos(\theta) \\ F_3(\theta) &= R_l[\sin(\theta+\beta) - \sin(\theta-\alpha)] \end{aligned} \tag{13}$$

The control inputs of the system are $v(t)$ and $T_u(t)$, where $v(t)$ is the strip speed difference which is controlled by an automatic speed regulator, and $T_u(t)$ is the looper actuator torque which is also controlled by an external actuator.

By considering the following control loops for the speed v of the strip and for the torque T_u of the looper's actuator, and by defining $T_{u,d}$ and v_d as setpoints for the torque of the looper's actuator and for the velocity of the strip, respectively, one has

$$\begin{aligned} \dot{T}_u(t) &= v_1 \quad v_1 = \dot{T}_{u,d}(t) - k_p^2(T_u(t) - T_{u,d}(t)) \\ \dot{v}(t) &= v_2 \quad v_2 = \dot{v}_d(t) - k_p^1(v(t) - v_d(t)) \end{aligned} \tag{14}$$

The control problem of the hot-steel rolling mill is thus formulated in two levels: (i) from (10) to (12) one computes the control inputs $v(t)$ and $T_u(t)$ that should be applied to the hot-steel roll mill system so as to assure that the angle of the looper and the tension on the strip will reach the desirable setpoints, (ii) the control inputs $v(t)$ and $T_u(t)$ become setpoints for the secondary control of (14). There, v_1 and v_2 stand for control currents/voltages that allow $T_u(t)$ and $v(t)$ to reach the values needed by the control loop of the hot-steel rolling mill.

Next, the dynamics of the hot-steel rolling mill is written in state-space form. By defining the following state variables: $x_1 = \theta$, $x_2 = \omega$ and $x_3 = \sigma$, and the control inputs: $u_1 = T_u(t)$, $u_2 = v(t)$, as well the outputs of the system: $y_1 = \theta$, $y_2 = \sigma$ one has

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{1}{J}[u_1 - F_1(x) - x_3 A F_3(x) + w_\omega(t)] \\ \dot{x}_3 &= \frac{E}{L}[F_3(x)x_2 - (u_2 + w_\xi(t))] \end{aligned} \tag{15}$$

By omitting from the previous state-space description, the additive-input disturbance terms $w_\omega(t)$ and $w_\xi(t)$, which are unknown and which are going to be compensated by the robustness of the control algorithm, one has the description of the state-space model into the following matrix form:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} x_2 \\ \frac{1}{J}(-F_1(x) - x_3 A F_3(x)) \\ \frac{E}{L}(F_3(x)x_2) \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ \frac{1}{J} & 0 \\ 0 & -\frac{E}{L} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \tag{16}$$

Equivalently, the system can be written into the following concise state-space description:

$$\dot{x} = f(x) + g(x)u \tag{17}$$

where $f(x) \in R^{3\times 1}$, $g(x) \in R^{3\times 2}$, $x \in R^{3\times 1}$ and $u \in R^{2\times 1}$.

3 Approximate linearisation of the hot-steel rolling mill dynamics

The hot-steel rolling mill system dynamics undergoes approximate linearisation around a temporary operating point $(\boldsymbol{x}^*, \boldsymbol{u}^*)$ which is recomputed at each time-step of the control method. This operating point consists of the present value of the system's state vector $\boldsymbol{x}^*$ and of the last value of the control inputs vector $\boldsymbol{u}^*$ that was applied on it. Thus, the initial non-linear state-space model of the system given in (17) is now written in the linearised form

$$\dot{x} = \boldsymbol{A}x + \boldsymbol{B}u + \tilde{\boldsymbol{d}} \tag{18}$$

where $\tilde{\boldsymbol{d}}$ is the aggregate disturbance vector due to modelling errors and external perturbations, while about the Jacobian matrices of the system it holds that

$$\begin{aligned} \boldsymbol{A} &= \nabla_x[f(x) + g(x)u]|_{(\boldsymbol{x}^*, \boldsymbol{u}^*)} \\ \Rightarrow \boldsymbol{A} &= \nabla_x[f(x)]|_{(\boldsymbol{x}^*, \boldsymbol{u}^*)} \end{aligned} \tag{19}$$

$$\begin{aligned} \boldsymbol{B} &= \nabla_u[f(x) + g(x)u]|_{(\boldsymbol{x}^*, \boldsymbol{u}^*)} \\ \Rightarrow \boldsymbol{B} &= g(x)|_{(\boldsymbol{x}^*, \boldsymbol{u}^*)} \end{aligned} \tag{20}$$

The computation of the Jacobian matrix $\nabla_x[f(x)]|_{(\boldsymbol{x}^*, \boldsymbol{u}^*)}$ proceeds as follows.

About the first row of $\nabla_x[f(x)]|_{(\boldsymbol{x}^*, \boldsymbol{u}^*)}$ one has: $\partial f_1/\partial x_1 = 0$, $\partial f_1/\partial x_2 = 1$ and $\partial f_1/\partial x_3 = 0$.

About the second row of $\nabla_x[f(x)]|_{(\boldsymbol{x}^*, \boldsymbol{u}^*)}$ one has

$$\frac{\partial f_2}{\partial x_1} = -\frac{1}{J}\frac{\partial F_1(x)}{\partial x_1} - \frac{1}{J}x_3\boldsymbol{A}\frac{\partial F_3(x)}{\partial x_1},$$
$$\frac{\partial f_2}{\partial x_2} = -\frac{1}{J}\frac{\partial F_1(x)}{\partial x_2} - \frac{1}{J}x_3\boldsymbol{A}\frac{\partial F_3(x)}{\partial x_2} \text{ and}$$
$$\frac{\partial f_2}{\partial x_3} = -\frac{1}{J}\frac{\partial F_1(x)}{\partial x_3} - \frac{1}{J}\boldsymbol{A}F_3(x) - x_3\boldsymbol{A}\frac{\partial F_3(x)}{\partial x_3}$$

About the third row of $\nabla_x[f(x)]|_{(\boldsymbol{x}^*,\boldsymbol{u}^*)}$ one has

$$\frac{\partial f_3}{\partial x_1} = \frac{E}{L}\frac{\partial F_3}{\partial x_1}x_2,$$
$$\frac{\partial f_3}{\partial x_2} = \frac{E}{L}\frac{\partial F_3}{\partial x_2}x_2 + \frac{E}{L}F_3(x) \text{ and}$$
$$\frac{\partial f_3}{\partial x_3} = \frac{E}{L}\frac{\partial F_3}{\partial x_3}x_2$$

Next, the following partial derivatives are computed:

$$\frac{\partial F_1}{\partial x_1} = 0.5 \cdot g \cdot \rho \cdot L \cdot h \cdot w \cdot R_l(-\sin(x_1)) + gM_L R_G(-\sin(x_1)),$$
$$\frac{\partial F_1}{\partial x_1} = 0 \text{ and } \frac{\partial F_1}{\partial x_3} = 0$$

and similarly

$$\frac{\partial F_3}{\partial x_1} = R_l[\cos(x_1+\beta)(1-\frac{\partial \beta}{\partial x_1}) + \cos(x_1-\alpha)(1-\frac{\partial \alpha}{\partial x_1})],$$
$$\frac{\partial F_3}{\partial x_2} = 0, \frac{\partial F_3}{\partial x_3} = 0.$$

Moreover, using (5) one finds that

$$\frac{\partial a}{\partial x_1} = \cos^2(\alpha)\frac{[R_l\cos(x_1)][L_1 + R_l\cos(x_1)] + [R_l\sin(x_1) - H_1 + R_r][R_l\sin(x_1)]}{[L_1 + R_l\cos(x_1)]^2} \quad (21)$$

while, using (6) one gets that

$$\frac{\partial \beta}{\partial x_1} = \cos^2(\beta)\frac{[R_l\cos(x_1)][L_2 - R_l\cos(x_1)] - [R_l\sin(x_1) - H_1 + R_r][R_l\sin(x_1)]}{[L_2 - R_l\cos(x_1)]^2} \quad (22)$$

4 Non-linear H-infinity control

The concept of the proposed control method is that it views the dynamics of the hot-steel rolling mill process as being linear around the local linearisation point which is updated at each time-step of the control algorithm. For the approximately linearised model, an H-infinity controller is computed, while the controller's gains are also updated at each iteration of the control scheme. Next, the stabilising control inputs are applied directly to the non-linear dynamics of the process and as proven by the related stability analysis, global asymptotic stability and elimination of the state variables tracking error is achieved. The dynamic model of the hot-steel rolling mill process was previously written in the non-linear multivariable form:

$$\dot{x} = f(x, u), \quad x \in R^n,\ u \in R^m \quad (23)$$

Linearisation of the hot-steel rolling mill dynamics is performed at each iteration of the control algorithm around its present operating point $(x^*, u^*) = (x(t), u(t - T_s))$. The linearised equivalent of the system is described by

$$\dot{x} = \boldsymbol{A}x + \boldsymbol{B}u + L\tilde{d}, \quad x \in R^n,\ u \in R^m,\ \tilde{\boldsymbol{d}} \in R^q \quad (24)$$

where matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are obtained from the computation of the Jacobians

$$\boldsymbol{A} = \left.\begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{pmatrix}\right|_{(\boldsymbol{x}^*,\boldsymbol{u}^*)},$$
$$\boldsymbol{B} = \left.\begin{pmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} & \cdots & \frac{\partial f_1}{\partial u_m} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} & \cdots & \frac{\partial f_2}{\partial u_m} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial u_1} & \frac{\partial f_n}{\partial u_2} & \cdots & \frac{\partial f_n}{\partial u_m} \end{pmatrix}\right|_{(\boldsymbol{x}^*,\boldsymbol{u}^*)} \quad (25)$$

and vector $\tilde{\boldsymbol{d}}$ denotes disturbance terms due to linearisation errors. Actually, the disturbance term $\tilde{\boldsymbol{d}}$ represents both the modelling error due to the approximate linearisation of the state-space model of the hot-steel rolling mill process and the external perturbations that may affect the system, as well as noise. After linearisation around its current operating point, the hot-steel rolling mill's dynamic model is written as

$$\dot{x} = \boldsymbol{A}x + \boldsymbol{B}u + d_1 \quad (26)$$

Parameter d_1 stands for the linearisation error in the hot-steel rolling mill's dynamic model appearing in (26). The reference setpoints are denoted by $\boldsymbol{x}_d = [x_1^d, x_2^d, x_3^d]^{\mathrm{T}}$. Tracking of this trajectory is achieved after applying the control input $\boldsymbol{u}^*$. At every time instant the control input $\boldsymbol{u}^*$ is assumed to differ from the control input u appearing in (26) by an amount equal to Δu, that is $\boldsymbol{u}^* = u + \Delta u$

$$\dot{x}_d = \boldsymbol{A}x_d + \boldsymbol{B}\boldsymbol{u}^* + d_2 \quad (27)$$

The dynamics of the controlled system described in (26) can be also written as

$$\dot{x} = \boldsymbol{A}x + Bu + \boldsymbol{B}\boldsymbol{u}^* - \boldsymbol{B}u^* + d_1 \quad (28)$$

and by denoting $d_3 = -\boldsymbol{B}\boldsymbol{u}^* + d_1$ as an aggregate disturbance term one obtains

$$\dot{x} = \boldsymbol{A}x + \boldsymbol{B}u + \boldsymbol{B}\boldsymbol{u}^* + d_3 \quad (29)$$

By subtracting (27) from (29) one has

$$\dot{x} - \dot{x}_d = \boldsymbol{A}(x - x_d) + \boldsymbol{B}u + d_3 - d_2 \quad (30)$$

By denoting the tracking error as $e = x - x_d$ and the aggregate disturbance term as $\tilde{\boldsymbol{d}} = d_3 - d_2$, the tracking error dynamics becomes

$$\dot{e} = \boldsymbol{A}e + \boldsymbol{B}u + \tilde{\boldsymbol{d}} \quad (31)$$

The problem of disturbance rejection for the previously linearised model that was described by (24) cannot be handled efficiently if the classical LQR control scheme is applied. This is because of the existence of the perturbation term $\tilde{\boldsymbol{d}}$. The disturbances' effects are incorporated in the following quadratic cost:

$$J(t) = \frac{1}{2}\int_0^T [y^T(t)y(t) + ru^T(t)u(t) - \rho^2\tilde{\boldsymbol{d}}^T(t)\tilde{d}(t)]\mathrm{d}t, \quad r, \rho > 0$$

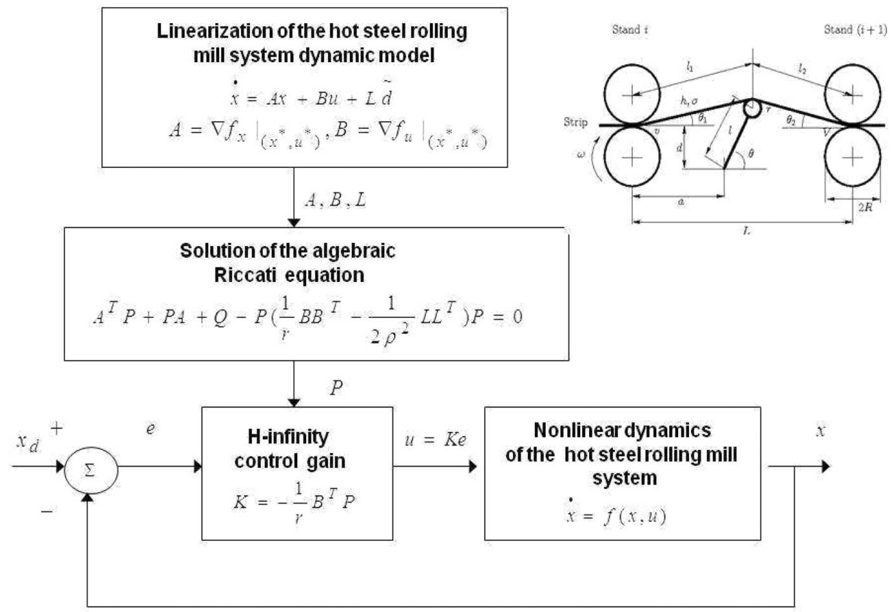


Fig. 2 *Diagram of the control scheme for the hot-steel rolling mill's system*

(32)

The significance of the negative sign in the cost function's term that is associated with the perturbation variable $\tilde{d}(t)$ is that the disturbance tries to maximise $J(t)$. This problem of min–max optimisation can be written as $\min_u \max_{\tilde{d}} J(u, \tilde{\boldsymbol{d}})$. The objective of the optimisation procedure is to compute a control signal $u(t)$ which can compensate for the worst possible disturbance that affects the system. The solution to the min–max optimisation problem is directly related to the value of the parameter ρ. This means that there is an upper bound in the disturbances magnitude that can be annihilated by the control signal.

For the linearised system given by (24), the cost function of (32) was defined, where the coefficient r determines the penalisation of the control input and the weight coefficient ρ determines the reward of the disturbances' effects. It is assumed that (i) the energy that is transferred from the disturbances signal $\tilde{d}(t)$ is bounded, that is $\int_0^\infty \tilde{\boldsymbol{d}}^T(t)\tilde{d}(t)\,\mathrm{d}t < \infty$, (ii) matrices $[\boldsymbol{A}, \boldsymbol{B}]$ and $[\boldsymbol{A}, \boldsymbol{L}]$ are stabilisable, (iii) matrix $[\boldsymbol{A}, \boldsymbol{C}]$ is detectable. Then, the optimal feedback control law is given by

$$u(t) = -Kx(t) \tag{33}$$

with $K = (1/r)\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}$ where $\boldsymbol{P}$ is a positive definite symmetric matrix which is obtained from the solution of the Riccati equation

$$\boldsymbol{A}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A} + \boldsymbol{Q} - \boldsymbol{P}\left(\frac{1}{r}\boldsymbol{B}\boldsymbol{B}^T - \frac{1}{2\rho^2}\boldsymbol{L}\boldsymbol{L}^T\right)\boldsymbol{P} = 0 \tag{34}$$

where $\boldsymbol{Q}$ is also a positive semi-definite symmetric matrix. The worst case disturbance is given by $\tilde{d}(t) = (1/\rho^2)\boldsymbol{L}^T\boldsymbol{P}x(t)$. This equation is obtained by solving the optimal control problem for the case that the system receives as input only the disturbance $\tilde{d}(t)$. The non-linear optimal control loop for the hot-steel rolling mill's system is given in Fig. 2.

The above present a generalised optimal control problem with the superposition of two control inputs, where the first input is u and the second input is $\tilde{\boldsymbol{d}}$. In such an optimal control problem, the Hamiltonian function of the system is modified accordingly. From the condition about minimising the Hamiltonian with respect to u one arrives at the relation of (33) about the optimal feedback control. From the condition about maximising the Hamiltonian with respect to $\tilde{\boldsymbol{d}}$ one arrives at the relation for the worst case disturbance the system can sustain $\tilde{\boldsymbol{d}} = (1/\rho^2)\boldsymbol{L}^T\boldsymbol{P}x$. Next by substituting these two equations in the Hamilton–Jacobi–Bellman equation of the system one can arrive at the algebraic Riccati equation given in (34).

5 Lyapunov stability analysis

Through Lyapunov stability analysis it will be shown that the proposed non-linear control scheme assures H_∞ tracking performance for the hot-steel rolling mill's system, and that in case of bounded disturbance terms asymptotic convergence to the reference setpoints is achieved. The tracking error dynamics for the hot-steel rolling mills system is written in the form

$$\dot{e} = \boldsymbol{A}e + \boldsymbol{B}u + \boldsymbol{L}\tilde{\boldsymbol{d}} \tag{35}$$

where in the hot-steel rolling mills case $\boldsymbol{L} = \boldsymbol{I} \in R^{3\times 3}$ with $\boldsymbol{I}$ being the identity matrix. Variable $\tilde{\boldsymbol{d}}$ denotes model uncertainties and external disturbances of the hot-steel rolling mill's model. The following Lyapunov equation is considered:

$$V = \frac{1}{2}e^T\boldsymbol{P}e \tag{36}$$

where $e = x - x_d$ is the tracking error. By differentiating with respect to time one obtains

$$\begin{aligned} \dot{V} &= \frac{1}{2}\dot{e}^T\boldsymbol{P}e + \frac{1}{2}e^T\boldsymbol{P}\dot{e} \Rightarrow \\ \dot{V} &= \frac{1}{2}[\boldsymbol{A}e + \boldsymbol{B}u + \boldsymbol{L}\tilde{\boldsymbol{d}}]^T P + \frac{1}{2}e^T P[\boldsymbol{A}e + \boldsymbol{B}u + \boldsymbol{L}\tilde{\boldsymbol{d}}] \Rightarrow \end{aligned} \tag{37}$$

$$\begin{aligned} \dot{V} &= \frac{1}{2}[e^T\boldsymbol{A}^T + u^T\boldsymbol{B}^T + \tilde{\boldsymbol{d}}^T\boldsymbol{L}^T]\boldsymbol{P}e \\ &+ \frac{1}{2}e^T\boldsymbol{P}[\boldsymbol{A}e + \boldsymbol{B}u + \boldsymbol{L}\tilde{\boldsymbol{d}}] \Rightarrow \end{aligned} \tag{38}$$

$$\begin{aligned} \dot{V} &= \frac{1}{2}e^T\boldsymbol{A}^T\boldsymbol{P}e + \frac{1}{2}u^T\boldsymbol{B}^T\boldsymbol{P}e + \frac{1}{2}\tilde{\boldsymbol{d}}^T\boldsymbol{L}^T\boldsymbol{P}e \\ &+ \frac{1}{2}e^T\boldsymbol{P}\boldsymbol{A}e + \frac{1}{2}e^T\boldsymbol{P}\boldsymbol{B}u + \frac{1}{2}e^T\boldsymbol{P}\boldsymbol{L}\tilde{\boldsymbol{d}} \end{aligned} \tag{39}$$

The previous equation is rewritten as

$$\dot{V} = \frac{1}{2}e^T(\boldsymbol{A}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A})e + \left(\frac{1}{2}u^T\boldsymbol{B}^T\boldsymbol{P}e + \frac{1}{2}e^T\boldsymbol{P}\boldsymbol{B}u\right) + \left(\frac{1}{2}\tilde{\boldsymbol{d}}^T\boldsymbol{L}^T\boldsymbol{P}e + \frac{1}{2}e^T\boldsymbol{P}\boldsymbol{L}\tilde{\boldsymbol{d}}\right) \tag{40}$$

Assumption: For given positive definite matrix $\boldsymbol{Q}$ and coefficients r and ρ there exists a positive definite matrix $\boldsymbol{P}$, which is the solution of the following matrix equation:

$$\boldsymbol{A}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A} = -\boldsymbol{Q} + \boldsymbol{P}\left(\frac{2}{r}\boldsymbol{B}\boldsymbol{B}^T - \frac{1}{\rho^2}\boldsymbol{L}\boldsymbol{L}^T\right)\boldsymbol{P} \tag{41}$$

Moreover, the following feedback control law is applied to the system:

$$u = -\frac{1}{r}\boldsymbol{B}^T\boldsymbol{P}e \tag{42}$$

By substituting (41) and (42) one obtains

$$\dot{V} = \frac{1}{2}e^T\left[-\boldsymbol{Q} + \boldsymbol{P}\left(\frac{2}{r}\boldsymbol{B}\boldsymbol{B}^T - \frac{1}{\rho^2}\boldsymbol{L}\boldsymbol{L}^T\right)\boldsymbol{P}\right]e + e^T\boldsymbol{P}\boldsymbol{B}\left(-\frac{1}{r}\boldsymbol{B}^T\boldsymbol{P}e\right) + e^T\boldsymbol{P}\boldsymbol{L}\tilde{\boldsymbol{d}} \Rightarrow \tag{43}$$

$$\dot{V} = -\frac{1}{2}e^T\boldsymbol{Q}e + \left(\frac{2}{r}\boldsymbol{P}\boldsymbol{B}\boldsymbol{B}^T\boldsymbol{P}e - \frac{1}{2rho^2}e^T\boldsymbol{P}\boldsymbol{L}\boldsymbol{L}^T\right)\boldsymbol{P}e - \frac{1}{r}e^T\boldsymbol{P}\boldsymbol{B}\boldsymbol{B}^T\boldsymbol{P}e + e^T\boldsymbol{P}\boldsymbol{L}\tilde{\boldsymbol{d}} \tag{44}$$

which after intermediate operations gives

$$\dot{V} = -\frac{1}{2}e^T\boldsymbol{Q}e - \frac{1}{2\rho^2}e^T\boldsymbol{P}\boldsymbol{L}\boldsymbol{L}^T\boldsymbol{P}e + e^T\boldsymbol{P}\boldsymbol{L}\tilde{\boldsymbol{d}} \tag{45}$$

or, equivalently

$$\dot{V} = -\frac{1}{2}e^T\boldsymbol{Q}e - \frac{1}{2\rho^2}e^T\boldsymbol{P}\boldsymbol{L}\boldsymbol{L}^T\boldsymbol{P}e + \frac{1}{2}e^T\boldsymbol{P}\boldsymbol{L}\tilde{\boldsymbol{d}} + \frac{1}{2}\tilde{\boldsymbol{d}}^T\boldsymbol{L}^T\boldsymbol{P}e \tag{46}$$

Lemma: The following inequality holds:

$$\frac{1}{2}e^T\boldsymbol{L}\tilde{\boldsymbol{d}} + \frac{1}{2}\tilde{\boldsymbol{d}}\boldsymbol{L}^T\boldsymbol{P}e - \frac{1}{2\rho^2}e^T\boldsymbol{P}\boldsymbol{L}\boldsymbol{L}^T\boldsymbol{P}e \le \frac{1}{2}\rho^2\tilde{\boldsymbol{d}}^T\tilde{\boldsymbol{d}} \tag{47}$$

Proof: The binomial $(\rho\alpha - (1/\rho b))^2$ is considered. Expanding the left part of the above inequality one gets

$$\rho^2a^2 + \frac{1}{\rho^2}b^2 - 2ab \ge 0 \Rightarrow \frac{1}{2}\rho^2a^2 + \frac{1}{2\rho^2}b^2 - ab \ge 0 \Rightarrow ab - \frac{1}{2\rho^2}b^2 \le \frac{1}{2}\rho^2a^2 \Rightarrow \frac{1}{2}ab + \frac{1}{2}ab - \frac{1}{2\rho^2}b^2 \le 12\rho^2a^2 \tag{48}$$

The following substitutions are carried out: $a = \tilde{d}$ and $b = e^TPL$ and the previous relation becomes

$$\frac{1}{2}\tilde{\boldsymbol{d}}^T\boldsymbol{L}^T\boldsymbol{P}e + \frac{1}{2}e^T\boldsymbol{P}\boldsymbol{L}\tilde{\boldsymbol{d}} - \frac{1}{2\rho^2}e^T\boldsymbol{P}\boldsymbol{L}\boldsymbol{L}^T\boldsymbol{P}e \le \frac{1}{2}\rho^2\tilde{\boldsymbol{d}}^T\tilde{\boldsymbol{d}} \tag{49}$$

Equation (49) is substituted in (46) and the inequality is enforced, thus giving

$$\dot{V} \le -\frac{1}{2}e^T\boldsymbol{Q}e + \frac{1}{2}\rho^2\tilde{\boldsymbol{d}}^T\tilde{\boldsymbol{d}} \tag{50}$$

Equation (50) shows that the H_∞ tracking performance criterion is satisfied. The integration of $\dot{V}$ from 0 to T gives

$$\int_0^T \dot{V}(t)\,\mathrm{d}t \le -\frac{1}{2}\int_0^T \parallel e \parallel_Q^2\,\mathrm{d}t + \frac{1}{2}\rho^2\int_0^T \parallel \tilde{\boldsymbol{d}} \parallel^2\,\mathrm{d}t \Rightarrow 2V(T) + \int_0^T \parallel e \parallel_Q^2\,\mathrm{d}t \le 2V(0) + \rho^2\int_0^T \parallel \tilde{\boldsymbol{d}} \parallel^2\,\mathrm{d}t \tag{51}$$

Moreover, if there exists a positive constant $M_d > 0$ such that

$$\int_0^\infty \parallel \tilde{\boldsymbol{d}} \parallel^2\,\mathrm{d}t \le M_d \tag{52}$$

then one gets

$$\int_0^\infty \parallel e \parallel_Q^2\,\mathrm{d}t \le 2V(0) + \rho^2M_d \tag{53}$$

Thus, the integral $\int_0^\infty \parallel e \parallel_Q^2\,\mathrm{d}t$ is bounded. Moreover, $V(T)$ is bounded and from the definition of the Lyapunov function V in (36) it becomes clear that $e(t)$ will be also bounded since $e(t) \in \Omega_e = \{e | e^TPe \le 2V(0) + \rho^2M_d\}$. According to the above and with the use of Barbalat's Lemma one obtains $\lim_{t \to \infty} e(t) = 0$. □

6 Robust state estimation with the use of the H_∞ Kalman filter

The control loop has to be implemented with the use of information provided by a small number of sensors and by processing only a small number of state variables. Actually, one can implement feedback control by measuring only the looper's angle. To reconstruct the missing information about the state vector of the hot-steel rolling mill's system, it is proposed to use a filtering scheme and based on it to apply state estimation-based control [29]. The recursion of the H_∞ Kalman filter, for the model of the hot-steel rolling mill, can be formulated in terms of a *measurement update* and a *time update* part

Measurement update:

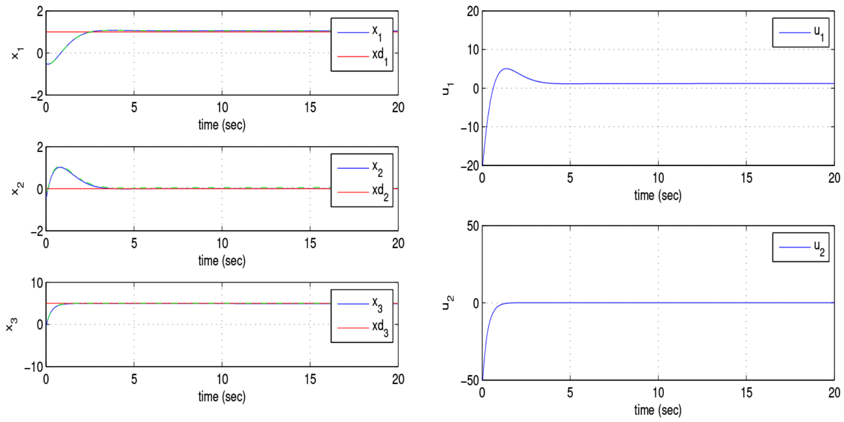
$$\begin{aligned} D(k) &= [\boldsymbol{I} - \theta\boldsymbol{W}(k)\boldsymbol{P}^-(k) + \boldsymbol{C}^T(k)\boldsymbol{R}(k)^{-1}\boldsymbol{C}(k)\boldsymbol{P}^-(k)]^{-1} \\ K(k) &= \boldsymbol{P}^-(k)\boldsymbol{D}(k)\boldsymbol{C}^T(k)\boldsymbol{R}(k)^{-1} \\ \hat{x}(k) &= \hat{x}^-(k) + \boldsymbol{K}(k)[y(k) - \boldsymbol{C}\hat{x}^-(k)] \end{aligned} \tag{54}$$

Time update:

$$\begin{aligned} \hat{x}^-(k+1) &= \boldsymbol{A}(k)x(k) + \boldsymbol{B}(k)u(k) \\ P^-(k+1) &= \boldsymbol{A}(k)\boldsymbol{P}^-(k)\boldsymbol{D}(k)\boldsymbol{A}^T(k) + \boldsymbol{Q}(k) \end{aligned} \tag{55}$$

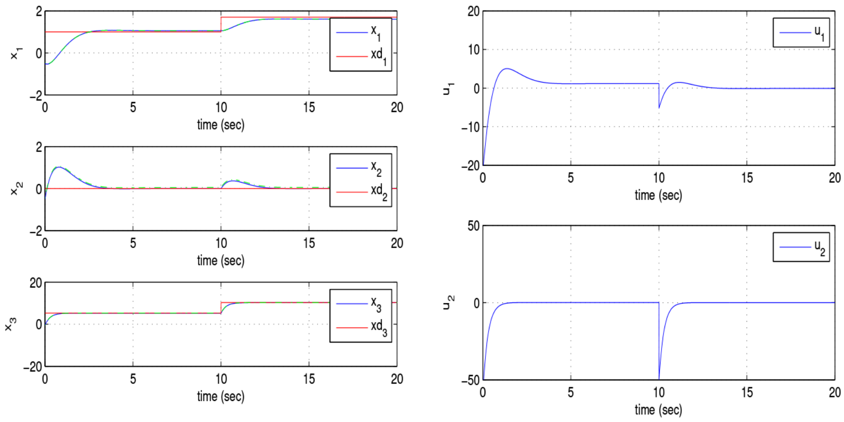
where it is assumed that parameter θ is sufficiently small to assure that the covariance matrix $\boldsymbol{P}^-(k)^{-1} - \theta\boldsymbol{W}(k) + \boldsymbol{C}^T(k)\boldsymbol{R}(k)^{-1}\boldsymbol{C}(k)$ will be positive definite. When $\theta = 0$ the H_∞ Kalman filter becomes equivalent to the standard Kalman filter. One can measure only a part of the state vector of the hot-steel rolling mill's system (e.g. the looper's angle θ), and can estimate through filtering the rest of the state vector elements.

The above equations provide the recursion of the H-infinity Kalman filter, which comprises two stages (i) the measurement update part and the time update part. To elaborate on the matrices which appear in the *Measurement update* part and in the *Time update part* of the H-infinity Kalman filter, the following can be noted: matrix $\boldsymbol{R}(k) \in R^{1\times 1}$ is the measurement noise covariance matrix, that is the covariance matrix of the measurement error vector of the system. Matrix $\boldsymbol{P}^-(k) \in R^{3\times 3}$ is the a-priori state vector estimation error covariance matrix of the system that is the covariance matrix of the state vector estimation error prior to receiving the updated measurement of the system's outputs. Matrix



a *b*

Fig. 3 *Test case 1*
(a) Tracking of reference setpoints by the state variables x_1 to x_3 of the hot-steel rolling mills system, ***(b)*** Control inputs u_1 and u_2 applied to the hot-steel rolling mills system



a *b*

Fig. 4 *Test case 2*
(a) Tracking of reference setpoints by the state variables x_1 to x_3 of the hot-steel rolling mills system, ***(b)*** Control inputs u_1 and u_2 applied to the hot-steel rolling mills system

$\boldsymbol{W}(k) \in R^{3\times 3}$ is a weight matrix which defines the significance to be attributed by the H-infinity Kalman filter to minimise the state vector's estimation error, relatively to the effects that the noise affecting the system may have. Actually, W is a diagonal matrix and the elements on its diagonal are given the value 10^{-3}. Finally, matrix $\boldsymbol{D}(k) \in R^{3\times 3}$ stands for a modified a-posteriori state vector estimation error covariance matrix that is the covariance matrix of the state vector estimation error after receiving the updated measurement of the system's outputs. Besides, about the process noise covariance matrix $\boldsymbol{Q} \in R^{3\times 3}$ that appears in the filter's time-update part it holds that this is also a diagonal matrix and the elements on its diagonal are given the value 10^{-4}. Conclusively, the H-infinity Kalman filter retains the structure of the typical Kalman filter, that is a recursion in discrete time comprising a Time update part (computation of variables prior to receiving measurements) and a Measurement update part (computation of variables after measurements have been received). There is a modified a-posteriori state vector estimation error covariance matrix, which in turn takes into account a weight matrix that defines the accuracy of the state estimation under the effects of elevated noise.

7 Simulation tests

The performance of the non-linear optimal control for the dynamic model of the hot-steel rolling-mill system was tested through simulation experiments. Nominal values for the parameters of the process are given in [1]. The parameters' values which have been used in the simulation tests are as follows: R_l: 0.611 m, h: 0.1 m, w: 2.5 m, M_L: 1.8×10^3 kg, R_G: 0.2 m, r: 0.08 m, E: 1.2×10^{10} Pa, L: 5.8 m, l_1: 2.34 m, l_2: 3.46 m, ρ: 8.0×10^3 kg/m^3, g: 9.8 m/s^2, H_1: 0.195 m and J: 600 kg·m^2. The obtained results are depicted in Figs. 3–9. The simulated parameters of the system are its state variables, that is $x_1 = \theta$ which is the turn angle of the looper (rad), ω which is the angular speed of the looper (rad/s) and σ which is the strip's tension (MPa). The control inputs of the system are the looper's actuator torque u_1 (kNt · m) and the strip's speed difference u_2 (m/s). The disturbances that affect the control loop were due to modelling errors and parametric changes in its state-space description.

The control scheme is a state estimation-based one and for its implementation requires measurement of only one state variable. The measured state variable is the turn angle $x_1 = \theta$ of the looper. The rest of the state variables $x_2 = \omega$ (angular speed of the looper) and $x_3 = \sigma$ (strip's tension) are not measured directly but are estimated through the processing of the measurements of $x_1 = \theta$ with the use of the H-infinity Kalman filter. There is measurement noise in x_1, while x_1 is considered to be the only measurable state variable. The real value of the state vector elements of the system is shown in blue, the estimated value as provided by the H-infinity Kalman filter is shown in green, while the associated reference setpoints are printed in red. The simulation tests show that fast and accurate convergence of the state variables to their setpoints was achieved, under moderate variations of the control inputs. The proposed control method not only achieves minimisation of the tracking error for the state variables of the hot-steel rolling mill system, but it also minimises the control effort or equivalently the cost of the electric power consumed by the rolling mill industrial units. Regarding the transient performance of the control method this is affected by the gains and parameters r, ρ and Q which appear in the Riccati equation of (41). Actually, the smallest value of the attenuation coefficient ρ, for which a valid solution of the Riccati equation can be obtained, is the one that provides the control loop with maximum robustness.

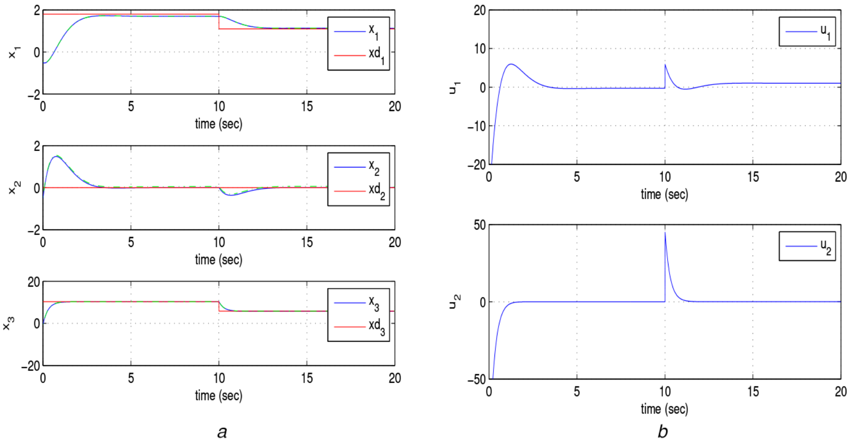


Fig. 5 *Test case 3*

(a) Tracking of reference setpoints by the state variables x_1 to x_3 of the hot-steel rolling mills system, ***(b)*** Control inputs u_1 and u_2 applied to the hot-steel rolling mills system

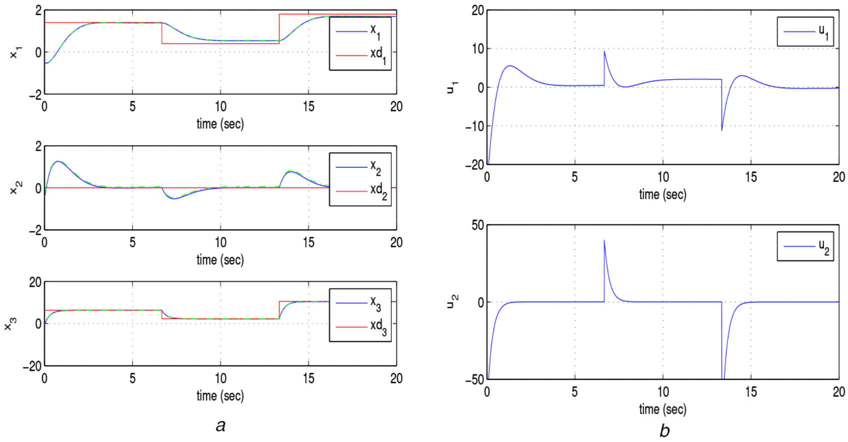


Fig. 6 *Test case 4*

(a) Tracking of reference setpoints by the state variables x_1 to x_3 of the hot-steel rolling mills system, ***(b)*** Control inputs u_1 and u_2 applied to the hot-steel rolling mills system

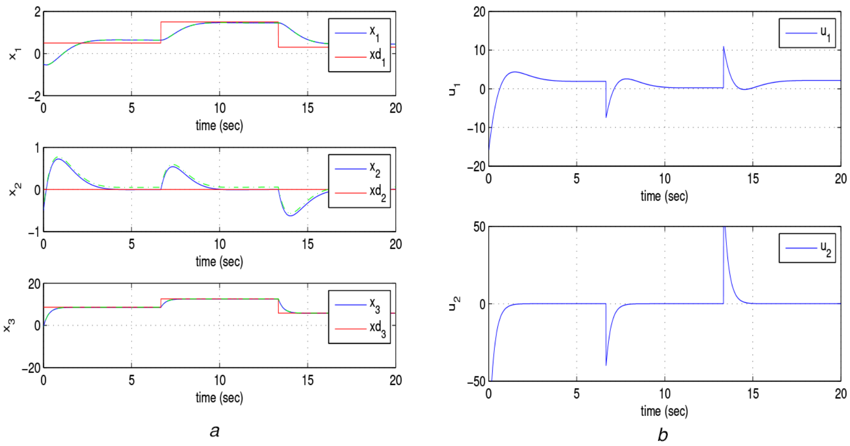


Fig. 7 *Test case 5*

(a) Tracking of reference setpoints by the state variables x_1 to x_3 of the hot-steel rolling mills system, ***(b)*** Control inputs u_1 and u_2 applied to the hot-steel rolling mills system

The advantages from the application of the proposed non-linear optimal control method are outlined as follows: (i) unlike global linearisation-based control schemes, the proposed non-linear optimal control method does not require changes of variables (diffeomorphisms) and application of complicated transformations of the system's state-space model, (ii) the new optimal control method is applied directly on the initial non-linear model of the hot-steel rolling mill system and avoids inverse transformations which are met in global linearisation-based control and which may come against singularities, (iii) the new control approach retains the advantages of typical optimal control, that is fast and accurate tracking of the reference setpoints, under moderate variations of the control inputs, (iv) unlike NMPC approaches the proposed control method is of proven convergence and stability, (v) unlike sliding-mode control approaches the proposed control method does not rely on intuitive definition of cost functions and does not need prior transformation of the state-space model into the canonical form, (vi) unlike proportional–integral–derivative control the proposed control method is of proven global stability and its functioning remains reliable after changes of operating points.

Finally, the accuracy of trajectory tracking by the state variables of the hot-steel rolling mill process $x_1 = \theta$, $x_2 = \omega$ and $x_3 = \sigma$ is depicted in Table 2. It can be confirmed that under the proposed flatness-based control and state-estimation scheme the tracking error for all state variables of the hot-steel rolling mill process was minimal. Moreover, in Table 3 the robustness of the control method

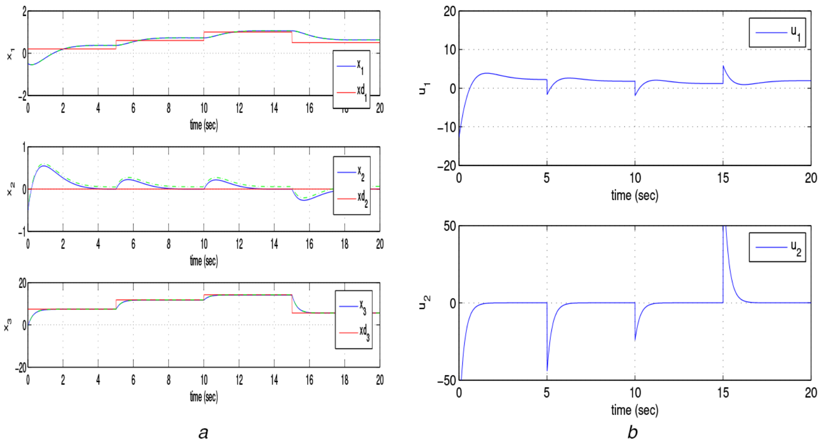


Fig. 8 *Test case 6*

(a) Tracking of reference setpoints by the state variables x_1 to x_3 of the hot-steel rolling mills system, ***(b)*** Control inputs u_1 and u_2 applied to the hot-steel rolling mills system

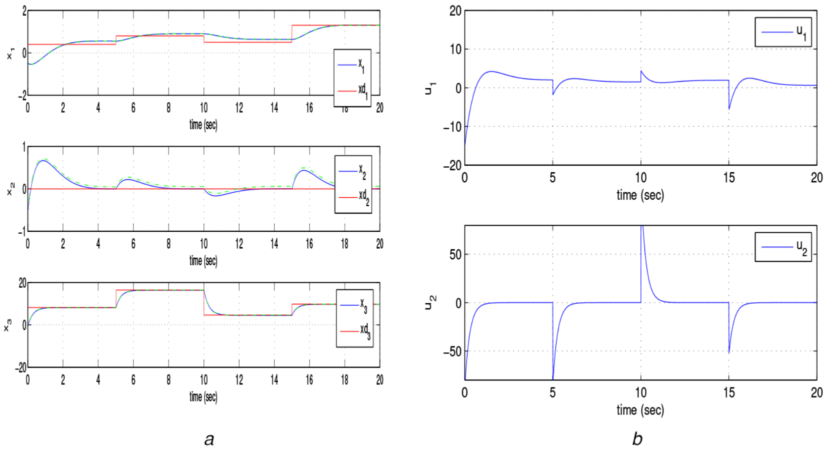


Fig. 9 *Test case 7*

(a) Tracking of reference setpoints by the state variables x_1 to x_3 of the hot-steel rolling mills system, ***(b)*** Control inputs u_1 and u_2 applied to the hot-steel rolling mills system

Table 2 RMSE of hot-steel rolling mill's state variables

No.	RMSE x_1	RMSE x_2	RMSE x_3
setpoint 1	0.0007	0.0001	0.0067
setpoint 2	0.0076	0.0001	0.0080
setpoint 3	0.0007	0.0001	0.0091
setpoint 4	0.0086	0.0001	0.0066
setpoint 5	0.0037	0.0001	0.0043
setpoint 6	0.0036	0.0009	0.0027
setpoint 7	0.0045	0.0016	0.0037

Table 3 RMSE under parametric changes

$\Delta a\%$	RMSE x_1	RMSE x_2	RMSE x_3
0%	0.0007	0.0001	0.0067
10%	0.0038	0.0001	0.0079
20%	0.0038	0.0001	0.0015
30%	0.0039	0.0001	0.0151
40%	0.0039	0.0001	0.0187
50%	0.0039	0.0001	0.0223
60%	0.0040	0.0001	0.0259

is shown in the case of parametric variations. Actually, the matrix presents the changes of the root mean square error (RMSE) of the state variables of the process under a change of $a\%$ is specific coefficients of the model, for instance the Young modulus E which defines the strip's tension.

Remark 1: As already analysed in the article, the approximate linearisation of the dynamic model of the hot-steel rolling mill process around the considered temporary operating point introduces a modelling error which is expected to be reasonably small provided that the control loop is sampled at a frequency that satisfies Nyquist's sampling theorem. The modelling error which is due to the truncation of higher-order terms in the performed Taylor series expansion is perceived as a perturbation that is asymptotically compensated by the robustness of the H-infinity controller. According to the article's stability analysis, the proposed control scheme is globally asymptotically stable. Actually, it is shown that the smallest value of the attenuation coefficient ρ for which a solution of the control system's Riccati equation can be reached is the one that provides the control loop with maximum robustness. Consequently, the article's stability analysis proves in an undebatable manner that despite the small linearisation error the global asymptotic stability of the control loop is attained. The proposed control scheme retains the advantages of the typical (linear) optimal control that is fast and accurate tracking of reference setpoints under moderate variations of the control inputs. In case that one applies global linearisation-based control methods to the dynamic model of the hot-steel rolling mill process (for instance Lie algebra-based methods) modelling errors due to approximate linearisation are avoided. The modelling achieved by the aforementioned methods is exact and relies on change variables transformations and the related state-space representation into equivalent linear forms. However, in such methods one has to take into account the following problems: (i) the state variables transformations (diffeomorphisms) can be complicated and the stages of computation for arriving at equivalent state-space descriptions can be elongated, (ii) the computation of the control signal that should be finally applied to the initial non-linear dynamics of the hot-steel rolling mill process requires inverse transformations that may come against singularities. On the other side, both problems (i) and (ii) are avoided in the design of the article's non-linear optimal (H-infinity) controller. For these reasons, the article's control method is in several aspects advantageous.

Remark 2: The main computations which are performed by the article's control algorithm are: (i) the computation of the elements of the Jacobian matrices that constitute the linearised state-space description of the system, (ii) the solution of an algebraic Riccati equation that finally allows for computing the control inputs that stabilise the hot-steel rolling mill process. The Riccati equation is solved with the use of Matlab's aresolv() function. Both stages (i) and (ii) have to be repeated at each time-step of the control algorithm. In a PC with an i7 Intel Processor at 2.0 GHz, the runtime for the completion of these stages is significantly smaller than a sampling period. The execution time can be further reduced if more powerful processors are used. Consequently, it can be stated that the proposed control algorithm can be safely run without ever risking that the computation of the control signal becomes computationally prohibitive.

Remark 3: From the essential concept of the proposed non-linear optimal (H-infinity) control method it is clear that energy consumption is minimised in the control loop of the hot-steel rolling mill process. This is because, according to (33) and the cost function that the control inputs minimise, the excessive variations of the control variables are penalised. Consequently, the control scheme tries to inhibit abrupt changes of the control inputs and to achieve accurate tracking of the reference setpoints under moderate variations of the control inputs. This finally signifies that less energy is spent for generating the stabilising control inputs of the process that is the looper's actuator torque $u_1 = T_u$ and the strip's speed difference $u_2 = v$. As noted above, unlike popular approaches for implementing optimal control in industry, such as MPC and NMPC, the method is of proven global stability and its performance does not depend on initialisation and on the selection of parameters.

8 Conclusions

A non-linear optimal (H-infinity) control method has been proposed for the dynamic model of the hot-steel rolling mill system. It was shown that due to strong non-linearities and the multivariable structure of the system's state-space model, the related control problem is of elevated difficulty. To achieve this problem's solution, the system's state-space model underwent first approximate linearisation around a temporary operating point which was recomputed at each time-step of the control algorithm. The linearisation relied on Taylor series expansion and on the computation of the associated Jacobian matrices. For the linearised description of the system, a stabilising H-infinity feedback controller was designed. To compute the controller's feedback gains, an algebraic Riccati equation had to be repetitively solved at each iteration of the control method.

The proposed control schemes provided a solution to the non-linear optimal control problem under model uncertainties and external perturbations. Actually, it represents the solution to a min–max differential game taking place between the controller and the model uncertainties or external perturbations. The stability properties of the control method were proven through Lyapunov analysis. First, it was demonstrated that the control loop satisfies the H-infinity tracking performance criterion, which signifies robustness against model imprecision and perturbations. Moreover, it was proven that the control scheme is globally asymptotically stable. The article's control method can contribute towards raising the quality levels of the production of hot-steel rolling mills, while also minimising the energy consumption of such industrial units.

9 References

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