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# Eðlisfræði 1R og 1V

## Lausnir á skiladæmum og tímadæmum

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# Efnisyfirlit

## Nokkrar athugasemdir:

Lausnirnar hér sýndar reyna að útskýra dæmin úr MasteringPhysics á heilsteyptan máta og vonandi geta aðstoðað nemendum sem eru fastir. Ég vill hinsvegar undirstrika að það er mikilvægt að reyna við dæmin áður en stokkið er í lausnirnar svo að fræðin haldist sem best í minnum.

Nokkur dæmi í Mastering Physics eru 'interactive' og byggjast á að nemandinn svari einföldum krossaspurningum, skilningsspurningum og/eða teikni vigurmyndir/gröf. Hér er oft hlaupið fram hjá þessum dæmum. Ef nemandi hefur einhverjar spurningar varðandi dæmi sem birtast ekki í þessu hefti, þá er velkomið að hafa samband við núverandi dæmatímakennara.

# 0 Vika

## 0.1 Æfingadæmi 1

### *Dæmi 1.14*

With a wooden ruler you measure the length of a rectangular piece of sheet metal to be $15\,\mathrm{mm}$ . You use micrometer calipers to measure the width of the rectangle and obtain the value $6.04\,\mathrm{mm}$. Give your answers to the following questions to the correct number of significant figures.

**A. What is the area of the rectangle?**

Flatarmálið er $A = 15\,\mathrm{mm} \cdot 6.04\,\mathrm{mm} = 91\,\mathrm{mm}^2$

**B. What is the ratio of the rectangle's width to its length?**

Hlutfall breiddar og lengdar er $\frac{b}{l} = \frac{6.04\,\mathrm{mm}}{15\,\mathrm{mm}} = 0.40$

**C. What is the perimeter of the rectangle?**

Ummál rétthyrningsins er $P = 2b + 2l = 2(b + l) = 2 \cdot (6.04\,\mathrm{mm} + 15\,\mathrm{mm}) = 42\,\mathrm{mm}$

**D. What is the difference between the length and width?**

Mismunur lengdar og breiddar er $l - b = 15\,\mathrm{mm} - 6.04\,\mathrm{mm} = 9.0\,\mathrm{mm}$

**E. What is the ratio of the length to the width?**

Hlutfall lengdar og breiddar er $\frac{l}{b} = \frac{15\,\mathrm{mm}}{6.04\,\mathrm{mm}} = 2.5$

### *Dæmi 1.21*

In Wagner's opera Das Rheingold, the goddess Freia is ransomed for a pile of gold just tall enough and wide enough to hide her from sight.

**A. Estimate the monetary value of this pile. The density of gold is $19.3\,\mathrm{g/cm^3}$, and its value is about \$10 per gram (although this varies).**

Hér þarf fyrst að áætla rúmmál Freyju (upp á stærðargráðu).
Segjum að rúmmál hennar sé $V = 0.5\,\mathrm{m} \cdot 0.5\,\mathrm{m} \cdot 2\,\mathrm{m} = 0.5\,\mathrm{m}^3 = 5 \cdot 10^5\,\mathrm{cm}^3$.
Massi gulls sem hefur samarúmmál er þá $m = \rho V = (19.3\,\mathrm{g/cm^3}) \cdot (5 \cdot 10^5\,\mathrm{cm}^3) = 9.65 \cdot 10^6\,\mathrm{g}$.
Verðmæti gullsins er þá $\$10/\mathrm{g} \cdot m = \$10/\mathrm{g} \cdot (9.65 \cdot 10^6\,\mathrm{g}) \approx \$10^8$.
Næsti svarmöguleiki gefur verðmæti upp á $\$10^9$.

### *Dæmi 1.30*

Vector $\mathbf{A}$ is in the direction $39.0°$ clockwise from the $-y$-axis. The $x$-component of $\mathbf{A}$ is $A_x = -17.0\,\mathrm{m}$ .

**A. What is the $y$-component of A ?**

Vigurinn liggur í *þriðja fjórðungi* $xy$-plansins. Þar sem $\tan\theta = \frac{A_x}{A_y}$, er $y$-þátturinn $A_y = \frac{A_x}{\tan\theta} = \frac{-17.0\,\mathrm{m}}{\tan 39°} = -21.0\,\mathrm{m}$.

**B. What is the magnitude of A ?**

Nú er $-\sin\theta = \frac{A_x}{A}$. Þá fæst lengd $\mathbf{A}$ með $A = -\frac{A_x}{\sin\theta} = \frac{17.0\,\mathrm{m}}{\sin 39°} = 27.0\,\mathrm{m}$

### *Resolving Vector Components with Trigonometry*

Often a vector is specified by a magnitude and a direction; for example, a rope with tension $\mathbf{T}$ exerts a force of magnitude $T$ in a direction $35°$ north of east. This is a good way to think of vectors; however, to calculate results with vectors, it is best to select a coordinate system and manipulate the components of the vectors in that coordinate system.

**A. Find the components of the vector A with length $a = 1.00$ and angle $\alpha = 15.0°$ with respect to the $x$ axis as shown.**

Vigurinn liggur í fyrsta fjórðungi $xy$-plansins.
Þá er $(A_x, A_y) = a \cdot (\cos\alpha, \sin\alpha) = 1.00 \cdot (\cos 15.0°, \sin 15.0°) = (0.966, 0.259)$.

**B. Find the components of the vector B with length $b = 1.00$ and angle $\beta = 10.0°$ with respect to the $x$ axis as shown.**

Vigurinn liggur í fyrsta fjórðungi $xy$-plansins.
Þá er $(B_x, B_y) = b \cdot (\cos\beta, \sin\beta) = 1.00 \cdot (\cos 10.0°, \sin 10.0°) = (0.985, 0.174)$.

**C. Find the components of the vector C with length $c = 1.00$ and angle $\phi = 25.0°$ as shown.**

Vigurinn liggur í öðrum fjórðungi $xy$-plansins.
Þá er $(C_x, C_y) = c \cdot (-\sin\phi, \cos\phi) = 1.00 \cdot (-\sin 25.0°, \cos 25.0°) = (-0.423, 0.906)$.

## *Dæmi 1.44*

**A. Given two vectors $\mathbf{A} = 4.00\hat{\imath} + 7.00\hat{\jmath}$ and $\mathbf{B} = 5.00\hat{\imath} - 2.00\hat{\jmath}$ , find the vector product $\mathbf{A} \times \mathbf{B}$ (expressed in unit vectors).**

Ritum krossfeldið sem ákveðuna:

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4.00 & 7.00 & 0 \\ 5.00 & -2.00 & 0 \end{vmatrix} = 0\hat{\imath} + 0\hat{\jmath} + \begin{vmatrix} 4.00 & 7.00 \\ 5.00 & -2.00 \end{vmatrix} \hat{k} = (4.00 \cdot (-2.00) - 7.00 \cdot 5.00)\hat{k} = -43.0\hat{k}$$

**B. What is the magnitude of the vector product?**

Lengd krossfeldisins er $|\mathbf{A} \times \mathbf{B}| = 43.0$

## *Dæmi 1.45*

Find the angle between each of the following pairs of vectors $\mathbf{A} = A_x\hat{\imath} + A_y\hat{\jmath}$ and $\mathbf{B} = B_x\hat{\imath} + B_y\hat{\jmath}$.

**A.** $A_{x1} = -2.00, A_{y1} = 7.60;\ B_{x1} = 2.90, B_{y1} = -2.50.$

Til að finna hornið milli tveggja vigra í plani er hægt að nota skilgreininguna á innfeldi.

$$\mathbf{A} \bullet \mathbf{B} = |\mathbf{A}||\mathbf{B}| \cos(\theta),$$

sem gefur okkur

$$\theta = \cos^{-1}\left(\frac{\mathbf{A} \bullet \mathbf{B}}{|\mathbf{A}||\mathbf{B}|}\right) = \cos^{-1}\left(\frac{A_x B_x + A_y B_y}{\sqrt{A_x^2 + A_y^2} \cdot \sqrt{B_x^2 + B_y^2}}\right) = 146°.$$

Sama aðferð er síðan notuð í B-lið og C-lið.

## *Vector Addition*

Consider the following three vectors:

$\mathbf{A} = (2, -1, 1)$, $\mathbf{B} = (3, 0, 5)$, and $\mathbf{C} = (1, 4, -2)$. Calculate the following combinations. Express your answers as ordered triplets [e.g., (9,4,-2)].

**A.** $\mathbf{A} + \mathbf{B}$

$$\mathbf{A} + \mathbf{B} = (A_x + B_x, A_y + B_y, A_z + B_z) = (2 + 3, -1 + 0, 1 + 5) = (5, -1, 6)$$

**E.** $2\mathbf{A} + 3\mathbf{B} + \mathbf{C}$

$$2\mathbf{A} + 3\mathbf{B} + \mathbf{C} = (2A_x + 3B_x + C_x, 2A_y + 3B_y + C_y, 2A_z + 3B_z + C_z) = (4 + 9 + 1, -2 + 0 + 4, 2 + 15 - 2) = (14, 2, 15)$$

## *Finding the Cross Product*

The figure shows two vectors $\mathbf{T}$ and $\mathbf{U}$ separated by an angle $\theta_{TU}$. You are given that $\mathbf{T} = (3, 1, 0)$, $\mathbf{U} = (2, 4, 0)$, and $\mathbf{T} \times \mathbf{U} = \mathbf{V}$ .

**A. Express V as an ordered triplet of values, separated by commas.**

$$\mathbf{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ T_x & T_y & T_z \\ U_x & U_y & U_z \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 0 \\ 2 & 4 & 0 \end{vmatrix} = \cancel{\begin{vmatrix} 1 & 0 \\ 4 & 0 \end{vmatrix}} \hat{i} - \cancel{\begin{vmatrix} 3 & 0 \\ 2 & 0 \end{vmatrix}} \hat{j} + \begin{vmatrix} 3 & 1 \\ 2 & 4 \end{vmatrix} \hat{k} = (3 \cdot 4 - 1 \cdot 2)\hat{k} = 10\hat{k}$$

**B. Find the magnitude of V .**

Lengdin á $\mathbf{V}$ er $V = 10$.

**C. Find the sine of the angle between T and U**

Lengd krossfeldis tveggja vigra er skilgreind sem $|\mathbf{T} \times \mathbf{U}| = |\mathbf{T}||\mathbf{U}| \sin\theta_{TU}$.
Þá er:

$$\sin\theta_{TU} = \frac{|\mathbf{T} \times \mathbf{U}|}{|\mathbf{T}||\mathbf{U}|} = \frac{10}{\sqrt{3^2 + 1^2} \cdot \sqrt{2^2 + 4^2}} = \frac{1}{\sqrt{2}} \approx 0.707$$

## *Exercise 2.8*

A bird is flying due east. Its distance from a tall building is given by $x(t) = 27.0\,\mathrm{m} + (12.3\,\mathrm{m/s})t - (0.0360\,\mathrm{m/s^3})t^3$.

**A. What is the instantaneous velocity of the bird when $t = 5.00\,\mathrm{s}$ ?**

Hraðinn fæst með því að taka tímaafleiðu af staðsetningunni

$$v(t) = \frac{dx}{dt} = 12.3\,\mathrm{m/s} - (0.108\,\mathrm{m/s^3})t^2$$
$$v(5.00\,\mathrm{s}) = 12.3\,\mathrm{m/s} - (0.108\,\mathrm{m/s^3}) \cdot (5.00\,\mathrm{s})^2 = 9.60\,\mathrm{m/s}$$

### *Exercise 2.18*

The position of the front bumper of a test car under microprocessor control is given by $x(t) = 2.17\,\mathrm{m} + (4.80\,{}^{\mathrm{m}}/_{\mathrm{s}^2})t^2 - (0.100\,{}^{\mathrm{m}}/_{\mathrm{s}^6})t^6$

**A. Find its position at the first instant when the car has zero velocity.**

Reiknum hraða og hröðun bílsins

$$v(t) = \frac{dx}{dt} = (9.60\,{}^{\mathrm{m}}/_{\mathrm{s}^2})t - (0.600\,{}^{\mathrm{m}}/_{\mathrm{s}^6})t^5$$
$$a(t) = \frac{d^2x}{dt^2} = 9.60\,{}^{\mathrm{m}}/_{\mathrm{s}^2} - (3.00\,{}^{\mathrm{m}}/_{\mathrm{s}^6})t^4$$

Hraðinn er núll þegar að

$$t = \begin{cases} 0\,\mathrm{s} \\ \sqrt[4]{\frac{9.60\,{}^{\mathrm{m}}/_{\mathrm{s}^2}}{0.600\,{}^{\mathrm{m}}/_{\mathrm{s}^6}}} = 2.00\,\mathrm{s} \end{cases}$$

Staðan er því $x(0\,\mathrm{s}) = 2.17\,\mathrm{m}$ þegar bíllinn er fyrst kyrrstæður.

**B. Find its acceleration at the first instant when the car has zero velocity.**

Hröðunin er $a(0\,\mathrm{s}) = 9.60\,{}^{\mathrm{m}}/_{\mathrm{s}^2}$ þegar bíllinn er fyrst kyrrstæður.

**C. Find its position at the second instant when the car has zero velocity.**

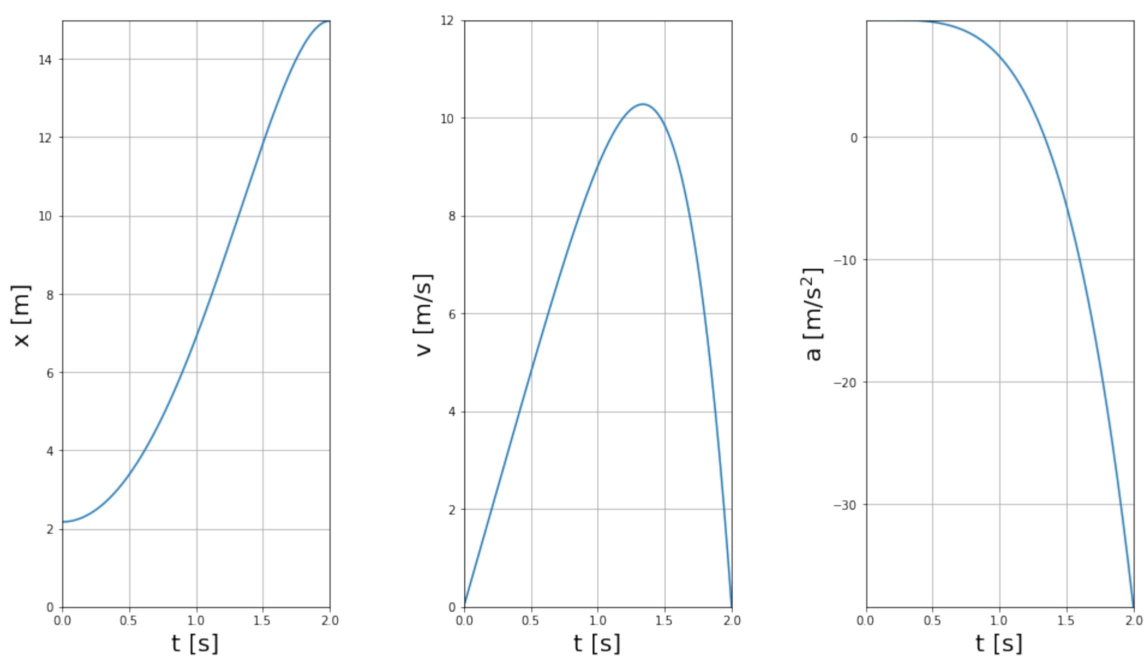
Staðan er $x(2.00\,\mathrm{s}) = 15.0\,\mathrm{m}$ þegar bíllinn er kyrrstæður í annað sinn.

**D. Find its position at the second instant when the car has zero velocity.**

Hröðunin er $a(2.00\,\mathrm{s}) = -38.4\,{}^{\mathrm{m}}/_{\mathrm{s}^2}$ þegar bíllinn er kyrrstæður í annað sinn.

**E. & F. & G.**

Draw $x - t$, $v - t$ and $a - t$ graphs for the motion of the bumper between $t = 0$ and $t = 2.00s$.



# 1 Vika

## 1.1 Skiladæmi 1

### *Advice for the Quarterback*

A quarterback is set up to throw the football to a receiver who is running with a constant velocity $vv_r$ directly away from the quarterback and is now a distance D away from the quarterback. The quarterback figures that the ball must be thrown at an angle $\theta$ to the horizontal and he estimates that the receiver must catch the ball a time interval $t_c$ after it is thrown to avoid having opposition players prevent the receiver from making the catch. In the following you may assume that the ball is thrown and caught at the same height above the level playing field. Assume that the y coordinate of the ball at the instant it is thrown or caught is $y = 0$ and that the horizontal position of the quarterback is $x = 0$. Use $g$ for the magnitude of the acceleration due to gravity, and use the pictured inertial coordinate system when solving the problem.

**A. Find $v_{0y}$ , the vertical component of the velocity of the ball when the quarterback releases it.**

Boltinn er gripinn í sömu hæð og honum er kastað upphaflega. Segjum að það sé í hæðinni $y = 0$. Notum tímaháðu jöfnuna fyrir lóðrétta færslu:

$$y(t_c) = -\frac{1}{2}gt_c^2 + v_{0y}t_c = 0$$

Einangrum $v_{0y}$:

$$v_{0y} = \frac{gt_c}{2}$$

**B. Find $v_{0x}$ , the initial horizontal component of velocity of the ball.**

Boltinn þarf að ferðast upphaflegu vegalengdina $D$ ásamt þeirri vegalengd sem móttakarinn hleypur á meðan boltinn er á lofti.

Láréttur hraðaþáttur boltans er því

$$v_{0x} = \frac{x(t_c)}{t_c} = \frac{D + v_r t_c}{t_c} = \frac{D}{t_c} + v_r$$

**C. Find the speed $v_0$ with which the quarterback must throw the ball.**

Báðir þættir upphafshraðans eru þekktir:

$$v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{\left(\frac{D}{t_c} + v_r\right)^2 + \left(\frac{gt_c}{2}\right)^2}$$

**D. Assuming that the quarterback throws the ball with speed $v_0$ , find the angle $\theta$ above the horizontal at which he should throw it.**

Skothornið fæst út frá $\tan\theta = \frac{v_{0y}}{v_{0x}}$.

$$\theta = \tan^{-1}\left(\frac{v_{0y}}{v_{0x}}\right)$$

## *Dæmi 3.16*

On level ground a shell is fired with an initial velocity of $83.3\,\mathrm{m/s}$ at $63.4\,°$ above the horizontal and feels no appreciable air resistance.

**A. Find the horizontal and vertical components of the shell's initial velocity.**

$x-$ og $y-$þættir upphafshraðans fást með:

$$(v_{0x}, v_{0y}) = v0 \cdot (\cos\theta, \sin\theta) = (83.3\,\mathrm{m/s}) \cdot (\cos(63.4\,°), \sin(63.4\,°)) = (37.3, 74.5)\,\mathrm{m/s}$$

**B. How long does it take the shell to reach its highest point?**

Notum tímaháðu hraðajöfnuna fyrir $v_{0y}$:

$$v_y = v_{0y} - gt = 0 \qquad \Rightarrow \qquad t = \frac{v_{0y}}{g} = \frac{74.5\,\mathrm{m/s}}{9.8\,\mathrm{m/s^2}} = 7.60\,\mathrm{s}$$

**C. Find its maximum height above the ground.**

Notum tímann í B lið og tímaháðu færslujöfnuna í lóðrétta stefnu:

$$y - y_0 = -\frac{1}{2}gt^2 + v_{0y}t = -\frac{1}{2}(9.8\,\mathrm{m/s^2}) \cdot (7.60\,\mathrm{s})^2 + (74.5\,\mathrm{m/s}) \cdot (7.60\,\mathrm{s}) = 283\,\mathrm{m}$$

**D. How far from its firing point does the shell land?**

Láréttur þáttur hraðans er fasti:

$$x = v_{0x}t = (37.3\,\mathrm{m/s}) \cdot (2 \cdot 7.60\,\mathrm{s}) = 567\,\mathrm{m}$$

**E. At its highest point, find the horizontal and vertical components of its acceleration.**

Hér er hröðunin fasti:

$$(a_x, a_y) = (0, -9.8)\,\mathrm{m/s^2}$$

**F. At its highest point, find the horizontal and vertical components of its velocity.**

Lóðréttur hraðaþáttur kúlunnar er núll í efstu stöðu, en lárétti þátturinn er fasti:

$$(v_x, v_y) = (37.3, 0)\,\mathrm{m/s}$$

## *Circular Launch*

A ball is launched up a semicircular chute in such a way that at the top of the chute, just before it goes into free fall, the ball has a centripetal acceleration of magnitude $2g$.

**A. How far from the bottom of the chute does the ball land?**

Ákvörðum skothraða og falltíma boltans.

Skothraðinn (láréttur) þarf að uppfylla jöfnuna fyrir miðsóknarhröðun hlutar á hringhreyfingu: $a_r ad = \frac{v_x^2}{R} = 2g \quad \Rightarrow \quad v = \sqrt{2gR}$.

Notum tímaháðu jöfnuna fyrir lóðrétta færslu til að ákvarða falltímann. Segjum að boltinn byrji í hæðinni $y_0 = 2R$ með láréttan upphafshraða ($v_{0y} = 0$): $y = 2R - \frac{1}{2}gt^2 = 0 \quad \Rightarrow \quad t = 2\sqrt{\frac{R}{g}}$.

Lárétt færsla boltans er því $x = v_x t = \sqrt{2gR} \cdot 2\sqrt{\frac{R}{g}} = 2\sqrt{2}R = \sqrt{8}R$.

## *Dæmi 3.28*

The radius of the earth's orbit around the sun (assumed to be circular) is $1.50 \cdot 10^8$ km, and the earth travels around this orbit in 365 days.

**A. What is the magnitude of the orbital velocity of the earth in $\mathrm{m/s}$?**

Brautarhraðinn fæst með $v = \frac{2\pi r}{T} = \frac{2\pi \cdot (1.50 \cdot 10^{11}\,\mathrm{m})}{(365 \cdot 24 \cdot 3600)\,\mathrm{s}} = 29.7\,\mathrm{km/s} = 2.97 \cdot 10^4\,\mathrm{m/s}$

**B. What is the radial acceleration of the earth toward the sun?**

Miðsóknarhröðunin er $a_{\mathrm{rad}} = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2(1.50 \cdot 10^{11}\,\mathrm{m})}{((365 \cdot 24 \cdot 3600)\,\mathrm{s})^2} = 5.91 \cdot 10^{-3}\,\mathrm{m/s^2}$

**C. What is the magnitude of the orbital velocity of the planet Mercury (orbit radius $= 5.79 \cdot 10^7$ km, orbital period $=$ 88.0 days)?**

$v = \frac{2\pi r}{T} = \frac{2\pi \cdot (5.97 \cdot 10^{10}\,\mathrm{m})}{(88 \cdot 24 \cdot 3600)\,\mathrm{s}} = 4.78 \cdot 10^4\,\mathrm{m/s}$

**D. What is the radial acceleration of the Mercury?**

$a_{\mathrm{rad}} = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2(5.97 \cdot 10^{10}\,\mathrm{m})}{((88 \cdot 24 \cdot 3600)\,\mathrm{s})^2} = 3.95 \cdot 10^{-2}\,\mathrm{m/s^2}$

## *Dæmi 3.38*

An airplane pilot wishes to fly due west. A wind of $86.0\,\mathrm{km/h}$ is blowing toward the south.

**A. If the airspeed of the plane (its speed in still air) is $405.0\,\mathrm{km/h}$ , in which direction should the pilot head?**

Summa lofthraða flugvélarinnar ($\mathbf{v}_a$) og vindhraðans ($\mathbf{v}_w$) gefur jarðhraða flugvélarinnar ($\mathbf{v}_g$). Þar sem $\mathbf{v}_a \perp \mathbf{v}_w$, þá gildir $\mathbf{v}_w = \mathbf{v}_a \sin\theta$. Stefna flugvélarinnar er $\theta = \sin^{-1}\left(\frac{\mathbf{v}_w}{\mathbf{v}_a}\right) = \sin^{-1}\left(\frac{86.0\,\mathrm{km/klst}}{405.0\,\mathrm{km/klst}}\right) = 12.3^\circ$ norðan við vesturstefnu.

**B. What is the speed of the plane over the ground?**

Þar sem $\mathbf{v}_a \perp \mathbf{v}_w$, þá getum við notað reglu Pýþagórasar til að finna jarðhraðann.

$$\mathbf{v}_g = \sqrt{\mathbf{v}_a^2 - \mathbf{v}_w^2} = \sqrt{(405.0\,\mathrm{km/klst})^2 - (86.0\,\mathrm{km/klst})^2} = 396\,\mathrm{km/klst}$$

## *Dæmi 4.4*

A man is dragging a trunk up the loading ramp of a mover's truck. The ramp has a slope angle of $20.0\,°$, and the man pulls upward with a force $\mathbf{F}$ whose direction makes an angle of $30.0\,°$ with the ramp .

**A. How large a force F is necessary for the component $F_x$ parallel to the ramp to be $60.0\,\mathrm{N}$?**

Hér gildir að $(F_x, F_y) = F(\cos\theta, \sin\theta)$, þar sem $\theta$ er hornið milli $\mathbf{F}$ og pallsins. Fyrir þekkt $F_x$ gildir:

$$F = \frac{F_x}{\cos\theta} = \frac{60.0\,\mathrm{N}}{\cos(30.0\,°)} = 69.3\,\mathrm{N}$$

**B. How large will the component $F_y$ perpendicular to the ramp then be?**

$$F_y = F\sin\theta = F_x\tan\theta = (60.0\,\mathrm{N})\cdot\tan(30.0\,°) = 34.6\,\mathrm{N}$$

## *A Gymnast on a Rope*

A gymnast of mass $53.0\,\mathrm{kg}$ hangs from a vertical rope attached to the ceiling. You can ignore the weight of the rope and assume that the rope does not stretch. Use the value $9.81\,\mathrm{m/s^2}$ for the acceleration of gravity.

**A. Calculate the tension $T$ in the rope if the gymnast hangs motionless on the rope.**

Á fimleikakappann verkar heildarkrafturinn $ma = T - mg$. Togkrafturinn er því $T = m(a+g)$.
Hér er $a = 0$, svo að $T = mg = 53.0\,\mathrm{kg}\cdot 9.81\,\mathrm{m/s^2} = 520\,\mathrm{N}$

**B. Calculate the tension $T$ in the rope if the gymnast climbs the rope at a constant rate.**

Togkrafturinn er ennþá $520\,\mathrm{N}$ því $a = 0$.

**C. Calculate the tension $T$ in the rope if the gymnast climbs up the rope with an upward acceleration of magnitude $0.600\,\mathrm{m/s^2}$ .**

$T = 53.0\,\mathrm{kg}\cdot(0.600\,\mathrm{m/s^2} + 9.81\,\mathrm{m/s^2}) = 552\,\mathrm{N}$

**D. Calculate the tension $T$ in the rope if the gymnast slides down the rope with a downward acceleration of magnitude $0.600\,\mathrm{m/s^2}$ .**

$T = 53.0\,\mathrm{kg}\cdot(-0.600\,\mathrm{m/s^2} + 9.81\,\mathrm{m/s^2}) = 488\,\mathrm{N}$

## 1.2 Tímadæmi 1

### *Dæmi 3.26*

A model of a helicopter rotor has four blades, each of length 3.40 m from the central shaft to the blade tip. The model is rotated in a wind tunnel at a rotational speed of 570 rev/min.

**A. What is the linear speed of the blade tip?**

Línulegur hraði endapunkts þyrluspaðans fæst með jöfnunni $v = \omega r$, þar sem $\omega$ er horntíðnin og $r$ er fjarlægð punktsins frá snúningsás.
Nú er $\omega = 2\pi f$, svo hraðinn er $v = 2\pi f r = 2\pi \left(\frac{570}{60}\,\mathrm{Hz}\right) \cdot (3.40\,\mathrm{m}) = 203\,\mathrm{m/s}$

**B. What is the radial acceleration of the blade tip expressed as a multiple of the acceleration of gravity, $g$?**

Miðsóknarhröðun endapunktsins er $a_r = \frac{v^2}{r} = 4\pi^2 f^2 r = 4\pi^2 \left(\frac{570}{60}\,\mathrm{Hz}\right)^2 \cdot (3.40\,\mathrm{m}) = 12114\,\mathrm{m/s^2} \cdot \frac{1g}{9.8\,\mathrm{m/s^2}} = 1236g$

### *Dæmi 3.33*

A canoe has a velocity of 0.54 m/s southeast relative to the earth. The canoe is on a river that is flowing 0.52 m/s east relative to the earth.

**A. Find the magnitude of the velocity of the canoe relative to the river.**

Látum $v_c = 0.54\,\mathrm{m/s}$ og $v_r = 0.52\,\mathrm{m/s}$. Hornið á milli hraðavigranna er $\theta_c = 45°$.
Ákvörðum ferð bátsins (miðað við ánna) með kósínus-reglunni:

$$v^2 = (\mathbf{v_c} - \mathbf{v_r}) \cdot (\mathbf{v_c} - \mathbf{v_r}) = v_c^2 - 2(\mathbf{v_c} \cdot \mathbf{v_r}) + v_r^2 = v_c^2 + v_r^2 - 2v_c v_r \cos\theta_c$$

$v = \sqrt{v_c^2 + v_r^2 - 2v_c v_r \cos\theta_c} = \sqrt{(0.54\,\mathrm{m/s})^2 + (0.52\,\mathrm{m/s})^2 - 2\,(0.54\,\mathrm{m/s}) \cdot (0.52\,\mathrm{m/s}) \cdot \cos(45°)} = 0.406\ldots\,\mathrm{m/s}$
$v \approx 0.41\,\mathrm{m/s}$

**B. Find the direction of the velocity of the canoe relative to the river.**

$y$-þættir $\mathbf{v_c}$ og $\mathbf{v}$ eru jafnir. Athugum að $\mathbf{v}$ liggur í 3. fjórðungi $xy$-plansins.

$$v \sin\theta_v = v_c \cdot \sin\theta_c \qquad \Rightarrow \qquad \theta_v = \sin^{-1}\left(\frac{v_c}{v}\sin\theta_c\right) = \sin^{-1}\left(\frac{0.54}{0.406\ldots}\sin(45°)\right) = 70° \text{ sunnan við vesturátt}$$

## *Dæmi 3.80*

Two students are canoeing on a river. While heading upstream, they accidentally drop an empty bottle overboard. They then continue paddling for 2.0 h , reaching a point 1.7 km farther upstream. At this point they realize that the bottle is missing and, driven by ecological awareness, they turn around and head downstream. They catch up with and retrieve the bottle (which has been moving along with the current) 4.1 km downstream from the turn-around point.

**A. Assuming a constant paddling effort throughout, how fast is the river flowing?**

Ferðalagi nemendanna er lýst með eftirfarandi jöfnum:

$$v_c - v_r = \frac{x_1}{t_1} \tag{*}$$

$$v_c + v_r = \frac{x_2}{t_2} \tag{**}$$

Í viðmiðunarkerfi flöskunnar mun báturinn báturinn hafa óþekkta hraðann $v'$ á fyrra tímabilinu ,$t_1$, og hraðan $-v'$ á seinna tímabilinu, $t_2$. Í því viðmiðunarkerfi eru vegalengdirnar $x'_1$ og $x'_2$ jafnlangar sem þýðir að $t_1 = t_2 = 2.0\,\mathrm{klst}$.

Drögum (*) frá (**) og fáum $2v_r = \frac{x_2 - x_1}{t_1}$. Hraði vatnsins miðað við bakkann er því $v_r = \frac{x_2 - x_1}{2t_1} = \frac{4.1\,\mathrm{km} - 1.7\,\mathrm{km}}{2\cdot(2.0\,\mathrm{klst})} = 0.600\,\mathrm{km/klst}$

**B. What would the canoe speed in a still lake be for the same paddling effort?**

Leggjum saman (*) og (**): $2v_c = \frac{x_2 + x_1}{t_1}$
$v_c = \frac{x_2 + x_1}{2t_1} = \frac{4.1\,\mathrm{km} + 1.7\,\mathrm{km}}{2\cdot(2.0\,\mathrm{klst})} = 1.45\,\mathrm{km/klst}$

## *Dæmi 4.8*

You walk into an elevator, step onto a scale, and push the "up"button. You also recall that your normal weight is $w = 660\,\mathrm{N}$.

**B. What does the scale read if the elevator has an acceleration of magnitude $a = 2.32\,\mathrm{m/s^2}$?**

Vigtin mælir þverkraftinn **n**, en túlkar hann sem þyngdarkraftinn **w**.

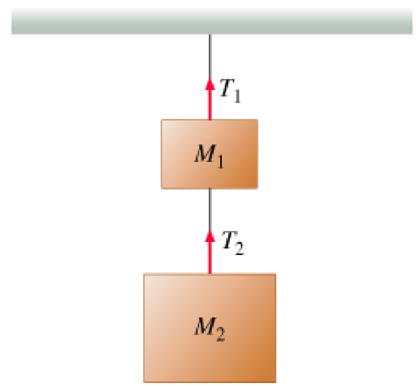
$$ma = n - w \qquad \Rightarrow \qquad n = ma + w = w \cdot \left(\frac{a}{g} + 1\right) = 660\,\mathrm{N} \cdot \left(\frac{2.32}{9.8} + 1\right) = 816\,\mathrm{N}$$

**D. If you start holding a 4.00 kg package by a light vertical string, what will be the tension in the string once the elevator begins accelerating?**

Togkrafturinn **T** þarf að yfirvinna þyngdarkraftinn **w**.

$$ma = T - w \qquad \Rightarrow \qquad T = ma + w = m(a + g) = 4.00\,\mathrm{kg} \cdot (2.32\,\mathrm{m/s^2} + 9.8\,\mathrm{m/s^2}) = 48.5\,\mathrm{N}$$

## *Two Hanging Masses*




Two blocks with masses $M_1$ and $M_2$ hang one under the other. For this problem, take the positive direction to be upward, and use $g$ for the magnitude of the acceleration due to gravity.

For Parts A and B assume the blocks are at rest. For Parts C and D the blocks are now accelerating upward (due to the tension in the strings) with acceleration of magnitude $a$.

**A. Find $T_2$, the tension in the lower rope.**

$$M_2 a = T_2 - M_2 g = 0 \qquad \Rightarrow \qquad T_2 = M_2 g$$

**B. Find $T_1$, the tension in the upper rope.**

$$M_1 a = T_1 - T_2 - M_1 g = 0 \qquad \Rightarrow \qquad T_1 = (M_1 + M_2) g$$

**C. Find $T_2$, the tension in the lower rope.**

$$M_2 a = T_2 - M_2 g \qquad \Rightarrow \qquad T_2 = M_2 (a + g)$$

**D. Find $T_1$, the tension in the upper rope.**

$$M_1 a = T_1 - T_2 - M_1 g \qquad \Rightarrow \qquad T_1 = (M_1 + M_2)(a + g)$$

## *Dæmi 4.21*

World-class sprinters can accelerate out of the starting blocks with an acceleration that is nearly horizontal and has magnitude $15\,\mathrm{m/s^2}$.

**A. How much horizontal force must a sprinter of mass 53 kg exert on the starting blocks during a start to produce this acceleration?**

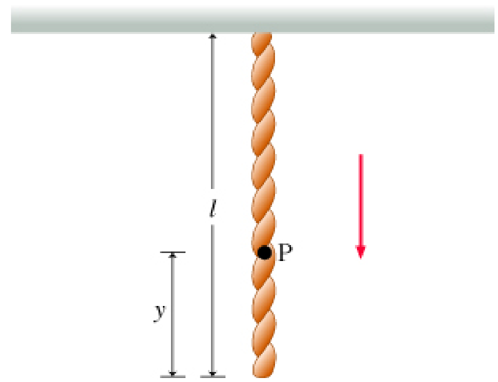
$$F = ma = (53\,\mathrm{kg}) \cdot (15\,\mathrm{m/s^2}) = 800\,\mathrm{N}$$

**B. Which body exerts the force that propels the sprinter, the blocks or the sprinter herself?**

Kubbarnir ýta spretthlauparanum áfram með kraftinum $F$.

## *Tension in a Hanging Massive Rope*

Consider a rope with length $l$, mass per unit length $\lambda$, experiencing a gravitational acceleration $g$ and hanging vertically as shown. Let $y$ refer to the height of a point $P$ above the bottom of the rope.




**A. The force exerted on the rope by the ceiling is in the ___________ direction.**

Reipið þarf að beita krafti sem verkar á móti þyngdarkraftinum.
Því verkar sá kraftur upp á við.

**B. Find $F$, the magnitude of the force exerted on the rope by the ceiling.**

Ákvörðum massa $m$ sem hangir út lofti: $m = \lambda l$.
Nú er $ma = F - mg = 0$. Þá er $F = mg = \lambda l g$

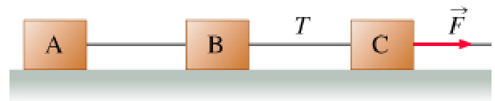
**C. What is the tension $T_P$ at point $P$ in the rope?**

Ákvörðum massa $m_p$ sem hangir úr punkti $P$: $m_P = \lambda y$. Nú er $m_p a = T_P - m_p g = 0$. Þá er $T_p = m_p g = \lambda y g$.

# 2 Vika

## 2.1 Skiladæmi 2

### *Pulling Three Blocks*




Three identical blocks connected by ideal strings are being pulled along a horizontal frictionless surface by a horizontal force **F**. The magnitude of the tension in the string between blocks B and C is $T = 3.00\,\mathrm{N}$. Assume that each block has mass $m = 0.400\,\mathrm{kg}$. **A. What is the magnitude** $F$ **of the force?**

Athugum að allir kassarnir hafa sömu hröðunina $a$ og allir hafa sama massann $m$ Finnum hröðun kerfisins með því að skoða kraftana sem verka á kassa A og B:

$$2ma = T \qquad \Rightarrow \qquad a = \frac{T}{2m}$$

Síðan ákvörðum við kraft $F$ með því að skoða kraftana á heildarkerfið:

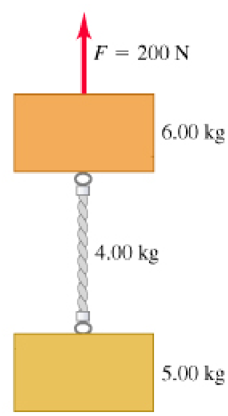
$$3ma = F \qquad \Rightarrow \qquad F = \frac{3T}{2} = \frac{3 \cdot 3.00\,\mathrm{N}}{2} = 4.50\,\mathrm{N}$$

**B. What is the tension $T_{\mathbf{AB}}$ in the string between block A and block B?**

Skoðum kraftana sem verka á kassa A:

$$ma = T_{\mathrm{AB}} \qquad \Rightarrow \qquad T_{\mathrm{AB}} = \frac{T}{2} = \frac{3.00\,\mathrm{N}}{2} = 1.50\,\mathrm{N}$$

### *Dæmi 4.48*




as The two blocks in the figure are connected by a heavy uniform rope with a mass of $4.00\,\mathrm{kg}$ . An upward force of $200\,\mathrm{N}$ is applied as shown. **A. What is the acceleration of the system?**

Skoðum kraftana á heildarkerfið:

$$M_{\mathrm{heild}}a = F - M_{\mathrm{heild}}g \qquad \Rightarrow \qquad a = \frac{F}{M_{\mathrm{heild}}} - g = \frac{200\,\mathrm{N}}{15.0\,\mathrm{kg}} - 9.8\,\mathrm{m/s^2} = 3.53\,\mathrm{m/s^2}$$

**B. What is the tension at the top of the heavy rope?**

Skoðum kraftinn á reipið og neðri kassann:

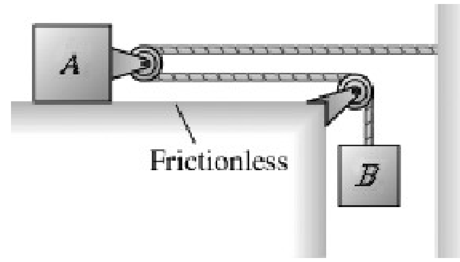
$$m_9 a = T - m_9 g \qquad \Rightarrow \qquad T = m_9(a+g) = \frac{m_9}{M_{\mathrm{heild}}}F = \frac{9.00\,\mathrm{kg}}{15.0\,\mathrm{kg}} \cdot 200\,\mathrm{N} = 120\,\mathrm{N}$$

**C. What is the tension at the midpoint of the rope?**

Skoðum kraftinn á neðri helming reipisins og neðri kassann:

$$m_7 a = T_{\mathrm{miðja}} - m_7 g \qquad \Rightarrow \qquad T_{\mathrm{miðja}} = m_7(a+g) = \frac{m_7}{M_{\mathrm{heild}}}F = \frac{7.00\,\mathrm{kg}}{15.0\,\mathrm{kg}} \cdot 200\,\mathrm{N} = 93.3\,\mathrm{N}$$

## *Dæmi 5.06*




**A. A wooden block A of mass 4.0 kg slides on a frictionless table when pulled using a massless string and pulley array by a hanging box B of mass 5.0 kg, as shown in the figure. What is the acceleration of block A as it slides on the frictionless table? Hint: Think carefully about the acceleration constraint.**

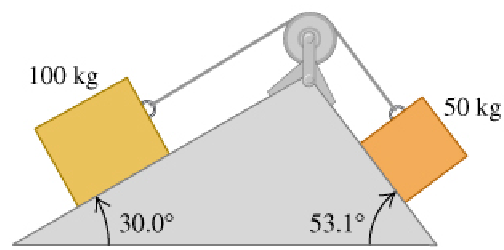
Ef trissuhjól við A snýst um $\theta$, þá halar það inn spotta við efri brún sem nemur $r\theta$. Spottinn er fastur í endann svo þá þarf miðja hjólsins að færast til hægri um þessa vegalengd. Neðri brún hjólsins gefur hins vegar út spotta sem nemur $r\theta$ líka, svo miðað við lokastöðuna hefur neðri spottinn færst um $2r\theta$, og þar með B líka. Kassi B ferðast tvöfalt lengri vegalengd en kassi A. Það sama gildir um allar tímaafleiður af staðsetningunni (hraðar og hraðanir). Því fæst $a_B = 2a_A$. Skoðum nú kraftana á sitthvorn massann:

$$m_A a_A = 2T$$
$$m_B a_B = 2m_B a_A = m_B g - T \qquad \Rightarrow \qquad T = m_B(g - 2a_A)$$

Leysum jöfnurnar saman og einangrum $a_A$:

$$m_A a_A = 2m_B(g - 2a_a) \qquad \Rightarrow \qquad a_A = \frac{2m_B}{m_A + 4m_B} g = \frac{10\,\mathrm{kg}}{24\,\mathrm{kg}} \cdot 9.8\,\mathrm{m/s^2} = 4.1\,\mathrm{m/s^2}$$

## *Dæmi 5.90*




Two blocks connected by a cord passing over a small, frictionless pulley rest on frictionless planes (the figure).

**A. Which way will the system move when the blocks are released from rest?**

Látum jákvæða hreyfistefnu vera til hægri. Heildarkrafturinn á kerfið er:

$$(m_1+m_2)a = m_2 g\sin(\theta_2) - m_1 g\sin(\theta_1) \qquad \Rightarrow \qquad a = \frac{m_2\sin(\theta_2) - m_1\sin(\theta_1)}{m_1+m_2} g = \frac{50\,\mathrm{kg}\sin(53.1\,^\circ) - 100\,\mathrm{kg}\sin(30.0\,^\circ)}{150\,\mathrm{kg}} \cdot 9.8\,\mathrm{m/s^2}$$

$$a = -0.658\,\mathrm{m/s^2}$$

Því hefur kerfið $0.658\,\mathrm{m/s^2}$ hröðun *til vinstri.*

**B. What is the acceleration of the blocks?**

Sjá svar við A-lið.

**C. What is the tension in the cord?**

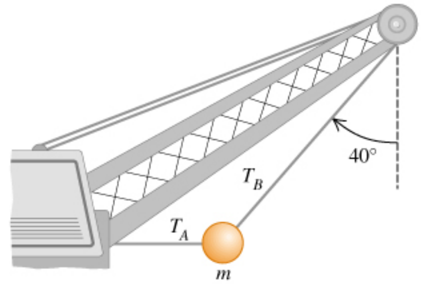
Skoðum kraftana á vinstri kubbinn:

$$m_1 a = T - m_1 g\sin\theta_1 \qquad \Rightarrow \qquad T = m_1(a + g\sin\theta_1) = m_1 g\left(\frac{m_2\sin(\theta_2) - m_1\sin(\theta_1)}{m_1+m_2} + \sin\theta_1\right) = \left(\frac{m_1 m_2}{m_1+m_2}\right) g(\sin\theta_2 + \sin\theta_1)$$

$$T = \left(\frac{(100\,\mathrm{kg})\cdot(50.0\,\mathrm{kg})}{150\,\mathrm{kg}}\right)\cdot(9.8\,\mathrm{m/s^2})\cdot(\sin(53.1\,^\circ) + \sin(30.0\,^\circ)) = 424\,\mathrm{N}$$

## *Dæmi 5.6*

A large wrecking ball is held in place by two light steel cables




**A. If the mass m of the wrecking ball is 4090 kg, what is the tension $T_B$ in the cable that makes an angle of $40°$ with the vertical?**

Þar sem vír $A$ er eftir $x$-ás þá fellur það á vír $B$ að halda kúlunni stöðugri eftir $y$-ás. Þetta þýðir að $mg = F_{By} = T_B \cos(40°)$ sem gefur að $T_B = 52323$ N.

**B. If the mass $m$ of the wrecking ball is 4090 kg, what is the tension $T_A$ in the horizontal cable?**

Vír $B$ myndar kraft í $x$ stefnu (til hægri) og sá kraftur er svaraður með mótkrafti í vír $A$, svo að $F_{Bx} = T_B \sin(40°) = F_{Ax} = T_A = 33633$ N.

## *Dæmi 5-16*

A 8.90 kg block of ice, released from rest at the top of a 1.12 m long frictionless ramp, slides downhill, reaching a speed of 2.65 m/s at the bottom.

**A. What is the angle between the ramp and the horizontal?**

Einungis samsíðaþáttur þyngdarkraftsins gefur ískubbnum hröðun niður eftir prammanum, sem gefur $ma_{\|} = mg\sin\theta$ (niður eftir pramma).

Finnum hröðunina með $v^2 = v_0^2 + 2a_{\|}s \quad \Rightarrow \quad a_{\|} = \frac{v^2 - v_0^2}{2s}$.

Skeytum jöfnunum saman $g\sin\theta = \frac{v^2-v_0^2}{2s}$.

Leysum fyrir $\theta$:

$$\theta = \sin^{-1}\left(\frac{v^2 - v_0^2}{2sg}\right) = \sin^{-1}\left(\frac{(2.65\,\mathrm{m/s})^2 - (0\,\mathrm{m/s})^2}{2\cdot(1.12\,\mathrm{m})\cdot(9.8\,\mathrm{m/s^2})}\right) = 18.7\,°$$

**B. What would be the speed of the ice at the bottom if the motion were opposed by a constant friction force of 10.2 N parallel to the surface of the ramp?**

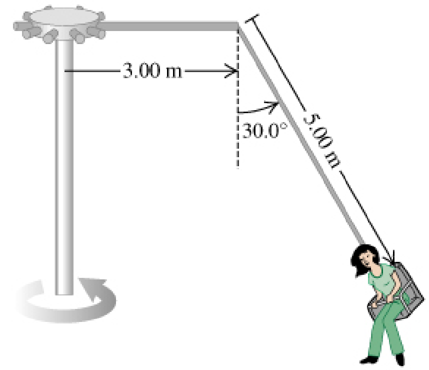
Nú er hröðunin $a_{\|} = g\sin\theta - \frac{f}{m} = \frac{v'^2 - v_0^2}{2s}$.

Einangrum $v'$:

$$v' = \sqrt{2sg\sin\theta - 2s\frac{f}{m} + v_0^2} = \sqrt{2\cdot(1.12\,\mathrm{m})\cdot(9.8\,\mathrm{m/s^2})\cdot\sin(18.7\,°) - 2\cdot(1.12\,\mathrm{m})\frac{10.2\,\mathrm{N}}{8.90\,\mathrm{kg}} + (0\,\mathrm{m/s})^2} = 2.11\,\mathrm{m/s}$$

## Dæmi 5.50

The "Giant Swing"at a county fair consists of a vertical central shaft with a number of horizontal arms attached at its upper end. Each arm supports a seat suspended from a cable 5.00 m long, the upper end of the cable being fastened to the arm at a point 3.00 m from the central shaft.




**A-B. Find the time of one revolution of the swing if the cable supporting a seat makes an angle of $30.0^\circ$ with the vertical. Does the angle depend on the weight of the passenger for a given rate of revolution?**

Til þess að farþeginn haldiðs í fastri hæð, verkar enginn lóðréttur kraftur á hann. Því fáum við að $F_y = F_{\text{tog}}\cos(\theta) - mg = 0$.
Því er $F_{\text{tog}} = \frac{mg}{\cos(\theta)}$. Farþeginn verður hins vegar fyrir miðsóknarhröðuninni $a_{\text{rad}} = \frac{v^2}{R} = \frac{4\pi^2 R}{T^2}$.
Eingöngu lárétti þáttur togkraftsins í kaplinum veldur þessari hröðun ($F_{\text{rad}} = m\frac{4\pi^2 R}{T^2} = F_{\text{tog}}\sin(\theta)$).
Stingum inn jöfnunni fyrir $F_{\text{tog}}$:

$$m\frac{4\pi^2 R}{T^2} = \frac{mg}{\cos(\theta)}\sin(\theta) = mg\tan(\theta)$$

Einangrum lotuna $T$:

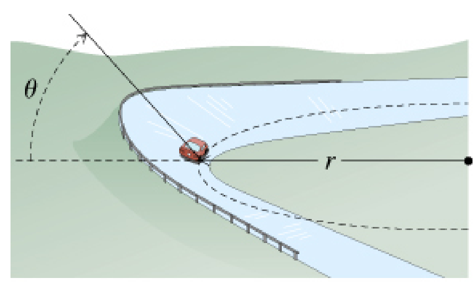
$$T = \sqrt{\frac{4\pi^2 R}{g\tan(\theta)}}$$

Hér er halli kapalsins $\theta = 30.0\,^\circ$ og brautarradíus farþegans $R = 3.0\,\text{m} + 5.0\,\text{m}\cdot\sin(30.0\,^\circ) = 5.5\,\text{m}$. Lotan er því $T = 6.19\,\text{s}$.

Athugið að við getum einnig einangrað halla kapalsins $\theta = \tan^{-1}\left(\frac{4\pi^2 R}{gT^2}\right)$. Þar sem enginn massaliður er í jöfnunni, er halli kapalsins <u>óháður</u> þyngd farþegans.

## Banked Frictionless Curve, and Flat Curve with Friction

A car of mass $M = 900\,\text{kg}$ traveling at $55.0\,{}^{\text{km}}/_{\text{hour}}$ enters a banked turn covered with ice. The road is banked at an angle $\theta$, and there is no friction between the road and the car's tires as shown in the following figure. Use $g = 9.80\,{}^{\text{m}}/_{\text{s}^2}$ throughout this problem.




**A. What is the radius $r$ of the turn if $\theta = 20.0\,^\circ$ (assuming the car continues in uniform circular motion around the turn)?**

Á bílinn verkar þyngdarkraftur og þverkraftur frá veginum. Eingöngu lárétti þáttur þverkraftsins heldur bílnum á jafnri hringhreyfingu í beygjunni. $m\frac{v^2}{r} = n\sin\theta$. Einnig er lóðrétti þáttur þverkraftsins styttur út af þyngdarkraftinum. $n\cos\theta = mg$. Deilum fyrri jöfnunni með þeirri seinni og fáum

$$\frac{v^2}{rg} = \tan\theta \qquad \Rightarrow \qquad r = \frac{v^2}{g\tan\theta} = \frac{(\frac{55}{3.6}\,{}^{\text{m}}/_{\text{s}})^2}{(9.80\,{}^{\text{m}}/_{\text{s}^2})\cdot\tan(20.0\,^\circ)} = 65.4\,\text{m}$$

**B. Now, suppose that the curve is level ($\theta = 0$) and that the ice has melted, so that there is a coefficient of static friction $\mu$ between the road and the car's tires as shown in the following figure. What is $\mu_{\text{min}}$, the minimum value of the coefficient of static friction between the tires and the road required to prevent the car from slipping? Assume that the car's speed is still $55.0\,{}^{\text{km}}/_{\text{hour}}$ and that the radius of the curve is $65.4\,\text{m}$.**

Nú þarf núningur dekkjanna ($f = \mu n = \mu mg\cos\theta = \mu mg$) að halda bílnum á jafnri hringhreyfingu.

$$m\frac{v^2}{r} = \mu_{\text{min}} mg \qquad \Rightarrow \qquad \mu_{\text{min}} = \frac{v^2}{rg} = \frac{(\frac{55}{3.6}\,{}^{\text{m}}/_{\text{s}})^2}{(65.4\,\text{m})\cdot(9.80\,{}^{\text{m}}/_{\text{s}^2})} = 0.364$$

### *Dæmi 5.74*

In the following figure $m_1 = 20.0\,\mathrm{kg}$ and $\alpha = 52.6\,^\circ$. The coefficient of kinetic friction between the block and the incline is $\mu_k = 0.40$.

**A. What must be the mass $m_2$ of the hanging block if it is to descend $15.0\,\mathrm{m}$ in the first $3.00\,\mathrm{s}$ after the system is released from rest?**

Ákvörðum $m_2$ út frá kraftajöfnunni fyrir heildarkerfið:

$$(m_1 + m_2)a = m_2 g - m_1 g(\sin\alpha + \mu_k \cos\alpha) \qquad \Rightarrow \qquad m_2 = \frac{g(\sin\alpha + \mu_k \cos\alpha) + a}{g - a} m_1$$

Hröðun kerfisins fæst með $y = \frac{1}{2}at^2 \qquad a = \frac{2y}{t^2}$. Þá fæst að lokum:

$$m_2 = \frac{g(\sin\alpha + \mu_k \cos\alpha) + \frac{2y}{t^2}}{g - \frac{2y}{t^2}} m_1 = \frac{(9.8\,\mathrm{m/s^2}) \cdot (\sin(52.6\,^\circ) + 0.40\cos(52.6\,^\circ)) + \frac{2\cdot(15.0\,\mathrm{m})}{(3.00\,\mathrm{s})^2}}{9.8\,\mathrm{m/s^2} - \frac{2\cdot(15.0\,\mathrm{m})}{(3.00\,\mathrm{s})^2}} \cdot (20.0\,\mathrm{kg}) = 42\,\mathrm{kg}$$

## 2.2 Tímadæmi 2

### *Dæmi 5.60*

An adventurous archaeologist crosses between two rock cliffs by slowly going hand-over-hand along a rope stretched between the cliffs. He stops to rest at the middle of the rope . The rope will break if the tension in it exceeds $2.85 \cdot 10^4\,\mathrm{N}$, and our hero's mass is $87.0\,\mathrm{kg}$ .

**A. If the angle between the rope and the horizontal is $\theta = 10.9\,^\circ$, find the tension in the rope.**

Hér skoðum við lóðrétta þætti þeirra krafta sem verka á fornleifafræðinginn.

$$ma_y = 2T\sin\theta - mg = 0 \qquad \Rightarrow \qquad T = \frac{mg}{2\sin\theta} = \frac{(87.0\,\mathrm{kg}) \cdot (9.8\,\mathrm{m/s^2})}{2\sin(10.9\,^\circ)} = 2250\,\mathrm{N}$$

**B. What is the smallest value the angle $\theta$ can have if the rope is not to break?**

Notum sömu upphaflegu jöfnu en einangrum $\theta$:

$$\theta = \sin^{-1}\left(\frac{mg}{2T}\right) = \sin\left(\frac{(87.0\,\mathrm{kg}) \cdot (9.8\,\mathrm{m/s^2})}{2 \cdot 2.85 \cdot 10^4\,\mathrm{N}}\right) = 0.857\,^\circ$$

### *Dæmi 5.45*

A small remote-control car with a mass of $1.63\,\mathrm{kg}$ moves at a constant speed of $v = 12.0\,\mathrm{m/s}$ in a vertical circle inside a hollow metal cylinder that has a radius of $5.00\,\mathrm{m}$.

**A. What is the magnitude of the normal force exerted on the car by the walls of the cylinder at point A (at the bottom of the vertical circle)?**

Fyrst að tölugildi hraða bílsins er fasti, þá er hröðunin miðsóknarhröðun. Látum jákvæða kraftstefnu vísa að miðju-hringbrautarinnar (svo að miðsóknarkrafturinn er jákvæður):

$$m\frac{v^2}{r} = n - mg \qquad \Rightarrow \qquad n = m(\frac{v^2}{r} + g) = 1.63\,\mathrm{kg} \cdot \left(\frac{(12.0\,\mathrm{m/s})^2}{5.00\,\mathrm{m}} + 9.8\,\mathrm{m/s^2}\right) = 62.9\,\mathrm{N}$$

**B. What is the magnitude of the normal force exerted on the car by the walls of the cylinder at point B (at the top of the vertical circle)?**

$$m\frac{v^2}{r} = n + mg \qquad \Rightarrow \qquad n = m(\frac{v^2}{r} - g) = 1.63\,\mathrm{kg} \cdot \left(\frac{(12.0\,\mathrm{m/s})^2}{5.00\,\mathrm{m}} - 9.8\,\mathrm{m/s^2}\right) = 31.0\,\mathrm{N}$$

## *Dæmi 5.40*

A baseball is thrown straight up. The drag force is proportional to $v^2$.

**A. In terms of $g$, what is the $y$-component of the ball's acceleration when its speed is half its terminal speed and it is moving up?**

Þegar að hraði boltans er $v = v_{\text{terminal}}$, þá er loftmótstaðan jöfn þyngdarkraftinum boltann, $Dv_{\text{terminal}}^2 = mgqquad$. Þá gildir að fyrir $v = \frac{v_{\text{terminal}}}{2}$, þá er $D\frac{v_{\text{terminal}}^2}{4} = \frac{mg}{4}$. Ef boltinn er á uppleið, þá vísar loftmótstaðan niður og

$$ma_y = -mg - \frac{mg}{4} = -\frac{5}{4}mg \qquad \Rightarrow \qquad a_y = -\frac{5}{4}g = -12.3\,\mathrm{m/s^2}$$

**B. In terms of $g$, what is the $y$-component of the ball's acceleration when its speed is half its terminal speed and it is moving back down?**

Nú vísar loftmótstaðan upp á við og við fáum:

$$ma_y = \frac{mg}{4} - mg = -\frac{3}{4}mg = -7.35\,\mathrm{m/s^2}$$

## *Dæmi 5.33*

You are lowering two boxes, one on top of the other, down the ramp shown in the figure by pulling on a rope parallel to the surface of the ramp. Both boxes move together at a constant speed of $11.0\,\mathrm{cm/s}$. The coefficient of kinetic friction between the ramp and the lower box is 0.428, and the coefficient of static friction between the two boxes is 0.788.

**A. What force do you need to exert to accomplish this?**

Heildarkerfið er í kraftajafnvægi. Halli prammans er $\theta = \tan^{-1}(\frac{2.50}{4.75}) = 27.76\,^\circ$

$$Ma = T - Mg\sin\theta - \mu_k Mg\cos\theta = 0 \quad \Rightarrow \quad T = Mg(\sin\theta - \mu_k\cos\theta) = (80.0\,\mathrm{kg})\cdot(9.8\,\mathrm{m/s^2})\cdot(\sin(27.76\,^\circ) - 0.428\cdot\cos(27.76\,^\circ)))$$

$$T = 68.2\,\mathrm{N}$$

**B. What is the magnitude of the friction force on the upper box?**

Kyrrstöðunúningurinn á efri kassann verkar á móti samsíða þætti þyngdarkraftsins:

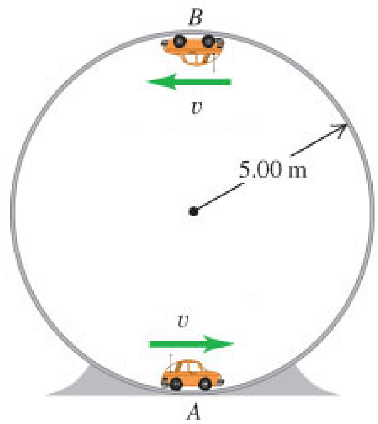
$$ma = f - mg\sin\theta = 0 \qquad \Rightarrow \qquad f = mg\sin\theta = (32.0\,\mathrm{kg})\cdot(9.8\,\mathrm{m/s^2})\cdot\sin(27.76\,^\circ) = 146\,\mathrm{N}$$

**C. What is the direction of the friction force on the upper box?**

Þar sem samsíðaþáttur þyngdarkraftsins verkar niður eftir prammanum, þá þarf kyrrstöðunúningurinn að vísa upp á við til að kassinn haldist kyrr á efri kassanum.

# 3 Vika

## 3.1 Skiladæmi 3




### *Dæmi 5-46*

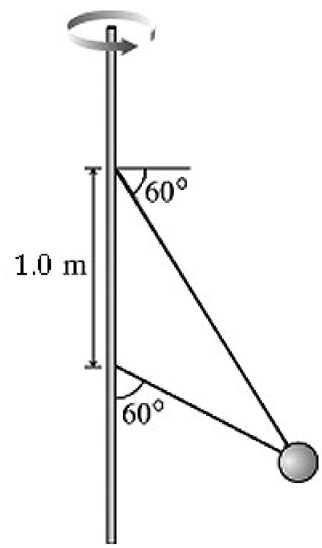
A small car with mass 0.710 kg travels at constant speed on the inside of a track that is a vertical circle with radius 5.00 m the following figure.

**A. If the normal force exerted by the track on the car when it is at the top of the track (point B) is $6.00\,\mathrm{N}$ , what is the normal force on the car when it is at the bottom of the track (point A)?**

Í punkti B fæst kraftajafnan $m\frac{v^2}{r} = mg + n_B$. Í punkti A fæst kraftajafnan $m\frac{v^2}{r} = n_A - mg$ Eingangrum $n_A$:

$$n_A = m\frac{v^2}{r} + mg = mg + n_B + mg = n_b + 2mg = 6.00\,\mathrm{N} + 2\cdot(0.710\,\mathrm{kg})\cdot(9.8\,\mathrm{m/s^2}) = 19.9\,\mathrm{N}$$

### *Dæmi 5.64*




**A. The figure shows two wires tied to a $9.5\,\mathrm{kg}$ sphere that revolves in a horizontal circle at constant speed. At this particular speed the tension is the SAME in both wires. What is the tension?**

Athugum að efra vírinn myndar $\theta_1 = 60^\circ$ horn við lárétt, en neðri vírinn myndar $\theta_2 = 60^\circ$ horn við stöngina.Skoðum lóðréttu þætti kraftanna sem verka á kúluna:

$$ma_y = T\sin\theta_1 + T\cos\theta_2 - mg = 0 \qquad \Rightarrow \qquad T = \frac{mg}{\sin\theta_1 + \cos\theta_2}$$

$$T = \frac{(9.5\,\mathrm{kg})\cdot(9.8\,\mathrm{m/s^2})}{\frac{\sqrt{3}}{2} + \frac{1}{2}} = 68\,\mathrm{N}$$

### *Dæmi 5.68*

A 9.00 kg box is suspended from the end of a light vertical rope. A time-dependent force is applied to the upper end of the rope, and the box moves upward with a velocity magnitude that varies in time according to $v(t) = (2.00\,\mathrm{m/s^2})t + (0.600\,\mathrm{m/s^3})t^2$ .

**A. What is the tension in the rope when the velocity of the box is $11.0\,\mathrm{m/s}$ ?**

Hröðun kassans er $a(t) = \frac{dv(t)}{dt} = 2.00\,\mathrm{m/s^2} + (1.20\,\mathrm{m/s^3})t$. Við þurfum að finna tímann $t$ þegar að $v(t) = 11.0\,\mathrm{m/s}$. Þá þarf að leysa annars stigs jöfnuna

$$(0.600\,\mathrm{m/s^3})t^2 + (2.00\,\mathrm{m/s^2})t - 11.0\,\mathrm{m/s} = 0 \qquad \Rightarrow \qquad t = \frac{(-2.00\,\mathrm{m/s^2}) \pm \sqrt{(2.00\,\mathrm{m/s^2}) - 4\cdot(0.600\,\mathrm{m/s^3})\cdot(-11.0\,\mathrm{m/s})}}{2\cdot(0.600\,\mathrm{m/s^3})} = 2.928\,\mathrm{s}$$

Finnum togkraftinn í bandinu á tímanum $t$ með því að skoða kraftana sem verka á kassann:

$$ma(t) = T - mg \qquad \Rightarrow \qquad T = m(a(t) + g) = (9.00\,\mathrm{kg})\cdot((2.00\,\mathrm{m/s^2} + (1.20\,\mathrm{m/s^3})\cdot(2.928\,\mathrm{s})) + 9.8\,\mathrm{m/s^2}) = 138\,\mathrm{N}$$

## *Dæmi 5.36*

A box of textbooks of mass 24.0 kg rests on a loading ramp that makes an angle $\alpha$ with the horizontal. The coefficient of kinetic friction is 0.25 and the coefficient of static friction is 0.36.

**A. As the angle $\alpha$ is increased, find the minimum angle at which the box starts to slip.**

Rétt áður en bókin rennur af stað, þá er hún í kraftajafnvægi. Skoðum þá krafta samsíða prammanum sem verka á bókina:

$$ma = \mu_s mg\cos\alpha - mg\sin\alpha = 0 \qquad \Rightarrow \qquad \alpha = \tan^{-1}(\mu_s) = \tan^{-1}(0.36) = 19.8\,^\circ \approx 20\,^\circ$$

**B. At this angle, find the acceleration once the box has begun to move.**

Á því augnabliki sem bókin rennur af stað verkar kyrrstöðunúningurinn ekki lengur á bókina, heldur hreyfinúningur.

$$ma = mg(\mu_k\cos\alpha - \sin\alpha) = \qquad \Rightarrow \qquad a = g(\mu_k\cos\alpha - \sin\alpha) = (9.8\,{}^{\mathrm{m}}/_{\mathrm{s}^2})\cdot(0.25\cdot\cos(19.8\,^\circ) - \sin(19.8\,^\circ)) = 1.0\,{}^{\mathrm{m}}/_{\mathrm{s}^2}$$

Hröðunin vísar niður eftir prammanum.

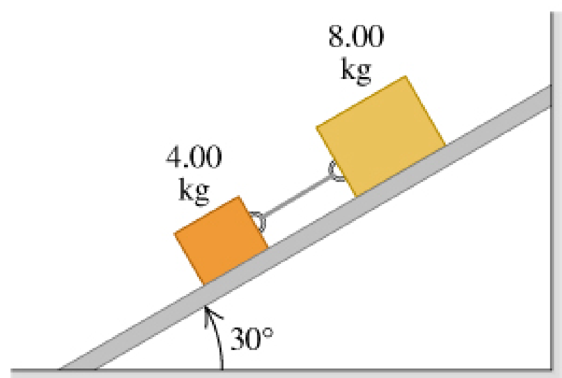
**C. At this angle, how fast will the box be moving after it has slid a distance $4.9\,\mathrm{m}$ along the loading ramp?**

Notum tímaóháðu hreyfijöfnuna $v^2 = v_0^2 + 2as$ til að finna ferð bókarinnar.

$$v = \sqrt{v_0^2 + 2as} = \sqrt{(0.0\,{}^{\mathrm{m}}/_{\mathrm{s}})^2 + 2\cdot(1.0\,{}^{\mathrm{m}}/_{\mathrm{s}^2})\cdot(4.9\,\mathrm{m})} = 3.2\,{}^{\mathrm{m}}/_{\mathrm{s}}$$

## *Dæmi 5.96*

Two blocks with masses 4.00 kg and 8.00 kg are connected by a string and slide down a 30.0° inclined plane (see the figure ). The coefficient of kinetic friction between the 4.00 kg block and the plane is 0.25, that between the 8.00 kg block and the plane is 0.35.




**AB. Calculate the acceleration of the blocks**

Við sjáum að stærri kassin, $m_2$ hefur stærri núningsstuðul en minni kassinn $m_1$, svo stærri kassinn mun ekki fara hraðar en sá minni $\rightarrow$ litli og stóri kassinn munu hafa sömu hröðun. Skilgreinum jákvæða stefnu niður kassan, þá fáum við fyrir minni kassan

$$m_1\cdot\sin\theta\cdot g - m_1\cdot\cos\theta\cdot\mu_1\cdot g - T = m_1\cdot a$$

og sama gildir fyrir þann stærri, nema togkrafturinn í reipinu verkar niður á við á efri kassan

$$m_2\cdot\sin\theta\cdot g - m_2\cdot\cos\theta\cdot\mu_2\cdot g + T = m_2\cdot a$$

Núna höfum við tvær óþekktar stærðir $a$ og $T$. Við byrjum á því að einangra $T$ fyrir báða kassana

$$T = m_1\cdot g\,(\sin\theta - \cos\theta\cdot\mu_1) - m_1\cdot a$$

Litli kassinn er þá

$$T = 11.11\ \mathrm{N}\ - (4.00\ \mathrm{kg})\cdot a$$

Stóri kassinn

$$T = -15.44\ \mathrm{N}\ + (8.00\ \mathrm{kg})\cdot a$$

Leysum þessar tvær jöfnur saman og einangrum $a$, þá fæst

$$a = 2.21\,{}^{\mathrm{m}}/_{\mathrm{s}^2}$$

**C. Calculate the tension in the string**

Notum jöfnuna úr A. lið til að finna togkraftinn í reipiu

$$T = 11.11 \text{ N } - (4.00 \text{ kg}) \cdot a$$

þar sem $a$ var fundið í A. lið, þá fæst að togkrafturinn sé

$$T = 2.27 \text{ N}$$

**D. What happens if the positions of the blocks are reversed, so the 4.00 kg block is above the 8.00 kg block?**

Strengurinn mun vera slakur, og hvor kassinn fyrir sig um fá hröðun sem eru óháðir hvor öðrum. 4 kg kassinn mun fá hröðun $a = 2.78\,{}^{\mathrm{m}}/_{\mathrm{s}^2}$ og 8 kg kassinn mun fá hröðun $a = 1.93\,{}^{\mathrm{m}}/_{\mathrm{s}^2}$ . Minni kassinn mun svo ná stærri kassanum og klessa á hann.

## *Dæmi 6.34*

To stretch a spring 7.00 cm from its unstretched length, 13.0 J of work must be done.

**A. What is the force constant of this spring?**

Hér fer vinnan í að auka stöðuorku gormsins, $W = \frac{1}{2}kx^2$. Þá er fjaðurstuðullinn:

$$k = \frac{2W}{x^2} = \frac{2 \cdot 13.0\,\mathrm{J}}{(0.0700\,\mathrm{m})^2} = 5310\,{}^{\mathrm{N}}/_{\mathrm{m}}$$

**B. What magnitude force is needed to stretch the spring** 7.00 cm **from its unstretched length?**

Til þess að yfirvinna fjaðurkraftinn $F = -kx$ þarf kraftinum $kx = (5310\,{}^{\mathrm{N}}/_{\mathrm{m}}) \cdot (0.0700\,\mathrm{m}) = 371\,\mathrm{N}$.

**C. How much work must be done to compress this spring** 4.00 cm **from its unstretched length?**

Hér væri hægt að reikna tölulega $\frac{1}{2}kx^2$ með fjaðurstuðlinum úr A-lið. Endurritum hins vegar jöfnuna $W_2 = \frac{1}{2}kx_2^2$ út frá $W_1 = \frac{1}{2}kx_1^2 = 13.0\,\mathrm{J}$ fyrir $x_1 = 7\,\mathrm{cm}$.

$$W_2 = \frac{1}{2}kx_1^2\left(\frac{x_2}{x_1}\right)^2 = W_1\left(\frac{x_2}{x_1}\right)^2 = 13.0\,\mathrm{J} \cdot \left(\frac{4.00\,\mathrm{cm}}{7.00\,\mathrm{cm}}\right)^2 = 4.24\,\mathrm{J}$$

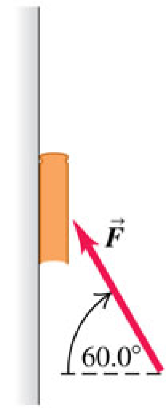
**D. What force is needed to stretch it this distance?**

Einnig getum við reiknað:

$$F_2 = F_1\frac{x_2}{x_1} = 371\,\mathrm{N} \cdot \frac{4.00\,\mathrm{cm}}{7.00\,\mathrm{cm}} = 212\,\mathrm{N}$$

### 3.2 Tímadæmi 3R

#### *Homemade problem based on 5.75*




(Ath. heimagert dæmi, svo MP svör virka ekki rétt!) You place a book of mass $m$ against a vertical wall. You apply a constant force $\mathbf{F}$ to the book, and the force is at angle $\alpha$ ( $\alpha = 60.0\,°$ in the figure) above the horizontal. The coefficient of static friction between the book and the wall is $\mu_s$

**A. The book is held still by the force. Find an expression for the force in terms of $m$, $g$, $\alpha$, and $\mu_s$.**

Núningskrafturinn getur verkað annað hvort upp á við eða niður á við, svo við þurfum að hafa bæði tilfellin í huga. Þverkrafturinn á bókina er $n = F\cos\alpha$. Skoðum kraftana sem verka í lóðrétta stefnu á bókina:

$$ma_y = F\sin\alpha - mg \pm \mu_s n = F\sin\alpha - mg \pm \mu_s F\cos\alpha = 0$$

Plúsinn gildir ef krafturinn $F$ er það lítill upp á við að bókin myndi renna niður án núnings. Mínusinn gildir svo ef krafturinn $F$ er of stór, þ.a. bókinn færi upp á við án núnings. Við getum einangrað $F$:

$$F = \frac{mg}{\sin\alpha \pm \mu_s\cos\alpha}$$

**B. Find the angle $\alpha$ that minimizes this force.**

Diffrum kraftinn $F$ m.t.t. hornsins $\alpha$:

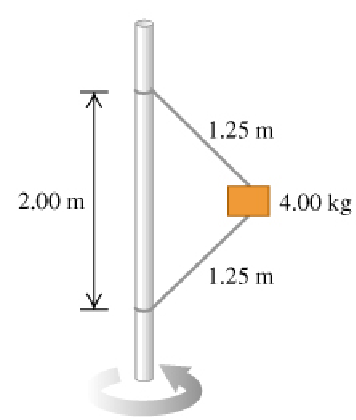
$$\frac{dF}{d\alpha} = -\frac{\cos\alpha \mp \mu_s\sin\alpha}{(\sin\alpha \pm \mu_s\cos\alpha)^2} = 0 \qquad \Rightarrow \qquad \cos\alpha \mp \mu_s\sin\alpha = 0 \qquad \Rightarrow \qquad \alpha = \tan^{-1}\left(\pm\frac{1}{\mu_s}\right)$$

**C. If $\alpha < 0\,°$ the force needed becomes large. Find the angle at which it is impossible to hold the book up any more in this way.**

Þegar ýtt er niður á bókina ($\alpha < 0\,°$), þá þarf að núningurinn að vísa upp á við. Það þýðir að plús formerkið gildir fyrir núningskraftinn jöfnunni $F = \frac{mg}{\sin\alpha \pm \mu_s\cos\alpha}$. Krafturinn verður óendanlega stór ef nefnarinn verður núll (sem myndi aldrei takast). Því gildir að $\sin\alpha + \mu_s\cos\alpha = 0$. Ómögulegt verður að halda bókinni uppi þegar að

$$\alpha = \tan^{-1}(-\mu_s)$$

#### *Dæmi 5.104*




The 4.00 kg block in the figure is attached to a vertical rod by means of two strings. When the system rotates about the axis of the rod, the strings are extended as shown in the diagram and the tension in the upper string is 81.0 N.

**A. What is the tension in the lower cord?**

Vírarnir mynda $\theta = \sin^{-1}(\frac{1.00\,\mathrm{m}}{1.25\,\mathrm{m}}) = 53.13\,°$ horn við lárétt. Skoðum lóðréttu þætti kraftanna á kubbinn.

$$ma_y = T_u\sin\theta - mg - T_l\sin\alpha = 0 \qquad \Rightarrow \qquad T_l = T_u - \frac{mg}{\sin\theta}$$

$$T_l = 81.0\,\mathrm{N} - \frac{(4.00\,\mathrm{kg})\cdot(9.8\,\mathrm{m/s^2})}{\frac{1.00\,\mathrm{m}}{1.25\,\mathrm{m}}} = 32.0\,\mathrm{N}$$

**B. How many revolutions per minute does the system make?**

Fjarlægð kubbsins frá stönginni er $R = \sqrt{(1.25\,\mathrm{m})^2 - (1.00\,\mathrm{m})^2} = 0.750\,\mathrm{m}$. Þá er $\cos\theta = \frac{0.750\,\mathrm{m}}{1.25\,\mathrm{m}} = 0.600$. Skoðum þætti kraftanna á kassann sem vísa í miðlæga stefnu.

$$m{\cdot}4\pi^2 R f^2 = (T_u{+}T_l)\cos\theta \qquad \Rightarrow \qquad f = \frac{1}{2\pi}\sqrt{\frac{(T_u+T_l)\cos\theta}{mR}} = \frac{1}{2\pi}\sqrt{\frac{(81.0\,\mathrm{N}+32.0\,\mathrm{N})\cdot 0.6}{(4.00\,\mathrm{kg})\cdot(0.750\,\mathrm{m})}}\cdot\frac{60\,{}^{\text{snún}}\!/\!{}_{\text{mín}}}{1\,\mathrm{Hz}} = 45.4\,{}^{\text{snún}}\!/\!{}_{\text{mín}}$$

**C. Find the number of revolutions per minute at which the lower cord just goes slack.**

Efri vírinn þarf að halda kubbnum uppi og togkrafturinn í neðri vírnum er $T_l = 0$. Athugum að $\tan\theta = \frac{1.00\,\mathrm{m}}{0.750\,\mathrm{m}}$. Skoðum aftur lóðrétta þætti kraftanna á kubbinn:

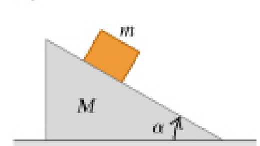
$$ma_y = F_u\sin\theta - mg = 0 \qquad \Rightarrow \qquad F_u = \frac{mg}{\sin\theta}$$

Notum sömu jöfnu og í B-lið:

$$f = \frac{1}{2\pi}\sqrt{\frac{F_u\cos\theta}{mR}} = \frac{1}{2\pi}\sqrt{\frac{g}{\tan\theta\cdot R}} = \frac{1}{2\pi}\sqrt{\frac{9.8\,{}^{\mathrm{m}}\!/\!{}_{\mathrm{s}^2}}{\frac{1.00\,\mathrm{m}}{\cancel{0.750\,\mathrm{m}}}\cdot\cancel{0.750\,\mathrm{m}}}}\cdot\frac{60\,{}^{\text{snún}}\!/\!{}_{\text{mín}}}{1\,\mathrm{Hz}} = 29.$$

## *Dæmi 5.122*

(a)




A wedge with mass $M$ rests on a frictionless horizontal table top. A block with mass $m$ is placed on the wedge. There is no friction between the block and the wedge. The system is released from rest.

**A. Calculate the acceleration of the wedge.**

Hér þarf að gera ráð fyrir að fleygurinn sé að hreyfast til baka vegna þverkrafts kassans. Þetta dæmi væri auðleysanlegt ef fleygurinn væri fastur, þá yrði dæmið alveg eins og kassi að renna eftir brekkur með horn $\alpha$. Stillum upp kraftajöfnum kerfisins

$$MA = -ma_x, \qquad ma_x = F_n\sin(\alpha), \qquad ma_y = F_n\cos(\alpha) - mg$$

Hér höfum við $A, a_x, a_y$ og $F_n$ sem óþekktar breytistærðir en aðeins þrjár óháðar jöfnur. Svo okkur vantar fjórðu jöfnuna. Hana getum við fengið með því að hverfa inn í viðmiðunarkerfi kassans sem skynjar ekki að fleygurinn ýtist til baka. Í því kerfi höfum við

$$ma'_x = F'_n\cos(\alpha), \qquad ma'_y = -F_n\sin(\alpha), \quad \Rightarrow \quad \frac{a'_y}{a'_x} = \frac{-a'\sin(\alpha)}{a'\cos(\alpha)} = -\tan(\alpha)$$

Nú spyrjum við okkur, hver eru tengslin milli hröðuninnar í viðmiðunarkerfi kassans og athuganda sem stendur hjá? Þau eru einfaldlega $a'_x = a_x - A$ og $a'_y = a_y$. Þ.e. kassinn fer hægar eftir $x$-ás í viðmiðunarkerfi athugandans (munið að $A < 0$). Þá höfum við

$$\frac{a_y}{a_x - A} = -\tan(\alpha)$$

Nú höfum við fjórar óháðar jöfnur og eftir smá algebru höfum við að

$$A = -\frac{mg\tan(\alpha)}{M + \tan^2(\alpha)\cdot(m+M)}$$

**B. Calculate the horizontal component of the acceleration of the block.**

Við höfum skrifað að $ma_x = -MA$ frá fyrri lið og því er ekkert mál að leysa þetta,

$$a_x = \frac{Mg\tan(\alpha)}{M + \tan^2(\alpha)\cdot(m+M)}$$

**C. Calculate the vertical component of the acceleration of the block.**

Við höfum að $ma_y = F_n \cos(\alpha) - mg$

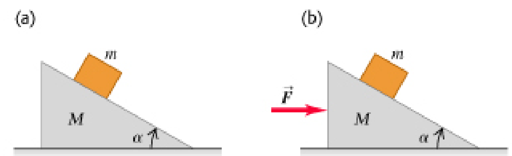
$$F_n = -\frac{MA}{\sin(\alpha)} = \frac{M}{\sin(\alpha)} \frac{mg\tan(\alpha)}{M + \tan^2(\alpha) \cdot (m+M)}$$

svo að

$$a_y = g\left\{\frac{M}{M + \tan^2(\alpha) \cdot (m+M)} - 1\right\}$$

### *Dæmi 5.113*

A wedge with mass $M$ rests on a frictionless horizontal tabletop. A block with mass $m$ is placed on the wedge and a horizontal force $\mathbf{F}$ is applied to the wedge.




**A. What must the magnitude of F be if the block is to remain at a constant height above the tabletop?**

Kubburinn og fleygurinn hafa sömu láréttu hröðun $a_x$:

$$(m+M)a_x = F$$

Eingöngu lárétti þáttur þverkraftsins á kubbinn ýtir honum áfram:

$$ma_x = n\sin\alpha \qquad \Rightarrow \qquad a_x = \frac{n\sin\alpha}{m}$$

Við getum ákvarðað þverkraftinn $n$ með því að skoða lóðréttan þátt kraftsins á kubbinn:

$$ma_y = n\cos\alpha - mg = 0 \qquad \Rightarrow \qquad n = \frac{mg}{\cos\alpha}$$

Þessi niðurstaða gefur okkur að $a_x = g\tan\alpha$, og því verður fyrsta jafnan:

$$F = (m+M)g\tan\alpha$$

## 3.3 Tímadæmi 3V

### *Dæmi 5.89*

Block A in the figure has a mass of 4.50 kg , and block B has mass 13.0 kg . The coefficient of kinetic friction between block B and the horizontal surface is 0.30.

**A. What is the mass of block C if block B is moving to the right and speeding up with an acceleration 2.10 m/s$^2$ ?**

Við sjáum út frá kraftmyndinni að heildar kraftur fyrir hvern hlut fyrir sig er

$$T_{AB} = m_A \cdot g + m_A \cdot a \qquad (1)$$

$$m_B \cdot a = T_{BC} - T_{AB} - f_k \quad \text{þar sem} \quad f_k = m_B \cdot g \cdot \mu \qquad (2)$$

$$T_{BC} = m_C \cdot g - m_C \cdot a \qquad (3)$$

Núna getum við einangrað $T_{AB}$ og $T_{BC}$ frá jöfnum (1) og (3) og stungið inní jöfnu (2). Þá fáum við

$$m_C = \frac{m_B \cdot a + (a+g)m_A + m_B \cdot g \cdot \mu}{g - a} = 15.5\text{kg}$$

**B. What is the tension in each cord when block B has this acceleration?**

Núna getum við notað jöfnu (1) og (3)

$$T_{AB} = m_A\,(g + a) = 53.6 \text{ N}$$

$$T_{BC} = m_C\,(g - a) = 119 \text{ N}$$

## *Dæmi 5.75*

You place a book of mass 5.00 kg against a vertical wall. You apply a constant force F to the book, where F = 95.0 N and the force is at angle of 60.0 ° above the horizontal. The coefficient of kinetic friction between the book and the wall is 0.300.

**A.If the book is initially at rest, what is its speed after it has traveled 0.400 m up the wall.**

Núningskrafturinn er

$$f_k = \mu \cdot F \cdot \cos\theta$$

og þá höfum við heildarkraftinn

$$F_\text{y} = F \cdot \sin\theta - m \cdot g - f_k$$

og þá er hröðunin gefin með

$$a = \frac{F_y}{m}$$

hraðinn er fundinn með

$$v^2 = v_0^2 + 2 \cdot a \cdot s$$

það er gefnið að upphafshraðinn er $v_0 = 0$

$$v = \sqrt{2 \cdot s \frac{F\sin\theta - m \cdot g - \mu F\cos\theta}{m}}$$

## *Dæmi 5.98*

Jack sits in the chair of a Ferris wheel that is rotating at a constant 0.130 rev/s. As Jack passes through the highest point of his circular part, the upward force that the chair exerts on him is equal to one-fourth of his weight.

**A. What is the radius of the circle in which Jack travels? Treat him as a point mass.**

Þá er þverkrafturinn

$$\hat{n} = 0.25 \cdot m \cdot g$$

heildarkrafturinn sem verkar á Jack er þá

$$m \cdot a = m \cdot g - 0.25 \cdot m \cdot g$$

þar sem miðsóknarhröðunin er

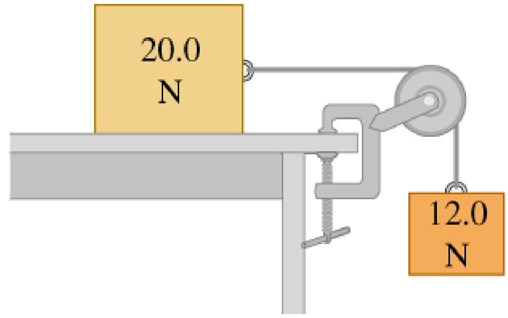
$$a = \frac{v^2}{R} = 4\pi^2 \cdot f^2 \cdot R$$

þá fáum við að

$$R = \frac{0.75 \cdot g}{4 \cdot \pi^2 f^2} = \frac{0.75 \cdot 9.8\,\mathrm{m/s^2}}{4\pi^2 \cdot (0.130\,\mathrm{Hz})^2} = 11.0\,\mathrm{m}$$

## *Dæmi 6.7*

Two blocks are connected by a very light string passing over a massless and frictionless pulley (the figure ). Traveling at constant speed, the 20.0 N block moves 76.0 cm to the right and the 12.0 N block moves 76.0 cm downward.




**AB. During this process, how much work is done on the 12.0-N block by gravity and the tension in the string?**

Til þess að neðri kassinn sé í kraftajafnvægi, þá er togkrafturinn í bandinu jafn þyngdarkraftinum á neðri kassann ($w = 12.0\,\mathrm{N}$).

$$W_A = w \cdot s = 12.0\,\mathrm{N} \cdot 0.760\,\mathrm{m} = 9.12\,\mathrm{J} \qquad W_B = -w \cdot s = -9.12\,\mathrm{J}$$

**CDEF. During this process, how much work is done on the 20.0-N block by gravity, the tension in the string, friction and the normal force?**

Núningurinn á efri kassann þarf að jafngilda togkraftinum í reipinu ($w = 12.0\,\mathrm{N}$) til þess að hann sé í kraftajafnvægi. Athugum einnig að þyngdarkrafturinn og þverkrafturinn eru hornréttir á færslu efri kassans, svo þeir framkvæma enga vinnu á hann.

$$W_C = 0\,\mathrm{J} \qquad W_D = w \cdot s = 9.12\,\mathrm{J} \qquad W_E = -w \cdot s = -9.12\,\mathrm{J} \qquad W_F = 0\,\mathrm{J}$$

**GH. Find the total work done on** 12.0 N **block and the** 20.0 N **block**

Hér má leggja saman vinnurnar sem voru reiknaðar í sitthvorum liðnum en skynsamlegra er að ákvarða heildarkraftinn á hvorn kassann fyrir sig og reikna vinnu hans. Þar sem báðir kassarnir eru í kraftajafnvægi, þá er engin heildarvinna framkvæmd á kassana.

$$W_{12} = 0\,\mathrm{J} \qquad W_{20} = 0\,\mathrm{J}$$

# 4 Vika

## 4.1 Skiladæmi 4

### *Dæmi 6-40*

As part of your daily workout, you lie on your back and push with your feet against a platform attached to two stiff springs arranged side by side so that they are parallel to each other. When you push the platform, you compress the springs. You do an amount of work of 77.0 J when you compress the springs a distance of 0.250 m from their uncompressed length.

**A. What magnitude of force must you apply to hold the platform in this position?**

Til þess að þjappa gormunum saman um lengdina $x$, þurfum við að framkvæma vinnuna $W = \frac{1}{2}kx^2 \quad \Rightarrow \quad k = \frac{2W}{x^2}$. Síðan þurfum við að halda pallinum kyrrum með kraftinum $F = kx = \frac{2W}{x^2} \cdot x = \frac{2W}{x}$:

$$F = \frac{2 \cdot (77.0\,\mathrm{J})}{0.250\,\mathrm{m}} = 616\,\mathrm{N}$$

**B. How much additional work must you do to move the platform a distance 0.250 m farther?**

Núna verður þjöppun gormsins $x' = 2x$ Við þurfum að framkvæma vinnuna $W' = \frac{1}{2}k(x'^2 - x^2) = \frac{1}{2}\frac{2W}{x^2}(3x^2) = 3W = 231\,\mathrm{J}$.

**C. What maximum force must you apply to move the platform to the position in Part B?**

Það þarf að beita mestum krafti á pallinn þegar þjöppun gormanna er í hámarki.

$$F = kx' = 2kx = \frac{4W}{x} = \frac{2 \cdot (77.0\,\mathrm{J})}{0.250\,\mathrm{m}} = 1230\,\mathrm{N}$$

### *Hooke's Law*

**A. A $61\,\mathrm{kg}$ driver gets into an empty taptap to start the day's work. The springs compress $1.8 \cdot 10^{-2}\,\mathrm{m}$. What is the effective spring constant of the spring system in the taptap?**

Viðbótar þyngd samsvarar auknum fjaðurkrafti í gormunum þegar þeir þjappast um lengdina $x$.

$$ma = kx - mg = 0 \qquad \Rightarrow \qquad k = \frac{mg}{x} = \frac{(61\,\mathrm{kg}) \cdot (9.8\,\mathrm{m/s^2})}{0.018\,\mathrm{m}} = 33\,\mathrm{kN/m}$$

**B. After driving a portion of the route, the taptap is fully loaded with a total of 25 people including the driver, with an average mass of $61\,\mathrm{kg}$ per person. In addition, there are three $15\,\mathrm{kg}$ goats, five $3\,\mathrm{kg}$ chickens, and a total of $25\,\mathrm{kg}$ of bananas on their way to the market. Assume that the springs have somehow not yet compressed to their maximum amount. How much are the springs compressed?**

Heildarmassi kerfisins er $M = 25 \cdot (61\,\mathrm{kg}) + 85\,\mathrm{kg} = 1610\,\mathrm{kg}$.

$$Ma = Mg - kx' = 0 \qquad \Rightarrow \qquad x' = \frac{Mg}{k} = \frac{Mgx}{mg} = \frac{M}{m}x = \frac{1610\,\mathrm{kg}}{61\,\mathrm{kg}} \cdot (0.018\,\mathrm{m}) = 0.48\,\mathrm{m}$$

þar sem $m = 61\,\mathrm{kg}$ er massi bílstjórans (og meðalmassi farþega í bílnum) og $x$ er þjöppun gormanna þegar einungis bílstjórinn situr í bílnum.

**C. Whenever you work a physics problem you should get into the habit of thinking about whether the answer is physically realistic. Think about how far off the ground a typical small truck is. Is the answer to Part B physically realistic?**

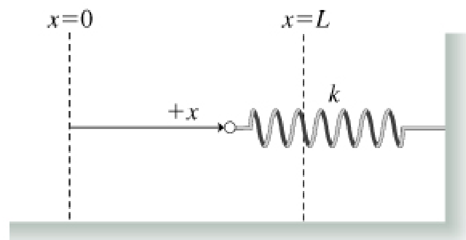
Svona gormar eru töluvert minni en 48 cm, svo það er óraunhæft að þjöppun gormanna sé lengri en upprunalega lengd þeirra.

**D. Now imagine that you are a Haitian taptap driver and want a more comfortable ride. You decide to replace the springs with new springs that can handle the typical heavy load on your vehicle. What spring constant do you want your new spring system to have?**

Gormarnir þurfa að vera töluvert stífari til þess að þeir ráði við háar hleðsluþyngdir.

## *Word Done by a Spring*

Consider a spring, with spring constant $k$, one end of which is attached to a wall. The spring is initially unstretched, with the unconstrained end of the spring at position $x = 0$.




**A. The spring is now compressed so that the unconstrained end moves from $x = 0$ to $x = L$. Using the work integral $W = \int_{x_i}^{x_f} \mathbf{F}(\mathbf{x}) \cdot d\mathbf{x}$, find the work done by the spring as it is compressed.**

Fyrir þjöppun gormsins, verður $\mathbf{F}(\mathbf{x}) \cdot d\mathbf{x} = -kx dx$.

$$W = \int_{x_i}^{x_f} \mathbf{F}(\mathbf{x}) \cdot d\mathbf{x} = -k \int_0^L x dx = \left[\frac{1}{2}kx^2\right]_0^L = -\frac{1}{2}kL^2$$

## *Dæmi 6.62*

A balky cow is leaving the barn as you try harder and harder to push her back in. In coordinates with the origin at the barn door, the cow walks from $x = 0$ to $x = 6.9\,\mathrm{m}$ as you apply a force with $x$-component $F_x = -[20.0\,\mathrm{N} + (3.0\,\mathrm{N/m})x]$.

**A. How much work does the force you apply do on the cow during this displacement?**

Hér borgar sig að heilda, fyrst að krafturinn **F** er ekki fasti.

$$W = \int_{x_i}^{x_f} \mathbf{F} \cdot d\mathbf{x} = -\int_0^{6.9\,\mathrm{m}} [20.0\,\mathrm{N} + (3.0\,\mathrm{N/m})x]dx = -\left[(20.0\,\mathrm{N})x + \frac{1}{2}(3.0\,\mathrm{N/m})x^2\right]_0^{6.9\,\mathrm{m}} = -209\,\mathrm{J}$$

## *Dæmi 6.58*

An elevator has mass 600 kg, not including passengers. The elevator is designed to ascend, at constant speed, a vertical distance of 20.0 m (five floors) in 16.0 s, and it is driven by a motor that can provide up to 40 hp to the elevator.

**A. What is the maximum number of passengers that can ride in the elevator? Assume that an average passenger has mass 65.0 kg.**

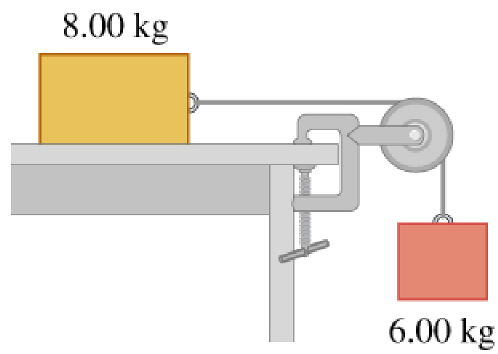
Mótorinn framkvæmir vinnuna $W = P \cdot \Delta t$ þegar hann flytur farþegana upp 5 hæðir. Sú vinna fer í að auka stöðuorku farþeganna (og lyftunnar): $W = (nm + M_{\mathrm{lyfta}})gy$, þar sem $n$ er fjöldi farþega í lyftunni og $m$ er meðalmassi eins farþega.

$$n = \frac{\frac{P\cdot\Delta t}{gy} - M_{\mathrm{lyfta}}}{m} = \frac{\frac{(40\cdot 746\,\mathrm{W})\cdot(16.0\,\mathrm{s})}{(9.8\,\mathrm{m/s^2})\cdot(20.0\,\mathrm{m})} - 600\,\mathrm{kg}}{65\,\mathrm{kg}} = 28.2$$

28 farþegar ættu að komast upp með lyftunni.

## Dæmi 6.82

Consider the system shown in the figure. The rope and pulley have negligible mass, and the pulley is frictionless. The coefficient of kinetic friction between the 8.00 kg block and the tabletop is $\mu_k = 0.250$. The blocks are released from rest.




**A. Use energy methods to calculate the speed of the 6.00 kg block after it has descended 1.50 m.**

Stöðuorka hangandi massans ($B$) breytist í hreyfiorku heildarkerfisins (að frádreginni vinnu núnings á efri kassann ($A$)). $m_B g y - \mu_k m_A g y = \frac{1}{2}(m_A + m_B)v^2$

$$v = \sqrt{\frac{2m_B g y - 2\mu_k m_A g y}{m_A + m_B}} = \sqrt{\frac{2 \cdot (6.00\,\mathrm{kg}) \cdot (9.8\,\mathrm{m/s^2}) \cdot (1.50\,\mathrm{m}) - 2 \cdot 0.250 \cdot (8.00\,\mathrm{kg}) \cdot (9.8\,\mathrm{m/s^2}) \cdot (1.50\,\mathrm{m})}{14.0\,\mathrm{kg}}} = 2.90\,\mathrm{m/s}$$

## Dæmi 7.10

A 24.0 kg child plays on a swing having support ropes that are 2.10 m long. A friend pulls her back until the ropes are 45.0 ° from the vertical and releases her from rest.

**A. What is the potential energy for the child just as she is released, compared with the potential energy at the bottom of the swing?**

Stöðuorkan (m.v. neðstu stöðu rólunnar) er $U_0 = mgy = mgL(1 - \cos\theta) = (24.0\,\mathrm{kg}) \cdot (9.8\,\mathrm{m/s^2}) \cdot (2.40\,\mathrm{m}) \cdot (1 - \cos(45.0\,°)) = 145\,\mathrm{J}$

**B. How fast will she be moving at the bottom of the swing?**

Notum varðveislu orkunnar, $\frac{1}{2}mv^2 = U_0$

$$v = \sqrt{\frac{2U_0}{m}} = \sqrt{\frac{2 \cdot (145\,\mathrm{J})}{24.0\,\mathrm{kg}}} = 3.47\,\mathrm{m/s}$$

**C. How much work does the tension in the ropes do as the child swings from the initial position to the bottom?**

Togkrafturinn vísar alltaf þvert á hreyfistefnu krakkans, svo vinna togkraftsins er alltaf núll.

## Drag on a Skydiver

A skydiver of mass $m$ jumps from a hot air balloon and falls a distance $d$ before reaching a terminal velocity of magnitude $v$. Assume that the magnitude of the acceleration due to gravity is $g$.

**A. What is the work $W_d$ done on the skydiver, over the distance $d$, by the drag force of the air?**

Hluti af stöðuorku stökkvarans tapast sem vinna loftmótstöðu, en afgangurinn skilar sér sem hreyfiorka. $mgd - W_d = \frac{1}{2}mv^2$.

$$W_d = \frac{1}{2}mv^2 - mgd$$

**B. Find the power $P_d$ supplied by the drag force after the skydiver has reached terminal velocity $v$.**

Loftmótstaðan verður fasti þegar stökkvarinn nær markhraðanum. Þá er stökkvarinn í kraftajafnvægi og mun framvegis öll stöðuorka tapast sem vinna loftmótstöðu: $dW_d = dU = -mgdy$. Afl loftmótstöðunnar er þá:

$$P_d = \frac{dW_d}{dt} = -mg\frac{dy}{dt} = -mgv$$

### *A Mass-Spring System with Recoil and Friction*

An object of mass $m$ is traveling on a horizontal surface. There is a coefficient of kinetic friction $\mu$ between the object and the surface. The object has speed $v$ when it reaches $x = 0$ and encounters a spring. The object compresses the spring, stops, and then recoils and travels in the opposite direction. When the object reaches $x = 0$ on its return trip, it stops.

**A. Find $k$, the spring constant.**

Við vitum að hreyfiorkan í upphafi breytist í stöðuorku gormsins þegar hluturinn stöðvast fyrst, en hluti af orkunni tapast sem vinna núnings á kubbinn.

$$\frac{1}{2}mv^2 - \mu mgx = \frac{1}{k}kx^2$$

Við einangrum $k$:

$$k = \frac{mv^2 - 2\mu mgx}{x^2}$$

Til þess að finna þjöppun gormsins verðum við að skoða heildarvinnuna sem núningurinn framkvæmir á kerfið. Kubburinn hefur tapað allri hreyfiorkunni sinni þegar hann stöðvast í annað sinn og þá er stöðuorka gormsins einnig núll ($x = 0$). Þá hefur kerfið ferðast vegalengdina $2x$.

$$\frac{1}{2}mv^2 - \mu mg(2x) = 0 \qquad \Rightarrow \qquad x = \frac{v^2}{4\mu g}$$

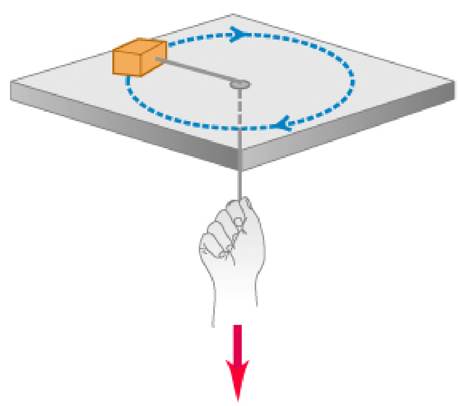
Við stingum inn í jöfnuna fyrir $k$ sem við fundum áður:

$$k = \frac{mv^2 - 2\mu mg\frac{v^2}{4\mu g}}{\left(\frac{v^2}{4\mu g}\right)^2} = \frac{8m\mu^2 g^2}{v^2}$$

## 4.2 Tímadæmi 4

### *Dæmi 6.71*

A small block with a mass of 0.0600 kg is attached to a cord passing through a hole in a frictionless, horizontal surface . The block is originally revolving at a distance of 0.41 m from the hole with a speed of 0.71 m/s . The cord is then pulled from below, shortening the radius of the circle in which the block revolves to 0.15 m . At this new distance, the speed of the block is 1.94 m/s.



**A. What is the tension in the cord in the original situation when the block has speed $v0 = 0.71\,\mathrm{m/s}$?**

Einungis togkrafturinn verkar að miðju hringhreyfingarinnar, svo við höfum $m\frac{v^2}{R} = T$:

$$T = (0.0600\,\mathrm{kg}) \cdot \frac{(0.71\,\mathrm{m/s})^2}{0.41\,\mathrm{m}} = 74\,\mathrm{mN}$$

**B. What is the tension in the cord in the final situation when the block has speed $v1 = 1.94\,\mathrm{m/s}$?**

Auka þarf togkraftinn til þess að auka hraða kubbsins og minnka brautarradíus hans:

$$T = (0.0600\,\mathrm{kg}) \cdot \frac{(1.94\,\mathrm{m/s})^2}{0.15\,\mathrm{m}} = 1.5\,\mathrm{N}$$

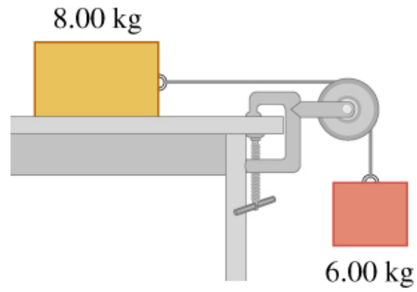
**C. How much work was done by the person who pulled on the cord?**

Vinnan samsvarar breytingunni á hreyfiorku kubbsins:

$$W = \Delta K = \frac{1}{2}m(v_2^2 - v_1^2) = \frac{1}{2} \cdot (0.0600\,\mathrm{kg}) \cdot ((1.94\,\mathrm{m/s})^2 - (0.71\,\mathrm{m/s})^2) = 98\,\mathrm{mJ}$$

## *Dæmi 6.81*

Consider the system shown in the figure . The rope and pulley have negligible mass, and the pulley is frictionless. Initially the 6.00 kg block is moving downward and the 8.00 kg block is moving to the right, both with a speed of 0.600 m/s . The blocks come to rest m




**A. Use the work-energy theorem to calculate the coefficient of kinetic friction between the 8.00 kg block and the tablet-op.**

Heildar vinnan sem verkar á kassana er

$$W = m_2 \cdot g \cdot a - m_1 \cdot g \cdot \mu \cdot a$$

Hreyfiorkan er gefin með

$$\Delta K = -\frac{1}{2}mv^2$$

þar sem kassinn hefur upphafs hraðan $v = 0.6$ m/s og lokahraða $v_0 = 0$ m/s, þá fæst

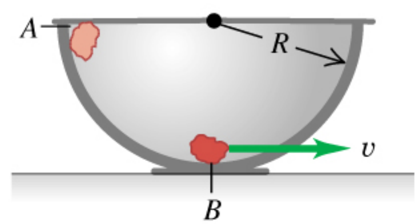
$$m_2 \cdot g \cdot a - m_1 \cdot g \cdot \mu \cdot a = -\frac{1}{2}mv^2$$

einangrum $\mu$

$$\mu = \frac{m_2 + \frac{m \cdot v^2}{2 \cdot g \cdot a}}{m_1} = 0.755$$

## *Dæmi 7.9*

A small rock with mass 0.22 kg is released from rest at point A, which is at the top edge of a large, hemispherical bowl with radius $R = 0.52$ m (the figure ). Assume that the size of the rock is small compared to $R$, so that the rock can be treated as a particle, and assume that the rock slides rather than rolls. The work done by friction on the rock when it moves from point $A$ to point $B$ at the bottom of the bowl has magnitude 0.22 J.




**A. Between points $A$ and $B$, how much work is done on the rock by the normal force?**

Þvervigurinn er alltaf hornréttur á stefnuna

$$\int F \cdot dS = 0$$

þar sem $\cdot$ í þessu tilfelli er innfeldi milli tveggja vigra, þá er

$$W = 0 \text{ J}$$

**B. Between points $A$ and $B$, how much work is done on the rock by gravity?**

Steinninn hefur fallið um hæð $R$

$$W = m \cdot g \cdot R = 1.1 \text{ J}$$

**C. What is the speed of the rock as it reaches point $B$?**

Hreyfiorkan neðst er

$$K = \frac{1}{2}mv^2$$

Vinnan neðst er

$$W = mgR - 0.22 \text{ J}$$

þá fæst

$$\frac{1}{2}mv^2 = mgR - 0.22 \text{ J}$$

eingangrum $v$

$$v = \sqrt{\frac{2\left(mgR - 0.22 \text{ J}\right)}{m}} = 2.9 \text{ m/s}$$

**D. the three forces acting on the rock as it slides down the bowl, which (if any) are constant and which are not? Explain.**

Þyngdakrafturinn, $m \cdot g$ er fasti. Þverkrafturinn og núningskrafturinn eru ekki fastar. (Núningskraftur og þverkraftur efst $= 0$, Núningskraftur og þverkraftur neðst $\neq 0$ )

**E. Just as the rock reaches point $B$, what is the normal force on it due to the bottom of the bowl?**

Þverkrafturinn neðst er fundinn með

$$ma = n - m \cdot g$$

þar sem $a$ er miðsóknarhröðunin, þverkafturinn er þá

$$n = m\left(g + \frac{v^2}{R}\right)$$

þekkjum $v$ úr lið C. þá fæst

$$n = 5.6 \text{ N}$$

## *Dæmi 7.28*

In an experiment, one of the forces exerted on a proton is $\hat{F} = -\alpha x \cdot 2\hat{\imath}$ where $\alpha = 12 \text{ N/m}^2$.

**A. How much work does $F$ do when the proton moves along the straight-line path from the point $(0.10m, 0)$ to the point $(0.10m, 0.40m)$?**

Vinnan er fundin með

$$W = \int_{(x_1,y_1)}^{(x_2,y_2)} \vec{F} \cdot d\vec{l} = \int_{x_1}^{x_2} \vec{F_x} dx + \int_{y_1}^{y_2} \vec{F_y} dy$$

þar sem $\vec{l}$ er fjarlægð. Vinnan er þá

$$W = -2\alpha \int_{0.10}^{0.10} x dx - 2\alpha \int_{0}^{0.40} 0 dy = 0 + 0 = 0 \text{ J}$$

þar sem $\vec{F} = (-2\alpha x, 0)$

**B. How much work does $\hat{F}$ do when the proton moves along the straight-line path from the point $(0.10m, 0)$ to the point $(0.30m, 0)$?**

$$W = \int_{x_1}^{x_2} \vec{F_x} dx + \int_{y_1}^{y_2} \vec{F_y} dy$$

$$W = -2\alpha \int_{0.10}^{0.30} x dx - 2\alpha \int_{0}^{0} 0 dy = -2\alpha \left[\frac{x^2}{2}\right]_{0.10}^{0.30} + 0 = -2\alpha \left(\frac{0.30^2}{2} - \frac{0.1^2}{2}\right) = -0.10 \text{ J}$$

**C. How much work does $\hat{F}$ do when the proton moves along the straight-line path from the point $(0.30m, 0)$ to the point $(0.10m, 0)$?**

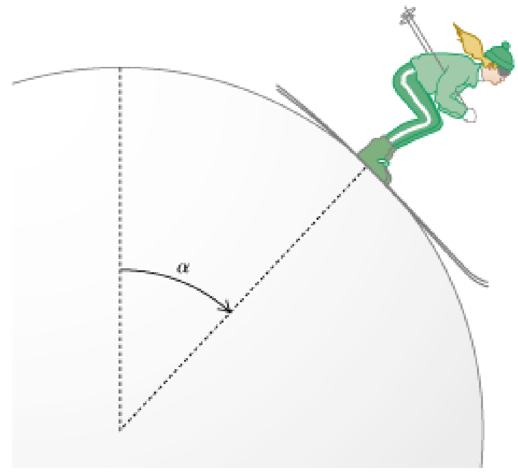
$$W = \int_{x_1}^{x_2} \vec{F_x} dx + \int_{y_1}^{y_2} \vec{F_y} dy$$

$$W = -2\alpha \int_{0.30}^{0.10} x dx - 2\alpha \int_{0}^{0} 0 dy = -2\alpha \left[\frac{x^2}{2}\right]_{0.30}^{0.10} + 0 = -2\alpha \left(\frac{0.10^2}{2} - \frac{0.30^2}{2}\right) = 0.10 \text{ J}$$

## *Dæmi 7.55*

A skier starts at the top of a very large, frictionless snowball, with a very small initial speed, and skis straight down the side

**A. At what point does she lose contact with the snowball and fly off at a tangent? That is, at the instant she loses contact with the snowball, what angle $\alpha$ does a radial line from the center of the snowball to the skier make with the vertical?**




Byrjum á því að finna heildar kraftinn sem hefur verkað á skíðamanneskjuna neðst

$$\sum F_x = m \cdot a$$

$$mg\cos\alpha - n = m \cdot a$$

þar sem $a$ er miðsóknarhröðunin $a = v^2/R$. Við viljum finna hornið þegar skíðarinn fer af snjóboltanum, þá er þverkrafturinn $n$ núll. þá er

$$mg\cos\alpha = m \cdot \frac{v^2}{R} \qquad (1)$$

Næst finnum við heildarorkuna í kerfinu

$$K_1 + U_1 = K_2 + U_2$$

Hreyfiorkan efst er $K_1 = 0$
Stöðuorkan efst er $U_1 = m \cdot g \cdot R$
Stöðuorkan neðst er $U_2 = m \cdot g \cdot \cos\alpha R$
Hreyfiorkan neðst er $K_2 = \frac{1}{2}mv^2$

Þá fæst

$$m \cdot g \cdot R = m \cdot g \cdot \cos\alpha R + \frac{1}{2}mv^2 \qquad (2)$$

Nú getum við stungið $v^2$ úr jöfnu (1) inn í jöfnu (2)

$$m \cdot g \cdot R = m \cdot g \cdot \cos\alpha R + \frac{1}{2}mRg\cos\alpha \qquad (2)$$

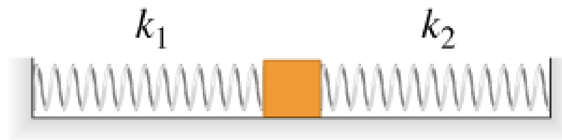
Einangrum $\alpha$ og fáum

$$\alpha = \cos^{-1}\left(\frac{2}{3}\right) = 48.19°$$

## *Dæmi 7.62*

A 3.00 kg block is connected to two ideal horizontal springs having force constants $k_1 = 26.0$ N/cm and $k_2 = 20.0$ N/cm. The system is initially in equilibrium on a horizontal, frictionless surface. The block is now pushed 15.0 cm to the right and released from rest.

**A. What is the maximum speed of the block?**




Kassinn sveiflast til og frá eins. Hámarkshraði kassans er þegar öll stöðuorkan er orðin að hreyfiorku, þetta gerist þegar kassinn er í miðjunni og gormarnir eru ekki strekktir. Við byrjum á því að færa kassan um 0.15 m til hægri. Þá mun gormur eitt færast um $x_{1A} = +0.15$ m og gormur 2 $x_{2A} = -0.15$ m. Þá er stöðuorkan fyrir

$$U_A = \frac{1}{2}k_1x_{1A}^2 + \frac{1}{2}k_2x_{2A}^2$$

Hreyfiorkan fyrir er $K_A = 0$
Stöðuorkan eftir (Í miðjunni) $U_B = 0$ og hreyfiorkan er

$$K_B = \frac{1}{2}mv^2$$

Heildar orkan er þá

$$\frac{1}{2}k_1x_{1A}^2 + \frac{1}{2}k_2x_{2A}^2 = \frac{1}{2}mv^2$$

Þá getum við einangrað $v$ og fáum

$$v = \sqrt{\frac{k_1 + k_2}{m}} \cdot x = 5.87 \text{ m/s}$$

**B. Where in the motion does the maximum speed occur?**

Í miðjunni, þegar báðir gormarnir eru slakir.

**C. What is the maximum compression of spring 1?**

Útslag hvors gorms fyrir sig verður $x = 0.15$ (í báðar áttir) úr jafnvægisstöðu.

$$\Delta x_1 = 0.15 \text{ m}$$

# 5 Vika

## 5.1 Skiladæmi 5

### *Dæmi 7.70*

A small block with mass 0.0450 kg slides in a vertical circle of radius 0.475 m on the inside of a circular track. During one of the revolutions of the block, when the block is at the bottom of its path, point A, the magnitude of the normal force exerted on the block by the track has magnitude 3.75 N. In this same revolution, when the block reaches the top of its path, point B, the magnitude of the normal force exerted on the block has magnitude 0.685 N.

**A. How much work was done on the block by friction during the motion of the block from point A to point B?**

Við reiknum heildarorkutap kubbsins þegar að kubburinn fer frá punkti A til punkts B:

$$W_{A\to B} = E_B - E_A = (K_B + U_B) - (K_A + U_A) = \frac{1}{2}m(v_B^2 - v_A^2) + mg(2R - 0)$$

Við þurfum að reikna hraða kubbsins í punktunum A og B. Við skoðum miðsóknarkraftinn sem verkar á kubbinn í báðum punktum, $m\frac{v_A^2}{R} = n_A - mg$ og $m\frac{v_B^2}{R} = n_B + mg$.

$$W_{A\to B} = \frac{R}{2}((n_B + mg) - (n_A - mg)) + 2mgR \quad = \quad R\left(\frac{n_B - n_A}{2} + 3mg\right)$$

$$W_{A\to B} = 0.475\,\mathrm{m} \cdot (\frac{0.685\,\mathrm{N} - 3.75\,\mathrm{N}}{2} + 3 \cdot (0.0450\,\mathrm{kg}) \cdot (9.8\,\mathrm{m/s^2})) = -98.9\,\mathrm{mJ}$$

### *Dæmi 7.72*

A small rock with mass 0.12 kg is fastened to a massless string with length 0.80 m to form a pendulum. The pendulum is swinging so as to make a maximum angle of 45° with the vertical. Air resistance is negligible.

**A. What is the speed of the rock when the string passes through the vertical position?**

Ákvörðum mestu hæð pendúlsins: $y_{\max} = l(1 - \cos\theta_{\max})$. Notum orkuvarðveislu:

$$\frac{1}{2}mv^2 = mgy_{\max} \qquad \Rightarrow \qquad v = \sqrt{2gy_{\max}} = \sqrt{2gl(1 - \cos\theta_{\max})} = \sqrt{2 \cdot (9.8\,\mathrm{m/s^2}) \cdot (0.80\,\mathrm{m}) \cdot (1 - \cos(45^\circ))} = 2.1\,\mathrm{m/s}$$

**B. What is the tension in the string when it makes an angle of $45^\circ$ with the vertical?**

Togkrafturinn í strengnum er jafn radíal-þætti þyngdarkraftsins í þessum punkti, þar sem miðsóknarkrafturinn er núll ($m\frac{v^2}{r} = T - mg\cos\theta = 0$).

$$T = mg\cos\theta = (0.12\,\mathrm{kg}) \cdot (9.8\,\mathrm{m/s^2}) \cdot \cos(45^\circ) = 0.83\,\mathrm{N}$$

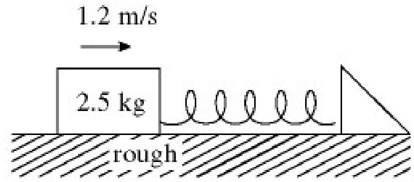
**C. What is the tension in the string as it passes through the vertical?**

Hér er $\theta = 0$ og $v = 2.1\,\mathrm{m/s}$.

$$m\frac{v^2}{r} = T - mg \qquad \Rightarrow \qquad T = m\left(\frac{v^2}{r} + g\right) = 0.12\,\mathrm{kg} \cdot \left(\frac{(2.1\,\mathrm{m/s})^2}{0.80\,\mathrm{m}} + 9.8\,\mathrm{m/s^2}\right) = 1.9\,\mathrm{N}$$

## Dæmi 7.32

A 2.5 kg box, sliding on a rough horizontal surface, has a speed of 1.2 m/s when it makes contact with a spring (see the figure). The block comes to a momentary halt when the compression of the spring is 5.0 cm. The work done by the friction, from the instant the block makes contact with the spring until is comes to a momentary halt, is −0.50 J.




**A. What is the spring constant of the spring?**

Hreyfiorkan í byrjun breytist í stöðuorku gormsins, en hluti af hreyfiorkunni tapaðist sem núningur, $\frac{1}{2}mv^2 + W = \frac{1}{2}kx^2$.

$$k = \frac{mv^2 + 2W}{x^2} = \frac{(2.5\,\mathrm{kg}) \cdot (1.2\,\mathrm{m/s})^2 2 \cdot (-0.50\,\mathrm{J})}{(0.050\,\mathrm{m})^2} = 1040\,\mathrm{N/m}$$

**B. What is the coefficient of kinetic friction between the box and the rough surface?**

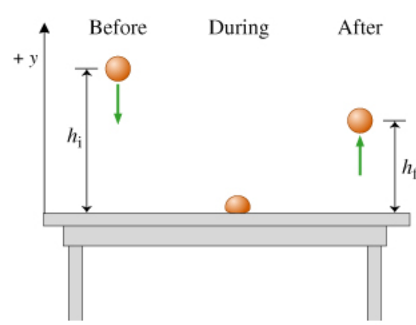
Vinna núningsins er $W = -\mu_k mgx$.

$$\mu_k = -\frac{W}{mgx} = -\frac{(-0.50\,\mathrm{J})}{(2.5\,\mathrm{kg}) \cdot (1.2\,\mathrm{m/s}) \cdot (0.050\,\mathrm{m})} = 0.41$$

## A Superball Collides Inelastically with a Table

As shown in the figure , a superball with mass m equal to 50 grams is dropped from a height of $h_i = 1.5$ m . It collides with a table, then bounces up to a height of $h_f = 1.0$ m . The duration of the collision (the time during which the superball is in contact with the table) is $t_c = 15$ m/s . In this problem, take the positive y direction to be upward, and use $g = 9.8m/s^2$ for the magnitude of the acceleration due to gravity. Neglect air resistance

As shown in the figure, a superball with mass $m$ equal to 50 grams is dropped from a height of $h_i = 1.5$ m. It collides with a table, then bounces up to a height of $h_f = 1.0$ m. The duration of the collision (the time during which the superball is in contact with the table) is $t_c = 15$ ms. In this problem, take the positive $y$ direction to be upward, and use $g = 9.8\ \mathrm{m/s^2}$ for the magnitude of the acceleration due to gravity. Neglect air resistance.




**A. Find the $y$ component of the momentum, $p_{\mathbf{before},y}$, of the ball immediately before the collision.**

Notfærum okkur að stöðuorka boltans fer öll í skriðorku þegar hann er lentur, svo að

$$mgh_i = \frac{1}{2}mv_i^2, \quad \Rightarrow \quad p_y = m\sqrt{2gh_i} = 0.2711\ \mathrm{kg\ m/s}$$

Munum að hér er boltinn á leið niður svo skriðþungi hans ætti að vera neikvæður.

**B. Find the $y$ component of the momentum of the ball immediately after the collision, that is, just as it is leaving the table.**

Beitum nákvæmlega sömu aðferð og í fyrri lið til að fá $p_y = 0.2214$ kg m/s.

**C. Find $J_y$, the $y$ component of the impulse imparted to the ball during the collision.**

Vitum að samkvæmt almennu jöfnunni fyrir atlag höfum við $J_y = p_{f,y} - p_{i,y} = 0.4925$ kg m/s.

**D. Find the $y$ component of the time-averaged force $F_{\mathbf{avg},y}$, in newtons, that the table exerts on the ball.**

Meðalkrafturinn sem boltinn fær frá borðinu er hægt að finna með

$$F_{\text{avg},y} = \frac{p_f - p_i}{\Delta t} = 32.83 \text{ N}$$

**E. Find $K_{\text{after}} - K_{\text{before}}$, the change in the kinetic energy of the ball during the collision, in joules.**

$$K_f - K_i = \frac{1}{2}\frac{p_f^2}{m} - \frac{1}{2}\frac{p_i^2}{m} = -0.2448 \text{ J}$$

## *Dæmi 8.32*

Two skaters collide and grab on to each other on frictionless ice. One of them, of mass 75.0 kg , is moving to the right at 4.00 m/s, while the other, of mass 61.0 kg , is moving to the left at 2.50 m/s.

**A. What is the magnitude of the velocity of these skaters just after they collide?**

Tökum saman skriðþunga fyrir og eftir árekstur

$$(m_1 + m_2) \cdot v = -m_2 \cdot v_2 + m_1 \cdot v_1$$

þá er hraðinn

$$v = \frac{-m_2 \cdot v_2 + m_1 \cdot v_1}{m_1 + m_2} = \frac{-61 \text{ kg} \cdot 2.5 \text{ m/s} + 75 \text{ kg} \cdot 4 \text{ m/s}}{75 \text{ kg} + 61 \text{ kg}} = 1.08 \text{ m/s}$$

**B. What is the direction of this velocity**

Hraðinn stefnir til hægri.

## *Dæmi 8.64*

A steel ball with mass 38.0 g is dropped from a height of 1.91 m onto a horizontal steel slab. The ball rebounds to a height of 1.60 m .

**A. Calculate the impulse delivered to the ball during impact.**

Þurfum að vinna hraðan fyrir og hraðan eftir (skilgreinum jákvæða stefnu upp)

$$v_1 = -\sqrt{2gh_1} \quad v_2 = \sqrt{2gh_2}$$

þá er atlagið

$$J_y = \Delta p = m\sqrt{2gh_2} + m\sqrt{2gh_1} = 0.445 \text{ N s}$$

**B. If the ball is in contact with the slab for a time of 1.50 ms , find the average force on the ball during impact.**

Meðtaltals krafturinn yfir tíma er fundinn með

$$F_{\text{avg}} = \frac{\Delta p}{\Delta t} = \frac{m\sqrt{2g}(\sqrt{h_2} + \sqrt{h_2})}{\Delta t} = 297 \text{ N}$$

## *Dæmi 8.46*

A 0.15 kg glider is moving to the right on a frictionless, horizontal air track with a speed of 0.800 m/s . It has a head-on collision with a 0.30 kg glider that is moving to the left with a speed of 2.20 m/s . Suppose the collision is elastic.

**A-D. Find the magnitude of the final velocity of the 0.15 kg glider.**

Glider $A$ ferðast til hægri, glider $B$ ferðast til vinstri. Engir ytri kraftar verka svo $P_x$ er varðveitt. Höfum fjaðrandi árekstur svo $K_1 = K_2$ gildir, þá höfum við að

$$v_{B2} - v_{A2} = -(v_{B1} - v_{A1})$$

$$v_{B2} - v_{A2} = -(-2.20 \text{ m/s} - 0.80 \text{ m/s})$$

$$v_{B2} - v_{A2} = 3 \text{ m/s} \qquad (1)$$

Skriðþunginn fyrir og eftir árekstur er varðveittur

$$m_A \cdot v_{A1} + m_B \cdot v_{B1} = m_A \cdot v_{A2} + m_B \cdot v_{B2}$$

$$0.150 \text{ kg} \cdot 0.80 \text{ m/s} - 0.30 \text{ kg} \cdot 2.20 \text{ m/s} = 0.150 \text{ kg} \cdot v_{A2} + 0.300 \text{ kg} \cdot v_{B2}$$

$$-3.60 \text{ m/s} = v_{2A} + 2v_{2B} \qquad (2)$$

Nú getum við lagt saman jöfnu (1) og (2) og fengið

$$-0.60 \text{ m/s} = 3v_{B2} \quad \rightarrow \quad v_{B2} = -0.20 \text{ m/s}$$

$$v_{A2} = v_{B2} - 3.00 \text{ m/s} = -3.20 \text{ m/s}$$

Þar sem $v_{B2}$ stefnir til vinstri, og $v_{A2}$ stefnir til hægri.

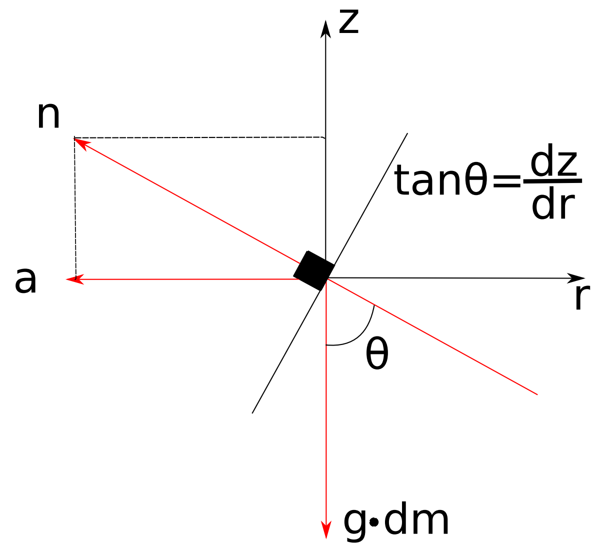
## 5.2 Tímadæmi 5R

### *Rotating liquid*

If a vertical cylinder of liquid (such as in a chemistry test tube) rotates about its axis, the surface forms a smooth curve that rises toward the edges of the tube. Let us call the rotation axis z and the radial direction r. Assume that all the liquid rotates with the same angular velocity $\omega$ , independent of position within the tube. Show that the surface is a parabola described by (g is the gravitational acceleration).

*Hint: Consider a liquid element at the top surface. The total force on this (or any other) element must be normal to the surface (perpendicular) in steady state. Otherwise the fluid would flow sideways and we wouldn't have a steady state.*

**Lausn:**




Byrjum á því að finna miðsóknarhröðun út frá hornhraðanum

$$a = \omega^2 r$$

Þyngdarkrafturinn er

$$dm \cdot g = n \cos\theta$$

sama fyrir

$$dm \cdot a = n \sin\theta$$

leysum þessar tvær saman og fáum

$$dm \cdot \omega^2 r = n \sin\theta$$

Notum $\tan\theta = \sin\theta / \cos\theta$

$$\tan\theta = \frac{\omega^2 r}{g}$$

Hallinn á vökvanum er gefinn með $\tan\theta = dz/dr$, þá fæst

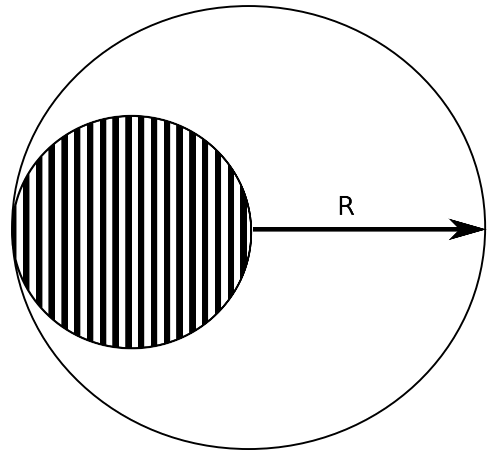
$$\frac{dz}{dr} = \frac{\omega^2 r}{g}$$

$$\int_0^z dz = \frac{\omega^2}{g} \int_0^r r dr$$

$$z = \frac{\omega^2 r^2}{2g}$$

## *Pizza*

From a uniform disk with radius R there has been cut a smaller disk, that has diameter R, equal to the radius of the initial disk. The centers of both the "disks"should be assumed to have the vertical coordinate 0. The two edges of the cutout disk touch the center and the edge of the original disk, respectively. Upon removal of the material the center of mass moves to the right.




**A. If the origin of the coordinate system is in the middle of the original (large) disk, where on the horizontal axis is the center of mass after the small disk is cut out. Give your answer in terms of R.**

Skiptum hringnum upp í þrjú tilvik

$$\text{Heil pizza: } \int_{\text{Pi}} \text{dm}$$
$$\text{Pizza án sneiðs: } \int_{\text{Án}} \text{dm}$$
$$\text{Sneið: } \int_{\text{Sn}} \text{dm}$$

Þá sjáum við að massamiðjan fyrir er sú sama og summan á massamiðjunni á sneiðinni og pizzu án sneiðar

$$\int_{\text{Pi}} \text{dm} = \int_{\text{Án}} \text{dm} + \int_{\text{Sn}} \text{dm}$$

Massamiðjan á pizzunni er í miðjunni svo

$$0 = \int_{\text{Án}} \text{dm} + \int_{\text{Sn}} \text{dm}$$

þar sem við getum skrifað

$$0 = m_{\text{Án}} \cdot x_{\text{Án}} + m_{\text{Sn}} \cdot x_{\text{Sn}}$$

Við viljum finna $x_{\text{Án}}$.
Massinn á pizzu sneiðinni er

$$m_{\text{Sn}} = \rho\pi \left(\frac{R}{2}\right)^2$$

þar sem $\rho$ er eðlismassi pizzunnar. Massamiðja pizzu sneiðinar er

$$x_{\text{Pi}} = -\frac{R}{2}$$

Massinn á pizzunni án sneiðar er þá flatarmálið á heilu pizzunni mínus pizza sneiðin

$$m_{\text{Án}} = \rho\left(\pi R^2 - \pi\left(\frac{R}{2}\right)\right)$$

Þá getum við fundið hvernig massamiðjan hliðrast eftir að sneiðin er fjarlægð

$$x_{\text{Án}} = -\frac{m_{\text{Sn}}}{m_{\text{Án}}.} \cdot x_{\text{Sn}}$$

$$x_{\text{Án}} = -\frac{\rho\pi\left(\frac{R}{2}\right)^2}{\rho\left(\pi R^2 - \pi\left(\frac{R}{2}\right)\right)} \cdot \left(-\frac{R}{2}\right)$$

$$x_{\text{Án}} = \frac{1/4}{1-1/4} \cdot \frac{R}{2} = \frac{R}{6}$$

### *Stream of water*

A stream of water 50 mm in diameter, moving 24 m/s horizontally to the right, strikes a flat vertical plate. After striking, the water moves parallel to the plate (spreads out in a circular pattern).

**A. What is the force extorted on the plate by the stream of water.**

Breytingin í þrýstingi er

$$\Delta p_x = v \cdot \mathrm{dm} \quad [\mathrm{Joule}]$$

þar sem dm er rúmmál

$$\mathrm{dm} = \rho \cdot v \cdot \mathrm{dt} \cdot A$$

þar sem $A$ er flatarmálið $A = \pi \cdot R^2$

$$p - p_0 = \vec{F}\mathrm{dt}$$

þá er

$$0 - \mathrm{dm} \cdot v = \vec{F}\mathrm{dt}$$

$$\rho \cdot v \cdot \mathrm{dt} \cdot \pi \cdot \frac{R^2}{4} = \vec{F}\mathrm{dt}$$

þá fáum við að

$$\vec{F} = -\rho \cdot v^2 \cdot A = 10^3 \ \mathrm{kg/m}^3 \cdot (24 \ \mathrm{m/s})^2 \cdot \pi \cdot 2.5 \cdot 10^{-3} \ \mathrm{m} = 1131 \ \mathrm{N}$$

### Two balls falling

Two balls fall to the ground, the smaller one (m) sitting on top of the larger (M), as demonstrated in class. Upon hitting the ground the smaller ball acquires what seems to be ȩxtra"velocity that shoots it high up into the air. To explain this effect, make the following assumptions: The event can be divided into first the collision of M with the ground (as if m weren't there), second, a collision between the upward traveling M with the still downward traveling m (assume that m is totally unaffected by M's collision with the ground). You may make the approximation that M, once it hits the ground has fallen a distance h=1 m, and that when m collides with M it ALSO has a velocity corresponding to having free-fallen a distance h=1 m.

**A. If you assume the collisions to be elastic, what is the maximum height that $m$ reaches?**

Þar sem boltarnir detta niður sömu vegalengd, ná þeir sama hraðanum $v = \sqrt{2gh}$ (m.v. jörðu) áður en massi $M$ skellur á jörðinu. Til þess að hreyfiorka $M$ varðveitist, þarf hann að skoppa upp með hraða $v$. Á sama augnabliki skella boltarnir saman (en eftir að bolti $M$ skoppar af jörðinni) í alfjaðrandi árekstri. Samanlagður skriðþungi og hreyfiorka boltanna varðveitist við þennan árekstur. Táknum hraða $m$ eftir árekstur með $u$ og hraða $M$ með $w$. Til þæginda ætti að skipta um viðmiðunarkerfi þar sem bolti $m$ er kyrrstæður rétt fyrir áreksturinn. Í því viðmiðunarkerfi hefur bolti $M$ hraðann $v' = v + v = 2v$ upp á við (m.v. bolta $m$) fyrir árekstur. Skoðum skriðþungavarðveislu í árekstri boltanna:

$$Mv' = Mw' + mu' \qquad \Rightarrow \qquad w' = \frac{Mv' - mu'}{M}$$

Skoðum síðan orkuvarðveisluna og stingum inn fyrir $w'$:

$$\frac{1}{2}Mv'^2 = \frac{1}{2}Mw'^2 + \frac{1}{2}mu'^2 \qquad \Rightarrow \qquad mu'^2 = M(v'^2 - w'^2) = M\left(v'^2 - \left(\frac{Mv' - mu'}{M}\right)^2\right)$$

Við fáum annars stigs jöfnu fyrir $u'$. Tökum saman annars stigs liði, fyrsta stigs liði og fasta:

$$u'^2\left(m + \frac{m^2}{M}\right) + u'(-2mv') + 0 = 0$$

Þessi jafna hefur tvær lausnir (sem er auðfundin með þáttun):

$$u' = \begin{cases} 0 \\ \frac{2m}{m+\frac{m^2}{M}}v' = \frac{2M}{M+m}v' = \frac{4M}{M+m}v \end{cases}$$

Núlllausnin gildir ef að enginn árekstur á sér stað (hraðar óbreyttir). Við verðum því að velja lausnina $u' = \frac{4M}{M+m}v$. Við þurfum að skipta til baka í viðmiðunarkerfi jarðarinnar:

$$u = u' - v = \frac{4M - (M+m)}{M+m}v = \frac{3M - m}{M+m}v = \frac{3M-m}{M+m} \cdot \sqrt{2gh}$$

Að lokum notum við orkuvarðveislu til að finna lokahæð boltans $m$:

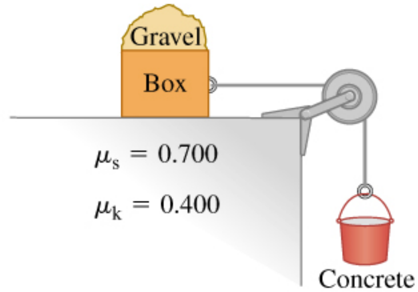
$$\frac{1}{2}mu^2 = mgy \qquad \Rightarrow \qquad y = \frac{u^2}{2g} = \frac{\left(\frac{3M-m}{M+m}\right)^2 \cdot 2gh}{2g} = \boxed{\left(\frac{3M-m}{M+m}\right)^2 h}$$

## 5.3 Tímadæmi 5V

### Dæmi 7.37

At a construction site, a 66.0 kg bucket of concrete hangs from a light (but strong) cable that passes over a light friction-free pulley and is connected to an 83.0 kg box on a horizontal roof (see the figure ). The cable pulls horizontally on the box, and a 48.0 kg bag of gravel rests on top of the box. The coefficients of friction between the box and roof are shown. The system is not moving.

**A. Find the friction force on the bag of gravel.**




Það verkar enginn núningskraftur á malarpokann

$$F = 0 \text{ N}$$

**B. Find the friction force on the box.**

Það er gefið að allt er kyrrstætt. Þá er núningskrafturinn sá sami og togkrafturinn sem verkar vegna fötunnar

$$M_C \cdot g = 647 \text{ N}$$

**C. Suddenly a worker picks up the bag of gravel. Use energy conservation to find the speed of the bucket after it has descended 2.10 m , from rest.**

$$U_1 + K_1 + W_f = U_2 + K_2$$

þar sem

$$U_1 = m_C \cdot g \cdot d \qquad K_1 = 0 \qquad W_f = -m_B \cdot g \cdot \mu_k \dot{d} \qquad K_2 = \frac{1}{2}mv^2 \qquad U_2 = 0$$

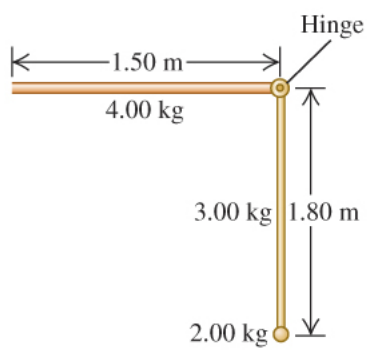
$$\frac{1}{2}mv^2 = m_C \cdot g \cdot d - m_B \cdot g \cdot \mu_k \dot{d}$$

$$v = \sqrt{\frac{2}{m_B + m_C}(m_c \cdot g \cdot d - m_B \cdot g\mu_k \cdot d)} = 3.01 \text{ m/s}$$

## *Dæmi 8.55*

A machine part consists of a thin, uniform 4.00-kg bar that is 1.50 m long, hinged perpendicular to a similar vertical bar of mass 3.00 kg and length 1.80 m. The longer bar has a small but dense 2.00-kg ball at one end

**A. By what distance will the center of mass of this part move horizontally and vertically if the vertical bar is pivoted counterclockwise through 90° to make the entire part horizontal? Find the magnitude of horizontal displacement.**




Tökum saman massamiðju fyrir og eftir færslu í lárétta stefnu

$$x_{\text{i cm}} = \frac{\sum_i m_i \cdot x_i}{\sum_i m_i}$$

Massamiðjan fyrir (initial):

$$x_{\text{i cm}} = \frac{4 \text{ kg} \cdot 0.750 \text{ m} + 0 + 0}{4 \text{ kg} + 3 \text{ kg} + 2 \text{ kg}} = 0.333 \text{ m}$$

Massamiðjan eftir (final):

$$x_{\text{f cm}} = \frac{0 + 3 \text{ kg} \cdot (-0.9 \text{ m}) + 2 \text{ kg} \cdot (-1.80 \text{ m})}{4 \text{ kg} + 3 \text{ kg} + 2 \text{ kg}} = -0.366 \text{ m}$$

Lárétta hliðrunin er þá

$$|\Delta x_{cm}| = |x_{\text{cm f}} - x_{\text{cm i}}| = 0.7 \text{ m}$$

**B. Find the direction of horizontal displacement.**

Hliðrunin er til hægri.

**C.Find the magnitude of vertical displacement.**

Tökum saman massamiðju fyrir og eftir færslu í lóðrétta stefnu

$$y_{\text{i cm}} = \frac{\sum_i m_i \cdot x_i}{\sum_i m_i}$$

Massamiðjan fyrir (initial):

$$y_{\text{i cm}} = \frac{0 + 3\text{ kg} \cdot 0.90\text{ m} + 2\text{ kg} \cdot 1.80\text{ m}}{4\text{ kg} + 3\text{ kg} + 2\text{ kg}} = 0.7\text{ m}$$

Massamiðjan eftir (final):

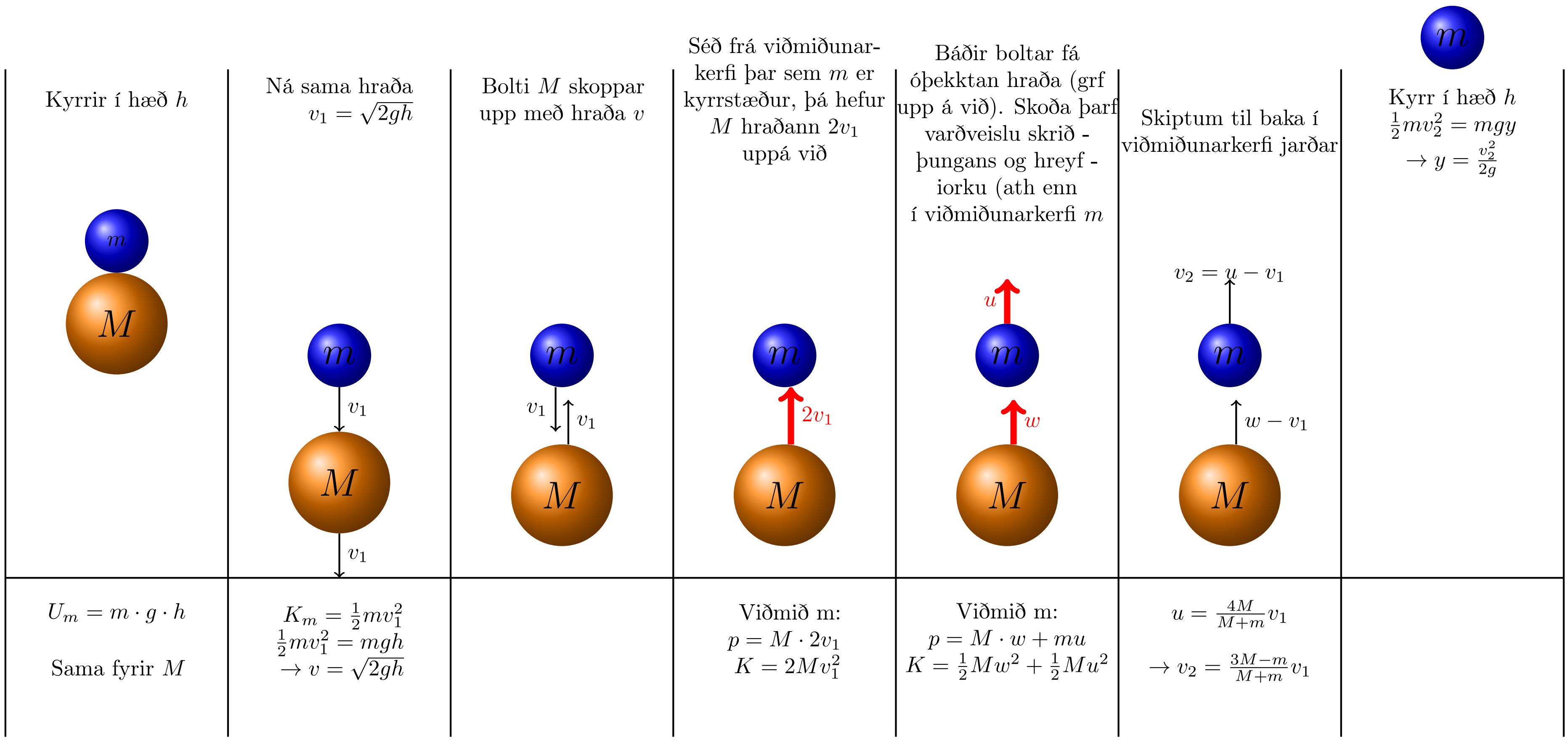
$$y_{\text{f cm}} = 0$$

Lóðrétta hliðrunin er þá

$$|\Delta y_{cm}| = |y_{\text{cm f}} - y_{\text{cm i}}| = 0.7\text{ m}$$

**D. Find the direction of horizontal displacement.**

Hliðrunin er upp.



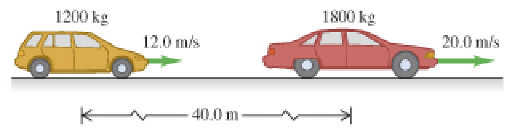

Mynd 1: Skýringarmynd fyrir dæmi: Two balls falling

# 6 Vika

## 6.1 Skiladæmi 6

### *Dæmi 8.54*

A 1200 kg SUV is moving along a straight highway at 12.0 m/s. Another car, with mass 1800 kg and speed 20.0 m/s, has its center of mass 40.0 m ahead of the center of mass of the SUV .




**A. Find the position of the center of mass of the system consisting of the two cars.**

Best er að skilgreina núllpunkt kerfisins út frá staðsetningu fremri bílsins og látum jákvæða stefnu vísa til hægri. Ákvörðum massamiðju heildarkerfisins (fyrir aftari bílinn ($m_1$) og þann fremri ($m_2$))

$$x_{\text{CM}} = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2} = \frac{-40.0\,\text{m} \cdot 1200\,\text{kg} + 0\,\text{m} \cdot 1800\,\text{kg}}{3000\,\text{kg}} = -16.0\,\text{m}$$

Massamiðjan er því 16.0 m fyrir aftan fremri bílinn

**B. Find the magnitude of the system's total momentum, by using the given data.**

Hér þurfum við að reikna skriðþungann með:

$$p = m_1 v_1 + m_2 v_2 = 1200\,\text{kg} \cdot 12.0\,\text{m/s} + 1800\,\text{kg} \cdot 20.0\,\text{m/s} = 5.04 \cdot 10^4\,\text{kg} \cdot \text{m/s}$$

**C. Find the speed of the system's center of mass.**

Í þessum lið er ætlast til að við tökum vegið meðaltal af hröðum bílanna:

$$v_{\text{CM}} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{1200\,\text{kg} \cdot 12.0\,\text{m/s} + 1800\,\text{kg} \cdot 20.0\,\text{m/s}}{3000\,\text{kg}} = 16.8\,\text{m/s}$$

**D. Find the system's total momentum, by using the speed of the center of mass.**

$$p_{\text{CM}} = (m_1 + m_2) \cdot v_{\text{CM}} = 3000\,\text{kg} \cdot 16.8\,\text{m/s}$$

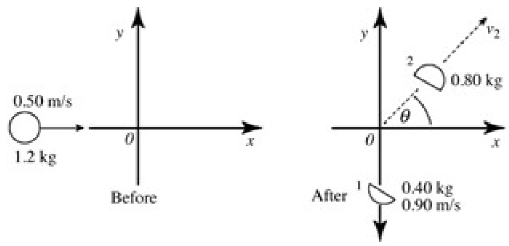
**E. Compare your results of parts B and D.**

Út frá nefnaranum í C lið sést að skriðþungi kerfisins er í raun skriðþungi massamiðjunnar:

$$p_{\text{CM}} = \cancel{(m_1 + m_2)} \cdot \frac{m_1 v_1 + m_2 v_2}{\cancel{m_1 + m_2}} = p$$

## *Dæmi 8.30*

**A. A $1.2\,\mathrm{kg}$ spring-activated toy bomb slides on a smooth surface along the $x$-axis with a speed of $0.50\,\mathrm{m/s}$. At the origin 0, the bomb explodes into two fragments. Fragment 1 has a mass of $0.40\,\mathrm{kg}$ and a speed of $0.90\,\mathrm{m/s}$ along the negative $y$-axis. In the figure, the energy released by the explosion is closest to**




Notum skriðþungavarðveislu til að ákvarða hraða á broti 2 (sem hefur massann $m_2 = 0.80\,\mathrm{kg}$).

$$(m_1 + m_2)\begin{pmatrix} v_{x0} \\ v_{y0} \end{pmatrix} = m_1 \begin{pmatrix} v_{x1} \\ v_{y1} \end{pmatrix} + m_2 \begin{pmatrix} v_{x2} \\ v_{y2} \end{pmatrix}$$

$$\begin{pmatrix} v_{x2} \\ v_{y2} \end{pmatrix} = \frac{m_1 + m_2}{m_2}\begin{pmatrix} v_{x0} \\ v_{y0} \end{pmatrix} - \frac{m_1}{m_2}\begin{pmatrix} v_{x1} \\ v_{y1} \end{pmatrix} = \frac{1.2\,\mathrm{kg}}{0.80\,\mathrm{kg}}\begin{pmatrix} 0.50\,\mathrm{m/s} \\ 0\,\mathrm{m/s} \end{pmatrix} - \frac{0.40\,\mathrm{kg}}{0.80\,\mathrm{kg}}\begin{pmatrix} 0\,\mathrm{m/s} \\ -0.90\,\mathrm{m/s} \end{pmatrix} = \begin{pmatrix} 0.75\,\mathrm{m/s} \\ 0.45\,\mathrm{m/s} \end{pmatrix}$$

Orkulosunin verður breytingin á hreyfiorku kerfisins:

$$\Delta K = \left(\frac{1}{2}m_1(v_{x1}^2 + v_{y1}^2) + \frac{1}{2}m_2(v_{x2}^2 + v_{y2}^2)\right) - \left(\frac{1}{2}(m_1 + m_2)(v_{x0}^2 + v_{y0}^2)\right)$$

$$\Delta K = \left(\frac{1}{2}\cdot(0.40\,\mathrm{kg})\cdot(-0.90\,\mathrm{m/s})^2 + \frac{1}{2}\cdot(0.80\,\mathrm{kg})\cdot((0.75\,\mathrm{m/s})^2 + (0.45\,\mathrm{m/s})^2)\right) - \left(\frac{1}{2}\cdot(1.2\,\mathrm{kg})\cdot(0.50\,\mathrm{m/s})^2\right) = 0.32\,\mathrm{J}$$

## *Dæmi 9.7*

The angle $\theta$ through which a disk drive turns is given by $\theta(t) = a + bt - ct^3$, where $a$,$b$ and $c$ are constants, $t$ is in seconds, and $\theta$ is in radians. When $t = 0$, $\theta = \frac{\pi}{4}\,\mathrm{rad}$ and the angular velocity is $3.40\,\mathrm{rad/s}$ , and when $1.70\,\mathrm{s}$ , the angular acceleration is $1.25\,\mathrm{rad/s^2}$.

**A. Find $a$ including their units.**

Upphafsskilyrðin gefa okkur að $\theta(0) = a = \frac{\pi}{4}\,\mathrm{rad}$

**B. Find $b$ including their units.**

Tökum tímaafleiðu af $\theta(t)$ til að fá $\omega(t) = b - 3ct^2$. Upphafsskilyrðin gefa okkur að $\omega(0) = b = 3.40\,\mathrm{rad/s}$

**C. Find $c$ including their units.**

Tökum tímaafleiðu af $\omega(t)$ til að fá $\alpha(t) = -6ct$. Notum þá upphafsskilyrðið $\alpha(1.70\,\mathrm{s}) = -6\cdot c\cdot(1.70\,\mathrm{s}) = 1.25\,\mathrm{rad/s^2}$. Einangrum $c$ og fáum $c = -\frac{1.25\,\mathrm{rad/s^2}}{6\cdot(1.70\,\mathrm{s})} = -0.123\,\mathrm{rad/s^3}$

**D. What is the angular acceleration when $\theta = \frac{\pi}{4}\,\mathrm{rad}$?**

Fyrsta upphafsskilyrðið gaf okkur að $t = 0\,\mathrm{s}$ þegar að $\theta(0) = \frac{\pi}{4}\,\mathrm{rad}$. Þá er $\alpha(0) = -6\cdot(-0.123\,\mathrm{rad/s^3})\cdot(0\,\mathrm{s}) = 0\,\mathrm{rad/s^2}$

**E. What is $\theta$ when the angular acceleration is $4.10\,\mathrm{rad/s^2}$?**

Einangrum $t$ úr jöfnunni fyrir $\alpha(t)$:

$$t = -\frac{\alpha}{6c} = -\frac{4.10\,\mathrm{rad/s^2}}{6\cdot -0.123\,\mathrm{rad/s^3}} = 5.556\,\mathrm{s}$$

Stingum þessu tímagildi í jöfnuna fyrir $\theta(t)$.

$$\theta(t) = \frac{\pi}{4}\,\mathrm{rad} + (3.40\,\mathrm{rad/s})\cdot(5.556\,\mathrm{s}) - (-0.123\,\mathrm{rad/s^3})\cdot(5.556\,\mathrm{s})^3 = 41.0\,\mathrm{rad}$$

**F. What is the angular velocity when the angular acceleration is $4.10\,\mathrm{rad/s^2}$?**

Notum sama tímagildi og stingum því inn í jöfnuna fyrir $\omega(t)$:

$$\omega(t) = 3.40\,\mathrm{rad/s} - 3\cdot(-0.123\,\mathrm{rad/s^3})\cdot(5.556\,\mathrm{s})^2 = 14.8\,\mathrm{rad/s}$$

## *Dæmi 9.25*

An advertisement claims that a centrifuge takes up only $0.127\,\mathrm{m}$ of bench space but can produce a radial acceleration of $2200g$ at $3670\,{}^{\mathrm{rev}}/_{\mathrm{min}}$.

**A. Calculate the required radius of the centrifuge.**

Skilvindan ætti að mynda miðsóknarhröðunina $a = \omega^2 r = 4\pi^2 f^2 r$ við jaðarinn.

$$r = \frac{a}{4\pi^2 f^2} = \frac{2200 \cdot 9.8\,{}^{\mathrm{m}}/_{\mathrm{s}^2}}{4\pi^2 \left(\frac{3670}{60}\,{}^{\mathrm{rev}}/_{\mathrm{s}}\right)^2} = 0.146\,\mathrm{m}$$

**B. Is the claim realistic?**

Radíus skilvindunnar er ekki nógu stór til þess að geta myndað svona mikla miðsóknarhröðun. (Radíusinn þarf að vera um 15% stærri til þess að staðhæfingin sé raunhæf)

## *Alternative Exercise 9.112*

The earth, which is not a uniform sphere, has a moment of inertia of $0.3308MR^2$ about an axis through its north and south poles. It takes the earth $86164\,\mathrm{s}$ to spin once about this axis.

**A. Use Appendix F in the textbook to calculate the earth's kinetic energy due to its rotation about this axis.**

Snúningsorka jarðar er $K_{\text{snún}} = \frac{1}{2}I\omega^2 = \frac{0.3308}{2}MR^2\left(\frac{2\pi}{T_{\text{sólarhringur}}}\right)^2 = \frac{0.3308}{2} \cdot 5.97 \cdot 10^{24}\,\mathrm{kg} \cdot (6.37 \cdot 10^6\,\mathrm{kg})^2 \left(\frac{2\pi}{86400\,\mathrm{s}}\right)^2$.

$$K_{\text{snún}} = 2.14 \cdot 10^{29}\,\mathrm{J}$$

**B. Use Appendix F in the textbook to calculate the earth's kinetic energy due to its orbital motion around the sun.**

Hreyfiorka jarðar vegna brautarhraða hennar í kringum sólina er $K_{\text{braut}} = \frac{1}{2}M\left(\frac{2\pi R_{\text{braut}}}{T_{\text{braut}}}\right)^2$

$$K_{\text{braut}} = \frac{1}{2} \cdot (5.97 \cdot 10^{24}\,\mathrm{kg}) \cdot \left(\frac{2\pi \cdot (1.5 \cdot 10^{11}\,\mathrm{m})}{3.156 \cdot 10^7\,\mathrm{s}}\right)^2 = 2.66 \cdot 10^{33}\,\mathrm{J}$$

**C. Explain how the value of the earth's moment of inertia tells us that the mass of the earth is concentrated toward the planet's center.**

Fyrir kúlulaga massa er fínt að nota einsleita kúludreifða massa til viðmiðunar. Hverfitregða einsleitrar kúlu um ás sem fer í gegnum massamiðju hennar er $I_{\text{kúla}} = \frac{2}{5}MR^2$. Fyrir jafnstóran massa fæst hærri hverfitregða um ás sem fer í gegnum massamiðju ef meginhluti massans er staðsettur sem fjærst massamiðjunni. Sama hverfitregða er lágmörkuð með því að færa sem mestan massa sem næst massamiðjunni. Þar sem $0.3308 < \frac{2}{5}$ má segja að miðhluti Jarðar er massameiri en ytri hlutinn.

## *Alternative Exercise 9.118*

Consider a rigid body that is a thin, plane sheet of arbitrary shape. Take the body to lie in the $xy$-plane and let the origin $O$ of coordinates be located at any point within or outside the body. Let $I_x$ and $I_y$ be the moments of inertia about the $x$- and $y$-axes, and let $I_0$ be the moment of inertia about an axis through $O$ perpendicular to the plane.

**A. By considering mass elements $m_i$ with coordinates $(x_i, y_i)$, show that $I_x + I_y = I_O$. This is called the perpendicular-axis theorem. Note that point $O$ does not have to be the center of mass.**

Hugsum okkur að platan sé samsett út strjálum massaeiningum $m_i$ sem eru í fjarlægðinni $r_i = \sqrt{x_i^2 + y_i^2}$ frá $O$. Hverfitregðan um $O$ er skilgreind sem $I_O = \sum_i m_i r_i^2 = \sum_i m_i(x_i^2 + y_i^2)$. Skoðum nú hverfitregðu plötunnar um $x$-ás og síðan $y$-ás. Hverfitregðan um $x$-ás er skilgreind sem $I_x = \sum_i m_i y_i^2$ og hverfitregðan um $y$-ás er skilgreind sem $I_y = \sum_i m_i x_i^2$. Summa hverfitregðanna er þá $I_x + I_y = \sum_i m_i(y_i^2 + x_i^2) = I_O$.

**B. For a thin washer with mass $M$ and with inner and outer radii $R_1$ and $R_2$, use the perpendicular-axis theorem to find the moment of inertia about an axis that is in the plane of the washer and that passes through its center. You may use the information in Table 9.2 in the textbook.**

Hér getum við skilgreint $I_O$ sem samhverfuás þvottatromlunnar. Þá er $I_O = I_x + I_y = 2I_x$ vegna samhverfu. Um þykkar hringgjarðir/tromlur gildir $I_O = \frac{1}{2}M(R_1^2 + R_2^2)$, þá er $I_x = \frac{I_O}{2} = \frac{1}{4}M(R_1^2 + R_2^2)$

**C. Use the perpendicular-axis theorem to show that for a thin, square sheet with mass $M$ and side $L$, the moment of inertia about any axis in the plane of the sheet that passes through the center of the sheet is $\frac{1}{12}ML^2$. You may use the information in Table 9.2 in the textbook.**

Um ferningslaga plötu með hliðarlengdir $a$ og $b$ gildir að hverfitregða um ás sem liggur þvert í gegnum massamiðju plötunnar er $I_O = \frac{1}{12}M(a^2 + b^2)^2$. Fyrir ferningslaga plötu $a = b = L$ verður $I_O = \frac{1}{6}ML^2$. Veljum nú einhvern ás sem liggur í plani plötunnar og fer í gegnum massamiðju plötunnar. Hverfitregða um þann ás er jöfn hverfitregðunni um annan hornréttan ás í sama plani. ($I_x = I_y$). Um upphaflegu hverfitregðuna gildir $I_O = I_x + I_y = 2I_x$, svo við höfum $I_x = \frac{I_O}{2} = \frac{1}{12}ML^2$.

## *Dæmi 9.17*

**A. A uniform solid sphere of mass $M$ and radius $R$ rotates with an angular speed $\omega$ about an axis through its center. A uniform solid cylinder of mass $M$, radius $R$, and length $2R$ rotates through an axis running through the central axis of the cylinder. What must be the angular speed of the cylinder so it will have the same rotational kinetic energy as the sphere?**

Snúningsorkur kúlunnar og sívalningsins eru jafnstórar. $\frac{1}{2}I_{\text{kúla}}\omega_{\text{kúla}}^2 = \frac{1}{2}I_{\text{sívalningur}}\omega_{\text{sívalningur}}^2$. Nú er $I_{\text{kúla}} = \frac{2}{5}MR^2$ og $I_{\text{sívalningur}} = \frac{1}{2}MR^2$. Þá eru orkurnar $\frac{1}{2} \cdot \frac{2}{5}MR^2\omega_{\text{kúla}}^2 = \frac{1}{2} \cdot \frac{1}{2}MR^2\omega_{\text{sívalningur}}^2$. Nú er $\omega_{\text{kúla}} = \omega$, einangrum $\omega_{\text{sívalningur}}$:

$$\omega_{\text{sívalningur}} = \sqrt{\frac{\frac{1}{5}MR^2\omega^2}{\frac{1}{4}MR^2}} = \frac{2}{\sqrt{5}}\omega$$

## *Dæmi 9.19*

**A. At any angular speed, a certain uniform solid sphere of diameter $D$ has half as much rotational kinetic energy as a certain uniform thin-walled hollow sphere of the same diameter when both are spinning about an axis through their centers. If the mass of the solid sphere is $M$, the mass of the hollow sphere is**

Notum sambærilega útleiðslu og áðan, skoðum hreyfiorkuna: $\frac{1}{2} \cdot \frac{2}{5}M\left(\frac{D}{2}\right)^2 = \frac{\frac{1}{2} \cdot \frac{2}{3}M_{\text{HS}}\left(\frac{D}{2}\right)^2}{2}$. Einangrum $M_{\text{HS}}$:

$$M_{\text{HS}} = \frac{2 \cdot \frac{2}{5}M}{\frac{2}{3}} = \frac{6}{5}M$$

## 6.2 Tímadæmi 6

### *Dæmi 9.92*

American eels (Anguilla rostrata) are freshwater fish with long, slender bodies that we can treat as uniform cylinders 1.0 m long and 10 cm in diameter. An eel compensates for its small jaw and teeth by holding onto prey with its mouth and then rapidly spinning its body around its long axis to tear off a piece of flesh. Eels have been recorded to spin at up to 14 revolutions per second when feeding in this way. Although this feeding method is costly in terms of energy, it allows the eel to feed on larger prey than it otherwise could.

**A. A field researcher uses the slow-motion feature on her phone's camera to shoot a video of an eel spinning at its maximum rate. The camera records at 120 frames per second. Through what angle does the eel rotate from one frame to the next?**

Állinn snýst 14 rev/s, svo á einni sekúndu fer hann

$$\theta = \frac{14\ \text{rev/s} \cdot 2\pi\ \text{rad/rev}}{120\ \text{frames/s}} = 0.73\ \text{rad}$$

Margföldum með $180°/\pi$ til að fá gráður, þá er $\theta = 42°$.

### *Dæmi 9.93*

American eels (Anguilla rostrata) are freshwater fish with long, slender bodies that we can treat as uniform cylinders 1.0 m long and 10 cm in diameter. An eel compensates for its small jaw and teeth by holding onto prey with its mouth and then rapidly spinning its body around its long axis to tear off a piece of flesh. Eels have been recorded to spin at up to 14 revolutions per second when feeding in this way. Although this feeding method is costly in terms of energy, it allows the eel to feed on larger prey than it otherwise could.

**A. The eel is observed to spin at 14 spins per second clockwise, and 10 seconds later it is observed to spin at 8 spins per second counterclockwise. What is the magnitude of the eel's average angular acceleration during this time?**

Hornhröðunin er

$$\alpha_{av} = \frac{\omega - \omega_0}{t} = (2\pi\ \text{rad/rev}) \cdot \frac{8\ \text{rev/s} - (14\ \text{rev/s})}{10\,\text{s}} = 4.4\pi\ \text{rad/s}^2$$

### *Alternative Exercise 9.135*

A totally inedible cafeteria meatball with mass 40.0 g is attached to the free end of a 2.50-m piece of string that is attached to the ceiling. The meatball is pulled to the left so that the string makes a 36.9° angle with the vertical and is then released.

**A. What is the magnitude of the angular velocity of the meatball the first time the angular acceleration is zero?**

Hornhröðunin er núll þegar kjötbollan er í neðstu stöðu, notum

$$mgy = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\omega l\right)^2$$

$$\omega = \frac{\sqrt{2gy}}{l}$$

þar sem $y$ er hæðin á kjötbollunni þegar hún er í 36°

$$\omega = \frac{\sqrt{2gl(1 - \cos\theta)}}{l} = 1.25\ \text{rad/s}$$

**B. What is the direction of the angular velocity of the meatball the first time the angular acceleration is zero?**

Hraðavigurinn stefnir útúr blaði skv. hægri handa reglunni.

**C. What is it the second time that $\alpha_z$=0?**

Næst þegar $\alpha_z = 0$ er næsta þegar kjötbollan fer í gegnum neðsta punktinn aftur.

**D-E. At the times described in parts (a) and (b), what is the magnitude of the meatball's radial acceleration?**

Geislalæga hröðunin er

$$a_{rad} = \omega^2 l$$

þar sem $\omega$ var fundinn í lið A, þá fæst

$$a_{rad} = 2g(1 - \cos\theta) = 3.93 \text{ m/s}^2$$

þar sem geislalægahröðunin stefnir inn að miðjunni.

Ath. Geislalægahröðunin (*e. radial acceleration* ) er ekki sú sama og hornhröðunin (*e. angular acceleration*). Geislalægahröðunin stefnir inn að miðjunni og er gefin með $a_{\text{rad}} = \omega l^2$ og hefur einingu m/s$^2$. Hornhröðunin $\alpha$ er gefin með td. $\omega^2 = \omega_0^2 + 2\alpha\theta$ og hefur einingu rad/s$^2$.

»»»> Stashed changes

## *Dæmi 9.12*

A piece of thin uniform wire of mass m and length 3b is bent into an equilateral triangle. Find the moment of inertia of the wire triangle about an axis perpendicular to the plane of the triangle and passing through one of its vertices.

**Lausn:**

Þetta er jafn arma þríhyrningur þar sem við erum að snúa honum um á sem gengur í gegnum eitt hornið. Armarnir hafa hver um sig lengd $b$ og massa $m/3$. Hverfitregða kerfisins er

$$I = 2I_{S,E} + I_{S,M,H}$$

þar sem

$$I_{S,E} = \frac{1}{3}\left(\frac{m}{3}\right)b^2$$

er hverfitregða stangar með snúningsás á endanum (sjá bls 311 í MP). $I_{S,M,H}$ er hverfitregða stangar í massa miðju + hliðrun (parallel axis theorem + sjá líka verklegt tilraun um hverfitregðu þegar hringnum er hliðrað).

$$I_{S,M,H} = I_{S,M} + \frac{m}{3}\left(b^2 - \left(\frac{b}{2}\right)^2\right)$$

þar sem $I_{S,M}$ er hverfitregða stangar um miðju hennar

$$I_{S,M,H} = \frac{1}{12}\frac{m}{3}b^2 + \frac{m}{3}\left(b^2 - \left(\frac{b}{2}\right)^2\right)$$

þá er heildar hverfitregðan

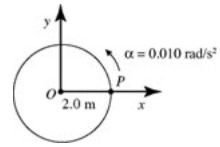
$$I = 2 \cdot I_{S,E} + I_{S,M,H}$$

$$I = 2\frac{1}{3}\left(\frac{m}{3}\right)b^2 + \frac{1}{12}\left(\frac{m}{3}\right)b^2 + \frac{m}{3}\left(b^2 - \left(\frac{b}{2}\right)^2\right)$$

$$I = \frac{1}{2}mb^2$$

## *Dæmi 9.09*

In the figure, point P is at rest when it is on the x-axis. The linear speed of point P when it reaches the y-axis is closest to




**Lausn:**

Jöfnur fyrir línulegan hraða er

$$v^2 = v_0^2 + 2as$$

að sama skapi er fyrir hornhraða

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

Við byrjum á því ð finna hornhraðan

$$\omega = \sqrt{2\alpha\theta}$$

þar sem línulegur hraði er gefinn með

$$v = r \cdot \omega = \sqrt{2\alpha\theta}$$

þar sem $\theta = \pi/2$ og $\alpha = 0.010$ rad/s$^2$, þá fáum við að

$$v = \sqrt{\alpha\pi} = 0.35 \text{ ms}$$

## *Dæmi 9.114*

As an intern with an engineering firm, you are asked to measure the moment of inertia of a large wheel, for rotation about an axis through its center. Since you were a good physics student, you know what to do. You measure the diameter of the wheel to be 0.740 m and find that it weighs 280 N. You mount the wheel, using frictionless bearings, on a horizontal axis through the wheel's center. You wrap a light rope around the wheel and hang a 8.00-kg mass from the free end of the rope. You release the mass from rest; the mass descends and the wheel turns as the rope unwinds. You find that the mass has speed 5.00 m/s after it has descended 2.00 m.

**A. What is the moment of inertia of the wheel for an axis perpendicular to the wheel at its center?**

Notum varðveislu orkunnar til að finna hverfitregða disksins

$$mgy = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

$$I\omega^2 = 2mgy - \frac{1}{2}mv^2$$

$$I = \frac{2mgy}{\omega^2} - m\left(\frac{v}{\omega}\right)^2$$

þar sem $\omega = v/r$

$$I = \frac{2mgy}{(v/(D/2))^2} - m\left(\frac{D}{2}\right)^2 = 0.6221 \text{ kg m}^2$$

**B. Your boss tells you that a larger I is needed. He asks you to design a wheel of the same mass and radius that has I=19.0 kg m$^2$. How do you reply?**

Massi disksins er 280 N/9.8 m/s = 28.6 kg. Hverfitregða fyrir þunnan sívalning er $I = MR^2$

$$I = 28.6 \text{ kg} \cdot (0.37 \text{ m})^2 = 3.92 \text{ kg m}$$

Ef við setjum allan massan á brúninga á disknum þá næst I = 3.92 kg m$^2$, sem er mjög fjarri I = 19 kg m$^2$. Svo svarið ætti að vera: Ekki hægt!

# 7 Vika

## 7.1 Skiladæmi 7

### *Dæmi 10.6*

A metal bar is in the $xy$-plane with one end of the bar at the origin. A force $\mathbf{F} = (6.52\,\mathrm{N})\hat{\imath} + (-3.27\,\mathrm{N})\hat{\jmath}$ is applied to the bar at the point $x = 2.25\,\mathrm{m}$ , $y = 3.17\,\mathrm{m}$.

**A. What is the position vector r for the point where the force is applied?**

Stöðuvigur átakspunktsins er $\mathbf{r} = x\hat{\imath} + y\hat{\jmath}$. Þá er $r_x = x$ og $r_y = y$.

**B. What are the magnitude of the torque with respect to the origin produced by F?**

Kraftvægið um núllpunktinn er $\tau = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ r_x & r_y & 0 \\ F_x & F_y & 0 \end{vmatrix} = (r_x F_y - r_y F_x)\hat{k}$. Því er tölugildi kraftvægisins:

$$\tau = |r_x F_y - r_y F_x| = |(2.25\,\mathrm{m}) \cdot (-3.27\,\mathrm{N}) - (3.17\,\mathrm{m}) \cdot (6.52\,\mathrm{N})| = |-28.0\,\mathrm{N} \cdot \mathrm{m}| = 28.0\,\mathrm{N} \cdot \mathrm{m}$$

Þar sem $\tau = (-28.0\,\mathrm{N} \cdot \mathrm{m})\hat{k}$, þá er stefnir kraftvægið í $-z$-átt.

### *Dæmi 10.10*

A cord is wrapped around the rim of a solid uniform wheel 0.210 m in radius and of mass 7.20 kg. A steady horizontal pull of 48.0 N to the right is exerted on the cord, pulling it off tangentially from the wheel. The wheel is mounted on frictionless bearings on a horizontal axle through its center.

**A. Compute the angular acceleration of the wheel.**

Kraftvægi er gefið með jöfnunni

$$\tau = \alpha I$$

þar sem

$$\tau = Fr \qquad \text{og} \qquad I = \frac{1}{2}mr^2$$

Þá fáum við

$$\alpha = \frac{Fr}{0.5mr^2} = 63.5 \text{ rad/s}^2$$

**B. Compute the acceleration of the part of the cord that has already been pulled off the wheel.**

Finnum hröðunina með

$$a = \alpha r$$

þar sem $\alpha$ er fundið í lið A.

$$a = 13.3 \text{ m/s}^2$$

**C + D. Find the magnitude and direction of the force that the axle exerts on the wheel.**

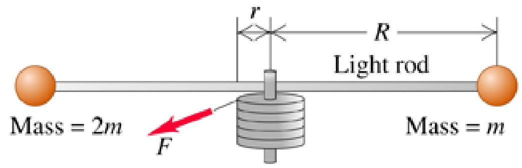
Krafturinn verkar á móti togkraftinum (í $x$-stefnu) og þyngdarkraftinum (-$y$-stefnu).
(Svo kraftvigurinn liggur í 2. fjórðungi $xy$-hnitakerfisins).

$$F_{\text{öxull}} = \sqrt{(-F_x)^2 + (+mg)^2} = 85.3\,\mathrm{N} \quad \text{og} \quad \theta = \tan^{-1}\left(\frac{|mg|}{|-F_x|}\right) = 55.8\,^\circ \text{ yfir -}x\text{-ás}$$

## *Rotating Spheres*

Two massive spheres are mounted on a light rod that can be rotated by a string wrapped around a central cylinder, forming a winch as shown in the figure. A force of magnitude $F$ is applied to the string to turn the system. With respect to the variables given in the figure, the equation for the magnitude of the angular acceleration $\alpha$ is

$$\alpha = \frac{rF}{3mR^2}$$




Assume that the spheres are small enough that they may be considered point masses and that the masses of the rod and cylinder can be neglected.

**A. If the sphere on the left is moved closer to the central cylinder and placed at a distance $R/2$ from the axis of rotation, what is the magnitude of the angular acceleration $\alpha$ of the modified system? Assume that the rest of the system doesn't change.**

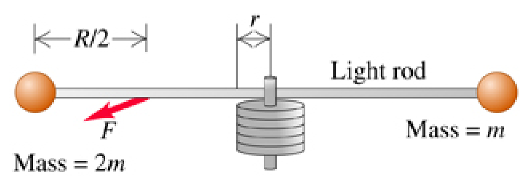
Athugum að einungis hverfitregðan $I$ minnkar. Hornhröðunin fæst út frá skilgreiningunni á kraftvægi $\tau = I\alpha$.

$$\alpha = \frac{\tau}{I} = \frac{Fr}{mR^2 + (2m)(R/2)^2} = \frac{2Fr}{3mR^2}$$

**B. Consider again the original system. Instead of applying the force to the string, a force with the same magnitude $F$ is applied to the rod at a point R/2 from the sphere of mass $2m$ and in a direction perpendicular to the rod, as shown in the figure.**

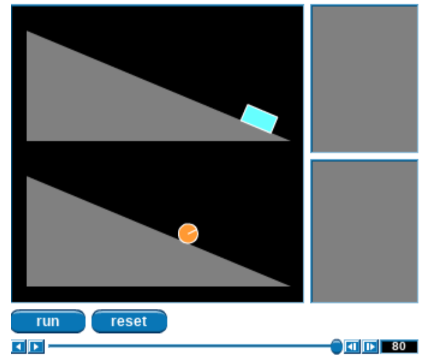
Hér er minnkar bara kraftvægið $\tau$:

$$\alpha = \frac{\tau}{I} = \frac{F\frac{R}{2}}{3mR^2} = \frac{F}{6mR}$$



### *An Unfair Race*

This applet shows the results of releasing a frictionless block and a rolling disk with equal masses from the top of identical inclined planes.




**A. Which of the following is the best explanation of the results shown in the applet?**

Báðir hlutir hafa upphaflega sömu stöðuorku. Þegar diskurinn rúllar af stað, fer hluti af stöðuorkunni í snúningsorku og restin af stöðuorkutapinu verður að línulega hreyfiorku disksins. Kubburinn breytir hins vegar allri stöðuorku sinni í línulega hreyfiorku þar sem hann rúllar ekki neitt.

**B. Assume that the box and disk each have mass $m$, the top of the incline is at height $h$, and the angle between the incline and the ground is $\theta$ (i.e., the incline is at an angle $\theta$ above the horizontal). Also, let the radius of the disk be $R$. How much sooner does the box reach the bottom of the incline than the disk?**

Ákvörðum lokahraða hlutanna. Fyrir kubbinn er $mgh = \frac{1}{2}mv_k^2 \quad \Rightarrow \quad v_k = \sqrt{2gh}$. Fyrir diskinn er $mgh = \frac{1}{2}mv_d^2 \frac{1}{2}\frac{mR^2}{2}\left(\frac{v_d}{R}\right)^2 \quad \Rightarrow \quad v_d = \sqrt{\frac{4}{3}gh}$. Meðalhraðar hlutanna eru $\bar{v}_k = \frac{v_k}{2}$ og $\bar{v}_d = \frac{v_d}{2}$ hvor um sig. Fyrir þekkta meðalhraða getum við notað eftirfarandi færslujöfnur til að finna falltímann:

$$h = \bar{v}_y t \qquad \Rightarrow \qquad t = \frac{h}{\bar{v}_y} = \frac{h}{\frac{1}{2}v\sin\theta}$$

Tímamismunurinn verður þá:

$$\Delta t = t_d - t_k = \frac{h}{\frac{1}{2}\sin\theta}\left(\frac{1}{\bar{v}_d} - \frac{1}{\bar{v}_k}\right) = \frac{h}{\frac{1}{2}\sin\theta}\left(\sqrt{\frac{3}{4gh}} - \sqrt{\frac{1}{2gh}}\right) = \frac{\sqrt{\frac{h}{g}}}{\sin\theta}\left(\sqrt{3} - \sqrt{2}\right)$$

## 7.2 Tímadæmi 7R

### *Class demo: Rods falling, one with mass on top end*

**A. Based on demonstration in class: Show that a uniform rod of length $L$ with mass $m$ falls faster (has greater angular acceleration at all angles, from a vertical rod (standing on end, angle=0) to horizontal (rod has fallen, angle= $\frac{\pi}{2}$ )) than an identical rod but with a point mass $m$ at the top end.**

Til eru tvær leiðir til að leysa þetta dæmi. Annað hvort með því að bera saman hornhraðanir stanganna fyrir gefið horn $\alpha$ eða að bera saman hornhraða stanganna fyrir sama horn.

Hornhröðun: Eingöngu þyngdarkrafturinn veldur kraftvægi um neðstu punkta stanganna. $\tau = |\mathbf{r}_{\text{CM}} \times m\mathbf{g}|$. Nú er:

$$\tau = I\alpha \qquad \Rightarrow \qquad \alpha = \frac{\tau}{I} = \frac{|\mathbf{r}_{\text{CM}} \times m\mathbf{g}|}{I} = \frac{r_{\text{CM}} mg\sin\alpha}{I}$$

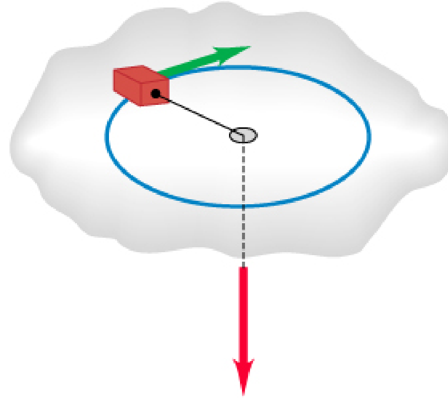
fyrir hvora stöng um sig. Köllum einsleitu stöngina stöng 1 og punktmassastöngina köllum við stöng 2. Finnum hlutfall hornhraðananna:

$$\frac{\alpha_1}{\alpha_2} = \frac{\frac{r_1 mg\sin\alpha}{I_1}}{\frac{r_2 mg\sin\alpha}{I_2}} = \frac{r_1 I_2}{r_2 I_1} = \frac{\frac{L}{2}\cdot mL^2}{L\cdot\frac{1}{3}mL^2} = \frac{3}{2}$$

Fyrir sömu upphafsskilyrði myndi því hornhröðun á einsleitu stönginni alltaf vera meiri en hornhröðun punktmassastangarinnar ($\alpha_1 > \alpha_2$). Því myndi einsleita stöngin falla hraðar niður.

## *Dæmi 10.40*

A block with mass $m$ is revolving with linear speed $v_1$ in a circle of radius $r_1$ on a frictionless horizontal surface (see the figure). The string is slowly pulled from below until the radius of the circle in which the block is revolving is reduced to $r_2$.



### A. Is the angular momenum concerved?

Í þessu dæmi er ekkert heildarkraftvægi sem verkar á kubbinn. Það er sannarlega vinna að færa hann frá einum stað yfir í annan, en hverfiþunginn varðveitist,

$$L_1 = L(r), \quad \Rightarrow \quad mv_1r_1 = mvr.$$

Jafnan nú fyrir togkraft spottans er

### B. What is the new angular speed?

Hverfiþunginn fyrir og eftir er sá sami

$$L_1 = L_2$$

þar sem $L = I\omega$, þá fæst

$$I_1\omega_1 = I_2\omega_2$$

þar sem $I = mr^2$

$$mr_1^2\omega_1 = mr_2^2\omega_2$$

$$\omega_2 = \frac{r_1^2}{r_2^2}\omega_2$$

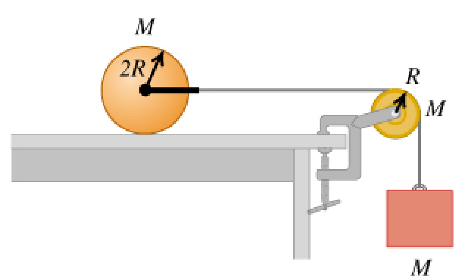
### C-D.Find the change in kinetic energy of the block.

Breytingin í skriðorkunni hlýtur að samsvara vinnunni sem fór í að breyta stöðu kassans úr $r_1$ í $r_2$.
Reiknum heildið beint af augum, athugið að togkrafturinn $\mathbf{T}$ er samsíða $\mathbf{r}$ en í öfuga stefnu (bentir í miðjuna) svo það bætist við einn mínus (munið að innfeldi er háð horninu $\cos(\theta)$ sem gefur mínus ef þeir eru samsíða en í öfuga stefnu),

$$W = -m(v_1r_1)^2 \int_{r_1}^{r_2} \frac{1}{r^3}\, dr = m\frac{v_1^2}{2}\left(\frac{r_1^2}{r_2^2} - 1\right)$$

## *Dæmi 10.75*

A uniform, solid cylinder with mass $M$ and radius $2R$ rests on a horizontal tabletop. A string is attached by a yoke to a frictionless axle through the center of the cylinder so that the cylinder can rotate about the axle. The string runs over a disk-shaped pulley with mass $M$ and radius $R$ that is mounted on a frictionless axle through its center. A block of mass $M$ is suspended from the free end of the string. The string doesn't slip over the pulley surface, and the cylinder rolls without slipping on the tabletop.




### A. Find the magnitude of the acceleration of the block after the system is released from rest.

Skoðum spennu spottans frá sívalingnum til trissunnar ($T_1$) og frá trissunni í lóðið ($T_2$). Þar sem sívalingurinn dregst ekki eftir borðinu hlýtur að verkar á hann kraftur eftir fletinum í öfuga stefnu sem samsvarar mótstöðu flatarins ($F_f$), þá er spennan

$$T_1 = Ma + F_f = Ma + I_s\frac{a}{4R^2}, \qquad T_2 = M(g-a)$$

þar sem $I_s$ er hverfitregða sívalingsins. Hér höfum við nýtt okkur að kraftvægið á sívalinginn er $\tau_s = 2RF_f = I\alpha = Ia/(2R)$.

Nú getum við skoðað trissuna sem uppfyllir sína eigin kraftvægisjöfnu

$$\tau_t = I_t \frac{a}{R} = T_2 R - T_1 R$$

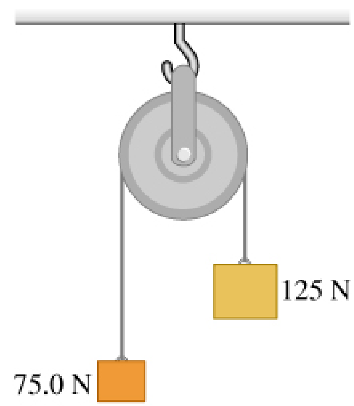
Notfærum okkur að trissan og sívalingurinn hafa sama form á hverfitregðu, þ.e. $I_s = M(2R)^2/2$ og $I_t = MR^2/2$. Stingum þessu öllu inn til að fá

$$a\left(\frac{I_t}{R} + MR + \frac{I_s}{4R^2}R + MR\right) = MgR$$

sem gefur okkur að $a = g/3$

## 7.3 Tímadæmi 7V

### *Dæmi 10.65*




**A. Two weights are connected by a very light flexible cord that passes over a 70.0 N frictionless pulley of radius 0.400 m . The pulley is a solid uniform disk and is supported by a hook connected to the ceiling (See the figure below ).**

Við höfum kynnst áður dæmum þar sem trissan er föst og hefur enga hverfitregðu. Í þeim tilfellum er spenna spottans sú sama allstaðar. Hér hinsvegar þarf að taka tillit til þess að kraftur fer í að snúa trissunni þegar hún hefur hverfitregðu og spenna spottans er mismunandi milli kassa $A$ (vinstra megin) og trissu $C$; eða trissu $C$ og kassa $B$ (hægra megin). Veljum jákvæða krafta sem valda réttsælis snúningi á trissu $C$.

$$T_A = m_A g + m_A a, \qquad T_B = m_B g - m_B a$$

Hér höfum við þrjár óþekktar stærðir $T_A$, $T_B$ og $a$.
Hverfitregða trissunar er gefin með

$$I = \frac{1}{2} m R^2$$

og ef kassi $A$ fær hröðun $a$, þá munt rissan snúast með hornhröðun $\alpha = \frac{a}{R}$. Þá getum við skrifað kraftvægi trissunar sem

$$\tau = I\alpha = I\frac{a}{r} = T_B R - T_A R$$

Nú getum við stungið inn fyrir $T_A$ og $T_B$:

$$a = \frac{(T_B - T_a)R^2}{\frac{1}{2} m_c R^2} = \frac{(m_B - m_A)g - (m_A + m_B)a}{\frac{1}{2} m_C}$$

Einangrum $a$:

$$a = \frac{w_B - w_A}{\frac{w_C}{2} + w_A + w_B} \cdot g$$

þar sem við höfum skrifað $w_i = m_i g$.
Nú getum við tekið saman heildarkraftinn sem verkar á krókinn

$$F = T_A + T_B + w_C$$

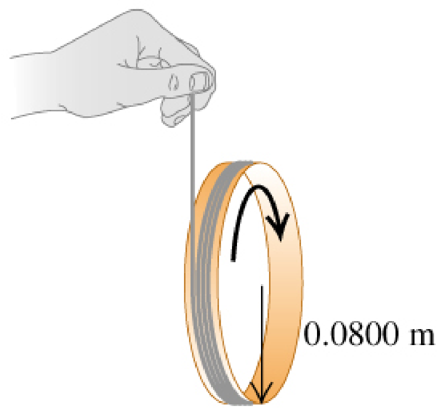
$$F = m_A g + m_A \left(\frac{w_B - w_A}{\frac{w_C}{2} + w_A + w_B} \cdot g\right) + m_B g - m_B \left(\frac{w_B - w_A}{\frac{w_C}{2} + w_A + w_B} \cdot g\right) + 70 \text{ N} = 259 \text{ N}$$

# 8 Vika

## 8.1 Skiladæmi 8

### *Dæmi 10.20*

A string is wrapped several times around the rim of a small hoop with radius 8.00 cm and mass 0.180 kg . The free end of the string is held in place and the hoop is released from rest (the figure ). After the hoop has descended 45.0 cm , calculate




#### A. the angular speed of the rotating hoop

Hornhraðinn fæst út frá tímaóháðu hreyfijöfnunni $\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$. Skoðum kraftvægið um neðri enda bandsins á lykkjunni: $\tau = mgR = I\alpha$. Hverfitregða lykkjunnar um miðju hennar er $I = mR^2$. Það gefur okkur að hornhröðun lykkjunnar er $\alpha = \frac{g}{R}$. Hornafærsla lykkjunnar við það að falla hæðina $\Delta y$ er $\Delta\theta = \frac{\Delta y}{2R}$ Þar sem lykkjan er upphaflega kyrr, verður lokahornhraðinn:

$$\omega = \sqrt{2\frac{g}{R}\frac{\Delta y}{2R}} = \frac{\sqrt{g\Delta y}}{R} = \frac{\sqrt{(9.8\,\mathrm{m/s^2})\cdot(0.450\,\mathrm{m})}}{0.08\,\mathrm{m}} = 26.3\,\mathrm{rad/s}$$

#### B. and the speed of its center.

Hraði miðjunnar fæst út frá $v = \omega R = \sqrt{g\Delta y} = \sqrt{(9.8\,\mathrm{m/s^2})\cdot(0.450\,\mathrm{m})} = 2.10\,\mathrm{m/s}$

### *Dæmi 10.44*

A solid wood door 1.00 m wide and 2.00 m high is hinged along one side and has a total mass of 45.0 kg . Initially open and at rest, the door is struck at its center by a handful of sticky mud with mass $0.700kg$, traveling perpendicular to the door at 14.0 m/s just before impact.

#### A. Find the final angular speed of the door.

Hverfiþungi kerfisins varðveitist, $mv_m\frac{x}{2} = I_{\text{heild}}\omega$, þar sem $m$ er massi moldarklessunnar, $v_m$ er upphaflegur hraði klessunnar og $x$ er breidd hurðarinnar. Ákvörðum hornhraðann eftir árekstur, $\omega$:

$$\omega = \frac{mv_m\frac{x}{2}}{I_{\text{heild}}} = \frac{mv_m\frac{x}{2}}{m\left(\frac{x}{2}\right)^2 + \frac{1}{3}Mx^2}$$

þar sem $M$ er massi hurðarinnar og er hverfitregða hurðarinnar um snúningsás sú sama og fyrir stöng með massa $M$ og lengd $x$. Hornhraðinn er:

$$\omega = \frac{(0.700\,\mathrm{kg})\cdot(14.0\,\mathrm{m/s})\cdot(0.500\,\mathrm{m})}{(0.700\,\mathrm{kg})\cdot(0.250\,\mathrm{m^2}) + \frac{1}{3}(45.0\,\mathrm{kg})\cdot(1.00\,\mathrm{m^2})} = 0.323\,\mathrm{rad/s}$$

#### B. Does the mud make a significant contribution to the moment of inertia?

Berum saman hverfitregðurnar tvær úr lausninni á undan:

$$\frac{I_{\text{klessa}}}{I_{\text{hurð}}} = \frac{m\frac{x^2}{4}}{\frac{1}{3}Mx^2} = \frac{m}{12M} = \frac{0.700\,\mathrm{kg}}{12\cdot 45.0\,\mathrm{kg}} = 1.30\cdot 10^{-3}$$

Við sjáum að hverfitregða klessunnar er umtalsvert minni en hverfitregða hurðarinnar fyrst að hlutfallið hér að ofan er svona lítið.

## *Dæmi 10.74*

A uniform rod of mass $3.20 \cdot 10^{-2}$ kg and length 0.370 m rotates in a horizontal plane about a fixed axis through its center and perpendicular to the rod. Two small rings, each with mass 0.200 kg , are mounted so that they can slide along the rod. They are initially held by catches at positions a distance $4.60 \cdot 10^{-2}$ m on each side from the center of the rod, and the system is rotating at an angular velocity $27.0\,{}^{\text{rev}}\!/\!_{\text{min}}$ . Without otherwise changing the system, the catches are released, and the rings slide outward along the rod and fly off at the ends.

**A. What is the angular speed of the system at the instant when the rings reach the ends of the rod?**

Hverfiþungi kerfisins varðveitist ($I\omega$ er fasti):

$$\omega_2 = \frac{I_1}{I_2}\omega_1 = \frac{\frac{1}{12}ML^2 + 2ml^2}{\frac{1}{12}ML^2 + 2m\left(\frac{L}{2}\right)^2}\omega_1$$

þar sem $M$ er massi stangarinnar, $L$ er lengd hennar og $l$ er fjarlægð stanganna frá snúningsás í upphafi.

$$\omega_2 = 2.33\,{}^{\text{rev}}\!/\!_{\text{min}}$$

**B. What is the angular speed of the rod after the rings leave it?**

Þótt að hringirnir fjarlægjast stöngina enn frekar, þá beita þeir ekki neinu "ytra"kraftvægi á stöngina. Því helst hverfiþunginn fasti á hverjum hlut fyrir sig. Stöngin hefur því ennþá sama hornhraða og þegar hringirnir runnu af henni.

$$\omega_2 = 2.33\,{}^{\text{rev}}\!/\!_{\text{min}}$$

## *Dæmi 10.80*

A large turntable with radius 6.00 m rotates about a fixed vertical axis, making one revolution in 8.00 s. The moment of inertia of the turntable about this axis is $1200\,\text{kg}\cdot\text{m}^2$. You stand, barefooted, at the rim of the turntable and very slowly walk toward the center, along a radial line painted on the surface of the turntable. Your mass is 79.0 kg . Since the radius of the turntable is large, it is a good approximation to treat yourself as a point mass. Assume that you can maintain your balance by adjusting the positions of your feet. You find that you can reach a point 3.00 m from the center of the turntable before your feet begin to slip.

**A. What is the coefficient of static friction between the bottoms of your feet and the surface of the turntable?**

Hér er $\mu$ kyrrstöðunúningsstuðull sem uppfyllir $mu \leq \mu_{\text{max}}$. Einangrum $\mu$:

$$\mu = \frac{r\omega^2}{g}$$

Hverfiþungi kerfisins varðveitist. Upphaflega er hverfiþungin $L = (I + mR^2)\frac{2\pi}{T}$. Nú er $L = (I + mr^2)\omega$ fasti, svo $\omega = \frac{(I+mR^2)\frac{2\pi}{T}}{I+mr^2}$. Þá fæst að lokum:

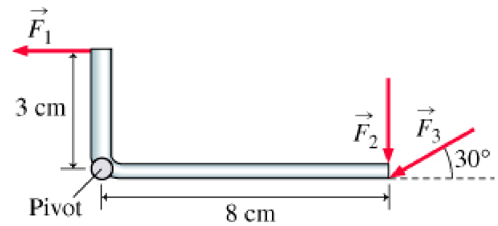
$$\mu_{\text{max}} = \frac{r\left(\frac{(I+mR^2)\frac{2\pi}{T}}{I+mr^2}\right)^2}{g}$$

Nú er $I = 1200\,\text{kg}\cdot\text{m}^2$, $m = 79.0\,\text{kg}$ , $R = 6.00\,\text{m}$, $r = 3.00\,\text{m}$ og $T = 8.00\,\text{s}$. Þá fæst núningsstuðullinn:

$$\mu = 0.846$$

## *Moments around a Rod*

A rod is bent into an L shape and attached at one point to a pivot. The rod sits on a frictionless table and the diagram is a view from above. This means that gravity can be ignored for this problem. There are three forces that are applied to the rod at different points and angles: $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$. Note that the dimensions of the bent rod are in centimeters in the figure, although the answers are requested in SI units (kilograms, meters, seconds).




**A. If $F_3 = 0$ and $F_1 = 12\,\mathrm{N}$, what does the magnitude of $\mathbf{F}_2$ have to be for there to be rotational equilibrium?**

Kraftvægin yrðu jöfn, en andstæð um snúningspunktinn:

$$F_1 x_1 = F_2 x_2 \qquad F_2 = \frac{x_1}{x_2} F_1 = \frac{3}{8} \cdot 12\,\mathrm{N} = 4.5\,\mathrm{N}$$

**B. If the L-shaped rod has a moment of inertia $I = 9\,\mathrm{kg} \cdot \mathrm{m}^2$, $F_1 = 12\,\mathrm{N}$, $F_2 = 27\,\mathrm{N}$, and again $F_3 = 0$, how long a time $t$ would it take for the object to move through $45^\circ$ ($\frac{\pi}{4}$ radians)? Assume that as the object starts to move, each force moves with the object so as to retain its initial angle relative to the object.**

Reiknum hornhröðunina út frá kraftvæginu um snúningsásinn:

$$|\alpha| = \frac{|\tau|}{I} = \frac{|F_1 x_1 - F_2 x_2|}{I}$$

Notum síðan tímaháðu hornfærslu jöfnuna (fyrir $\omega_0 = 0$):

$$|\Delta\theta| = \frac{1}{2}|\alpha| t^2 \qquad \Rightarrow \qquad t = \sqrt{\frac{2|\Delta\theta|}{|\alpha|}} = \sqrt{\frac{2 \cdot |\Delta\theta| \cdot I}{|F_1 x_1 - F_2 x_2|}} = \sqrt{\frac{2 \cdot \frac{\pi}{4} \cdot (9\,\mathrm{kg} \cdot \mathrm{m}^2)}{|(12\,\mathrm{N}) \cdot (0.03\,\mathrm{m}) - (27\,\mathrm{m}) \cdot (0.08\,\mathrm{m})|}} = 2.8\,\mathrm{s}$$

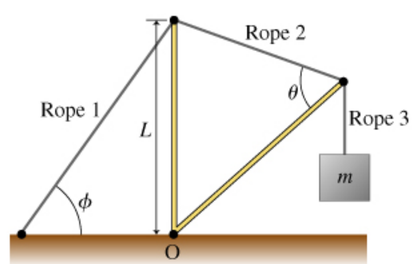
**C. Now consider the situation in which $F_1 = 12\,\mathrm{N}$ and $F_2 = 0$, but now a force with nonzero magnitude $F_3$ is acting on the rod. What does $F_3$ have to be to obtain equilibrium?**

Eingöngu þátturinn $F_3 \sin(30^\circ)$ veldur kraftvægi sem kemur kerfinu í snúningsjafnvægi.

$$F_3 = \frac{x_1}{x_3 \sin(30^\circ)} F_1 = \frac{3}{8 \cdot \frac{1}{2}} \cdot 12\,\mathrm{N} = 9.0\,\mathrm{N}$$

## *Quarry Crane*

A quarry crane is used to lift massive rocks from a quarry pit. Consider the simplified model of such a crane shown in the figure. The ends of two poles are anchored to the ground at the same point (point O). From this point, one pole rises vertically and the second pole rises at an angle. The vertical pole has its free end connected to the ground via an unstretchable, massless rope labeled rope 1. A second rope, labeled rope 2, connects the free ends of the two poles. The angle between the tilted pole and rope 2 is $\theta$. Both poles have length $L$ and can be considered massless for the purposes of this problem. Hanging from the end of the second pole, via rope 3, is a granite block of mass $m$. Throughout this problem use $g$ for the magnitude of the acceleration due to gravity.




**A. Find $T_3$, the tension in rope 3.**

Athugum að kassinn er í kraftajafnvægi ($ma = T_3 - mg = 0$), svo togkrafturinn í reipinu sem heldur kassanum kyrrum er $T_3 = mg$.

**B. Find $T_2$, the tension in rope 2.**

Hornið milli stanganna er $\pi - 2\theta$, því þær mynda jafnarma þríhyrning með bandi númer 2. Því myndar band 3 hornið $\pi - 2\theta$ við hægri stöngina. Hægri stöngin er í snúningsjafnvægi, svo ekkert ytra kraftvægi verkar neins staðar á hana. Skoðum kraftvægið um neðri enda stangarinnar:

$$\tau_{\text{hægri stöng}} = \sum_i F_i r_i \sin(\theta_i) = T_3 L \sin(\pi - 2\theta) - T_2 L \sin(\theta) = 0 \qquad \Rightarrow \qquad T_2 = T_3 \frac{\sin(2\theta)}{\sin(\theta)} = 2mg\cos\theta$$

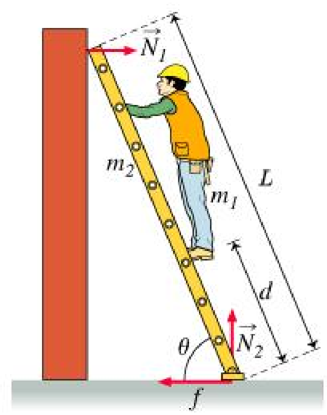
þar sem við höfum nýtt okkur að $\sin(\pi - 2\theta) = \sin(2\theta) = 2\cos(\theta)\sin(\theta)$.

**C. Find $T_1$, the tension in rope 1.**

Vinstri stöngin er líka í snúningsjafnvægi. Skoðum kraftvægið um neðri enda hennar:

$$\tau_{\text{vinstri stöng}} = T_2 L \sin(\theta) - T_1 L \sin(\frac{\pi}{2} - \phi) = 0 \qquad \Rightarrow \qquad T_1 = T_2 \frac{\sin(\theta)}{\sin(\frac{\pi}{2} - \phi)} = mg\frac{\sin(2\theta)}{\cos(\phi)}$$

## *A Person Standing on a Leaning Ladder*




A uniform ladder with mass $m_2$ and length $L$ rests against a smooth wall. A do-it-yourself enthusiast of mass $m_1$ stands on the ladder a distance $d$ from the bottom (measured along the ladder). The ladder makes an angle $\theta$ with the ground. There is no friction between the wall and the ladder, but there is a frictional force of magnitude $f$ between the floor and the ladder. $N_1$ is the magnitude of the normal force exerted by the wall on the ladder, and $N_2$ is the magnitude of the normal force exerted by the ground on the ladder. Throughout the problem, consider counterclockwise torques to be positive. None of your answers should involve $\pi$ (i.e., simplify your trig functions).

**A. What is the minimum coeffecient of static friction $\mu_{\text{min}}$ required between the ladder and the ground so that the ladder does not slip?**

Til þess að stiginn og maðurinn eru í kraftajafnvægi þarf $N_1 = f$ og $N_2 = (m_1 + m_2)g$. Nú er núningskrafturinn á neðri enda stigans $f = \mu_{\text{min}} N_2 = \mu_{\text{min}}(m_1 + m_2)g$, svo við viljum finna núningsstuðulinn út frá $\mu_{\text{min}} = \frac{N_1}{(m_1+m_2)g}$. Kerfið er einnig í snúningsjafnvægi (ekkert ytra kraftvægi verkar neins staðar á kerfið). Skoðum kraftvægið um neðri enda stigans.

$$-N_1 L \sin\theta + (m_2 \frac{L}{2} + m_1 d) g \cos\theta = 0 \qquad \Rightarrow \qquad N_1 = \frac{(m_2 \frac{L}{2} + m_1 d) g \cos\theta}{L \sin\theta}$$

Þá fæst að lokum að:

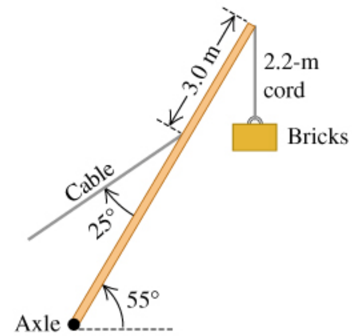
$$\mu_{\text{min}} = \frac{(m_2 \frac{L}{2} + m_1 d)\cos\theta}{(m_1 + m_2) L \sin\theta}$$

**B. Suppose that the actual coefficent of friction is one and a half times as large as the value of $\mu_{\text{min}}$. That is, $\mu_s = (3/2)\mu_{\text{min}}$. Under these circumstances, what is the magnitude of the force of friction $f$ that the floor applies to the ladder?**

Þótt að núningsstuðulinn er stærri en $\mu_{\text{min}}$, þá er núningskraftur alltaf í lágmarki (svo lengi sem að stiginn haldis kyrr). Því verkar núningskrafturinn $f = \mu_{\text{min}}(m_1 + m_2)g = \frac{(m_2 \frac{L}{2} + m_1 d) g \cos\theta}{L \sin\theta}$ á stigann.
Hér hefði alveg eins mátt reikna núningskraftinn með því að reikna kraftvægið um neðri enda stigans, fá að $N_1 = (\frac{m_2}{2} + m_1 \frac{d}{L})\cot\theta$ og notfært okkur síðan að $f = N_1$.

### *Exercise 11.18*

A 15000 N crane pivots around a friction-free axle at its base and is supported by a cable making a 25° angle with the crane (the figure ). The crane is 16 m long and is not uniform, its center of gravity being 7.0 m from the axle as measured along the crane. The cable is attached 3.0 m from the upper end of the crane.




**A. When the crane is raised to 55° above the horizontal holding an 11000 N pallet of bricks by a 2.2 m very light cord, find the tension in the cable.**

Reiknum kraftvægið um öxulinn:

$$Tr_T\sin\theta_T - w_m r_m\cos\theta_m - w_M r_M\cos\theta = 0 \qquad \Rightarrow \qquad T = \frac{(w_m r_m + w_M r_M)\cos\theta_m}{r_T\sin\theta_T}$$

Nú er $w_m = 11000\,\mathrm{N}$, $w_M = 15000\,\mathrm{N}$, $r_m = 16\,\mathrm{m}$, $r_M = 7.0\,\mathrm{m}$, $r_T = 13\,\mathrm{m}$, $\theta_m = 55°$ og $\theta_T = 25°$. Þá fæst að togkrafturinn í kaplinum er:

$$T = 2.9\cdot 10^4\,\mathrm{N}$$

**B. Find the horizontal component of the force that the axle exerts on the crane.**

Lárétti þáttur kraftsins frá öxlinum þarf að stytta út lárétta þátt togkraftsins í kaplinum.

$$F_H = T\sin(\frac{\pi}{2} + \theta_m - \theta_T) = T\cos(\theta_m - \theta_T) = 2.5\cdot 10^4\,\mathrm{N}$$

**C. Find the vertical component of the force that the axle exerts on the crane.**

Þessi lóðrétti þáttur styttir út lóðréttan þátt togkraftsins í kaplinum og styttir einnig út þyngarkraftinn á kranann $M$ og hlassið $m$:

$$F_V = T\sin(\theta_m - \theta_T) + w_m + w_M = 4.1\cdot 10^4\,\mathrm{N}$$

### *A Wire under Stress*

A steel wire of length 1.98 m with circular cross section must stretch no more than 0.260 cm when a tensile (stretching) force of 450 N is applied to each end of the wire.

**A. What minimum diameter $d_{\min}$ is required for the wire?**

Young-stuðullinn er $Y = \frac{F_\perp/A}{\Delta L/L}$, með þversniðsflatarmálið $A = \pi\left(\frac{d}{2}\right)^2$.

$$Y = \frac{4F_\perp L}{\pi\Delta L d^2} \qquad \Rightarrow \qquad d = \sqrt{\frac{4F_\perp L}{\pi\Delta L Y}} = 0.00148\,\mathrm{m} = 1.48\,\mathrm{mm}$$

## 8.2 Tímadæmi 8R

### *Dæmi 10.85*

A 510.0 g bird is flying horizontally at 2.30 m/s , not paying much attention, when it suddenly flies into a stationary vertical bar, hitting it 25.0 cm below the top (the figure). The bar is uniform, 0.760 m long, has a mass of 1.90 kg , and is hinged at its base. The collision stuns the bird so that it just drops to the ground afterward (but soon recovers to fly happily away).

**A. What is the angular velocity of the bar just after it is hit by the bird?**

Hverfiþungi fuglsins breytist í hverfitregðu stangarinnar við áreksturinn (vegna varðveislu).

$$L_{\text{fyrir}} = m_{\text{fugl}}v_{\text{fugl}}(l_{\text{stöng}} - h) = L_{\text{eftir}} = I_{\text{stöng}}\omega_0 \qquad \Rightarrow \qquad \omega_0 = \frac{m_{\text{fugl}}v_{\text{fugl}}(l_{\text{stöng}} - h)}{\frac{1}{3}m_{\text{stöng}}l^2_{\text{stöng}}} = 1.64\,\mathrm{rad/s}$$

**B. What is the angular velocity of the bar just as it reaches the ground?**

Stöðuorkutap stangarinnar veldur aukinni snúningsorku:

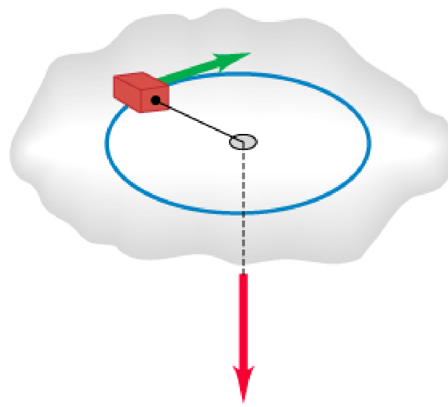
$$\frac{1}{2}I_{\text{stöng}}\omega^2 = \frac{1}{2}I_{\text{stöng}}\omega_0^2 + m_{\text{stöng}}g\frac{l_{\text{stöng}}}{2}$$

Lokahornhraðinn verður því:

$$\omega = \sqrt{\omega_0^2 + \frac{m_{\text{stöng}}g\frac{l_{\text{stöng}}}{2}}{\frac{1}{2}\cdot\frac{1}{3}m_{\text{stöng}}l_{\text{stöng}}^2}} = \sqrt{\omega_0^2 + \frac{3g}{l_{\text{stöng}}}} = 6.43\,\text{rad/s}$$

## *Dæmi 10.91*

A block with mass $m$ is revolving with linear speed $v_1$ in a circle of radius $r_1$ on a frictionless horizontal surface (see the figure ). The string is slowly pulled from below until the radius of the circle in which the block is revolving is reduced to $r_2$.



**A. Calculate the tension $T$ in the string as a function of $r$, the distance of the block from the hole.**

Kubburinn verður fyrir miðsóknarkraftinum $T = m\frac{v^2}{r}$. Nú er $v^2 = \frac{l^2}{m^2r^2}$, þar sem $l$ er hverfiþungi kerfisins. Togkrafturinn veldur engu kraftvægi, svo við getum sagt að $l$ sé fasti.
Veljum $l = mv_1r_1 \quad \Rightarrow \quad l^2 = m^2v_1^2r_1^2$. Þá er togkrafturinn:

$$T = m\frac{\frac{l^2}{m^2r^2}}{r} = \frac{mv_1^2r_1^2}{r^3}$$

**B. Use $W = \int_{r_1}^{r_2}\mathbf{T}\cdot\mathbf{r}$, to calculate the work done by T when $r$ changes from $r_1$ to $r_2$.**

Þar sem togkrafturinn verkar inn að miðju hringhreyfingarinnar, verður innfeldið $\mathbf{T}\cdot d\mathbf{r} = -|T\cdot dr|$, (andsamsíða vigrar). Framkvæmum heildið:

$$W = -mv_1^2r_1^2\int_{r_1}^{r_2}\frac{dr}{r^3} = \frac{mv_1^2r_1^2}{2}\left(\frac{1}{r_2^2} - \frac{1}{r_1^2}\right)$$

**C. Find the change in the kinetic energy of the block, when $r$ changes from $r_1$ to $r_2$.**

Eins og við tókum fram í a-lið, þá er hverfiþunginn $l$ fasti. $mv_1r_1 = mv_2r_2 \quad \Rightarrow \quad v_2 = \frac{r_1}{r_2}v_1$. Reiknum breytinguna á hreyfiorkunni:

$$\Delta K = \frac{1}{2}m(v_2^2 - v_1^2) = \frac{1}{2}mv_1^2\left(\frac{r_1^2}{r_2^2} - 1\right) = \frac{mv_1^2r_1^2}{2}\left(\frac{1}{r_2^2} - \frac{1}{r_1^2}\right)$$

**D. Compare the results of part (b) to the result of part (c).**

b- og c-liður gefa að $W = \Delta K = \frac{mv_1^2r_1^2}{2}\left(\frac{1}{r_2^2} - \frac{1}{r_1^2}\right)$

## 8.3 Tímadæmi 8V

### *Dæmi 10.77*

A 5.10 kg ball is dropped from a height of 12.0 m above one end of a uniform bar that pivots at its center. The bar has mass 5.50 kg and is 6.80 m in length. At the other end of the bar sits another 4.30 kg ball, unattached to the bar. The dropped ball sticks to the bar after the collision.

**A. How high will the other ball go after the collision?**

Fyrri boltinn ($m_1$) nær hraðanum $v_1 = \sqrt{2gy_1}$ (vegna stöðuorkutaps) og hefur hverfiþungann $L_1 = m_1\frac{l}{2}v_1$ um snúningsás stangarinnar. Í þessum árekstri varðveitist heildarhverfiþunginn (stangarinnar ($M$) og massanna tveggja ($m_1$ og $m_2$)):

$$L = I\omega = \left(\frac{M}{12}l^2 + (m_1+m_2)\left(\frac{l}{2}\right)^2\right)\omega$$

Við getum sagt að á því augnabliki þegar að áreksturinn á sér stað, þá séu hlutirnir þrír límdir saman. Eftir áreksturinn hafa því allir hlutirnir sama hornhraðann $\omega$ umhverfis snúningsás stangarinnar.
Seinni boltinn skoppar því upp með hraðanum $v_2 = \omega\frac{l}{2}$.
Þegar að við tökum þessar stærðir saman fæst að:

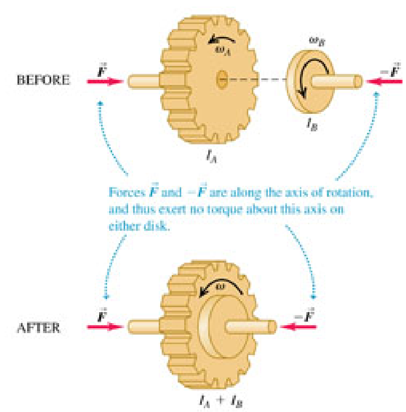
$$v_2 = \frac{m_1\frac{l}{2}v_1\frac{l}{2}}{\left(\frac{M}{12}l^2 + (m_1+m_2)\left(\frac{l}{2}\right)^2\right)} = \frac{m_1}{\left(\frac{M}{3} + (m_1+m_2)\right)}v_1$$

Þá fæst lokahæð seinni boltans með:

$$y_2 = v_2^2 \cdot \frac{1}{2g} = v_2^2 \cdot \frac{y_1}{v_1^2} = \frac{m_1^2}{\left(\frac{M}{3} + (m_1+m_2)\right)^2}y_1 = 2.47\,\mathrm{m}$$

### *Dæmi 10.81*

In your job as a mechanical engineer you are designing a flywheel and clutch-plate system like the one in the figure. Disk A is made of a lighter material than disk B, and the moment of inertia of disk A about the shaft is one-third that of disk B. The moment of inertia of the shaft is negligible. With the clutch disconnected, A is brought up to an angular speed $\omega_0$; B is initially at rest. The accelerating torque is then removed from A, and A is coupled to B. (Ignore bearing friction.) The design specifications allow for a maximum of 2800 J of thermal energy to be developed when the connection is made.




**A. What can be the maximum value of the original kinetic energy of disk A so as not to exceed the maximum allowed value of the thermal energy?**

Hverfitregða kerfisins varðveitist (en hluti af hreyfiorkunnar tapast sem varmi $Q$).

$$I_A\omega_0 = (I_A + I_B)\omega \qquad \Rightarrow \omega = \frac{I_A}{I_A+I_B}\omega_0$$

Skoðum orkuvarðveisluna:

$$\frac{1}{2}I_A\omega_0^2 = \frac{1}{2}(I_A+I_B)\omega^2 + Q \qquad \Rightarrow \qquad \frac{1}{2}I_A\omega_0^2(1 - \frac{I_A}{I_A+I_B}) = Q$$

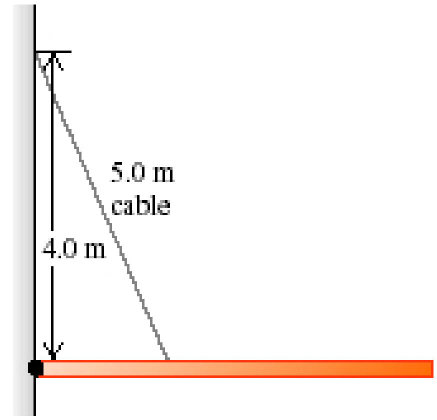
Hreyfiorka disks A uppfyllir því:

$$\frac{1}{2}I_A\omega_0^2 \le \frac{Q_{\max}}{(1-\frac{I_A}{I_A+I_B})} = \frac{I_A+I_B}{I_B}Q_{\max} = \frac{4}{3}Q_{\max}$$

því $I_B = 3I_A$. Þá er $\frac{1}{2}I_A\omega_0^2 \le \frac{4}{3}\cdot 2800\,\mathrm{J} = 3700\,\mathrm{J}$.

## *Dæmi 11.17*

A 9.00 m uniform beam is hinged to a vertical wall and held horizontally by a 5.00 m cable attached to the wall 4.00 m above the hinge, as shown in the figure below . The metal of this cable has a test strength of 1.10 kN , which means that it will break if the tension in it exceeds that amount.




**A. What is the heaviest beam that the cable can support with the given configuration?**

Stöngin er í fullkomnu jafnvægi (bæði krafta- og snúningsjafnvægi). Táknum stefnuhorn togkraftsins $T$ í bandinu með $\alpha$ og stefnuhorn kraftsins frá veggnum $F$ með $\theta$:

$$\begin{aligned}
F_{\text{heild},x} = 0 \quad &\Rightarrow \quad F\cos\theta - T\cos\alpha &= 0\\
F_{\text{heild},y} = 0 \quad &\Rightarrow \quad F\sin\theta + T\sin\alpha - mg &= 0\\
\tau = 0 \quad &\Rightarrow \quad F\sin\theta\cdot 0 + T\sin\alpha\cdot(3.0\,\mathrm{m}) - mg\cdot(4.5\,\mathrm{m}) &= 0
\end{aligned}$$

Notum síðustu jöfnuna til að ákvarða $m$:

$$m = \frac{(1100\,\mathrm{N})\cdot\frac{4}{5}\cdot(3\,\mathrm{m})}{(9.8\,\mathrm{m/s^2})\cdot(4.5\,\mathrm{m})} = 59.9\,\mathrm{kg}$$

þar sem $\sin\alpha = \frac{4}{5}$.

**B. Find the horizontal component of the force the hinge exerts on the beam.**

Notum jöfnuna fyrir láréttu kraftaþættina:

$$F_x = F\cos\theta = T\cos\alpha = (1100\,\mathrm{N})\cdot\frac{3}{5} = 660\,\mathrm{N}$$

þar sem $\cos\alpha = \frac{3}{5}$.

**C. Find the vertical component of the force the hinge exerts on the beam.**

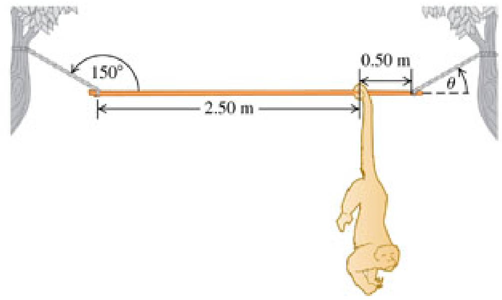
Notum jöfnuna fyrir lóðréttu kraftaþættina:

$$F_y = F\sin\theta = mg - T\sin\alpha = (59.9\,\mathrm{kg})\cdot(9.8\,\mathrm{m/s^2}) - (1100\,\mathrm{N})\cdot\frac{4}{5} = -293\,\mathrm{N}$$

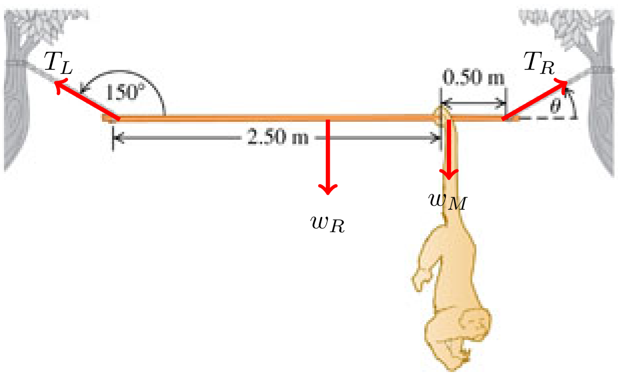
Svo $F_x$ vísar til hægri, og $F_y$ vísar niður á við.

## *Dæmi 11.19*

A 3.00 m long, 180 N, uniform rod at the zoo is held in a horizontal position by two ropes at its ends. The left rope makes an angle of 150 ° with the rod and the right rope makes an angle $\theta$ with the horizontal. A 90 N howler monkey (Alouatta seniculus) hangs motionless 0.50 m from the right end of the rod as he carefully studies you.




**A. Make a free-body diagram of the rod.**




**B. Calculate the tension in the left rope.**

Kraftvægi stangarinnar er núll (um hvaða punkt sem er). Miðum við hægri enda stangarinnar.

$$T_L \sin(150\,°)\cdot(3.00\,\mathrm{m}) - w_R\cdot(1.50\,\mathrm{m}) - w_M\cdot(0.5\,\mathrm{m}) = 0 \qquad \Rightarrow \qquad T_L = \frac{(180\,\mathrm{N})\cdot(1.50\,\mathrm{m}) + (90\,\mathrm{N})\cdot(0.5\,\mathrm{m})}{\sin(150\,°)\cdot(3.00\,\mathrm{m})} = 210\,\mathrm{N}$$

**C. Calculate the tension in the right rope.**

Heildarkraftinn á stöngina er núll:

$$T_R = \sqrt{T_{R;x}^2 + T_{R;y}^2} = \sqrt{(-T_L\cos(150\,°))^2 + (W_R + W_M - T_L\sin(150\,°))^2} = 246\,\mathrm{N}$$

**D. Calculate the angle $\theta$.**

Hér væri hægt að skoða $x$- og $y$-þætti $T_R$ með $\theta = \tan^{-1}\left(\frac{W_R+W_M-T_L\sin(150\,°)}{-T_L\cos(150\,°)}\right) = 42\,°$.
Hins vegar mætti alveg eins endurskoða kraftvægið með því að velja vinstri endann sem viðmiðspunktinn:

$$\tau = W_R\cdot(1.50\,\mathrm{m}) + W_M\cdot(2.50\,\mathrm{m}) - T_R\cdot(3.0\,\mathrm{m})\cdot\sin\theta = 0 \qquad \Rightarrow \qquad \sin\theta = \frac{W_R\cdot(1.50\,\mathrm{m}) + W_M\cdot(2.50\,\mathrm{m})}{T_R\cdot(3.0\,\mathrm{m})}$$

Þá fæst að $\theta = \sin^{-1}\left(\frac{W_R\cdot(1.50\,\mathrm{m})+W_M\cdot(2.50\,\mathrm{m})}{T_R\cdot(3.0\,\mathrm{m})}\right) = 42\,°$

## Dæmi 11.28

A nylon rope used by mountaineers elongates 1.10 m under the weight of an 65.0 kg climber.

**A. If the rope is 45.0 m in length and 7.0 mm in diameter, what is Young's modulus for this material?**

Þar sem efri endi reipisins er festur, þá má halda því fram að sitthvor endi reipisins verði fyrir kraftinum $F_\perp = mg$, sem er þyngd klifrarans. Þá fæst Young-stuðullinn:

$$Y = \frac{F_\perp/A}{\Delta L/L} = \frac{(mg)/(\pi\left(\frac{d}{2}\right)^2)}{\Delta L/L} = \frac{((65.0\,\mathrm{kg})\cdot(9.8\,\mathrm{m/s^2}))/(\pi\left(\frac{0.007\,\mathrm{m}}{2}\right)^2)}{(1.10\,\mathrm{m})/(45.0\,\mathrm{m})} = 6.8\cdot 10^8\,\mathrm{Pa}$$

## Dæmi 11.37

In lab tests on a 9.25 cm cube of a certain material, a force of 1375 N directed at 8.50 ° to the cube, as shown in the figure, causes the cube to deform through an angle of 1.24 °.

**A. What is the shear modulus of the material?**

Skerstuðullinn fæst með jöfnunni $S = \frac{F_\parallel/A}{x/h}$, þar sem $F_\parallel$ er þáttur kraftsins sem er samsíða bjöguninni og $x/h$ er tangensinn af bjögunarhorninu $\phi$. Látum $l = 0.0925\,\mathrm{m}$ vera hliðarlengdir teningsins og $\theta$ vera hornið milli kraftsins $F$ og flatarins sem hann verkar á. Þá er:

$$S = \frac{F\cos\theta/l^2}{\tan\phi} \simeq= \frac{F\cos\theta/l^2}{\phi}$$

þar sem $\tan\phi \approx \phi$, fyrir lítil horn $\phi$.

$$S \approx \frac{(1375\,\mathrm{N})\cdot\cos(\frac{17}{360}\pi)/(0.0925\,\mathrm{m})^2}{0.02164} = 7.34\cdot 10^6\,\mathrm{Pa}$$

þar sem $8.5\,^\circ = \frac{17}{360}\pi\,\mathrm{rad}$ og $1.24\,^\circ \approx 0.02164\,\mathrm{rad}$.

## Dæmi 11.41

A steel cable with cross-sectional area $2.90\,\mathrm{cm}^2$ has an elastic limit of $e_0 = 2.40\cdot 10^8\,\mathrm{Pa}$.

**A. Find the maximum upward acceleration that can be given an elevator of mass 1150 kg supported by the cable if the stress is not to exceed one-third of the elastic limit.**

Lyftan verður fyrir kraftinum $ma = T - mg$, þar sem $T$ er togkrafturinn í bandinu. Ef að togkrafturinn í kaplinum er $T_{\max} = e_0 A$, þá slitnar hann. Af öryggisástæðum ætti kapallinn ekki að fara yfir $T = \frac{1}{3}T_{\max} = \frac{1}{3}e_0 A$. Fyrir þessi efri (öryggis)mörk á togkraftinum, verður hröðun lyftunnar:

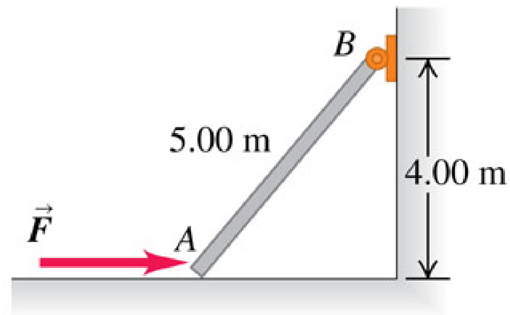
$$a = \frac{e_0 A}{3m} - g = \frac{(2.40\cdot 10^8\,\mathrm{Pa})\cdot(2.90\cdot 10^{-4}\,\mathrm{m^2})}{3\cdot(1150\,\mathrm{kg})} - 9.8\,\mathrm{m/s^2} = 10.4\,\mathrm{m/s^2}$$

# 9 Vika

## 9.1 Skiladæmi 9

### *Dæmi 11.50*

End A of the bar AB in rests on a frictionless horizontal surface, and end B is hinged. A horizontal force **F** of magnitude 250 N is exerted on end A. Ignore the weight of the bar.




**A. What is the horizontal component of the force exerted by the bar on the hinge at B?**

Látum $l = 5.00\,\mathrm{m}$ og $y = 4.00\,\mathrm{m}$. Þá er $\sqrt{l^2 - y^2} = \sqrt{(5.00\,\mathrm{m})^2 - (4.00\,\mathrm{m})^2} = 3.00\,\mathrm{m}$ fjarlægð A frá veggnum. Stöngin er í kraftajafnvægi. Það þýðir að $m\mathbf{a}_x = F - F_{\mathrm{hjör},x} = 0$ svo $F_{\mathrm{hjör},x} = 250\,\mathrm{N}$. Athugum að hér er $F_{\mathrm{hjör},x}$ kraftur hjarar á stöngina.
Skv. 3. lögmáli Newtons ýtir stöngin á hjörina með jafnmiklum, en andstæðum krafti (til hægri).

**B. What is the vertical component of the force exerted by the bar on the hinge at B?**

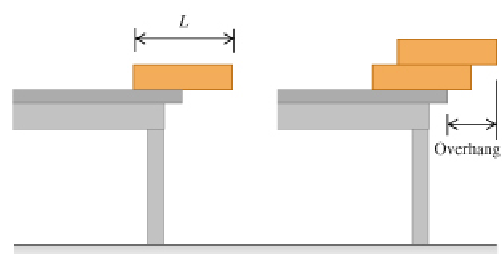
Fyrst við hunsum þyngd stangarinnar, vitum við að $ma_y = n - F_{\mathrm{hjör},y} = 0$.
Við getum skoðað kraftvægið um hjörina til að ákvarða $n$ og notfært okkur síðan jöfnuna $F_{\mathrm{hjör},y} = n$.

$$\tau = Fy - n\sqrt{l^1 - y^2} = 0 \qquad \Rightarrow \qquad n = F_{\mathrm{hjör},y} = F\frac{y}{\sqrt{l^2 - y^2}} = 250\,\mathrm{N} \cdot \frac{4.00\,\mathrm{m}}{3.00\,\mathrm{m}} = 330\,\mathrm{N}$$

$F_{\mathrm{hjör},y}$ vísar niður á við (á stöngina). Samkvæmt 3. lögmáli Newtons ýtir stöngin til baka upp á við með 330 N krafti á hjörina.

### *Dæmi 11.74*

If you put a uniform block at the edge of a table, the center of the block must be over the table for the block not to fall off.




**A. If you stack two identical blocks at the table edge, the center of the top block must be over the bottom block, and the center of gravity of the two blocks together must be over the table. In terms of the length L of each block, what is the maximum overhang possible?**

Við byrjum á því að skoða einn kubb, massamiðjan hennar þarf að liggja ofan á borðinu. Við getum sett kubbinn svo að hann standi hálfur fram af, án þess að hann detti, þá er $x = 1/2L$.
Næst setjum við annan kubb ofan á. Þegar við setjum næsta kubb ofan á, þá þurfum við að passa uppá að massamiðjan fari ekki út fyrir borð brúnina. Við setjum upp jöfnuna

$$x_2(2M) = x_1 M$$

þar sem $M$ er massi kubbsins, $x_1$ er hliðrun á kubbi 1, og $x_2$ er hliðrun á kubbi 2. $x_1 = 1/2$ og þá fæst

$$x_2 = 1/4L$$

þá er hliðrun á næsta kubbi $x_2 = 1/4$ fram af kubbi 1. Heildar hliðrun er þá $x = 1/2L + 1/4L = 3/4L$

**B. Repeat part (A) for three identical blocks.**

Til að bæta þriðja kubbnum við, þá endurtökum við það sem við gerðum í A.

$$x_3(3M) = x_1 M$$

$$x_3 = 1/6L$$

Heildar hliðrunin er þá $x = 1/2L + 1/4L + 1/6L = 11/12L$

**C. Repeat part (A) for four identical blocks.**

Endurtökum enn og aftur fyrir fjórða kubbinn

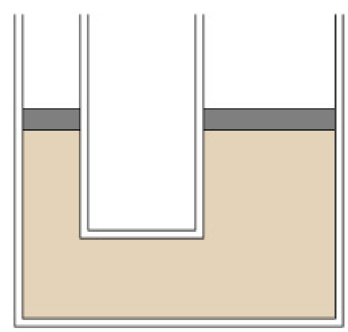
$$x_3(4M) = x_1 M$$

$$x_3 = 1/8L$$

Heildar hliðrunin er þá $x = 1/2L + 1/4L + 1/6L + 1/8L = 25/24L$ og þá erum við búin að hliðra þessu svo mikið að efsti kubburinn nær út fyrir brún borðsins, og það tók einungis 4 kubba.

## ***Fluid Pressure in a U-Tube***

A U-tube is filled with water, and the two arms are capped. The tube is cylindrical, and the right arm has twice the radius of the left arm. The caps have negligible mass, are watertight, and can freely slide up and down the tube.



**A. A one-inch depth of sand is poured onto the cap on each arm. After the caps have moved (if necessary) to reestablish equilibrium, is the right cap higher, lower, or the same height as the left cap?**

Aukna þyngdin sitt hvorum megin dreifist jafn yfir sitthvorn flötinn. Sand- og vatnssúlurnar þurfa að vera jafnháar til þess að þrýstingurinn á gefinni dýpt sé jafn í báðum súlum.

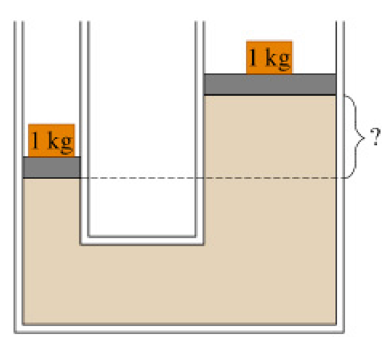
**B. The sand is removed and a 1.0 kg mass block is placed on each cap. After the caps have moved (if necessary) to reestablish equilibrium, is the right cap higher, lower, or the same height as the left cap?**

Jafnmikil þyngd hvílir á báðum lokum, svo þrýstingurinn verður minni á stærra lokinu (hægra lokið). Því þarf sú súla að hækka til þess að þrýstingurinn verði jafn í báðum súlu á gefinni dýpt.

**C. If a 1.0-kg-mass block is on the left cap, how much total mass must be placed on the right cap so that the caps equilibrate at equal height?**

Flatarmál hægra loksins er fjórfalt stærra en flatarmál vinstra loksins, þarf fjórfalt meiri þyngd að hvíla á því loki til að sami þrýsingur fáist í gefinni hæð. Það þyrfti því 4.00 kg lóð á hægra lokið.

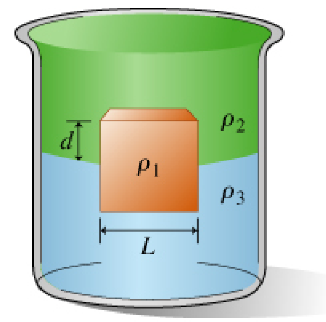
**D. The locations of the two caps at equilibrium are now as given in this figure. The dashed line represents the level of the water in the left arm. What is the mass of the water located in the right arm between the dashed line and the right cap?**




Vökva/lóðasúlurnar þurfa að að beita sama þrýstingi á gefnu dýpi í rörinu. Þar sem þyngd vinstri súlunnar samsvarar þyngd vinstra lóðsins (1.00 kg), þá þarf þyngd súlunnar yfir sama dýpi í hægri súlunni að samsvara 4.00 kg. Fyrst að lóðið í hægri súlunni er 1.00 kg, mun vatnssúlan vega 3.00 kg.

## Submerged Block

A beaker contains a thick layer of oil (shown in green) of density $\rho_2$ floating on water (shown in blue), which has density $\rho_3$. A cubical block of wood of density $\rho_1$ with side length $L$ is gently lowered into the beaker, so as not to disturb the layers of liquid, until it floats peacefully between the layers, as shown.




**A. What is the distance $d$ between the top of the wood cube (after it has come to rest) and the interface between the oil and water?**

Teningurinn er í kraftajafnvægi. Á hann verkar þyngdarkraftur og flotkraftar frá vatni og olíu.

$$\rho_1 L^3 a = \rho_1 L^3 g - \rho_2 d L^2 g - \rho_3 (L-d) L^2 g = 0$$

Einangrum liði sem innihalda $d$:

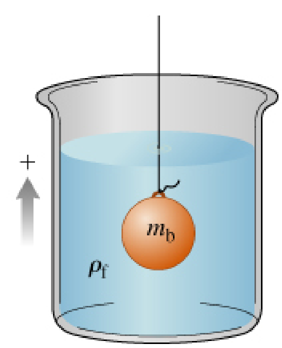
$$\rho_2 d L^2 g - \rho_3 d L^2 g = \rho_1 L^3 g - \rho_3 L^3 g$$

og að lokum sjálft $d$:

$$d = \frac{\rho_1 L^3 g - \rho_3 L^3 g}{\rho_2 L^2 g - \rho_3 L^2 g} = \frac{L(\rho_3 - \rho_1)}{\rho_3 - \rho_2}$$

## A Submerged Ball

A ball of mass $m_b$ and volume $V$ is lowered on a string into a fluid of density $\rho_f$. Assume that the object would sink to the bottom if it were not supported by the string.




**A. What is the tension $T$ in the string when the ball is fully submerged but not touching the bottom, as shown in the figure?**

Kúlan er í kraftajafnvægi. Kúlan er "léttari"á kafi þar sem flotkrafturinn verkar upp á kúluna.

$$ma = T + \rho_f V g - m_b g = 0 \qquad \Rightarrow \qquad T = (m_b - \rho_f V) g$$

## A Water Tank That Needs Cleaning

A cylindrical open tank needs cleaning. The tank is filled with water to a height $h_0 = 1.00\,\mathrm{m}$, so you decide to empty it by letting the water flow steadily from an opening at the side of the tank, located near the bottom. The cross-sectional area of the tank is $A_1 = 0.785\,\mathrm{m}^2$, while that of the opening is $A_2 = 0.002\,\mathrm{m}^2$.

**A. How much time $t_{1/2}$ does it take to empty half the tank? (Note: A useful antiderivative is $\int x^{-1/2} dx = 2x^{1/2}$.)**

Fyrst notum við samleitnijöfnuna til að finna vensl $v_1$ (hraða vatnsyfirborðsins) og $v_2$ (hraða vants út úr gati). $v_1 A_1 = v_2 A_2 \quad \Rightarrow \quad v_2 = \frac{A_1}{A_2} v_1$. Út frá jöfnu Bernoulli (fastur loftþrýstingur) er $\frac{1}{2}\rho v_1^2 + \rho g h = \frac{1}{2}\rho v_2^2$, þar sem $h$ er hæð vatnsyfirborðsins. Við getum skeytt saman þessum jöfnum til að fá hraðann út úr gatinu:

$$\frac{1}{2} v_1^2 + gh = \frac{1}{2}\left(\frac{A_1}{A_2}\right)^2 v_1^2 \qquad \Rightarrow \qquad v_1 = -\sqrt{\frac{2gh}{\left(\frac{A_1}{A_2}\right)^2 - 1}} = \frac{dh}{dt}$$

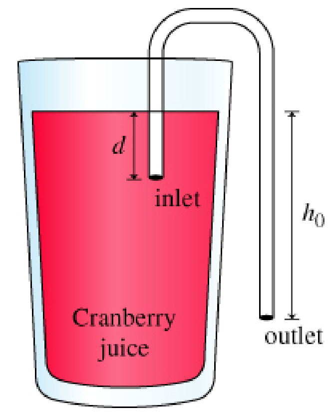
Við völdum neikvæðu rótina því við viljum að hraðinn $v_1$ aukist fyrir minnkandi $h$. Við endurröðum þessari fyrsta stigs diffurjöfnu og heildum:

$$\int_0^t dt' = -\sqrt{\frac{\left(\frac{A_1}{A_2}\right)^2 - 1}{2g}} \int_{h_0}^{h_0/2} h^{-1/2} dh = \sqrt{\frac{\left(\frac{A_1}{A_2}\right)^2 - 1}{2g}} \cdot \left(2\sqrt{\frac{h_0}{2}} - 2\sqrt{h_0}\right)$$

Tíminn sem það tekur er að tæma hálfann tankinn er:

$$t = \sqrt{\frac{2h_0\left(\left(\frac{A_1}{A_2}\right)^2 - 1\right)}{g}} \cdot \left(\frac{\sqrt{2}-1}{\sqrt{2}}\right) = 51.9\,\mathrm{s}$$

## *A Siphon at the Bar*




Jane goes to a juice bar with her friend Neil. She is thinking of ordering her favorite drink, 7/8 orange juice and 1/8 cranberry juice, but the drink is not on the menu, so she decides to order a glass of orange juice and a glass of cranberry juice and do the mixing herself. The drinks come in two identical tall glasses; to avoid spilling while mixing the two juices, Jane shows Neil something she learned that day in class. She drinks about 1/8 of the orange juice, then takes the straw from the glass containing cranberry juice, sucks up just enough cranberry juice to fill the straw, and while covering the top of the straw with her thumb, carefully bends the straw and places the end over the orange juice glass. After she releases her thumb, the cranberry juice flows through the straw into the orange juice glass. Jane has successfully designed a siphon. Assume that the glass containing cranberry juice has a very large diameter with respect to the diameter of the straw and that the cross-sectional area of the straw is the same at all points. Let the atmospheric pressure be $p_a$ and assume that the cranberry juice has negligible viscosity.

**A. Consider the end of the straw from which the cranberry juice is flowing into the glass containing orange juice, and let $h_0$ be the distance below the surface of cranberry juice at which that end of the straw is located: . What is the initial velocity $v$ of the cranberry juice as it flows out of the straw? Let $g$ denote the magnitude of the acceleration due to gravity.**

Við þurfum að bera saman tvo punkta á straumlínu með jöfnu Bernoulli. Þægilegast er að velja annan þeirra sem er á yfirborð vökvans í glasinu. Þar má áætla að vökvinn sé kyrrstæður því við þurfum að uppfylla samleitniskilyrðið $A_{\text{yfirborð}}v_{\text{yfirborð}} = A_{\text{rör}}v_{\text{rör}} \quad \Rightarrow \quad v_{\text{yfirborð}} \approx 0$ (því þversniðflatarmál glassins er talsvert stærra en þversniðsflatarmál rörsins).
Stöðorkan á vatnsyfirborðinu ($\rho g h_0$) í glasinu breytist í hreyfiþrýsting $\frac{1}{2}\rho v^2$. Ytri andrúmsloftsþrýstingur $p_0$ er fasti.

$$p_0 + \rho g h_0 + 0 = p_0 + 0 + \frac{1}{2}\rho v^2$$

Þá er $v = \sqrt{2gh_0}$

**B. Given the information found in Part A, find the time $t$ it takes to Jane to transfer enough cranberry juice into the orange juice glass to make her favorite drink if $h_0 = 10.0\,\mathrm{cm}$. Assume that the flow rate of the liquid is constant, and that the glasses are cylindrical with a diameter of $7.0\,\mathrm{cm}$ and are filled to height $14.0\,\mathrm{cm}$. Take the diameter of the straw to be $0.4\,\mathrm{cm}$.**

Við þurfum að reikna tímann sem það tekur að fylla rúmmálið $V = \frac{1}{8}\pi(d_{\text{glas}}/2)^2 h_{\text{glas}}$ ef að vökvaflæðið er $\frac{dV}{dt} = vA = \sqrt{2gh_0}\pi(d_{\text{rör}}/2)^2$. Nú er $V = \frac{dV}{dt} \cdot t$, svo við þurfum að eftirfarandi jöfnu fyrir $t$:

$$\frac{1}{8}\pi\frac{d_{\text{glas}}^2}{4}h_{\text{glas}} = \sqrt{2gh_0}\pi\frac{d_{\text{rör}}^2}{4} \cdot t$$

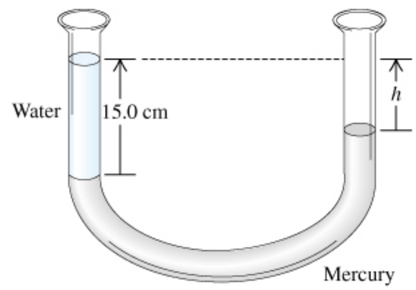
Við fáum:

$$t = \frac{h_{\text{glas}}}{8\sqrt{2gh_0}} \cdot \left(\frac{d_{\text{glas}}}{d_{\text{rör}}}\right)^2 = \frac{0.140\,\mathrm{m}}{8 \cdot \sqrt{2 \cdot (9.8\,\mathrm{m/s^2}) \cdot (0.100\,\mathrm{m})}} \cdot \left(\frac{0.07\,\mathrm{m}}{0.004\,\mathrm{m}}\right)^2 = 3.8\,\mathrm{s}$$

## 9.2 Tímadæmi 9

### *Dæmi 12.59*

A U-shaped tube open to the air at both ends contains some mercury. A quantity of water is carefully poured into the left arm of the U-shaped tube until the vertical height of the water column is 15.0 cm.




**A. What is the gauge pressure at the water-mercury interface?**

Þrýstingurinn við kvikasilfurs-vatns mörkin er einunings háð rúmmáli vökvans fyrir ofan

$$p = \rho \cdot 9.8 \text{ m/s}^2 \cdot 1000 \text{ kg/m}^3 0.15 \text{ m} = 1470 \text{ Pa}$$

**B. Calculate the vertical distance $h$ from the top of the mercury in the right-hand arm of the tube to the top of the water in the left-hand arm.**

Fyrst sjáum við að þrýstingurinn er sá sami hægra og vinstra megin í hæð $h$. Við þekkjum þrýstinginn í hæð 0.150 m

$$p_1 = \rho_w \cdot g \cdot 0.15 \text{ m}$$

þrýstingurinn í sama punkti hægra megin er

$$p_2 = \rho_{Hg} g (0.15 \text{ m} - h)$$

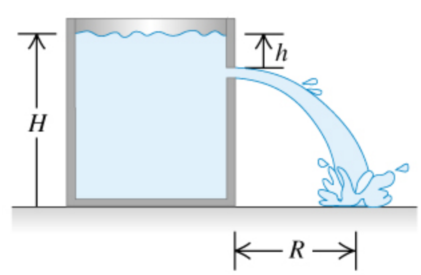
þá fæst

$$\rho_w \cdot g \cdot 0.15 \text{ m} = \rho_{Hg} \cdot g \cdot (0.15 \text{ m} - h)$$

$$h = 0.15 \text{ m} - \frac{\rho_w}{\rho_{Hg}} \cdot 0.15 \text{ m} = 13.9 \text{ cm}$$

### *Dæmi 12.77*

Water stands at a depth $H$ in a large open tank whose side walls are vertical . A hole is made in one of the walls at a depth $h$ below the water surface.




**A. At what distance $R$ from the foot of the wall does the emerging stream strike the floor?**

Látum $p_0$ vera andrúms þrýsting. Efst í tankinum og í fjarðlægð $h$ frá eftir brún tanksins er þrýstingurinn $p_0$. Notum að

$$\frac{\rho v^2}{2} + \rho g h + p_0 = fasti$$

þar sem efst í tankinum er $v = 0$ og $\rho g h$, og við gatið þar sem vatnið streymir út $v = v_0$ og $h = 0$, þá fáum við

$$\rho g h + p_0 = \frac{\rho v^2}{2} + p_0$$

$$v_0^2 = mgh$$

þar sem $v_0$ er hraðinn á vatnsbununni þegar hún kemur út um gatið. Þá er vatnsbunan í frjálsu falli. Vegalengdin sem vatnsbunan mun fara áður en hún lendir á jörðinni er

$$R = v \cdot t$$

þar sem $t$ er falltíminn. Falltíman getum við fundið með

$$H - h = \frac{1}{2} g t^2 \qquad \rightarrow \qquad t = \sqrt{\frac{2(H-h)}{g}}$$

fá fæst að

$$R = \sqrt{mgh}\sqrt{\frac{2\,(H-h)}{g}} = 2\sqrt{h(H-h)}$$

**B. How far above the bottom of the tank could a second hole be cut so that the stream emerging from it could have the same range as for the first hole?**

Nú eigum við að finna hvar við gætum gert gat, svo nýja bunan lendi á sama stað og sú fyrri. Ef við hugsum okkur að við borum tvö göt á sama stað í miðju tanksins, þá er augljóst að þær tvær bunur lendi á sama stað $R$ frá tankinum. Svo ef við borum gat efst á brún tanksins, og neðst þá mun bunan efst hafa litla stöðuorku, og því lítinn hraða þegar hún fer útum gatið. Bunan neðst mun hafa mikla stöðuorku, en lítinn (engan) falltíma. Þá sjáum við að frá miðjunni og í sitthvora átt frá henni, er fjarðlægðin $R$ sem bunan mun hafa samhverf. Þá getum við stokkið beint í lausnina og gert það sama og við gerum fyrir A lið, nema nú er stöðuorka bununnar gefin með $mg(H-h)$, þá fæst

$$\rho gh + p_0 = \frac{\rho v^2}{2} + p_0$$

$$v_0^2 = mg(H-h)$$

falltíminn er núna

$$h = \frac{1}{2}gt^2 \qquad \rightarrow \qquad t = \sqrt{\frac{2\,(h)}{g}}$$

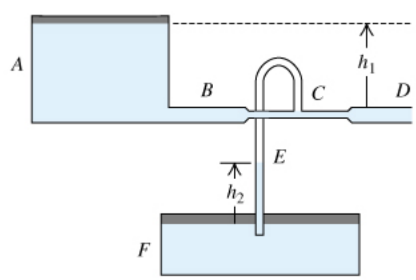
og þá fæst

$$R = \sqrt{mg(H-h)}\sqrt{\frac{2\,(h)}{g}} = 2\sqrt{(H-h)h}$$

sem er sama lausn og við fengum í A. lið.

## *Dæmi 12.83*

Two very large open tanks $A$ and $F$ (the figure ) contain the same liquid. A horizontal pipe $BCD$, having a constriction at $C$ and open to the air at $D$, leads out of the bottom of tank $A$, and a vertical pipe $E$ opens into the constriction at $C$ and dips into the liquid in tank $F$. Assume streamline flow and no viscosity. **A. If the cross-sectional area at $C$ is one-half the area at $D$ and if $D$ is a distance $h_1$ below the level of the liquid in $A$, to what height $h_2$ will liquid rise in pipe $E$?**




Látum $p_0$ vera andrúms þrýsting og $p_x$ og $v_x$ í punkti $x$ (C eða D). Sístætt núningslaust flði óþjappanlegs vökva án seigju, svo

$$\frac{\rho v^2}{2} + \rho gh + p_0 = fasti$$

í öllum vökvum.
Efst í tankinum er $v = 0$ og í punkti $D$ er þrýstingurinn $P_D$ (vökvi í snertingu við andrúmsloft). Efst í tankinum er hraði 0 svo,

$$\frac{\rho v_0^2}{2} + \rho gh + p_0 = \rho gh_1 + p_0 \qquad \rightarrow \qquad 2gh_1$$

Samfelldnis jafnan

$$A_c v_c = A_D v_D \quad \rightarrow \quad v_c = 2v_D$$

Þrýstingur í $C$fæst með Bernoulli

$$p_C = p_0 - \frac{\rho}{2}\left(v_C^2 - v_D^2\right) = p_0 - \frac{\rho}{2}3v_D^2$$

$$p_C = p_0 - 3\rho gh_1$$

Þrýstingurinn í $E$ er sá sami

$$p_0 - 3\rho gh_1 = p_E = p_0 - 3\rho gh_2 \quad \rightarrow \quad h_2 = 3h_1$$

# 10 Vika

## 10.1 Skiladæmi 10

### *Dæmi 13.16*

Jupiter's moon Io has active volcanoes (in fact, it is the most volcanically active body in the solar system) that eject material as high as 500 km (or even higher) above the surface. Io has a mass of $8.93 \times 10^{22}$ kg and a radius of 1821 km . For this calculation, ignore any variation in gravity over the 500-km range of the debris.

**A. How high would this material go on earth if it were ejected with the same speed as on Io?**

Byrjum á því að finna hver þyngdahröðunin er á Io, það gerum við með jöfnunni

$$g_{\mathrm{Io}} = \frac{Gm}{R^2}$$

Við yfirborðið er hröðunin þá

$$g_{\mathrm{Io}} = \frac{6.673 \times 10^{-11}\ \mathrm{m} \cdot 8.93 \times 10^{22}\ \mathrm{kg}}{(1821\ \mathrm{m})^2} = 1.797\ \mathrm{m/s}^2$$

Sagt í dæminum að við eigum að gera ráð fyrir fastri hröðun þar sem lokahraðinn $v_2 = 0$ , og þá gildir

$$v^2 = v_0^2 + 2g_{\mathrm{Io}}(y - y_0) \quad \rightarrow v_0 = \sqrt{-2a(y - y_0)} = 1.3405 \times 10^3\ \mathrm{m/s}$$

Nú vitum við hver hraðinn á öskunni er þegar hún sleppur út. Nú getum við reikna hversu hátt askan færi á jörðinni

$$v^2 = v_0^2 g_{\mathrm{Jord}}(y - y_0) \quad \rightarrow \quad y = \frac{v_0}{2g_{\mathrm{jord}}} = 91.7\ \mathrm{km}$$

### *Dæmi 13.24*

The International Space Station makes 15.65 revolutions per day in its orbit around the earth.

**A. Assuming a circular orbit, how high is this satellite above the surface of the earth?**

Þriðja lögmál Keplers segir að

$$mr\omega^2 = G\frac{mM_r}{r^2} \qquad (1)$$

þar sem við getum skrifað $\omega$ sem

$$\omega = 2\pi/T$$

þar sem $T$ er ein lota, T = 24klst/15.65(rev/24klst) = 1.533klst = 5520.76sek.
Radíusinn í jöfnu (1) er $r = r_e + h$, þ.e. radíus jarðar og fjarlægð geimstöðvarinnar frá jörðinni. Þá höfum við að

$$r =^3 \sqrt{T^2 G\frac{M_r}{4\pi^2}}$$

þar sem

$$r = r_e + h \qquad r_e = 6.378 \times 10^6\ \mathrm{m}$$

$$h = 370\ \mathrm{km}$$

### *Dæmi 13.26*

Suppose that a planet were discovered between the sun and Mercury, with a circular orbit of radius equal to 2/3 of the average orbit radius of Mercury. (Such a planet was once postulated, in part to explain the precession of Mercury's orbit. It was even given the name Vulcan, although we now have no evidence that it actually exists. Mercury's precession has been explained by general relativity.) The orbital period of Mercury is 88.0 days.

**A. What would be the orbital period of such a planet?**

Massi sólarinnar er $m_s = 1.99 \times 10^{30}$ kg og radíus brautar Merkúrusar um Sólina er $r_m = 5.79 \times 10^{10}$ m, þar sem braut Vulkan um Sólina er $2/3$ henni, er hún $r_b = 3.86 \times 10^{10}$ m. Nú notum við

$$F = m \cdot a \qquad F = G\frac{m_s m}{r^2} \qquad a = \frac{v^2}{r}$$

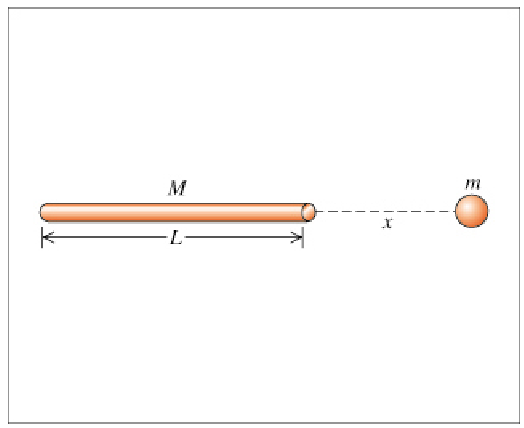
$$G\frac{m_s m}{r^2} = m\frac{v^2}{r} \quad \rightarrow \quad v = \sqrt{\frac{Gm_s}{r}}$$

Tíminn sem að tekur plánetuna að fara eina umferð í kringum Sólina er brautar radíusinn deilt með hraðanum

$$T = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{Gm_s}{r}} = 4.13 \times 10^6 \text{ s} = 47.9 \text{ dagar}$$

## *Dæmi 13.34*

A thin, uniform rod has length L and mass M. A small uniform sphere of mass m is placed a distance x from one end of the rod, along the axis of the rod .




**A. Calculate the gravitational potential energy of the rod-sphere system. Take the potential energy to be zero when the rod and sphere are infinitely far apart. (Hint: Use the power series expansion for ln(1+x).)**

Við getum litið á litlu kúluna sem punktpassa, en getum ekki gert þá einföldun fyrir stöngina. Til að reikna þetta, þá skiptum við stönginni, sem hefur lengd $L$ og massa $M$ upp í þunnar skífur sem hafa þykktina $dl$ og massa $dM$, þar sem $L \cdot dM = M \cdot dl$ . Nú finnum við mættið milli allra þessa litlu skífa og punktpassan. Milli kúlunnar og einnar skífu er mættið

$$dU = G\frac{dM \cdot m}{x + l}$$

þar sem $l$ er fjarlægðin frá hægri enda stangarinnar að skífunni. Við stingum inn fyrir $dM = dl \cdot M/L$, og þá höfum við

$$dU = G\frac{Mm}{x + l}\frac{dl}{L}$$

Nú heildum við yfir alla stöngina $L$

$$\int dU = \int_0^L G\frac{Mm}{L(x + l)}dl$$

$$U = -G\frac{mM}{L} \cdot |\ln(x + l)|_0^L$$

$$U = -G\frac{mM}{L} \cdot (\ln(x + L) - \ln(x)) = -G\frac{mM}{L} \cdot \ln((x + L)/x)$$

og þá fáum við

$$U = -G\frac{mM}{L} \cdot \ln(1 + L/x)$$

**B. Use $F_x = -dU/dx$ to find the magnitude of the gravitational force exerted on the sphere by the rod.**

Í lið A fundum við $U$

$$U = -G\frac{mM}{L} \cdot \ln(1 + L/x)$$

Þar sem kerfið er í raun einvítt, þá er nóg að diffra $U$ með tilliti til $x$ til að finna hver krafturinn er í punkti $x$.

$$F_x = -\frac{dU}{dx} = G\frac{mM}{L} \cdot \frac{d}{dx}\left(\ln(1 + L/x)\right)$$

Notum keðjuregluna til að diffra

$$\frac{d}{dx}\left(\ln(1 + L/x)\right) = \frac{1}{1 + L/x} \cdot \frac{x \cdot 0 - 1 \cdot L}{x^2} = -\frac{L/x^2}{1 + L/x}$$

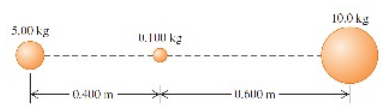
þá fáum við

$$F_x = -\frac{dU}{dx} = -G\frac{mM}{x^2 + Lx}$$

Athugum að tölugildið verður $|F_x| = G\frac{mM}{x^2+Lx}$.

## *Dæmi 13.58*

The 0.100 kg sphere is released from rest at the position shown in the sketch, with its center 0.400 m from the center of the 5.00-kg mass. Assume that the only forces on the 0.100 kg sphere are the gravitational forces exerted by the other two spheres and that the 5.00-kg and 10.0 kg spheres are held in place at their initial positions.




**A. What is the speed of the 0.100-kg sphere when it has moved 0.200 m to the left from its initial position?**

Það verkar eitthvað þyngdarmætti á 0.100 kg kúluna, og við byrjum á því að finna hvað þetta mætti er. Notum til þess orkuvarðveislu

$$K_1 + U_1 = K_2 + U_2$$

þar sem $K = 1/2 \cdot mv^2$ og $U = -GmM/r$ Í upphafi er $K_1 = 0$

$$U_1 = -\frac{Gm_A \cdot m_B}{r_{B1}} - \frac{Gm_A \cdot m_C}{r_{C1}}$$

$$U_2 = -\frac{Gm_A \cdot m_B}{r_{B2}} - \frac{Gm_A \cdot m_C}{r_{C2}}$$

þar sem $m_A = 0.100$ kg, $m_B = 5.00$ kg, $m_C = 10.0$ kg, $r_{B1} = 0.400$ m, $r_{C1} = 0.600$ m, $r_{B2} = 0.200$ m, og $r_{C2} = 0.800$ m.
Þá fáum við að

$$-1.946 \times 10^{-10} \text{ J} = -2.501 \times 10^{-10} \text{ J} + K_2$$

þar sem $K_2 = 1/2mv^2 \quad \rightarrow v = \sqrt{2K_2/m}$ og þá fáum við hraðan

$$v = 3.33 \times 10^{-5} \text{ m/s}$$

## *Dæmi 14.8*

In a physics lab, you attach a 0.200-kg air-track glider to the end of an ideal spring of negligible mass and start it oscillating. The elapsed time from when the glider first moves through the equilibrium point to the second time it moves through that point is 2.60 s.

**A. Find the spring's force constant.**

Hér gildir

$$F = m \cdot a$$

þar sem við umskrifum $m \cdot a = m \cdot \ddot{x}$. Fyrir gorm gildir

$$F = -kx$$

þá fáum við

$$\ddot{x} + \frac{k}{m}x = 0$$

þar sem $\omega^2 = k/m$. Getum umskrifað $\omega = 2\pi/T$. Þá fáum við að

$$T = 2\pi\sqrt{m/k}$$

Það er gefið að það tekur 2.60 sek fyrir massan að ljúka hálfri lotu, þá er T = 5.2 s

$$k = m \cdot (2\pi/T)^2 = 0.292 \text{ N/m}$$

## *Kepler's 3rd law*

A planet moves in an elliptical orbit around the sun. The mass of the sun is $M_s$. The minimum and maximum distances of the planet from the sun are $R_1$ and $R_2$, respectively.

**A. Using Kepler's 3rd law and Newton's law of universal gravitation, find the period of revolution $P$ of the planet as it moves around the sun. Assume that the mass of the planet is much smaller than the mass of the sun.**

Þriðja lögmál Keplers er

$$mr\omega^2 = G\frac{mM_s}{r^2} \qquad (1)$$

Við byrjum á því að finna hálf-langás (*e. semi-major axis*), það gerum við með því að taka meðtaltal af stærstu og minnstu fjarlægð frá sólinni

$$a = \frac{R_1 + R_2}{2}$$

Þar sem fjarlægðin $a$ er þá meðalfjarlægð frá sólu.
Nú skoðum við aftur jöfnu (1), þar getum við umskrifað $\omega$ sem

$$\omega = 2\pi/T$$

þar sem $T$ er tíminn sem það tekur plánetuna að fara einn hring í kringum sólina. Við stingum þessu inn fyrir $\omega$ í jöfnu 1 og einangrum $T$

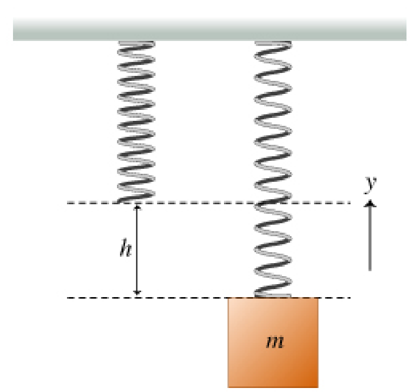
$$T = \frac{2\pi r^3}{GM_s}$$

þar sem við stingum meðalfjarlægðina $a$ inn fyrir $r$ í jöfnunni.

$$T = \frac{2\pi\left(\frac{R_1+R_2}{2}\right)^3}{GM_s}$$

## *Vertical Mass-and-spring oscillator*

A block of mass m is attached to the end of an ideal spring. Due to the weight of the block, the block remains at rest when the spring is stretched a distance h from its equilibrium length. The spring has an unknown spring constant k.




**A. What is the spring constant k?**

Gormurinn er kyrrstæður, svo heildarkrafturinn $F_y = 0$.

$$\sum F_y = h \cdot k - m \cdot g = 0$$

þá fáum við að

$$k = \frac{m \cdot g}{h}$$

þar sem $h$ er hliðrunin, $m$ massinn og $g$ þyngdahröðunin.

**B. Suppose that the block gets bumped and undergoes a small vertical displacement. Find the resulting angular frequency $\omega$ of the block's oscillation about its equilibrium position.**

Kassanum er hliðrað örlítið lárétt, þá mun hann byrja að sveiflast fram og tilbaka. Hér ætlum við að finna horn tíðnina $\omega$.
Setjum fyrst upp hreyfijöfnu fyrir pendúlinn. Á pendúlinn verkar krafturinn

$$F = m \cdot a$$

Fyrir gorm gildir

$$F = -k \cdot x$$

þar sem $k$ er gormstuðullinn og $x$ hliðrunin. Setjum þessar tvær jöfnur saman, og skrifum $a = \ddot{x}$ (hröðunin er önnur afleiða af staðsetningunni)

$$-k \cdot x = m \cdot \ddot{x}$$

$$\ddot{x} + \frac{k}{m} \cdot x = 0$$

þar sem $\omega^2$ er liðurinn sem stendur fyrir framan $x$

$$\omega^2 = \frac{k}{m} \quad \rightarrow \quad \omega = \sqrt{\frac{k}{m}}$$

Við vitum hvað $k$ er úr A lið.

$$\omega = \sqrt{\frac{m \cdot g}{h \cdot m}} = \sqrt{\frac{g}{h}}$$

## *A satellite in orbit*

A satellite used in a cellular telephone network has a mass of 2340 kg and is in a circular orbit at a height of 670 km above the surface of the earth.

**A. What is the gravitational force Fgrav on the satellite?**

Gefið að

$$G = 6.66 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$$

$$m_e = 5.97 \times 10^{24} \text{ kg}$$

$$r_e = 6.38 \times 10^6 \text{ m}$$

Við notum

$$F_{\text{grav}} = \frac{G \cdot m_1 \cdot m_2}{R^2}$$

þar sem $G$ er þyngdarfastinn, $m_1$ og $m_2$ eru massar og $R$ er fjarlægðin á milli $m_1$ og $m_2$. Þá fáum við að

$$F_{\text{grav}} = \frac{6.66 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \cdot 5.97 \times 10^{24} \text{ kg} \cdot 2340 \text{ kg}}{(6.38 \times 10^6 \text{ m} + 670000 \text{ m})^2} = 1.87 \times 10^4 \text{ N}$$

**B. What fraction is this of the satellite's weight at the surface of the earth? Take the free-fall acceleration at the surface of the earth to be $g$ = 9.80 m/s$^2$ .**

Við erum búin að finna $F_{\text{grav}}$. Nú viljum við finna hversu þungur gervidiskurinn er miðað við ef hann væri á jörðinni. Á yfirborði jarðar verkar krafturinn

$$F_g = m_{\text{gervitungl}} \cdot g$$

Nú finnum við

$$\frac{F_{\text{grav}}}{F_g} = \frac{G \cdot m_e}{g \cdot (h + r_e)^2} = 0.819$$

## 10.2 Tímadæmi 10

### *Dæmi 14.38*

A proud deep-sea fisherman hangs a 65.0-kg fish from an ideal spring having negligible mass. The fish stretches the spring 0.180 m.

**A. Find the force constant of the spring.**

Notum Hooks law

$$F = -kx \quad \rightarrow \quad k = 65 \text{ kg} \cdot 9.8 \text{ m/s}^2 / 0.180 \text{ m}$$

fáum þá

$$k = 3540 \text{ N/m}$$

**B. The fish is now pulled down 5.00 cm and released. What is the period of oscillation of the fish?**

Hreyfijafnan er fundið með

$$F = m \cdot a$$

þar sem $F = -kx$ og $a = \ddot{x}$

$$m\ddot{x} + kx = 0$$

$$\ddot{x} + \frac{k}{m}x = 0$$

þar sem liðurinn fyrir framan $x$ er $\omega^2 = k/m$ þá er sveiflutíminn fundinn með

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$

þá er

$$T = 2\pi\sqrt{\frac{m}{k}} = 0.852 \text{ s}$$

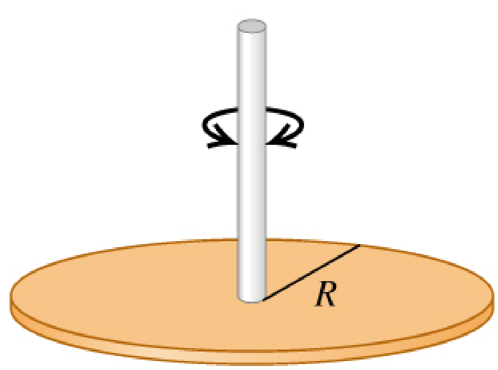
**C. What is the maximum speed it will reach?**

Sveiflutíminn er tíminn sem það tekur pendúl að fara eina umferð fram og tilbaka. Það þýðir að sama hvað útslagið er, þá mun sá tími sem það tekur pendúlinn að fara eina umferð vera sá sami. Hámarkshraðinn er því fundinn með því að deila útslaginu með sveiflutímanum

$$v_{max} = 2\pi\frac{A}{T} = 0.369 \text{ m/s}$$

### *Dæmi 14.42*

A thin metal disk mass $2.00 \times 10^{-3}$ kg and radius 2.20 cm is attached at its center to a long fiber. The disk, when twisted and released, oscillates with a period of 1.00s. **A. Find the torsion constant of the fiber.**




Við byrjum dæmið á því að leiða út jöfnuna fyrir horn tíðni einfalds pendúls sem snýst um sjálfan sig um ás $z$. Frá þeirri jöfnu fáum við svo vindingsfastann$\kappa$ (*e. torsion constant*). Á þráðinn verkar eitthvað kraftvægi

$$\tau_z = \kappa \cdot \theta$$

við þekkjum jöfnu sem tengir kraftvægi við hverfiþunga

$$\tau_z = I \cdot \alpha_z$$

og getum stungið inn fyrir fyrstu jöfnuna, þá fáum við að

$$I \cdot \alpha_z = \kappa \cdot \theta$$

þar sem $\alpha_z$ er hornhröðunin. Horn hröðunina getum við líka skrifað sem

$$\alpha_z = \frac{d^2\theta}{dt^2}$$

þá fáum við

$$\frac{d^2\ddot{\theta}}{dt^2} - \frac{\kappa}{I}\theta = 0$$

Hér höfum við hreyfijöfnu fyrir kerfið okkar, þar sem liðurinn sem stendur fyrir framan $\theta$ er

$$\omega^2 = \frac{\kappa}{I} \qquad \omega = 2\pi f$$

þar sem $f$ er tíðni Hverfitregða disksins með snúningsás í miðju er $I = \frac{1}{2}mR^2$, þá fæst

$$(2\pi f)^2 = \frac{2\kappa}{mR^2} \qquad \rightarrow \qquad \kappa = (2\pi f)^2 \frac{mR^2}{2} = 1.9 \times 10^{-5} \text{ m/rad}$$

## *Dæmi 14.54*

We want to support a thin hoop by a horizontal nail and have the hoop make one complete small-angle oscillation each 2.0 s.

**A. What must the hoop's radius be?**

Hreyfijafna fyrir einfaldan pendúl er

$$\ddot{\theta} + \frac{g}{l}\sin\theta = 0$$

þar sem fyrir lítil horn gildir $\sin\theta \simeq \theta$.

$$\ddot{\theta} + \frac{g}{l}\theta = 0$$

Þar sem liðurinn sem stendur fyrir framan $\theta$ er $\omega^2 = g/l$ Þá er sveiflutíminn

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{g}{l}}$$

og þar með

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Hægt er að sýna að radíus sveiflu fyrir hvern pendúl er

$$L = \frac{I}{md}$$

þar sem $d$ er fjarlægð að massamiðju, hér er $d = R$. Þá fæst

$$T = 2\pi\sqrt{\frac{I}{mgR}}$$

Nú þurfum við að finna hverfitregðu lykkjunnar. Við þekkjum hverfitregðu fyrir holan sívalning (lykkjan er stuttur holur sívalningur).

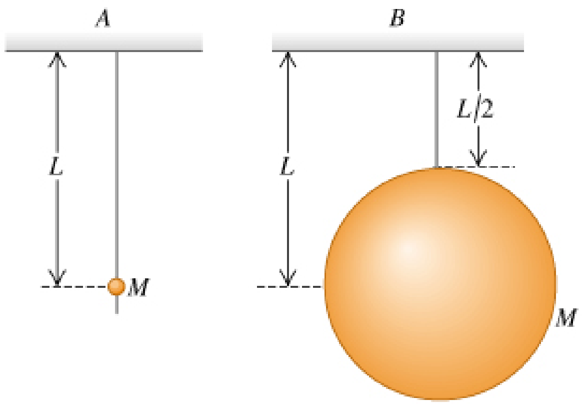
$$I = mR^2 + mR^2$$

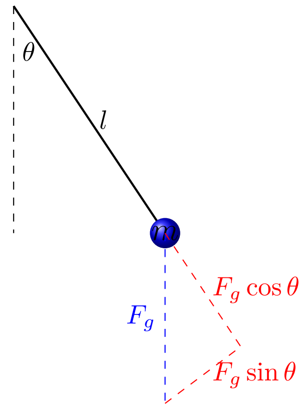
þar sem seinni liðurinn kemur frá parallel axis theorem, þar sem snúningsásinn hefur fjarlægð R frá massa miðju lykkjunnar.
Þá fáum við

$$T = 2\pi\sqrt{\frac{2mR^2}{mgR}} = 2\pi\sqrt{\frac{2R}{g}}$$

eingangrum $R$

$$R = \left(\frac{T}{2\pi}\right)^2 g\frac{1}{2} = 0.496 \text{ m}$$




## *Dæmi 14.57*

The two pendulums shown in the figure each consist of a uniform solid ball of mass M supported by a massless string, but the ball for pendulum A is very tiny while the ball for pendulum B is much larger.

**A. Find the period of pendulum A for small displacements.**

Lítum á litla pendúlinn sem punktmassa. ATH jafnan fyrir $\omega$ er gefin á formúlublaðinu. Athugið að jafna (1) er sértilvik, þar sem massinn er punktmassi. Jafna (2) er almenn jafna (líka á formúlublaðinu). Við byrjum á því að finna hreyfijöfnu fyrir einfaldan pendúl. Við skoðum myndina af pendúl hér fyrir ofan. Þá er kraftvægi pendúlsins

$$\tau = F_g \sin\theta \cdot l$$

og

$$\tau = I\alpha$$

þá höfum við að

$$-m \cdot g \cdot l \cdot \sin\theta = I\alpha$$

þar sem $\alpha = \partial^2\theta/\partial t^2 = \ddot{\theta}$

$$\ddot{\theta} + \frac{m \cdot g \cdot l}{I} \sin\theta = 0$$

þar sem $I = ml^2$ (parallel axis theorem) þá fæst

$$\ddot{\theta} + \frac{g}{l} \sin\theta = 0$$

þar sem fyrir lítil horn gildir $\sin\theta \simeq \theta$.

$$\ddot{\theta} + \frac{g}{l}\theta = 0$$

Þar sem liðurinn sem stendur fyrir framan $\theta$ er $\omega^2 = g/l$ Þá er sveiflutíminn

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{g}{l}} \qquad (1)$$

og þar með

$$T = 2\pi\sqrt{\frac{l}{g}}$$

**B. Find the period of pendulum B for small displacements.**

Nú getum við ekki litið á pendúlinn sem punktmassa. Byrjum á því að finna hveriftregðu kúlunnar. Vitum að hverfitregða gegnheillar kúlu er

$$I = \frac{2}{5}mR^2 + Mh^2$$

$R$ er radíus kúlunnar og $h$ er hliðrun frá snúningsás (seinni liðurinn kemur frá parallel axis theorem). Hægt er að sýna að radíus sveiflu fyrir hvern pendúl er

$$L = \frac{I}{md}$$

Þar sem $d$ er staðsetning massa miðju þegar kerfið er í jafnvægi. Núna er $d = L$. þá er sveiflutíminn gefinn með

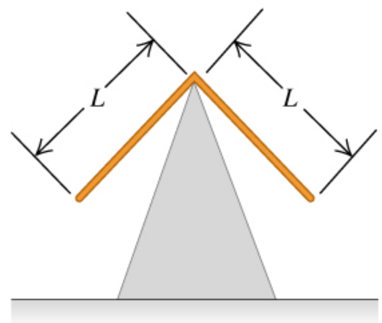
$$T = 2\pi\sqrt{\frac{I}{mLg}} \qquad (2)$$

$$T = 2\pi\sqrt{\frac{2/5m(L/2)^2 + ML^2}{mLg}} = 2\pi\sqrt{\frac{11L}{10g}}$$

Þá sjáum við að það tæki pendúl $B$ lengur að sveiflast eina umferð en pendúl $A$.

### *Dæmi 14.88*

Two identical, thin rods, each with mass m and length L, are joined at right angles to form an L-shaped object. This object is balanced on top of a sharp edge . If the L-shaped object is deflected slightly, it oscillates.




**A. Find the frequency of the oscillation**

Byrjum á því að finna hverfitregðu stanganna, vitum að hverfitregða fyrir heila stöng sem sveiflast um enda er

$$I = \frac{1}{3}mL^2$$

Við höfum tvær stangir svo

$$I = \frac{2}{3}mL^2$$

Sveiflutími einfalds pendúls er

$$\frac{1}{T} = f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

Hægt er að sýna að radíus sveiflu fyrir hvern pendúl er

$$L = \frac{I}{md} \quad \text{Hjá okkur eru tvær stangir svo} \quad L = \frac{I}{2md}$$

Þar sem $d$ er staðsetning massa miðju þegar kerfið er í jafnvægi $d = L/(2\sqrt{2})$

þá fáum við að tíðnin er

$$f = \frac{1}{2\pi}\sqrt{\frac{g2md}{I}} = \frac{1}{2\pi}\sqrt{\frac{g2mL/(2\sqrt{2})}{2/3 \cdot mL^2}}$$

$$f = \frac{1}{4\pi}\sqrt{\frac{3\sqrt{2}\ g}{L}}$$

# 11 Vika

## 11.1 Skiladæmi 11

### *Dæmi 14.18*

A 0.400-kg object undergoing SHM has $a_x$ = -1.40 m/s$^2$ when $x$ = 0.300 m.

**A. What is the time for one oscillation?**

Skv. 2. lögmáli Newtons verður hluturinn fyrir kraftinum $ma_x = -kx$ (í átt að jafnvægisstöðu). Nú er horntíðni hlutarins $\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{a_x}{-x}}$ og því er lotan $T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{-x}{a_x}} = 2\pi\sqrt{\frac{-0.300\,\mathrm{m}}{-1.40\,\mathrm{m/s^2}}} = 2.91\,\mathrm{s}$

### *Dæmi 14.22*

In February 2004, scientists at Purdue University used a highly sensitive technique to measure the mass of a vaccinia virus (the kind used in smallpox vaccine). The procedure involved measuring the frequency of oscillation of a tiny sliver of silicon (just 32.0 nm long) with a laser, first without the virus and then after the virus had attached itself to the silicon. The difference in mass caused a change in the frequency. We can model such a process as a mass on a spring.

**A. Find the ratio of the frequency with the virus attached ( $f_{\mathrm{S+V}}$) to the frequency without the virus ($f_{\mathrm{S}}$) in terms of $m_{\mathrm{V}}$ and $m_{\mathrm{S}}$, where $m_{\mathrm{V}}$ is the mass of the virus and $m_{\mathrm{S}}$ is the mass of the silicon sliver. Notice that it is not necessary to know or measure the force constant of the spring.**

Við viljum ákvarða hlutfallið $\frac{f_{\mathrm{S+V}}}{f_{\mathrm{S}}} = \frac{\omega_{\mathrm{S+V}}}{\omega_{\mathrm{S}}}$. Fyrir hreintóna hreyfingu massa gildir $\omega = \sqrt{\frac{k}{m}}$ þar sem $k$ er einhver almennur fjaðurstuðull hreyfingarinnar. Hér er fjaðurstuðullinn fastur í báðum tilfellum svo við fáum að hlutfallið verður $\frac{f_{\mathrm{S+V}}}{f_{\mathrm{S}}} = \sqrt{\frac{\cancel{k}}{m_{\mathrm{S}}+m_{\mathrm{V}}} \cdot \frac{m_{\mathrm{S}}}{\cancel{k}}} = \frac{1}{\sqrt{1+\frac{m_{\mathrm{V}}}{m_{\mathrm{S}}}}}$

**B. In some data, the silicon sliver has a mass of $2.10 \cdot 10^{-16}$ g and a frequency of $2.04 \cdot 10^{15}$ Hz without the virus and $2.88 \cdot 10^{14}$ Hz with the virus. What is the mass of the virus in grams?**

Einangrum $m_{\mathrm{V}}$ úr jöfnuninni úr liðnum á undan.

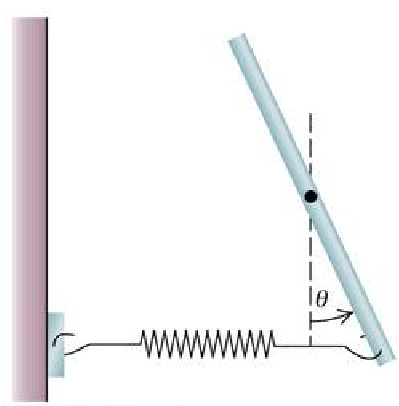
$$m_{\mathrm{V}} = m_{\mathrm{S}} \cdot \left( \left( \frac{f_{\mathrm{S}}}{f_{\mathrm{S+V}}} \right)^2 - 1 \right) = 2.10 \cdot 10^{-16}\,\mathrm{g} \cdot \left( \left( \frac{20.4}{2.88} \right)^2 - 1 \right) = 1.03 \cdot 10^{-14}\,\mathrm{g}$$

**C. What is the mass of the virus in femtograms?**

Nú er $1\,\mathrm{fg} = 10^{-15}\,\mathrm{g}$, svo við höfum:

$$m_{\mathrm{V}} = 10.3\,\mathrm{fg}$$

### *A Pivoting Rod on a Spring*



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A slender, uniform metal rod of mass $M$ and length $l$ is pivoted without friction about an axis through its midpoint and perpendicular to the rod. A horizontal spring, assumed massless and with force constant $k$, is attached to the lower end of the rod, with the other end of the spring attached to a rigid support.

**A. We start by analyzing the torques acting on the rod when it is deflected by a small angle $\theta$ from the vertical. Consider first the torque due to gravity. Which of the following statements most accurately describes the effect of gravity on the rod?**

Hér væri hægt að skoða kraftvægi þyngdarkraftins á sitthvorn helming stangarinnar og áttað sig á því að þau eru jöfn en andstæð. Hins vegar er fljótlegast að átta sig á því að þyngdarkrafturinn verkar á massamiðju stangarinnar (sem snúningsás stangarinnar fer í gegnum). Því er armlengdin núll og þar með kraftvægið núll ($\tau = mg \cdot 0 = 0$).

**B. Find the torque $\tau$ due to the spring. Assume that $\theta$ is small enough that the spring remains effectively horizontal and you can approximate $\sin(\theta) \approx \theta$ (and $\cos(\theta) \approx 1$).**

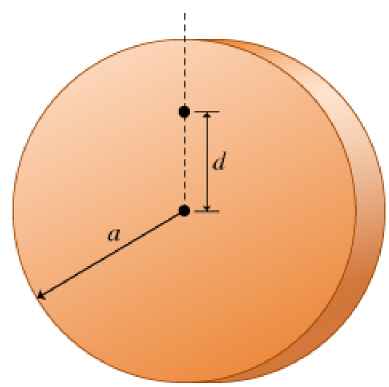
Gerum ráð fyrir að gormurinn er í jafnvægisstöðu þegar $\theta = 0$. Armlengd fjaðurkrafts um snúningsásinn samsvarar hæð massamiðju stangarinnar frá gorminum ($h = \frac{l}{2}\cos\theta \approx \frac{l}{2}$). Lenging/þjöppun gormsins er $x = \frac{l}{2}\sin\theta \approx \frac{l}{2}\theta$. Kraftvægið er því

$$\tau = -kx \cdot h \approx -k\left(\frac{l}{2}\right)^2 \theta$$

**C. What is the angular frequency $\omega$ of oscillations of the rod?**

Munum að $\tau = I\alpha = I\frac{d^2\theta}{dt^2}$, þar sem $I = \frac{1}{12}Ml^2$ er hverfitregða stangarinnar um snúningsás (massamiðju). Þar sem við reiknuðum kraftvægið í fyrri liðnum, fáum við annars stigs diffurjöfnuna $\frac{d^2\theta}{dt^2} = -\frac{k\left(\frac{l}{2}\right)^2}{I}\theta$ fyrir hreintóna sveiflur. Allar diffurjöfnur á forminu $\frac{d^2x}{dt^2} = -\omega^2 x$ samsvara hreintóna sveifli með horntíðni $\omega$. Því er $\omega^2 = \frac{k\left(\frac{l}{2}\right)^2}{I} = \frac{k\left(\frac{l}{2}\right)^2}{\frac{1}{12}Ml^2} = \frac{12}{4} \cdot \frac{k}{M}$. Horntíðnin er því $\omega = \sqrt{\frac{3k}{M}}$.

## *Extreme Period for a Physical Pendulum*




**A. What is $I_{\text{cm}}$, the moment of inertia of the disk around its center of mass? You should know this formula well.**

Þetta hefur verið leitt út í fyrirlestri (úr kafla 9). Þar var sýnt fram á að $I_{\text{cm}} = \frac{1}{2}ma^2$.

**B. If you use this disk as a pendulum bob, what is $T(d)$, the period of the pendulum, if the axis is a distance $d$ from the center of mass of the disk?**

Notum reglu Steiner til að ákvarða hverfitregðuna um hengipunkt: $I = I_{\text{cm}} + md^2 = m\left(\frac{a^2}{2} + d^2\right)$.
Þá er lota pendúlsins

$$T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{\frac{a^2}{2} + d^2}{gd}}$$

**C. The period of the pendulum has an extremum (a local maximum or a local minimum) for some value of $d$ between zero and infinity. Is it a local maximum or a local minimum?**

Það er fljótséð að lotan $T$ stefnir í óendanlegt þegar að $d$ stefnir á annað hvort útgildanna. Útgildið hlýtur því að vera lággildi.

**D. What is $T_{\min}$, the minimum period of the pendulum?**

Tökum tímaafleiðu af lotunni:

$$\frac{dT}{dd} = \frac{2\pi}{\sqrt{g}} \frac{\left(-\frac{a^2}{2d^2} + 1\right)}{2\sqrt{\frac{\frac{a^2}{2}+d^2}{d}}} = 0 \qquad \Rightarrow \qquad d = \frac{a}{\sqrt{2}} \qquad \Rightarrow \qquad T_{\min} = 2\pi\sqrt{\frac{a^2}{g\frac{a}{\sqrt{2}}}} = 2\pi\sqrt{\frac{\sqrt{2}a}{g}}$$

## *Dæmi 15.6*

A fisherman notices that his boat is moving up and down periodically, owing to waves on the surface of the water. It takes 3.4 s for the boat to travel from its highest point to its lowest, a total distance of 0.52 m . The fisherman sees that the wave crests are spaced 5.2 m apart.

**A. How fast are the waves traveling?**

Það tekur $3.4\,\mathrm{s}$ að fara úr hæsta punkti í lægsta, sem er einungis helmingurinn af einni umferð með lotuna $T = 2{\cdot}(3.4\,\mathrm{s})$. Við notum því

$$v = \lambda/T = \frac{5.2\,\mathrm{m}}{2\cdot(3.4\,\mathrm{s})} = 0.76\,{}^{\mathrm{m}}\!/\!_{\mathrm{s}}$$

**B. What is the amplitude of each wave?**

Útslag sjávarbylgnanna, $A$, er lóðrétt fjarlægð öldutoppa frá miðju aldanna. Lóðrétt fjarlægð milli bylgjutopps og bylgjubotns er því

$$2A = 0.52\,\mathrm{m} \qquad \Rightarrow \qquad A = 0.26\,\mathrm{m}$$

**C. If the total vertical distance traveled by the boat were $0.40\,\mathrm{m}$ , but the other data remained the same, how fast are the waves traveling ?**

Útslagið hefur minnkað þar sem lóðrétt fjarlægð milli bylgjutopps og bylgjubotns hefur minnkað í $2A = 0.40\,\mathrm{m}$. Útslagið hefur hins vegar ekkert að gera með hraða bylgnanna (í lárétta stefnu). Ef allar aðrar stærðir eru óbreyttar, þarf því bylgjuhraðinn að vera óbreyttur: $v = 0.76\,{}^{\mathrm{m}}\!/\!_{\mathrm{s}}$.

**D. If the total vertical distance traveled by the boat were 0.40 m , but the other data remained the same, what is the amplitude of each wave?**

Útslagið, $A$, fæst út frá:

$$2A = 0.40\,\mathrm{m} \qquad \Rightarrow \qquad A = 0.20\,\mathrm{m}$$

## *Dæmi 15.10*

A water wave traveling in a straight line on a lake is described by the equation

$$y(x,t) = (2.75\ \mathrm{cm})\cos(0.410\ \mathrm{rad/cm}\ x + 6.20\ \mathrm{rad/s}\ t)$$

where $y$ is the displacement perpendicular to the undisturbed surface of the lake.

**A. How much time does it take for one complete wave pattern to go past a fisherman in a boat at anchor?**

Bylgjujafnan fyrir bylgju sem fer í $-x$ stefnu er

$$y(x,t) = A\cos(kx + \omega t)$$

til að að finna lotuna notum við svo

$$T = \frac{2\pi\ \mathrm{rad}}{\omega} = \frac{2\pi\ \mathrm{rad}}{6.20\ \mathrm{rad/s}} = 1.0134\ \mathrm{s}$$

**B. What horizontal distance does the wave crest travel in that time?**

Lárétta fjarlægðin er

$$\lambda = \frac{2\pi\ \mathrm{rad}}{k} = \frac{2\pi\ \mathrm{rad}}{0.140\ \mathrm{rad/cm}} = 15.3\ \mathrm{cm}$$

**C-F. What is the wave number? What is the number of waves per second that pass the fisherman? How fast does a wave crest travel past the fisherman?What is the maximum speed of his cork floater**

**as the wave causes it to bob up and down?**

Bylgjutalan er

$$k = 0.410 \text{ rad/cm}$$

og bylgjur á sekúndu er tíðni

$$f = 1/T = 1/1.0134 \text{ s} = 0.9868 \text{ Hz}$$

og hraðinn er

$$v = f\lambda = (0.9868 \text{ Hz})(0.15325 \text{ m}) = 0.151 \text{ m/s}$$

Hámarkshraðinn er

$$v_{\text{max}} = \omega A = (6.20 \text{ rad/s})(2.75 \text{ cm}) = 0.171 \text{ m/s}$$

## ***Ant on a Thightrope***

A large ant is standing on the middle of a circus tightrope that is stretched with tension $T_s$. The rope has mass per unit length $\mu$. Wanting to shake the ant off the rope, a tightrope walker moves her foot up and down near the end of the tightrope, generating a sinusoidal transverse wave of wavelength $\lambda$ and amplitude $A$. Assume that the magnitude of the acceleration due to gravity is $g$.

**A. What is the minimum wave amplitude Amin such that the ant will become momentarily "weightless"at some point as the wave passes underneath it? Assume that the mass of the ant is too small to have any effect on the wave propagation.**

Byrjum á því að skoða bylgjujöfnu fyrir bylju sem ferðast í $+x$ stefnu

$$y(x,t) = A\sin(\omega t - kx)$$

þar sem

$$v_y = \frac{dy}{dt} = -\omega A \sin(kx - \omega t)$$

og

$$a_y = \frac{dv_y}{dt} = -\omega^2 A \sin(kx - \omega t)$$

Hámarkshröðunin á sér stað þegar $\sin(kx - \omega t) = 1$, svo

$$a_{\text{max}} = -\omega^2 A$$

til þess að maurinn sé "massalaus"þá þarf hröðunin að vera sá sami og þyngdarhröðunin $g$

$$-g = -\omega^2 A \quad \rightarrow \quad A = \frac{g}{\omega^2}$$

þar sem $\omega$ er

$$v = \frac{\omega}{k} \quad \rightarrow \quad \omega = vk$$

þar sem $k = 2\pi/\lambda$ við notum svo

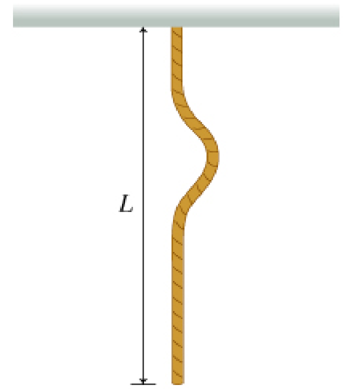
$$v = \sqrt{F/\mu}$$

Þá minnsta útslagið

$$A = \frac{\mu\,(\lambda/2\pi)^2\, g}{T_s}$$

## ***Wave in a Dangling Rope***

A uniform rope of length $L$ and negligible stiffness hangs from a solid fixture in the ceiling .




**A. The free lower end of the rope is struck sharply at time $t = 0$. What is the time $t$ it takes the resulting wave on the rope to travel to the ceiling, be reflected, and return to the lower end of the rope?**

Látum $z$ tákna fjarlægð bylgjutoppsins frá neðri enda reipisins. Hér þarf að leysa eftirfarandi diffurjöfnu:

$$v = \frac{dz}{dt} = \sqrt{\frac{F(z)}{\mu}}$$

þar sem $F(z)$ er togkrafturinn í punkti $z$ í reipinu og $\mu = \frac{m}{L}$ er massi reipisins á lengdareiningu. Togkrafturinn er jafngildir þyngdarkraftinum sem verkar á hangandi hluta reipisins *undir* fyrir neðan bylgutoppinn: $F(z) = z\mu g$.

Þegar við endurröðum diffurjöfnunni, fæst að:

$$dt = \frac{1}{\sqrt{g}} \frac{dz}{\sqrt{z}}$$

Ákvörðum fyrst tímann sem það tekur bylgjuna að fara upp eftir reipinu með því að heilda báðar hliðar:

$$\int_0^{t_L} dt = \frac{1}{\sqrt{g}} \int_0^L \frac{dz}{\sqrt{z}} \qquad \Rightarrow \qquad t_L = \frac{1}{\sqrt{g}} \cdot 2\sqrt{L}$$

Bylgjan er jafnlengi að fara niður eftir reipinu eftir að hún kastast til baka. Þá er heildartíminn

$$t_{\text{heild}} = 2t_L = 4\sqrt{\frac{L}{g}}$$

### *Dæmi 15.22*

A piano wire with mass 3.05 g and length 76.0 cm is stretched with a tension of 29.0 N . A wave with frequency 120 Hz and amplitude 1.80 mm travels along the wire.

**A. Calculate the average power carried by the wave.**

Meðal aflið finnum við með því að nota jöfnuna

$$P_{\text{av}} = \frac{1}{2}\mu v \omega^2 A^2 = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2 = 0.314 \text{ W}$$

þar sem

$$P_{\text{av}} = \frac{1}{2} P_{\text{max}}$$

**B. What happens to the average power if the wave amplitude is halved?**

Þá höfum við $A/2$ í stað $A$

$$P_{\text{av}} = \frac{1}{2}\mu v \omega^2 (A/2)^2 = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2 = 7.85 \times 10^{-2} \text{ W}$$

## 11.2 Tímadæmi 11

### *Dæmi 15.7*

Transverse waves on a string have wave speed $v = 8.00$ m/s, amplitude $A = 0.0700$ m, and wavelength $\lambda = 0.320$ m. The waves travel in the $-x$ direction, and at $t = 0$ the $x = 0$ end of the string has its maximum upward displacement.

**A. Find the frequency of these waves**

Tíðnin er fundin með

$$f = \frac{v}{\lambda} = \frac{8 \text{ m/s}}{0.320 \text{ m}} = 25 \text{ Hz}$$

**B. Find the period of these waves.**

Lotan er

$$T = \frac{1}{f} = \frac{1}{25 \text{ Hz}} = 0.04 \text{ s}$$

**C. Find the wave number of these waves**

Bylgjutalan er gefin með

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.320 \text{ m}} = 19.6 \text{ rad/m}$$

**D. Write a wave function describing the wave**

Bylgjufall er gefið með

$$y(x,t) = A\cos\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right)$$

í dæminu er sagt að bylgjurnar ferðist í $-x$ stefnu, svo

$$y(x,t) = A\cos\left(2\pi\left(-\frac{x}{\lambda} - \frac{t}{T}\right)\right)$$

og þar sem $\cos(\theta)$ er jafnstætt fall ($f(x) = f(-x)$), getum við þá skrifað

$$y(x,t) = A\cos\left(2\pi\left(\frac{x}{\lambda} + \frac{t}{T}\right)\right)$$

og stigum inn fyrir $\lambda$,$A$ og $T$

$$y(x,t) = (0.07\text{ m})\cos\left(2\pi\left(\frac{x}{0.320\text{ m}} + \frac{t}{0.04\text{ s}}\right)\right)$$

Hér væri líka hægt að skrifa

$$y(x,t) = A\cos\left(2\pi\left(kx + \omega t\right)\right)$$

**E. Find the transverse displacement of a particle at $x = 0.360$ m at time $t = 0.150$ s .**

Stingum inn fyrir $x$ og $t$

$$y(x,t) = (0.07\text{ m})\cos\left(2\pi\left(\frac{0.360\text{ m}}{0.320\text{ m}} + \frac{0.150\text{ s}}{0.04\text{ s}}\right)\right) = 4.95\times 10^{-2}\text{ m}$$

**F. How much time must elapse from the instant in part E until the particle at $x = 0.360$ m next has maximum upward displacement?**

Hér á að finna hvenær í punkti $x = 0.360$ m eftir atvikið í lið E, hvenær mun eind á þessum stað næst upplifa hámarksútlsag, þ.e. $y(x,t) = 0.07$ m. Frá bylgjujöfnunni

$$y(0.360,t) = (0.07\text{ m})\cos\left(2\pi\left(\frac{0.360\text{ m}}{0.320\text{ m}} + \frac{t}{0.04\text{ s}}\right)\right)$$

þurfum við nú að leysa fyrir $t$ þannig að $y(0.360,?) = 0.07$. Það gerist þegar að

$$\cos\left(2\pi\left(\frac{0.360\text{ m}}{0.320\text{ m}} + \frac{t}{0.04\text{ s}}\right)\right) = 1$$

sem gerist þegar að

$$2\pi\left(\frac{0.360\text{ m}}{0.320\text{ m}} + \frac{t}{0.04\text{ s}}\right) = 2\pi n$$

þar sem $n = 1,2,3,4,....$ Einangrum $t$

$$t = 0.07\text{ m}\left(n - \frac{0.360\text{ m}}{0.320\text{ m}}\right)$$

nú stingum við inn $n = 4$ og $n = 5$, þá fæst

$$t_4 = 0.1150\text{ s}$$

sem er andartakið áður en atvikið í lið E gerist.

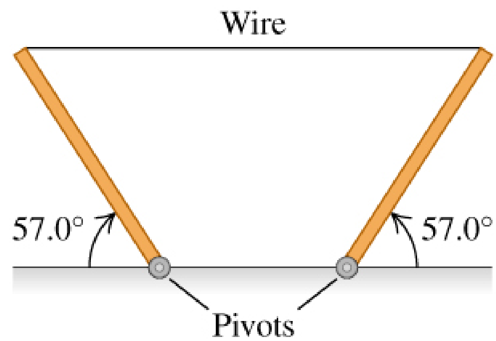
$$t_5 = 0.1550\text{ s}$$

þetta er rétt eftir að atvikið í lið E gerist. Svo næsta hámark eftir E lið gerist á t = 0.1550 s. Þá er tíminn sem líður milli E og F

$$\Delta t = 0.155\text{ s} - 0.150\text{ s} = 0.005\text{ s}$$

***Dæmi 15.55***

A 5.00-m, 0.736-kg wire is used to support two uniform 236-N posts of equal length (the figure ). Assume that the wire is essentially horizontal and that the speed of sound is 344 m/s. A strong wind is blowing, causing the wire to vibrate in its 5th overtone.




**A. What is the frequency of the sound this wire produces?**

Notum jöfnuna

$$v = \sqrt{F/\mu} \qquad (1)$$

Byrjum á því að finna togkraftinn í bandinu. Togkraftinn finnum við með

$$\sum \tau = 0$$

þar sem lengdin á stönginn er $x$, þá er

$$w \cos(57°) \frac{x}{2} - T \sin(57°)\, x = 0$$

þar sem togkrafturinn er

$$T = \frac{w}{2 \tan(57°)}$$

Við setjum

$$v = f\lambda \quad \text{og} \quad \mu = m/L$$

og þá verður jafna (1)

$$f = \frac{1}{T}\sqrt{\frac{w \cdot L}{m \cdot 2\tan(57°)}} = 13.69 \text{ Hz}$$

**B. What is the wavelength of the sound this wire produces?**

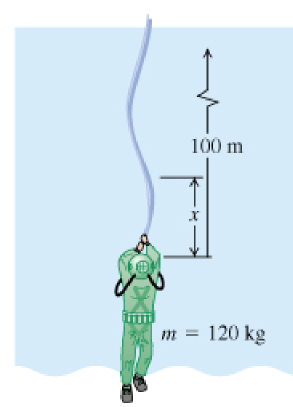
Notum

$$\lambda = \frac{v_s}{f} = \frac{344 \text{ m/s}}{13.69 \text{ Hz}} = 25.1 \text{ Hz}$$

þar sem $v_s$ er hraði hljóðs í lofti.

## *Dæmi 15.77*

A deep-sea diver is suspended beneath the surface of Loch Ness by a cable of length h = 100 m that is attached to a boat on the surface . The diver and his suit have a total mass of m = 120 kg and a volume of V = $8.10 \times 10^{-2}$ m$^3$ . The cable has a diameter of 2.05 cm and a linear mass density of $\mu = 1.11$ kg/m . The diver thinks he sees something moving in the murky depths and jerks the end of the cable back and forth to send transverse waves up the cable as a signal to his companions in the boat.




**A. What is the tension in the cable at its lower end, where it is attached to the diver? Do not forget to include the buoyant force that the water (density $\rho_{\text{water}} = 1000$ kg/m$^3$ ) exerts on him.**

Togkrafturinn í reipinu neðst er

$$F_k = m \cdot g - V \cdot \rho_{\text{water}} \cdot g = 382 \text{ N}$$

**B. Calculate the tension in the cable a distance x above the diver. The buoyant force on the cable must be included in your calculation.**

Í punkti $x$ fyrir ofan kafarann er togkrafturinn

$$F = F_k + m \cdot g \cdot x - \rho_{\text{water}} \cdot \pi \left(\frac{D}{2}\right)^2 xg$$

**C. The speed of transverse waves on the cable is given by $v = \sqrt{F/\mu}$. The speed therefore varies along the cable, since the tension is not constant. (This expression neglects the damping force that the water exerts on the moving cable.) Integrate to find the time required for the first signal to reach the surface.**

Heildum yfir tíman

$$\int \mathrm{d}t = t \qquad (1)$$

þar sem

$$v = \frac{dx}{dt}$$

Við margföldum jöfnu (1) með $dx/dx$

$$\int \frac{\mathrm{d}x \cdot \mathrm{d}t}{\mathrm{d}x} = \int \frac{\mathrm{d}x \cdot}{\mathrm{d}x/\mathrm{d}t} = \int \frac{1}{v}\mathrm{d}x$$

þá heildum við

$$\int_0^L \frac{\sqrt{\mu}}{\sqrt{F}}\mathrm{d}t$$

Við umskrifum

$$F = F_k + x\left(m \cdot g \cdot -\rho_{\text{water}} \cdot \pi \left(\frac{D}{2}\right)^2 g\right) = F_k + x\alpha$$

þar sem $F_k$ og $\alpha$ eru fastar.

$$\int_0^L \frac{\sqrt{\mu}}{\sqrt{F_k + \alpha x}}\mathrm{d}t$$

notum breytuskiptin

$$u = F_k + \alpha x \qquad du = \alpha dx$$

$$t = \frac{\sqrt{\mu}}{\alpha}\int_0^L \frac{1}{\sqrt{u}}\mathrm{d}u = \frac{\sqrt{\mu}}{\alpha}\left|\frac{\sqrt{u}}{1/2}\right|_0^L = \frac{2\sqrt{\mu}}{\alpha}\left(\sqrt{F_k + L} - \sqrt{F_k}\right) = 3.95 \text{ s}$$

## *Dæmi 15.67*

A thin string 2.50 m in length is stretched with a tension of 90.0 N between two supports. When the string vibrates in its first overtone, a point at an antinode of the standing wave on the string has an amplitude of 3.50 cm and a maximum transverse speed of 26 m/s .

**A. What is the string's mass?**

Byrjum á því að finna tíðnina út frá $v_{\max} = 26$ m/s

$$f = \frac{v_{\max}}{2\pi A}$$

og svo bylgjuhraðan út frá tíðninni

$$v = f\lambda$$

Svo notum við jöfnuna

$$v = \sqrt{F/\mu} \quad \rightarrow \quad m = \frac{F \cdot L}{(f \cdot \lambda)^2} = 2.58 \text{ gr}$$

þar sem $\mu = m/l$.

**B. What is the magnitude of the maximum transverse acceleration of this point on the string?**

Notum

$$a = A\omega^2 = 1.93 \times 10^4 \text{ m/s}^2$$

# 12 Varmafræði

## Skammtur 1R:varmafraedi-2

### *Dæmi 19.26*

Five moles of monatomic ideal gas have initial pressure $2.50 \cdot 10^3\,\mathrm{Pa}$ and initial volume $2.10\,\mathrm{m}^3$. While undergoing an adiabatic expansion, the gas does 1480 J of work.

**A. What is the final pressure of the gas after the expansion?**

Fyrir kjörgas sem verður fyrir óverminni útþenslu, gildir:

$$p_1 V_1^\gamma = p_2 V_2^\gamma \qquad \Rightarrow \qquad p_2 = p_1 \left(\frac{V_1}{V_2}\right)^\gamma$$

Einnig gildir:

$$W = \frac{1}{\gamma - 1}(p_1 V_1 - p_2 V_2) \qquad \Rightarrow$$

Við getum endurskrifað svigann með $(p_1 V_1 - p_2 V_2) = p_1 V_1^\gamma (V_1^{1-\gamma} - V_2^{1-\gamma})$ út frá fyrstu jöfnunni. Einangrum $V_2$:

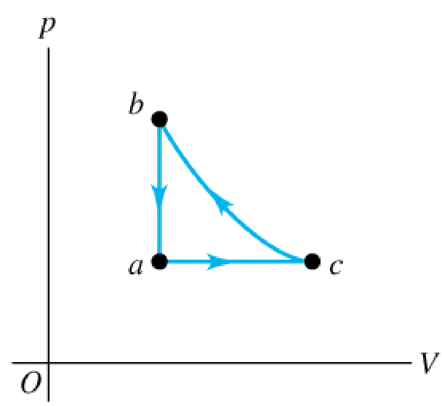
$$(V_1^{1-\gamma} - V_2^{1-\gamma}) = \frac{W(\gamma - 1)}{p_1 V_1^\gamma} \qquad \Rightarrow \qquad V_2^{1-\gamma} = \frac{W(1-\gamma)}{p_1 V_1^\gamma} + V_1^{1-\gamma} \qquad \Rightarrow \qquad V_2 = \left(\frac{W(1-\gamma)}{p_1 V_1^\gamma} + V_1^{1-\gamma}\right)^{\frac{1}{1-\gamma}}$$

Stingum þessu inn í jöfnuna fyrir $p_2$:

$$p_2 = p_1 \left(\frac{V_1}{\left(\frac{W(1-\gamma)}{p_1 V_1^\gamma} + V_1^{1-\gamma}\right)^{\frac{1}{1-\gamma}}}\right)^\gamma = 2500\,\mathrm{Pa} \cdot \left(\frac{2.1\,\mathrm{m}^3}{\left(\frac{1480\,\mathrm{J}\cdot\left(-\frac{2}{3}\right)}{(2500\,\mathrm{Pa})\cdot(2.1\,\mathrm{m}^3)^{\frac{5}{3}}} + (2.1\,\mathrm{m}^3)^{-\frac{2}{3}}\right)^{\frac{1}{1-\frac{5}{3}}}}\right)^{\frac{5}{3}} = 1490\,\mathrm{Pa} = 1.49\,\mathrm{kPa}$$

### *Dæmi 19.42*

Three moles of an ideal gas are taken around the cycle abc shown in the figure . For this gas, $C_p = 29.1\,{}^{\mathrm{J}}/_{<\mathrm{mol}\cdot\mathrm{K}}$. Process ac is at constant pressure, process ba is at constant volume, and process cb is adiabatic. The temperatures of the gas in states a, c, and b are $T_a = 300\,\mathrm{K}$, $T_c = 492\,\mathrm{K}$, and $T_b = 600\,\mathrm{K}$.




**A. Calculate the total work $W$ for the cycle.**

Vinnan er flatarmálið sem ferllinn afmarkar, þar sem $b \to a$ gerist við fast rúmmál (*isochoric process*). Þegar rúmmál varmafræðilegs ástands er fasti, þá er vinnan

$$W_{ba} = 0$$

$a \to c$ gerist við fastan þrýsting (*isobaric process*). Þá er vinnan

$$W_{ac} = p(V_2 - V_1)$$

þar sem kjörgasjafnan

$$pV = nRT \quad \to \quad p\Delta V = nR\Delta T$$

og þá fæst að vinnan er

$$W = nR\Delta T = nR\,(492\ \mathrm{K} - 300\ \mathrm{K}) = 4.79 \times 10^3\ \mathrm{J}$$

$c \to b$ þá hækkar þrýstingurinn á sama tíma og rúmmálið minnkar. Enginn hiti eða efni bætist við í kerfið, svo $Q = 0$ (*Adiabatic*). Þá notum við

$$W_{cb} = Q - \Delta U = -\Delta U$$

Þar sem $\Delta U$ er breyting á innriorkunni kerfisins, og þar sem $\Delta U$ er skilgreint sem

$$W_{cb} = -nC_v\Delta T$$

Nú þurfum við að finna varmarýmd við fast rúmmál $C_v$, það er

$$C_v = C_p - R$$

þar sem $R = 8.315$ J/molK er gasfastinn, þá er vinnan

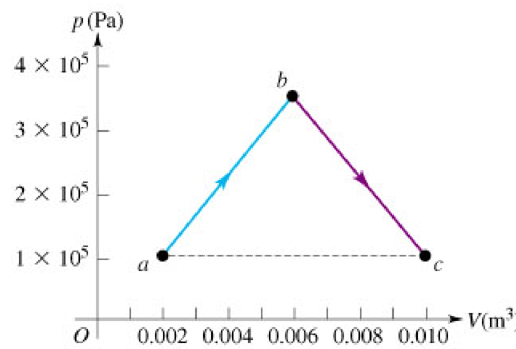
$$W_{cb} = -6.74 \times 10^3 \text{ J}$$

Heildar vinnan er þá

$$W = 4.79 \times 10^3 \text{ J} - 6.74 \times 10^3 \text{ J} = -1.95 \times 10^3 \text{ J}$$

## *Dæmi 19.44*

One-third of a mole of He gas is taken along the path abc shown as the solid line in the figure.




**A. Assume that the gas may be treated as ideal. How much heat is transferred into or out of the gas?**

Heildarvarminn sem fer í gasið er $Q = \Delta U + W$. Vinnuna ákvörðum við út frá flatarmálinu undir ferlinum á $p - V$ grafinu:

$$W = \int_{V_a}^{V_c} pdV = \left(\frac{(p_b - p_a)}{2} + p_a\right) \cdot (V_c - V_a) = 1800\,\text{J}$$

Við reiknum aukningu innri orkunnar með $\Delta U = \int nC_V dT = nC_V \Delta T$. Notum kjörgasjöfnuna til að finna $T_i = \frac{p_i V_i}{nR}$, þar sem $i \in \{a, b, c\}$. Hér er nóg að skoða heildar hitastigsbreytinguna $\Delta T \equiv T_c - T_a = \frac{p_a(V_c - V_a)}{nR}$, þar sem við höfum nýtt okkur að $p_a = p_c$ í þessu ferli. Þá er:

$$\Delta U = nC_v \frac{p_a(V_c - V_a)}{nR} = (12.47\,{}^{\text{J}}/_{\text{mol}\cdot\text{K}}) \cdot \frac{(1.0 \cdot 10^5\,\text{Pa}) \cdot (0.008\,\text{m}^3)}{8.315\,{}^{\text{J}}/_{\text{mol}\cdot\text{K}}} = 1200\,\text{J}$$

Heilarvarminn sem fer í gasið er $Q = \Delta U + W = 1200\,\text{J} + 1800\,\text{J} = 3000\,\text{J}$.

**B. If the gas instead went from state a to state c along the horizontal dashed line in the figure, how much heat would be transferred into or out of the gas?**

Við þurfum bara að endurreikna vinnuna $W$, þar sem flöturinn undir ferlinum er ferhhyrningur með flatarmálið $W = 800\,\text{J}$. Þá er $Q = 1200\,\text{J} + 800\,\text{J} = 2000\,\text{J}$.

**C. How does $Q$ in part (b) compare against $Q$ in part (a)? Explain.**

Töluvert meiri vinna er framkvæmd við það að auka þrýstinginn og minnka hann, þótt að upphafs- og lokaástand gassins er það sama. Því gildir $Q_a > Q_b$ fyrir aukna vinnu.