

Hafið þann föstudagur, 23. febrúar 2018, 9:03 eh

Staða Lokið

Lokið þann fimmtudagur, 8. mars 2018, 7:32 eh

Tími 12 dagar 22 klukkustundir

## Spurning 1

Að hluta til rétt

Marked out of 4,00

The following optimal Simplex table is given for a linear programming problem where the decision variables are  $x_1$ ,  $x_2$ ,  $x_3$  and the slack variables are denoted by  $x_4$ , ...:

$$\max x \rightarrow \zeta = 5 - 14x_2 - 9x_3 - 5x_5$$

subject to

$$x_4 = 6 + 5x_2 + 1x_3 + 2x_5$$

$$x_1 = 1 - 4x_2 - 2x_3 - 1x_5$$

and  $x_1$ ,  $x_2$ ,  $x_3 \geq 0$ . Now let us assume we need to add the following constraint to the problem:

$$5x_1 + 6x_2 + 1x_3 \leq 4$$

Now answer the following:

- Add this new constraint to the current Simplex table, is this table optimal? (answer: 0 for no, 1 for yes)  ✓
- What is the optimal solution for this modified problem  $\zeta =$   ✗
- If the table is non-optimal, which variable should leave the basis? (answer: 0 for none, else the index of the variable)  ✓
- Assume we decide to add a very large number of constraints to the problem should we solve the dual problem (answer: 0) or the primal problem (answer: 1)?  ✓

The augmented problem considered was the following:

$$\max x \rightarrow \zeta = 5x_1 + 6x_2 + 1x_3$$

subject to

$$2x_1 + 3x_2 + 3x_3 \leq 8$$

$$1x_1 + 4x_2 + 2x_3 \leq 1$$

$$5x_1 + 6x_2 + 1x_3 \leq 4$$

and  $x_1$ ,  $x_2$ ,  $x_3 \geq 0$

Some notes:

The first pivot is a dual pivot, since the dual solution remains feasible (was primal optimal), but the primal problem is now infeasible (dual non-optimal).

When adding constraints to problems, we switch to the dual simplex method and usually only a few iterations are required when starting with the current Simplex table.

If we add a huge number of constraints to the problem, such that  $m \gg n$ , then solving the dual problem would be wiser since the dual problem will have  $m$  variables and only  $n$  constraints and so the number of corner point feasible solutions will be fewer and the Simplex method will require fewer iterations.

Notice also that the optimal solution  $\zeta$  to the augmented problem is worse than the original one, why is that?

The answers:

- Add this new constraint to the current Simplex table, is this table optimal? (answer: 0 for no, 1 for yes) 0
- What is the optimal solution for this modified problem  $\zeta =$  4
- If the table is non-optimal, which variable should leave the basis? (answer: 0 for none, else the index of the variable) 6
- Assume we decide to add a very large number of constraints to the problem should we solve the dual problem (answer: 0) or the primal problem (answer: 1)? 0

**Important: the equations will only render correctly using the Firefox browser!**

The following optimal Simplex table is given for a linear programming problem where the decision variables are  $x_1$ ,  $x_2$ ,  $x_3$  and the slack variables are denoted by  $x_4$ ,  $x_5$ ,  $x_6$ :

$$\max x \rightarrow \zeta = 3 - 2x_6 - 6x_2 - 1x_4$$

subject to

$$x_3 = 1 + 1x_6 - 3x_2 - 2x_4$$

$$x_5 = 4 - 6x_6 + 11x_2 + 11x_4$$

$$x_1 = 0 - 1x_6 + 0x_2 + 1x_4$$

and  $x_1, x_2, x_3 \geq 0$ .

The cost coefficients for the problem are  $c_1 = 5$ ,  $c_2 = 3$  and  $c_3 = 3$  and the resource constraints are bounded by  $b_1 = 1$ ,  $b_2 = 9$  and  $b_3 = 1$ . We are interested in knowing the bounds of some of these variables such that the current basis holds. In other words we want to perform sensitivity analysis.

Now compute allowable changes  $\Delta c$  for the different variables given below. In the case where you would like to enter plus infinity use the value 9999 and in the case you would like to use minus infinity then enter the value -9999. Note also that the decrease values are negative. Furthermore, report the values to 3 decimal places.

- a. What is the allowable decrease for  $c_3$  ?  ✖
- b. What is the allowable increase for  $c_3$  ?  ✖
- c. What is the allowable decrease for  $c_2$  ?  ✔
- d. What is the allowable increase for  $c_2$  ?  ✔

Some notes:

- The variable  $c_3$  is a decision variable and is in the basis, while  $c_2$  also decision variable but not within the current basis.
- The inverse of the basis matrix  $B$  times the non-basis matrix  $N$  is found by taking the coefficient in front of the variables that are not in the basis (where all variable are on the left hand side or the  $=$ , or if you like multiplied by  $-1$ ). That is,  $B^{-1}N =$

$$-132 \ 6 - 11 - 11 \ 10 - 1$$

Look the Simplex table above to confirm this.

- This matrix is then used to compute the allowable increase or decrease by the fact that:

$$z \rightarrow N + \Delta z \rightarrow N = (B^{-1}N)^T (c \rightarrow B + \Delta c \rightarrow B) - (c \rightarrow N + \Delta c \rightarrow N) \geq 0$$

That is, the changes  $\Delta c \rightarrow$  in  $c$  may not be such that dual solution becomes infeasible (negative).

The answers (recall that here 9999 denotes Infinity):

- a. What is the allowable decrease for  $c_3$  ? -0.5
- b. What is the allowable increase for  $c_3$  ? 2.0
- c. What is the allowable decrease for  $c_2$  ? -9999.0
- d. What is the allowable increase for  $c_2$  ? 6.0

**Important: the equations will only render correctly using the Firefox browser!**

The following optimal Simplex table is given for a linear programming problem where the decision variables are  $x_1$ ,  $x_2$ ,  $x_3$  and the slack variables are denoted by  $x_4$ ,  $x_5$ ,  $x_6$ :

$$\max x \rightarrow \zeta = 3x_1 - 2x_2 - 6x_3 - x_4$$

subject to

$$x_3 = 1 + x_6 - 3x_2 - 2x_4$$

$$x_5 = 4 - 6x_6 + 11x_2 + 11x_4$$

$$x_1 = 0 - 1x_6 + 0x_2 + 1x_4$$

and  $x_1, x_2, x_3 \geq 0$ .

The cost coefficients for the problem are  $c_1 = 5$ ,  $c_2 = 3$  and  $c_3 = 3$  and the resource constraints are bounded by  $b_1 = 1$ ,  $b_2 = 9$  and  $b_3 = 1$ . We are interested in knowing the bounds of some of these variables such that the current basis holds. In other words we want to perform sensitivity analysis.

Now compute allowable changes  $\Delta b$  for the different resource bounds given below. In the case where you would like to enter plus infinity use the value 9999 and in the case you would like to use minus infinity then enter the value -9999. Note also that the decrease values are negative. Furthermore, report the values to 3 decimal places.

- What is the allowable decrease for  $b_1$ ?  ✖
- What is the allowable increase for  $b_1$ ?  ✖
- What is the allowable decrease for  $b_2$ ?  ✖
- What is the allowable increase for  $b_2$ ?  ✖

Some notes:

- The resource variable  $b_1$  belongs to an active constraint, while  $b_2$  to an inactive constraint.
- The inverse of the basis matrix  $B$  is found by taking the coefficient for the slack variables in the primal Simplex table (where all variables are on the left hand side or the =). That is,  $B^{-1} =$

$$20 \quad -1 \quad 11 \quad 16 \quad -101$$

Look the Simplex table given above to confirm.

- This matrix is then used to compute the allowable increase or decrease by simply knowing the fact that:

$$x \rightarrow B + \Delta x \rightarrow B = B^{-1} (b \rightarrow +\Delta b \rightarrow) \geq 0$$

That is, the changes  $\Delta b \rightarrow$  in  $b$  may not be such that  $x \rightarrow B + \Delta x \rightarrow B$  becomes negative.

The answers (recall that here 9999 denotes Infinity):

- What is the allowable decrease for  $b_1$ ? -0.5
- What is the allowable increase for  $b_1$ ? 0.0
- What is the allowable decrease for  $b_2$ ? -4.0
- What is the allowable increase for  $b_2$ ? 9999.0