

*Read each question thoroughly and motivate all of your answers with arguments and/or computations.*

*This is still a closed book exam. The use of a simple calculator is allowed. The use of graphical calculators, phones, watches and other electrical devices is prohibited. You can earn up to 50 points in this part.*

1. **[5 points]** Given the following situation:

Anti-spam techniques are used to prevent email spam and may be based, among all, on word checking.

- Explain how words in an email could be transformed from data into information and then into knowledge.
- Express the knowledge generated at the previous point into a rule which could be implemented into a knowledge based system.

2. **[10 points]** Let  $z = 1 + i$  and  $w = 2 - i$ .

- Compute  $\bar{z}$
- Compute  $|z|$
- Express  $z$  in trigonometric form and plot it in the complex plane.
- Compute  $\frac{\bar{z}}{w}$
- Find  $x \in \mathbb{C}$  such that:  $\bar{x} = i(x - 1)$

$\theta$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\infty$	0	$-\infty$	0

3. **[10 points]** Given the three time series:

$$x = \{1, 0, 1, 1\}, \quad y = \{0, 2, 1, -1\}, \quad z = \{1, 0, 0, 4\}$$

- Compute the absolute distances  $d(x, y)$ ,  $d(x, z)$ , and  $d(y, z)$ . What are the most similar time series?
  - Compute the infinity distances  $d(x, y)$ ,  $d(x, z)$ , and  $d(y, z)$ . What are the most similar time series?
  - Compute the Edit distances  $d(x, y)$ ,  $d(x, z)$ , and  $d(y, z)$ . What are the most similar time series?
4. **[10 points]** In a lab experiment four different measurements  $y_k$  of a certain reaction are obtained, depending on a variable  $x_k$ :

$x_k$	-1	0	1	2
$y_k$	0	1	1	3

The physicist who collected those is interested in fitting these measurements to a model of the form:  $y_k = ax_k + b$

Compute the optimal values for  $a$  and  $b$  according to the chosen model and the data collected during the experiment.

5. [5 points] Consider the experiment of picking a fruit at random from a bag, noting the type of fruit, and putting it back. The bag has 8 apples, 6 oranges, 3 bananas, and 5 pears.

- a. Complete the following table summarizing the probability distribution of this experiment:

Fruit type	Probability	Cumulative probability

- b. Write a rule that would allow you to simulate this experiment with a computer.

- c. Consider the following data set of four values  $\{-1, 0, 2, 2\}$ . Generate three bootstrap samples with replacement of the original data set, each containing four values (same amount of values as in the original dataset).

6. [10 points] Given the following game:

Grab the Dollar

A dollar is placed on a table between two players, A and B, who must each decide whether to grab it or not. If both grab, then they are both fined 1 dollar; if only one grabs, he keeps the dollar.

- a. Construct the payoff table.  
b. Is this a zero-sum game?  
c. Is there a Nash equilibrium for this game?

**Success!**