SOLUTIONS LECTURE GAME THEORY

PROBLEM 1

1. PAYOFF TABLE

PLAYER 2

PLAYER 1

STRATEGY	1	2	3	
1	1	-2	-3	-3
2	-1	2	-3	-3
3	-1	$\boxed{-2}$	3	$\boxed{-2}$
	1	2	3	

2. NO DOMINATED STRATEGIES

3. If we apply the minimax criterion

   * for player 1 : maximize the minimum payoff (maximum value) □
   * for player 2 : minimize the maximum payoff to player 1 (minimum value) ○

maximum and minimum values do not coincide, then there is no obvious strategy for either player in the game.

Any stable solution involves randomization.

4. Possible evolution of the game:

steps	Player 1	Player 2	Comments
1	3	1	Player 1 would lose 1, which makes player 1 unhappy, hence, player 1 switches to strategy 1
2	1	1	Player 1 wins 1 from player 2; player 2 switches to strategy 3
3	1	3	Player 2 wins 3 from player 1; player 1 switches to strategy 3
4	3	3	player 1 wins 3; player 2 switches to strategy 2
5	3	2	player 2 wins 2; player 1 switches to strategy 2
6	2	2	player 1 wins 2 player 2 switches to strategy 3

steps	player 1	player 2	comments
7	2	3	player 1 loses 3; player 1 switches to strategy 3
8	3	3	back to step 4 and infinite loop

PROBLEM 2:

		player 2		
player 1	strategy	1	2	3
	1	0, 4	$\boxed{4}$, 0	5, 3
	2	$\boxed{4}$, 0	0, 4	5, 3
	3	3, 5	3, 5	$\boxed{6}$, 6

1. By looking at the payoff table, it is clear that there is no dominated strategy for neither player 1 nor player 2.
Hence, dominated strategies cannot be used to find a solution to this game.

2. The Nash equilibrium of this game can be found by looking at the best response of one player to each strategy of the other player

- for player 1 (focus on columns - or strategies of player 2). □
- for player 2 (focus on rows - or strategies of player 1): ○

Then, Nash equilibrium is given by strategies (3, 3), which provide a payoff of (6, 6) to players 1 and 2, respectively

PROBLEM 3

1.

player 2

player 1			
strategy	1	2	
1	1	-1	-1
2	-1	1	-1
	1	1	

By applying minimax criterion:

* maximize minimum payoff player 1
* minimize maximum payoff player 2 to player 1

→ it can be noticed that there is no obvious solution (strategy) for either player to this game. Any stable solution involves randomization.

2. Since a stable solution involves randomization, this game does not have a pure Nash equilibrium.

If we tried to maximize the responses for both players, we will see that a combination of strategies that provides an optimal (obvious) choice for both players is missing.