

operations with complex numbers

1) algebraic addition

$$z = a + ib \quad w = c + id$$

$$z + w = a + ib + c + id = (a + c) + i(b + d)$$

2) multiplication

$$z \cdot w = (a + ib) \cdot (c + id) =$$

$$= ac + iad + ibc + i^2 bd =$$

$$= ac + iad + ibc - bd = (ac - bd) + i(ad + bc)$$

3) division

$$\frac{z}{w} = \frac{a + ib}{c + id} \quad \left\{ \begin{array}{l} \text{multiply both numerator and} \\ \text{denominator by complex conjugate} \\ \text{of denominator, in this case} \\ c - id \end{array} \right.$$

$$= \frac{a + ib}{c + id} \cdot \frac{c - id}{c - id} = \frac{ac - iad + ibc - i^2 bd}{c^2 - icd + icd + i^2 d^2} =$$

$$= \frac{ac - iad + ibc + bd}{c^2 + d^2} = \frac{(ac + bd)}{c^2 + d^2} + i \frac{(bc - ad)}{c^2 + d^2}$$

* complex conjugate

$$z = a + ib \rightarrow \text{complex conjugate } \bar{z} = a - ib$$

$$z \bar{z} = (a + ib)(a - ib) = a^2 + b^2 = \rho^2 = |z|^2$$