EXERCISES ON COMPLEX NUMBERS - SOLUTIONS

1. $z = 3+4i$

a. $Re(z) = 3 \qquad Im(z) = 4$

b. $\bar{z} = 3-4i$

c. $|z| = \sqrt{Re(z)^2 + Im(z)^2} = \sqrt{9+16} = \sqrt{25} = 5$

2. $z = 3+4i \quad , w = 2-i$

a. $z+w = 3+4i+2-i = 5+3i$

$z-w = 3+4i-2+i = 1+5i$

$z+\bar{w} = 3+4i+2+i = 5+5i$

$\overline{z+w} = \overline{(3+4i+2-i)} = \overline{(5+3i)} = 5-3i$

b. $zw = (3+4i)(2-i) = 6-3i+8i-4i^2 =$ $[i^2=-1]$

$= 6+5i+4 = 10+5i$

$$\frac{z}{w} = \frac{3+4i}{2-i} \cdot \frac{2+i}{2+i} = \frac{(3+4i)(2+i)}{5} = \frac{6+3i+8i+4i^2}{5} =$$

$$= \frac{6+11i-4}{5} = \frac{2+11i}{5} = \frac{2}{5} + \frac{11}{5}i$$

3. $z = \dfrac{i-4}{-3+2i} = \dfrac{i-4}{-3+2i} \cdot \dfrac{-3-2i}{-3-2i} = \dfrac{(i-4)(-3-2i)}{13} =$

$= \dfrac{-3i-2i^2+12+8i}{13} = \dfrac{-3i+2+12+8i}{13} =$

$= \dfrac{14+5i}{13} = \dfrac{14}{13} + \dfrac{5}{13}i \qquad Re(z) = \dfrac{14}{13} \qquad Im(z) = \dfrac{5}{13}$

4. $w = i^{17} = i \cdot i^{16} = i \cdot (i^4)^4 = i \cdot (1)^4 = i$

$|w| = \sqrt{0+1} = 1$

$\overline{w} = -i$

$$\left[\begin{array}{l} i = \sqrt{-1} \\ i^2 = -1 \\ i^3 = -1\sqrt{-1} \\ i^4 = 1 \end{array}\right]$$

5. a. $(1+2i)z + 3 + 4i = 0$

$\Rightarrow (1+2i)z = -3-4i \Rightarrow z = \dfrac{-3-4i}{1+2i} = \dfrac{-3-4i}{1+2i} \cdot \dfrac{1-2i}{1-2i} =$

$= \dfrac{(-3-4i)(1-2i)}{5} = \dfrac{-3+6i-4i+8i^2}{5} = \dfrac{-3+6i-4i-8}{5} =$

$= \dfrac{-11+2i}{5} = -\dfrac{11}{5} + \dfrac{2}{5}i$

b. $z + \overline{z} = 6 \Rightarrow Re(z) + \cancel{i\,Im(z)} + Re(z) - \cancel{i\,Im(z)} = 6$

$\Rightarrow 2Re(z) = 6 \Rightarrow Re(z) = 3$

$\Rightarrow z = 3 + ib \quad \forall b \in \mathbb{R}$

6. $|z| - z = i$

$$|z| = i + z = i + Re(z) + i\,Im(z) = i + Re(z) - i = Re(z)$$

since $|z|$ is a real number:

$$i + i\,Im(z) = 0 \Rightarrow Im(z) = -1$$

$$\Rightarrow |z| = Re(z) \Rightarrow \sqrt{Re^2(z) + Im^2(z)} = Re(z) \Rightarrow$$

$$\Rightarrow \sqrt{Re(z)^2 + 1} = Re(z) \Rightarrow Re(z)^2 + 1 = Re(z)^2 \Rightarrow 1 = 0$$

IMPOSSIBLE!

7. $z = \sqrt{2} + \sqrt{2}\,i$, $w = -i$

a.

z: $\rho = \sqrt{2+2} = 2$

$\theta = \text{atan}\,1 = \pi/4$

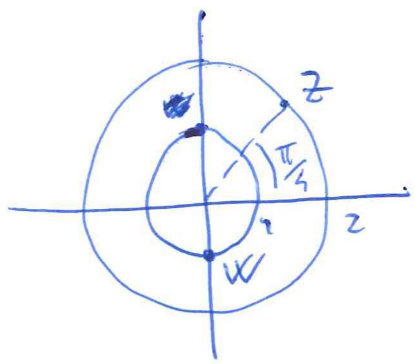
$z = \rho e^{i\pi/4}$

$= 2e^{i\pi/4}$

w: $\rho = \sqrt{0+1} = 1$

$\theta = \text{atan}\,{-\infty} \Rightarrow \theta = -\pi/2$

$w = e^{-i\pi/2}$



b. $zw = 2e^{i\pi/4} \cdot e^{-i\pi/2} = 2e^{i\pi/4 - i\pi/2} = 2e^{-i\pi/4}$

$$\frac{z}{w} = \frac{2e^{i\pi/4}}{e^{-i\pi/2}} = 2e^{i\pi/4}e^{i\pi/2} = 2e^{i\left(\pi/4 + \pi/2\right)} = 2e^{i\,3\pi/4}$$

$zw = 2e^{-i\pi/4}$

$2 = \sqrt{a^2+b^2} \Rightarrow a^2+b^2 = 4 \Rightarrow a^2 + (-a)^2 = 4 \Rightarrow a^2 = 2$

$\Rightarrow a = \pm\sqrt{2}$

$\tan(-\pi/4) = b/a \Rightarrow b/a = -1 \Rightarrow [b = -a]$

$\Rightarrow b = \mp\sqrt{2}$

$\Rightarrow \theta = -\pi/4$ then $zw = \sqrt{2} - i\sqrt{2}$

$\frac{z}{w} = 2e^{i3\pi/4} \Rightarrow \sqrt{a^2+b^2} = 2 \Rightarrow a^2+b^2 = 4$ $\quad a = \pm\sqrt{2}$

$\tan(3\pi/4) = b/a \Rightarrow b/a = -1 \Rightarrow b = \mp\sqrt{2}$

$\Rightarrow \theta = 3\pi/4$ then $\frac{z}{w} = -\sqrt{2} + i\sqrt{2}$