SOLUTIONS EXERCISE LECTURE 4 – LINEAR REGRESSION

X_k	-1	0	2	3
Y_k	1	2	2	4

model : $Y_k = a X_k + b$

- estimate optimal values of a and b according to the chosen model and the data collected during the experiment

→ given all the errors between the observations and the output of the model :

$$e_k = Y_k - (X_k a + b)$$

$$e_1 = 1 - (-a + b)$$

$$e_2 = 2 - (b)$$

$$e_3 = 2 - (2a + b)$$

$$e_4 = 4 - (3a + b)$$

minimize $V = \sum_{i=1}^{4} e_i^2$: $\min_{a,b} V$

$$V = [1-(-a+b)]^2 + [2-b]^2 + [2-(2a+b)]^2 + [4-(3a+b)]^2$$

$$= [1+a-b]^2 + [2-b]^2 + [2-2a+b]^2 + [4-3a-b]^2$$

$$= 1 + a^2 + b^2 + 2a - 2b - 2ab +$$
$$4 + b^2 - 4b +$$
$$4 + 4a^2 + b^2 - 8a - 4b + 4ab +$$
$$16 + 9a^2 + b^2 - 24a - 8b + 6ab$$

$$= 14a^2 + 4b^2 - 30a - 18b + 8ab$$

$$\frac{\partial V}{\partial a} \text{ (b const)} = 28a - 30 + 8b = 0$$

$$\frac{\partial V}{\partial b} \text{ (a const)} \quad 8b - 18 + 8a = 0$$

$$\begin{cases} 28a + 8b - 30 = 0 \\ 8a + 8b - 18 = 0 \end{cases} \quad \begin{cases} 28a + 18 - 8a - 30 = 0 \\ 8b = 18 - 8a \end{cases} \quad \rightarrow$$

$$\rightarrow \begin{cases} 20a = 12 \\ 8b = 18 - 8a \end{cases} \rightarrow \begin{cases} a = \frac{12}{20} = \frac{3}{5} \\ b = \frac{18}{8} - \frac{8 \cdot \frac{3}{5}}{8} = \frac{9}{4} - \frac{3}{5} = \frac{45-12}{20} = \end{cases}$$

$$= \frac{33}{20}$$

$$\rightarrow \text{optimal model:}$$

$$\boxed{Y_k = \frac{3}{5} X_k + \frac{33}{20}}$$