SOLUTIONS MOCK EXAM IKE

1\. a. data: a person's blood pressure value

information: blood pressure is normal or not

b. data: blood pressure from a person measured over so many days

information: blood pressure over so many days stays high, then it's an indication of hypertension

knowledge: if a person is showing signs of hypertension, then they should be given medications to reduce blood pressure

c. IF blood pressure over so many days > threshold, THEN give medications to reduce it

2. $z = 1+3i \quad , w = 2-3i$

a $\bar{z} = 1-3i$ (algebraic inverse of the imaginary part)

b. $|z| = \sqrt{1+9} = \sqrt{10}$

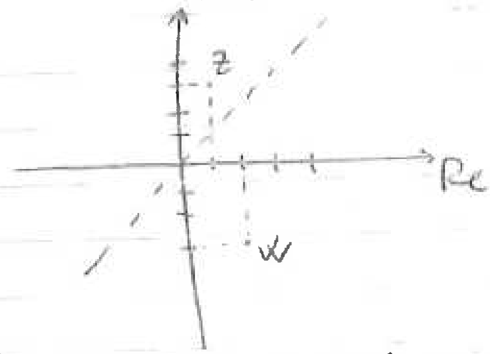
c. $z = 1+3i = \rho e^{i\theta}$ $\qquad$ $w = 2-3i = \rho e^{i\theta}$

$\rho = |z| = \sqrt{10}$ $\qquad$ $\rho = |w| = \sqrt{4+9} = \sqrt{13}$

$\theta = \operatorname{atan}\frac{3}{1} = \operatorname{atan} 3$ $\qquad$ $\theta = \operatorname{atan}\left(-\frac{3}{2}\right)$

given:
$z = a+ib$
$\rho = \sqrt{a^2+b^2}$
$\theta = \operatorname{atan}\left(\frac{b}{a}\right)$

$\simeq 1.3 > \pi/4$ $\qquad$ $\simeq -0.98 > -\pi/4$



d. $\frac{\bar{z}}{w} = \frac{1-3i}{2-3i} \cdot \frac{2+3i}{2+3i} = \frac{2+3i-6i+9}{13} = \frac{11-3i}{13}$

$= \frac{11}{13} - \frac{3}{13}i$

$\left[z\bar{z} = (a+ib)(a-ib) = \right.$
$\left. = a^2 \cancel{+iab} \cancel{+iab} - i^2b^2 = a^2+b^2 = \rho^2 = |z|^2 \right]$

e. $z^2 = -1 \Rightarrow z = \pm\sqrt{-1} = \pm i$

by definition $i = \sqrt{-1}$

$\Rightarrow z_1 = i$ and $z_2 = -i$

3) $x = \{2, 1, 2, 0\}$

$y = \{0, 1, 2, 5\}$

$z = \{-1, 2, 3, 0\}$

Euclidean distance

$$d(x,y) = \sqrt{\sum_{i=1}^{N} (y_i - x_i)^2}$$

a. $d(x,y) = \sqrt{(2-0)^2 + (1-1)^2 + (2-2)^2 + (0-5)^2} =$

$= \sqrt{4 + 0 + 0 + 25} = \sqrt{29}$

$d(x,z) = \sqrt{(2+1)^2 + (1-2)^2 + (2-3)^2 + (0-0)^2} =$

$= \sqrt{9 + 1 + 1 + 0} = \sqrt{11}$

$d(y,z) = \sqrt{(0+1)^2 + (1-2)^2 + (2-3)^2 + (5-0)^2} =$

$= \sqrt{1 + 1 + 1 + 25} = \sqrt{28}$

smallest distance : $d(x,z)$, time series x and z are the most similar time series, w.r.t. the Euclidean distance

b. $d(x,y) = 2\cdot 0 + 1\cdot 1 + 2\cdot 2 + 0\cdot 5 = 5$

$d(x,z) = 2\cdot(-1) + 1\cdot 2 + 2\cdot 3 + 0\cdot 0 = 6$

$d(y,z) = 0\cdot(-1) + 1\cdot 2 + 2\cdot 3 + 5\cdot 0 = 8$

$d(x,y)$ is the smallest, then x and y are the most similar time series, w.r.t. the distance defined at this point

c. $d(z,w) = \sum_{i=1}^{N} z_i w_i$

1. $d(z,w) \geq 0$ NOT FULFILLED

2. $d(z,w) = d(w,z)$ FULFILLED

3. $d(z,w) = 0$ iff $z = w$ NOT FULFILLED

4.

t_k	1	2	3	4
y_k	-1	1	0	1

model : $y_k = a t_k + b$ error $e_k = y_k - (a t_k + b)$

$e_1 = -1 - (a + b) = -1 - a - b$

$e_2 = 1 - (2a + b) = 1 - 2a - b$

$e_3 = 0 - (3a + b) = -3a - b$

$e_4 = 1 - (4a + b) = 1 - 4a - b$

$V = e_1^2 + e_2^2 + e_3^2 + e_4^2$

$\min_{a,b} V$

$$V = (-1-a-b)^2 + (1-2a-b)^2 + (-3a-b)^2 + (1-4a-b)^2$$

$$= 1 + a^2 + b^2 + 2a + 2b + 2ab$$

$$+ 1 + 4a^2 + b^2 - 4a - 2b + 4ab$$

$$+ 9a^2 + b^2 + 6ab$$

$$+ 1 + 16a^2 + b^2 - 8a - 2b + 8ab$$

$$\frac{\partial V}{\partial a} \text{ (b const)} = 2a + 2 + 2b$$

$$+ 8a - 4 + 4b$$

$$+ 18a + 6b$$

$$+ 32a - 8 + 8b = 60a - 10 + 20b = 0$$

$$\frac{\partial V}{\partial b} \text{ (a const)} = 2b + 2 + 2a$$

$$+ 2b - 2 + 4a$$

$$+ 2b + 6a$$

$$+ 2b - 2 + 8a = 8b - 2 + 20a = 0$$

$$\begin{cases} 60a - 10 + 20b = 0 & \rightarrow b = -3a + \frac{1}{2} \\ 8b - 2 + 20a = 0 & \rightarrow 24a - 4 - 2 + 20a = 0 \end{cases}$$

$b = -3a + 1/2$

$-24a + 4 - 2 + 20a = 0$

$-4a = -2 \Rightarrow a = \frac{1}{2}$

$b = -\frac{3}{2} + \frac{1}{2} = -1$

optimal model. $\boxed{y_k = \frac{1}{2} t_k - 1}$

5.

a

Outcome value	Probability	Cumulative Prob.
1	1/6	1/6
2	1/6	2/6 = 1/3
3	1/6	3/6 = 1/2
4	1/6	4/6 = 2/3
5	1/6	5/6
6	1/6	6/6 = 1

b. Generate a random number from a uniform distribution [0, 1)

If the random number is less than $\frac{1}{6}$, then the outcome is 1;
if it is between $\frac{1}{6}$ and $\frac{1}{3}$, then the outcome is 2;
if it is between $\frac{1}{3}$ and $\frac{1}{2}$, then the outcome is 3
if it is between $\frac{1}{2}$ and $\frac{2}{3}$, than the outcome is 4
if it is between $\frac{2}{3}$ and $5/6$, than the outcome is 5
if it is between $5/6$ and 1, then the outcome is 6

7.

		P2		
P1	strategy	1	2	3
	1	$\boxed{5}$, 1	0, 4	1, [illegible]
	2	3, 1	0, 2	$\boxed{3}$, 5 ← Nash equilibrium
	3	3, 1	4, 4	2, 5

· Nash equilibrium:

* Best response player 1 to each strategy of player 2 (focus on columns) $\square$

* Best response player 2 to each strategy player 1 (focus on rows) ○

⇒ Nash equilibrium: (2, 3) which gives payoff P1 of 3 and to P2 of 5

- dominated strategies

	1	2	3
1	~~5,1~~	0,4	1,0
2	~~3,1~~	0,2	3,5
3	~~2,1~~	4,4	2,5

- strategy 1 of player 2 is dominated by strategy 2
(4>1, 2>1, 4>1)

	2	3
~~1~~	~~0,4~~	~~1,0~~
2	0,2	3,5
3	4,4	2,5

- strategy 1 of player 1 dominated by strategy 2

	2	3
2	~~0,2~~	3,5
3	~~4,4~~	2,5

- strategy 2 of player 2 dominated by strategy 3

	3
2	3,5
~~3~~	~~2,5~~

- strategy 3 of player 1 dominated by strategy 2

$\Rightarrow$ dominant strategy (2,3) with payoffs (3,5)