CS3081 Assignment 3

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Question 1 (Problem 4.26)

(i) (a) = 4, (b)= 7
(ii) (a)=2.2, (b)=7
(iii) (a)=4, (b)=2.2
(iv) (a)=7, (b)=4

Your Answer ((i)-(iv)): (i) : (a) = 4, (b)= 7

Matlab Code:

```
A1 = [
      -2, 1, 0;
      1, -2, 1;
      0, 1, -1.5;
   ];


N = InfinityNorm(A1);
disp(N);


A2 = [
      4, -1, 0, 1, 0;
      -1, 4, -1, 0, 1;
      0, -1, 4, -1, 0;
      1, 0, -1, 4, -1;
      0, 1, 0, -1, 4;
   ];
```

```
N = InfinityNorm(A2);
disp(N);


function [ N ] = InfinityNorm( A )
    N = 0;
    [m, n] = size(A);
    for i = 1:m
        sum = 0;
        for j = 1:n
            sum = sum + abs(A(i,j));
        end
        if sum > N
            N = sum;
        end
    end
end
```

Question 2 (Problem 6.13)

(i) 420W
(ii) 420KW
(iii) 530W
(iv) 580KW

Your Answer ((i)-(iv)): (iii) : 530W

Wind Speed(mph)	14	22	30	38	46
Electric Power (W)	320	490	540	500	480

$$f(x) = \frac{(x-22)(x-30)(x-38)(x-46)}{(14-22)(14-30)(14-38)(14-46)}320 + \frac{(x-14)(x-30)(x-38)(x-46)}{(22-14)(22-30)(22-38)(22-46)}490 + \frac{(x-14)(x-22)(x-38)(x-46)}{(30-14)(30-22)(30-38)(30-46)}540 + \frac{(x-14)(x-22)(x-30)(x-46)}{(38-14)(38-22)(38-30)(38-46)}500 + \frac{(x-14)(x-22)(x-30)(x-38)}{(46-14)(46-22)(46-30)(46-38)}480$$

$$f(x) = \frac{(x-22)(x-30)(x-38)(x-46)}{98304}320 + \frac{(x-14)(x-30)(x-38)(x-46)}{-24576}490 + \frac{(x-14)(x-22)(x-38)(x-46)}{16384}540 + \frac{(x-14)(x-22)(x-30)(x-46)}{-24576}500 + \frac{(x-14)(x-22)(x-30)(x-38)}{98304}480$$

$$f(26) = \frac{-3840}{98304}320 + \frac{-11520}{-24576}490 + \frac{11520}{16384}540 + \frac{3840}{-24576}500 + \frac{2304}{98304}480$$

$$f(26) = -12.5 + 229.6875 + 379.6875 + (-78.125) + 11.25$$

$$f(26) = 530 \text{ W}$$

Question 3 (Problem 8.7)

The truncation error is:

(i) O(h)
(ii) O(h^2)
(iii) O(h^3)
(iv) O(h^4)

Your Answer ((i)-(iv)): (ii) O(h^2)

Taylor series expansion for point xi+1:

$$f(x_{i+1}) = f(x_i) + f'(x_i)h + \frac{f''(x_i)h^2}{2!} + \frac{f'''(x_i)h^3}{3!}$$

Taylor series expansion for point xi-1:

$$f(x_{i-1}) = f(x_i) - f'(x_i)2h + \frac{f''(x_i)(2h)^2}{2!} - \frac{f'''(x_i)(2h)^3}{3!}$$

Add them

$$f(x_{i+1}) + f(x_{i-1}) = 2f(x_i) - f'(x_i)(h) + 5\frac{f''(x_i)h^2}{2!} - 7\frac{f'''(x_i)(h)^3}{3!}$$

Solve for f''(xi) and introduce truncation error

$$5\frac{f''(x_i)h^2}{2!} = -f(x_{i+1}) - f(x_{i-1}) - 2f(x_i) + f'(x_i)(h) + 7\frac{f'''(x_i)(h)^3}{3!}$$

$$f''(x_i) = (2!)\left(\frac{-f(x_{i+1}) - f(x_{i-1}) - 2f(x_i) + f'(x_i)(h) + 7\frac{f'''(x_i)(h)^3}{3!}}{5h^2}\right)$$

$$f''(x_i) = (2!)\left(\frac{-f(x_{i+1}) - f(x_{i-1}) - 2f(x_i) + f'(x_i)(h) + 7\frac{f'''(x_i)(h)^3}{3!}}{5}\right) + O(h^2)$$

Question 4 (Problem 8.9)

(i) **f'_male(2006)=4965;**
f'_female(2006)=10681;
Predicted_Males(2008)=673601;
Error_Males=0.62%;
Predicted_Females(2008)=277990;
Error_Females=0.58%

(ii) **f'_male(2006)=4940;**
f'_female(2006)=10681;
Predicted_Males(2008)=673601;
Error_Males=0.62%;
Predicted_Females(2008)=277987;
Error_Females=0.57%

(iii) **f'_male(2006)=4940;**
f'_female(2006)=10681;
Predicted_Males(2008)=673601;
Error_Males=0.68%;
Predicted_Females(2008)=277987;
Error_Females=0.42%

(iv) **f'_male(2006)=4965;**
f'_female(2006)=10670;
Predicted_Males(2008)=673601;
Error_Males=0.68%;
Predicted_Females(2008)=277987;
Error_Females=0.52%

Your Answer ((i)-(iv)): (ii)

Year	1980	1990	2000	2002	2003	2006	2008
# Males	413,395	511,227	618,182	638,182	646,493	665,647	677,807
# Females	54,284	104,194	195,537	215,005	225,042	256,257	276,417

(a)

$$f'(x_{i+2}) = \frac{x_{i+2}-x_{i+1}}{(x_i-x_{i+1})(x_i-x_{i+2})}y_i + \frac{x_{i+2}-x_i}{(x_{i+1}-x_i)(x_{i+1}-x_{i+2})}(y_{i+1}) + \frac{x_{i+2}-x_i}{(x_{i+1}-x_i)(x_{i+1}-x_{i+2})}(y_{i+2})$$

$$x_{i+2} = 2006, x_{i+1} = 2003, x_i = 2002$$

$$f'(2006) = \frac{3}{4}y_i + \frac{4}{3}(y_{i+1}) + \frac{7}{12}(y_{i+2})$$

$$f'_{female}(2006) = 10681, \quad f'_{male}(2006) = 4940$$

(b)

$$f'(x_{i+2}) = \frac{x_{i+2}-x_{i+1}}{(x_i-x_{i+1})(x_i-x_{i+2})}y_i + \frac{x_{i+2}-x_i}{(x_{i+1}-x_i)(x_{i+1}-x_{i+2})}(y_{i+1}) + \frac{x_{i+2}-x_i}{(x_{i+1}-x_i)(x_{i+1}-x_{i+2})}(y_{i+2})$$

$$x_{i+2} = 2006, x_{i+1} = 2003, x_i = 2002$$

$$f'(2006) = -\frac{2}{15}y_i - \frac{1}{6}(y_{i+1}) + \frac{3}{10}(y_{i+2})$$

Predicted Female 2008 :

$$10681 = -\frac{2}{15}(225042) - \frac{1}{6}(256257) + \frac{3}{10}(y)$$

$$y = 277987$$

$$ERROR: \left|1 - \frac{276419}{277987}\right| = 0.00564 = 0.564\%$$

Predicted Male 2008 :

$$4940 = -\frac{2}{15}(646493) - \frac{1}{6}(665647) + \frac{3}{10}(y)$$

$$y = 673601$$

$$ERROR: \left|1 - \frac{677807}{673601}\right| = 0.006244 = 0.6244\%$$