1.4

vibratory system
- storing potential energy (e.g. spring)
- 、、 kinetic energy (e.g. mass)
- lost energy (e.g. damper)

有限自由度的 system : discrete or lump parameter system

∞ 、、 : continuous or distributed system

現實皆是 ∞自由度 system (∵彈性变形)

1.5

Free vibration : system受初始擾动後，任其震动。

Force vibration : system 受外力(通常是重覆性的力)產生震动

if 外力頻率 = system的固有頻率(natural frequencies)

"resonance"(共振)

Undamped vibration : 振动过程中，無能量的損失
(無阻尼)

Linear vibration : 振动 system 中的所有組件,都呈線性
(spring · mass · damper)

Deterministic vibration : 作用在振动 system 上的 excitation (激勵)
(確定性) (force · motion)
在 any time 都是已知。而這種 excitation called determinstic

Random vibration : excitation 是 random e.g. 風

1.6

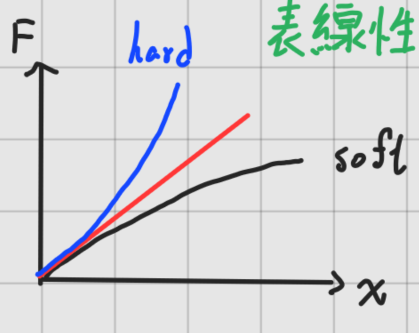
excitations (input) 、 responses (output)

1.7

linear spring : $F = kx$ 、 $U = \frac{1}{2}kx^2$

當 spring 变形較大時,會呈現非線性的 force - deflection

nonlinear spring : $F = \underbrace{ax}_{\text{表線性}} + \underbrace{bx^3}_{\text{表非線性}}$, $a > 0$



spring "hard" : $b > 0$
"linear" : $b = 0$
"soft" : $b < 0$

多個彈簧也可能展示出 nonlinear 的 force - deflection relation

1.7.2

spring force : $F + \Delta F = F(x^* + \Delta x)$

Taylor's series expansion : $= F(x^*) + \left.\frac{dF}{dx}\right|_{x^*} \Delta x + \cancel{\frac{1}{2!} \left.\frac{d^2F}{dx^2}\right|_{x^*} \Delta x^2} \cdots$

0 $\because \Delta x$ small

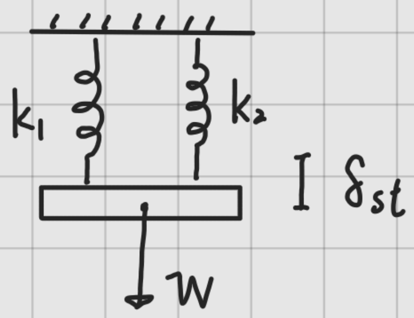
$\Rightarrow \cancel{F} + \Delta F = \cancel{F(x^*)} + \left.\frac{dF}{dx}\right|_{x^*} (\Delta x)$

$\Rightarrow \quad \Delta F = k \Delta x \qquad \Rightarrow \qquad k = \left.\frac{dF}{dx}\right|_{x^*}$

ex

1.7.4 Combination of spring

Case 1 : Springs in Parallel



$$W = k_1\delta_{st} + k_2\delta_{st} = \underbrace{(k_1+k_2)}\delta_{st}$$

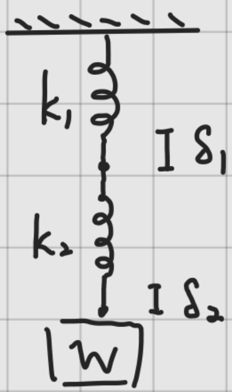
if set k_{eq}，使 ： $W = \underbrace{k_{eq}}\delta_{st}$ 成立

$$k_{eq} = k_1 + k_2$$

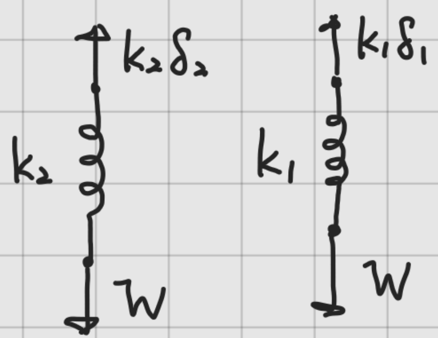
⇒ 有 n spring in Parallel ：

$$k_{eq} = k_1 + k_2 + \cdots + k_n$$

Case 2 : Springs in Series



total deflection : $\delta_{st} = \delta_1 + \delta_2$



$$W = k_1\delta_1 = k_2\delta_2$$

if set k_{eq}，使 : $W = k_{eq}\delta_{st}$ 成立

$$\Rightarrow W = k_{eq}\delta_{st} = k_1\delta_1 = k_2\delta_2$$

$$\Rightarrow \delta_1 = \frac{k_{eq}}{k_1}\delta_{st} , \quad \delta_2 = \frac{k_{eq}}{k_2}\delta_{st}$$

$$\delta_1 + \delta_2 = \delta_{st} = \left(\frac{k_{eq}}{k_1} + \frac{k_{eq}}{k_2}\right)\delta_{st}$$

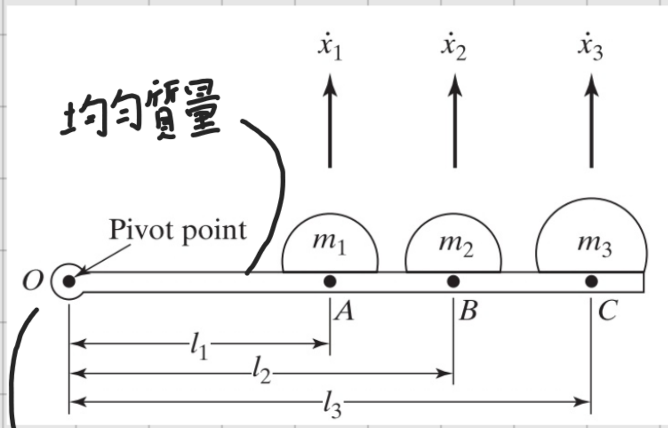
$$\Rightarrow \frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

n spring in Series : $\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \cdots \frac{1}{k_n}$

1.8 Combination of Masses

Case 1 :

均匀質量

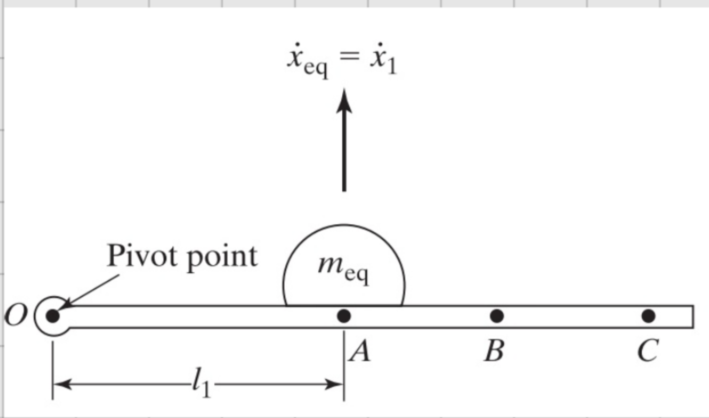


$$\dot{x}_2 = \frac{l_2}{l_1}\dot{x}_1$$

$$\dot{x}_3 = \frac{l_3}{l_1}\dot{x}_1$$

小角位移

等效:



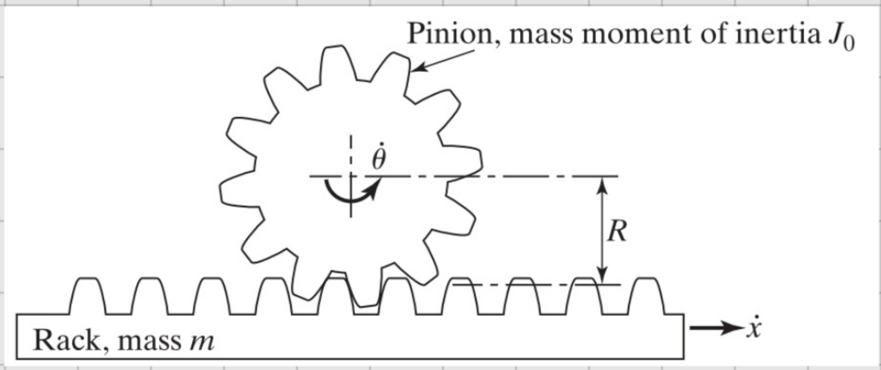
$$\dot{x}_1 = \dot{x}_{eq}$$

energy equaction :

$$\frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2 + \frac{1}{2}m_3\dot{x}_3^2 = \frac{1}{2}m_{eq}\dot{x}_{eq}^2$$

$$\Rightarrow m_{eq} = m_1 + \left(\frac{l_2}{l_1}\right)^2 m_2 + \left(\frac{l_3}{l_1}\right)^2 m_3$$

Case 2 :



1. Equivalent translational mass

$$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} J_0 \dot{\theta}^2 = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} J_0 \left(\frac{\dot{x}}{R}\right)^2$$

$$\parallel$$

$$T_{eq} = \frac{1}{2} m_{eq} \dot{x}_{eq}^2 \qquad ※ \; \dot{x}_{eq} = \dot{x}$$

$$\Rightarrow \quad m_{eq} = m + \frac{J_0}{R}$$

2. Equivalent rotational mass

$$\frac{1}{2} J_{eq} \dot{\theta}_{eq}^2 = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} J_0 \dot{\theta}^2 \qquad ※ \; \dot{\theta}_{eq} = \dot{\theta}$$

$$\Rightarrow \quad \frac{1}{2} J_{eq} \dot{\theta}^2 = \frac{1}{2} m (\dot{\theta} R)^2 + \frac{1}{2} J_0 \dot{\theta}^2$$

$$\Rightarrow \quad J_{eq} = J_0 + m R^2$$

1.9 Damping element

damping ： 振动能量逐漸轉化成 heat or sound 的机制

Type :

Viscous Damping : use 氣体、水、油等介質做為阻尼

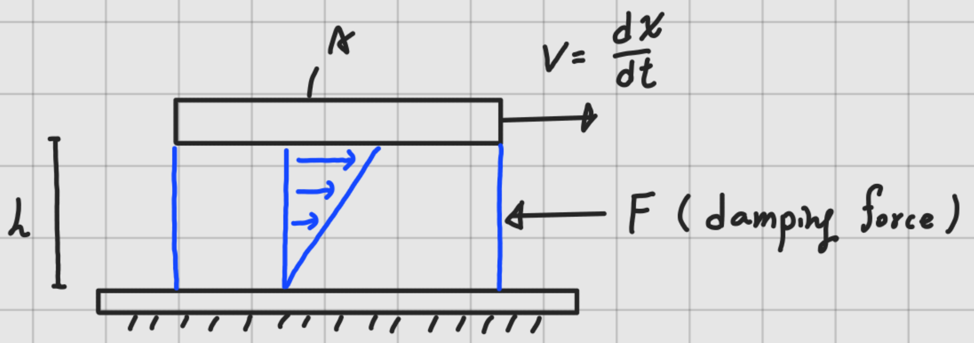
Coulomb or Dry-Friction Damping : use 摩擦力

Material or Solid or Hysteretic Damping :

當材料變形時,能量被材料吸收耗散

1.9.1 Viscous Damping

ex 1.13



$$\frac{du}{dy} = \frac{v}{h}$$

$$\tau = \mu \frac{du}{dy} \quad , \quad F = \tau A = \frac{\mu v}{h} A$$

set : $F = cv$, $c = \frac{\mu A}{h}$

Nonlinear Damper : $F = F(v)$, $c = \left.\frac{dF}{dv}\right|_{v^*}$

Equivalent damping const :

parallel dampers : $c_{eq} = c_1 + c_2$

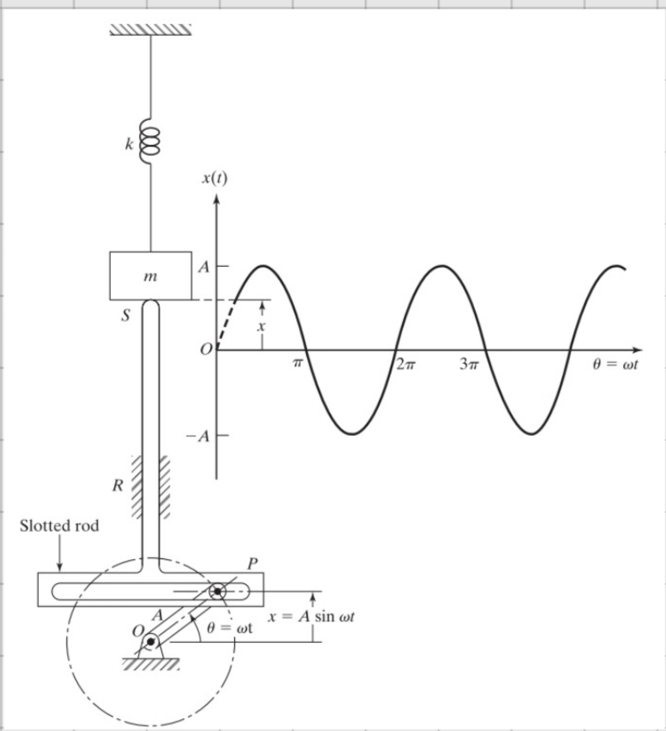
series dampers : $\frac{1}{c_{eq}} = \frac{1}{c_1} + \frac{1}{c_2}$

1.10 Harmonic motion

periodic motion : 運動在相等的時間間隔重覆出現

The simplest type of periodic motion is harmonic motion

e.g.



$x = A\sin\theta = A\sin\omega t$

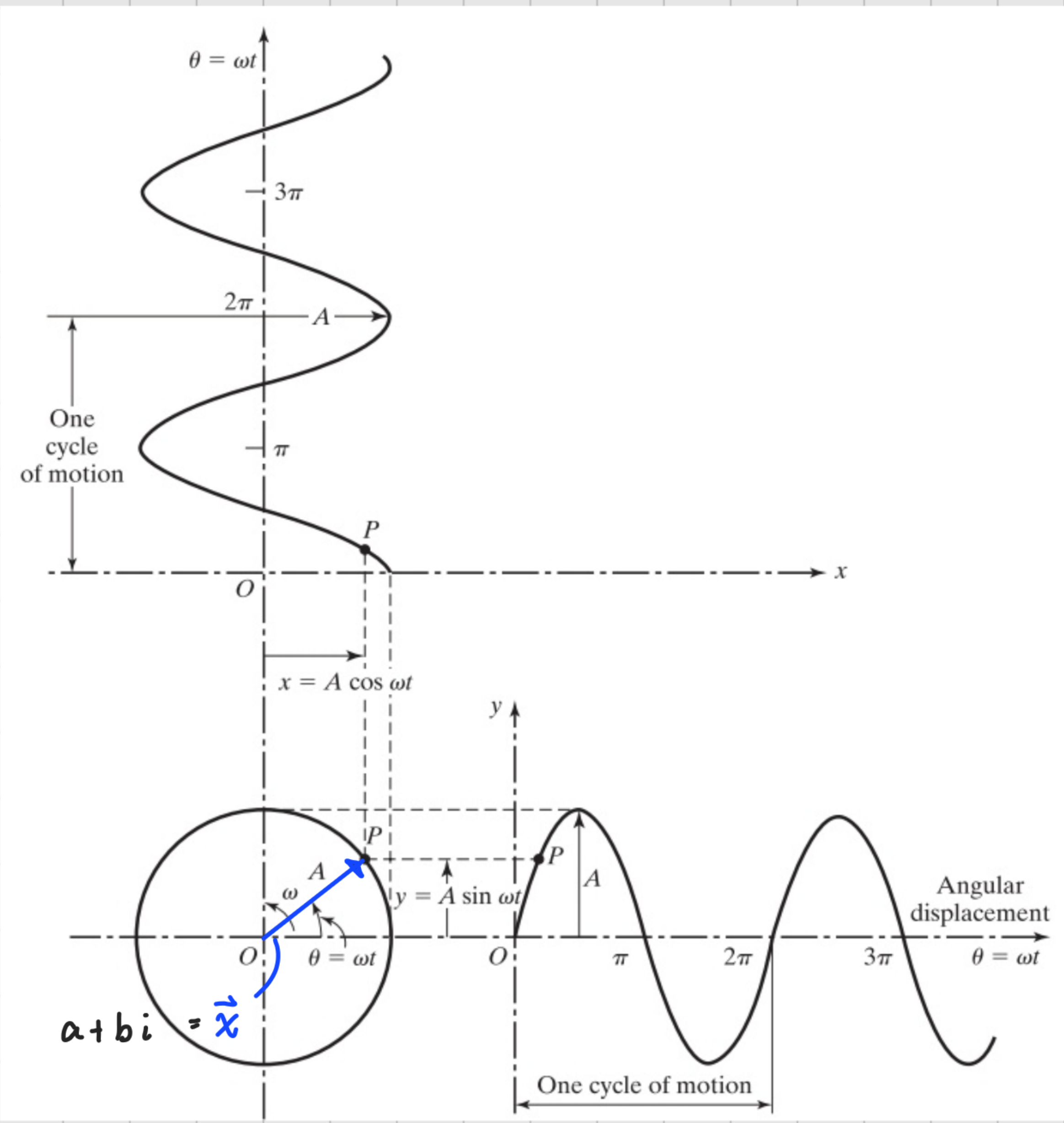
$\frac{dx}{dt} = \omega A\cos\omega t$

$\frac{d^2x}{dt^2} = -\omega^2 A\sin\omega t$

$= -\omega^2 x$

simple harmonic motion :

物体的加速度與位移成正比，且指向平衡位置



$$\vec{x} = A\cos\theta + iA\sin\theta \quad , \quad A = (a^2+b^2)^{1/2}$$

$$\theta = \tan^{-1}\frac{b}{a}$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots = 1 + \frac{(i\theta)^2}{2!} + \frac{(i\theta)^4}{4!} + \cdots$$

$$i\sin\theta = i\left[\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots\right] = i\theta + \frac{(i\theta)^3}{3!} + \cdots$$

$$\Rightarrow (\cos\theta + i\sin\theta) = e^{i\theta}$$

$$\Rightarrow \vec{x} = A(\cos\theta + i\sin\theta) = Ae^{i\theta}$$

※ 可用複救計算

set $\vec{X} = Ae^{i\omega t}$

displacement $= Re\left[Ae^{i\omega t}\right] = A\cos\omega t$

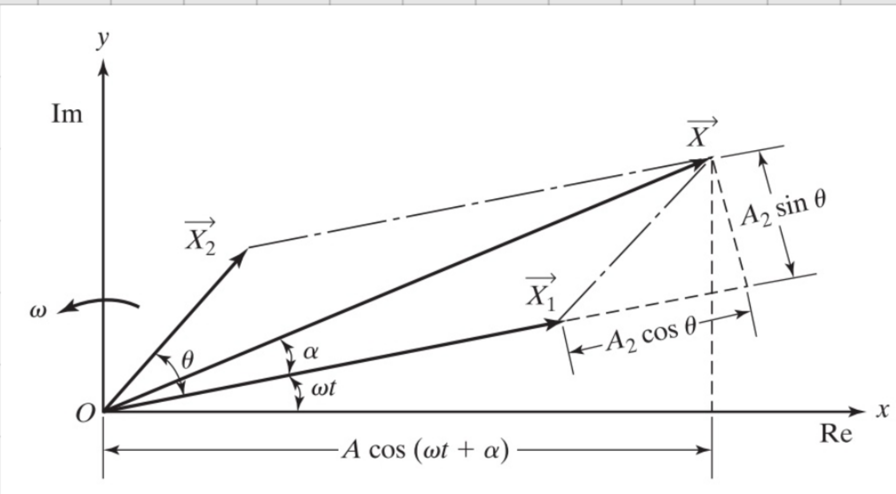
velocity $= Re\left[i\omega Ae^{i\omega t}\right] = -\omega A\sin\omega t$

acceleration $= Re\left[-\omega^2 Ae^{i\omega t}\right] = -\omega^2 A\cos\omega t$

if set $Re[\vec{x}_1] = A_1 \cos\omega t$

$$Re[\vec{x}_2] = A_2 \cos(\omega t + \theta)$$

则 $\vec{X} = \vec{x}_1 + \vec{x}_2$, $Re[\vec{X}] = A\cos(\omega t + \alpha)$



$$A = \sqrt{(A_1 + A_2\cos\theta)^2 + (A_2\sin\theta)^2}$$

$$\alpha = \tan^{-1}\left(\frac{A_2\sin\theta}{A_1 + A_2\cos\theta}\right)$$

Cycle

Amplitude

Period of oscillation : $\tau = \frac{2\pi}{\omega}$

Frequency of oscillation : $f = \frac{1}{\tau}$

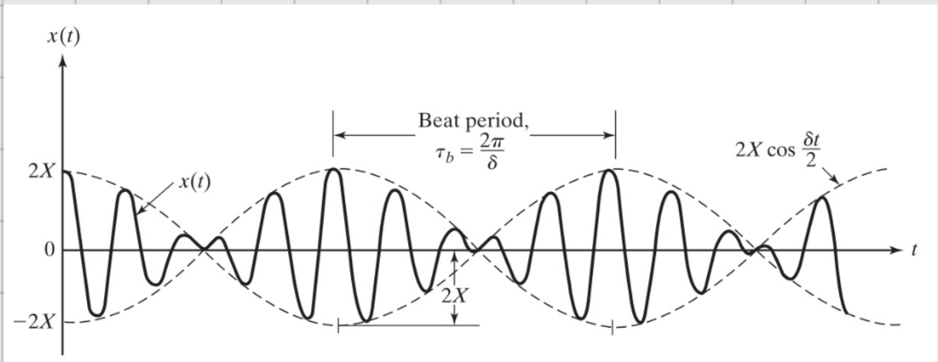
⋮

Beats : 兩頻率相近的 harmonic motion 相加

$$x_1(t) = X\cos\omega t$$

$$x_2(t) = X\cos(\omega+\delta)t$$

$$x(t) = x_1 + x_2 = X\left[\cos\omega t + \cos(\omega+\delta)t\right]$$



Octava : 一範圍的頻率，其 $f_{max} = 2 \cdot f_{min}$

e.g. 150 ~ 300 Hz

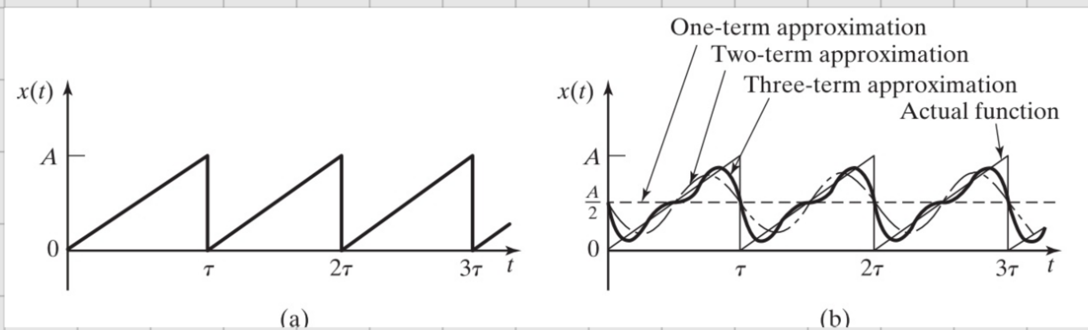
Decibel (分贝)：电功率的比值

$$dB = 10\log\left(\frac{P}{P_0}\right) = 20\log\left(\frac{X}{X_0}\right)$$

电压

1.11

Fourier Series 近似



$$x(t) = \frac{a_0}{2} + a_1\cos\omega t + a_2\cos 2\omega t + \cdots$$

$$+ b_1\sin\omega t + b_2\sin 2\omega t + \cdots$$

$$x(t) = \frac{a_0}{2} + \sum(a_n\cos n\omega t + b_n\sin n\omega t)$$

$$a_0 = \frac{\omega}{\pi}\int_0^{2\pi/\omega} x(t)\,dt = \frac{2}{\tau}\int_0^{\tau} x(t)\,dt$$

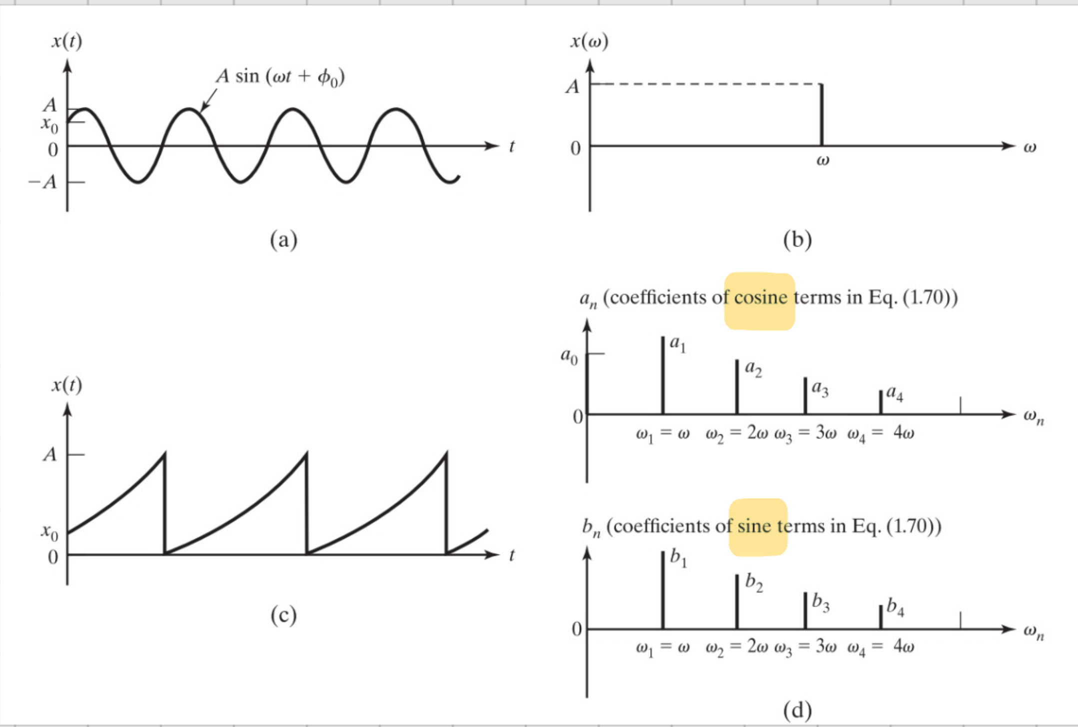
$$a_n = \frac{\omega}{\pi}\int_0^{\frac{2\pi}{\omega}} x(t)\cos n\omega t \, dt$$

$$b_n = \frac{\omega}{\pi}\int_0^{2\pi/\omega} x(t)\sin n\omega t$$

Gibbs' Phenomenon：用傅里叶近似時，隨 $n \to \infty$

振幅誤差仍會保持在 9% 左右

1.11.3 Frequency Spectrum



1.11.5 Even and Odd function

even : $x(-t) = x(t)$

$$\Rightarrow x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega t$$

odd : $x(-t) = -x(t)$

$$\Rightarrow x(t) = \sum_{n=1}^{\infty} b_n \sin n\omega t$$

1.11.6 Half - range