EDA

1) $f(x) = 2(1-x)$, $0 < x < 1$ Find variance & Sd

$$f(x) = 2 - 2x$$

$E(X) =$

$$= \int_0^1 (2-2x)x \, dx$$

$$= \int_0^1 2x - 2x^2 \, dx$$

$$= \left[\frac{2x^2}{2} - \frac{2x^3}{3} \right]_0^1$$

$$= \frac{2(1)^2}{2} - \frac{2(1)^3}{3} - \left[\frac{2(0)^2}{2} - \frac{2(0)^3}{3} \right]$$

$$= \frac{2}{2} - \frac{2}{3} - [0 - 0]$$

$$= \frac{\cancel{2}}{2} - \frac{2}{3}$$

$$= \frac{1}{1} - \frac{2}{3}$$

$$= \frac{3}{6} - \frac{2}{6}$$

$$E(X) = \frac{1}{3}$$

$$\mu^2 = E(x)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9} \quad -(1)$$

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$$E(x^2) = \int_0^1 (2-2x)x^2\,dx$$

$$= \int_0^1 2x^2 - 2x^3\,dx$$

$$= \left[\frac{2x^3}{3} - \frac{2x^4}{4}\right]_0^1$$

$$= \frac{2(1)^3}{3} - \frac{2(1)^4}{4} - \left[\frac{2(0)^3}{3} - \frac{2(0)^4}{4}\right]$$

$$= \frac{2}{3} - \frac{2}{4} - [0 - 0]$$

$$= \frac{8}{12} - \frac{6}{12}$$

$$= \frac{8-6}{12}$$

$$E(x^2) = \frac{1}{6} \quad -②$$

By ① & ②

Varionce $= \sigma^2 = E(x^2) - \mu^2$

$$= \frac{1}{6} - \frac{1}{9}$$

$$= \frac{9-6}{54}$$

$$= \frac{1}{18}$$

St.dev $= \sqrt{\sigma^2}$

$$= \sqrt{\frac{1}{18}} = 0.235$$

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2) $f(x,y) = K(x^2+y^2) \quad 30 \leq x < 50, \; 30 \leq y < 50$

(i) Find K

$$\int_{30}^{60}\int_{30}^{50} K(x^2+y^2)\,dx\,dy = 1$$

$$\int_{30}^{50}\int_{30}^{50} \left[Kx^2 + Ky^2\right] dx\,dy = 1$$

$$\int_{30}^{50} \left[\frac{Kx^3}{3} + Ky^2x\right]_{30}^{50} dy = 1$$

$$\int_{30}^{50} \left[\frac{K(50)^3}{3} + Ky^2(50) - \left[\frac{K(30)^3}{3} + Ky^2(30)\right]\right] dy = 1$$

$$\int_{30}^{50} \left[\frac{K(50)^3}{3} - \frac{K(30)^3}{3} + Ky^2(50) - Ky^2(30)\right] dy = 1$$

$$\int_{30}^{50} \left[\frac{K(50)^3}{3} - \frac{K(30)^3}{3} + Ky^2 20\right] dy = 1$$

$$\left[\frac{K(50)^3}{3}y - \frac{K(30)^3}{3}y + \frac{Ky^3}{3}20\right]_{30}^{50} = 1$$

$$1 = \frac{K(50)^3 50}{3} - \frac{K(30)^3 50}{3} + \frac{K(50)^3 20}{3} - \frac{K(50)^3 30}{3}$$
$$+ \frac{K(30)^3 30}{3} - \frac{K(30)^3 20}{3}$$

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$$K(2083333.3333 - 450000 - 1250000 + 270000 + 653333.33333 + 653333.3333) = 1$$

$$K(1306666.6666) = 1$$

$$K = \frac{1}{1306666.6666}$$

$$K = 7.653061224 \times 10^{-7}$$

(ii) $P(30 \le X \le 45 \ \& \ 40 \le Y < 50)$

$$\int_{40}^{50}\int_{30}^{45} Kx^2 + Ky^2 \, dx\, dy$$

$$\int_{40}^{50} \left[\frac{Kx^3}{3} + Ky^2x\right]_{30}^{45} dy$$

$$\int_{40}^{50} \left[\frac{K(45)^3}{3} + Ky^2(45) - \frac{K(30)^3}{3} - Ky^2(30)\right] dy$$

$$\int_{40}^{50} \left[\frac{K(45)^3}{3} - \frac{K(30)^3}{3} + Ky^2(45) - Ky^2(30)\right] dy$$

$$\int_{40}^{50} \left[0.0232461735 - 0.006887755 + Ky^2(45) - Ky^2(30)\right] dy$$

$$\int_{40}^{50} \left[0.0163584184 + Ky^2(45) - Ky^2(30)\right] dy$$

$$= \left[0.0163584184y + \frac{Ky^3(45)}{3} - \frac{Ky^3(30)}{3} \right]_{40}^{50}$$

$$= 0.0163584184(50) - 0.0163584184(40) + \frac{K(50)^3(45)}{3}$$
$$- \frac{K(50)^3(30)}{3} - \frac{K(40)^3(45)}{3} + \frac{K(40)^3(30)}{3}$$

$$= 1.4349489765 - 0.734693877\,5 - 0.956632653 + 0.4897959183$$

$$= 0.3970025483$$

c) $\int_{30}^{40}\int_{30}^{40} K(x^2+y^2)\,dx\,dy \quad 30 < x < 40,\ 30 < y < 40;$

$$K\int_{30}^{40} \left[\frac{x^3}{3} + y^2x \right]_{30}^{40} dy$$

$$K\int_{30}^{40} \left[\frac{(40)^3}{3} + y^2(40) - \frac{(30)^3}{3} - y^2(30) \right] dy$$

$$K\int_{30}^{40} \left[\frac{40^3}{3} - \frac{30^3}{3} + y^2(40) - y^2(30) \right] dy$$

$$K\int_{30}^{40} \left[\frac{40^3 - 30^3}{3} + y^2(40-30) \right] dy$$

$$K\left[y\left(\frac{40^3-30^3}{3}\right) + \frac{10y^3}{3} \right]_{30}^{40}$$

$$= K\left[40\left(\frac{40^3-30^3}{3}\right)+\frac{10(40)^3}{3}-\left[30\left(\frac{40^3-30^3}{3}\right)+\frac{10(30)^3}{3}\right]\right]$$

$$= K\left[40\left(\frac{40^3-30^3}{3}\right)-30\left(\frac{40^3-30^3}{3}\right)+\frac{10(40)^3}{3}-\frac{10(30)^3}{3}\right]$$

$$= K\left[\left(\frac{40^3-30^3}{3}\right)(40-30)+\frac{(40)^3-(30)^3}{3}(10)\right]$$

$$= K\left[\frac{40^3-30^3}{3}(10)+\frac{(40)^3-(30)^3}{3}(10)\right]$$

$$= K\left[(10+10)\left(\frac{40^3-30^3}{3}\right)\right]$$

$$= K\left[20\left(\frac{40^3-30^3}{3}\right)\right]$$

$= \text{W.K.T } K = 7.653061224 \times 10^{-7}$

$$= 7.653061224 \times 10^{-7}\left[20\left(\frac{40^3-30^3}{3}\right)\right]$$

$= 0.189$

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③ $f(x,y) = \frac{x+y}{30}$

Y/x	0	1	2	3
0	0	0.03333333	0.0666667	0.1
1	0.03333333	0.06666667	0.1	0.13333333
2	0.0666667	0.1	0.13333333	0.1666667

After substituting value of x, y into the equation $f(x,y) = \frac{x+y}{30}$ the above table was generated

(i) $P(x \leq 2 \quad y=1)$

$P(0,1) + P(1,1) + P(2,1)$

$0.03333333 + 0.06666667 + 0.1$

$= 0.2$

(ii) $P(x > 2, y \leq 1)$

$P(3,0) + P(3,1)$

$0.1 + 0.13333333$

$= 0.23333333$

(iii) $P(x > y)$

$P(1,0) + P(2,0) + P(3,0) + P(2,1) + P(3,1) + P(3,3)$

$0.03333333 + 0.06666667 + 0.1 + 0.1 + 0.13333333 + 0.16666667$

$= \underline{\underline{0.6}}$

(iv) $P(x + y = 3)$

$P(3,0) = 0.1$, $P(2,1) = 0.1$, $P(2,2) = 0.1$

$= 0.1 + 0.1 + 0.1 = \boxed{0.3}$

4)

Station	A	B	C	Total
Problem with electricity supplies	2	1	1	$\boxed{4}$
Computer malfunction	4	3	2	$\boxed{9}$
Malfunctioning electrical equip	5	4	2	$\boxed{11}$
Caused by other human error	7	7	5	$\boxed{19}$
Total	$\boxed{18}$	$\boxed{15}$	$\boxed{10}$	

(i) $\frac{5}{19} = 0.26315789$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

where A → Malfunction caused by Human error

B → Event that already occurred

(ii) $\frac{3}{9} = \frac{1}{3} = 0.33333$

$P(A/B) = \frac{P(A \cap B)}{P(B)}$

A → Computer Malfunction

B → Event that already caused

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5) <u>Given</u>

Defect in Brake = 0.25
Defect in transmition = 0.18
Defect in fuel = 0.17
other = 0.40

W.K.T $P(\text{Brake}) = 0.25$
$P(\text{Fuel} = 0.17$
$P(\text{Brake} \cap \text{Fuel}) = 0.15$ {Given}

$P(B) + P(F) - P(B \cap F)$

$= 0.25 + 0.17 - 0.15$

$= 0.27$