Week 1

The Fault in Our Stars

and Infinity

Introduction

Welcome to Math 88S: Mathematics & Movies!

- Thursday, 3:00 - 3:50
- MS 6201
- 1 unit P/NP
- Part of the Undergraduate Student Initiated Education (USIE)

Introduction

Goal of course: make you believe Math is...

Introduction

Goal of course: make you believe Math is...

Cool

Introduction

Goal of course: make you believe Math is...

Cool

Not Scary

Introduction

Goal of course: make you believe Math is...

Used to do more than split checks during dinner

Cool

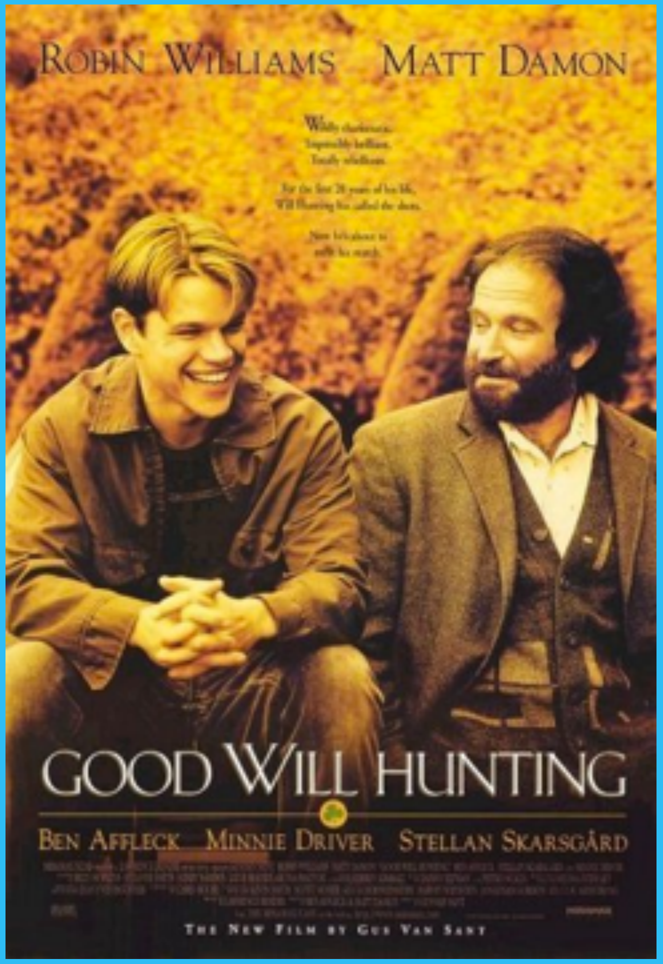
Not Scary

How to achieve this goal

Show students topics in higher level mathematics through the context of movies. Through the use of movies as visual aids and pop-culture references, framing mathematics in this context makes it more relatable and more easily understood.

Introduction

What we will NOT be discussing



Very classic, and well done, but not during the course

Introduction

Logistics:

- CCLE is up and running, but personally prefer using a personal website
  - all material will be posted on CCLE, but a more in-depth/interactive material is on public site
- Enrollment: I will never turn any student away from attending the seminars
- Actually enrolling: Waitlist and those who came out of interest (i.e. not enrolled or waitlist), we can talk to math department to see if we can officially enroll you

Introduction

Expectations:

- Attendance/participation: hopefully can attend all (or 9/10) sessions
- No laptops
- Food is allowed/encouraged
- Submit “final essay”
- **Learn something new**

Introduction

Final Essay:

"What is your perspective on math?"

- 1 page length
- written, typed, pictures, drawn, creative mediums of expression
- hand-in/email by Week 10 session

Introduction

Syllabus:

http://ruthjohnson95.github.io/Math88S/syllabus.pdf

Movie

Plot of the Fault in Our Stars

Hazel Grace Lancaster (Shailene Woodley), a 16-year-old cancer patient, meets and falls in love with Gus Waters (Ansel Elgort), a similarly afflicted teen from her cancer support group. Hazel feels that Gus really understands her. They both share the same acerbic wit and a love of books, especially Grace's touchstone, "An Imperial Affliction" by Peter Van Houten. When Gus scores an invitation to meet the reclusive author, he and Hazel embark on the adventure of their brief lives.

Movie


0:01 / 2:58
CC
HD
The Fault in Our Stars: Pre-funeral speech
Lokesh Pande
Subscribe
143
402,389
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More
2,477
27

Movie

"There are infinite numbers between 0 and 1. There's .1 and .12 and .112 and an infinite collection of others. Of course, there is a bigger infinite set of numbers between 0 and 2, or between 0 and a million. **Some infinities are bigger than other infinities."**

What does it mean to be “bigger” than others?

$2 < 3$?

What does it mean to be “bigger” than others?

$2 < 3$ ✓

Math

What does it mean to be "bigger" than others?

$$\infty > \infty ?$$

How do we **know** that $2 < 3$?

Math

Normally, we decide whether a number is greater than another number based on their **value**

$2 < $1 $1 $1

Notice that value and size are not always synonymous, as seen above

Math

But Infinity doesn't follow this rule…

.3333333… = 1/3

(multiply by 3)

.999999… = 1

Not true!

Math

But Infinity doesn't follow this rule…

$$.3333333\ldots = 1/3$$

(multiply by 3)

$$.999999\ldots = 1$$

Not true!

Thus, we can't use our usual notion of value to evaluate infinities

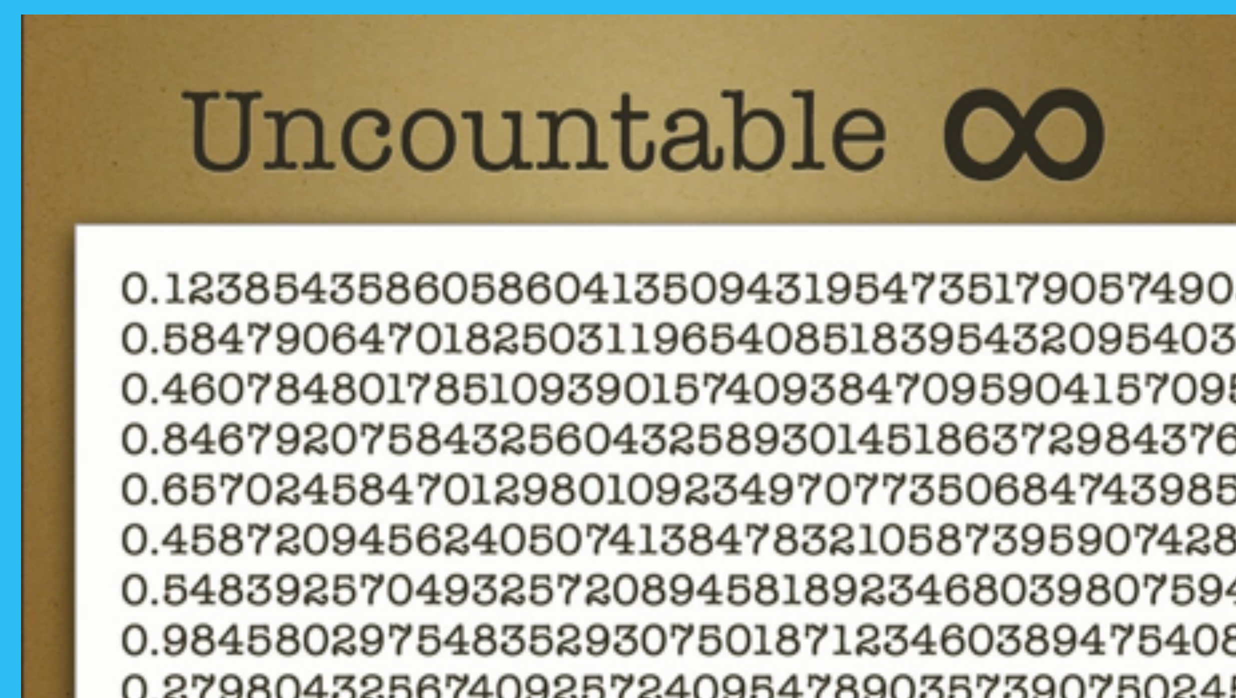
Math

Let's look at "size" of infinities:

Recall, from the Hotel Paradox, countable infinities. What about those infinities that we can't count?

Math

a countable set is any set in which all the terms can be associated with a natural number



Math

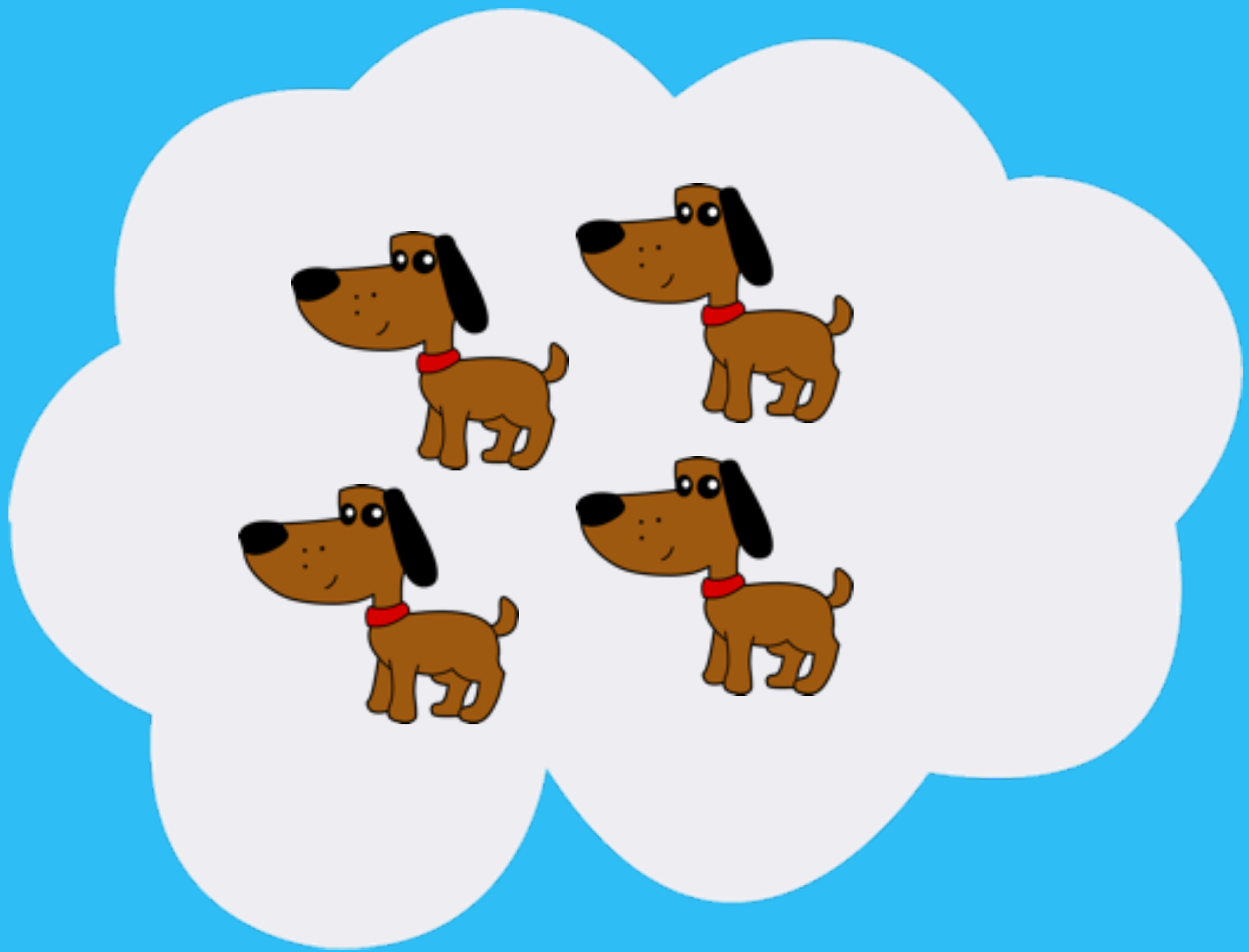
We don’t need to count! Just need to know how big. Are these notions the same?

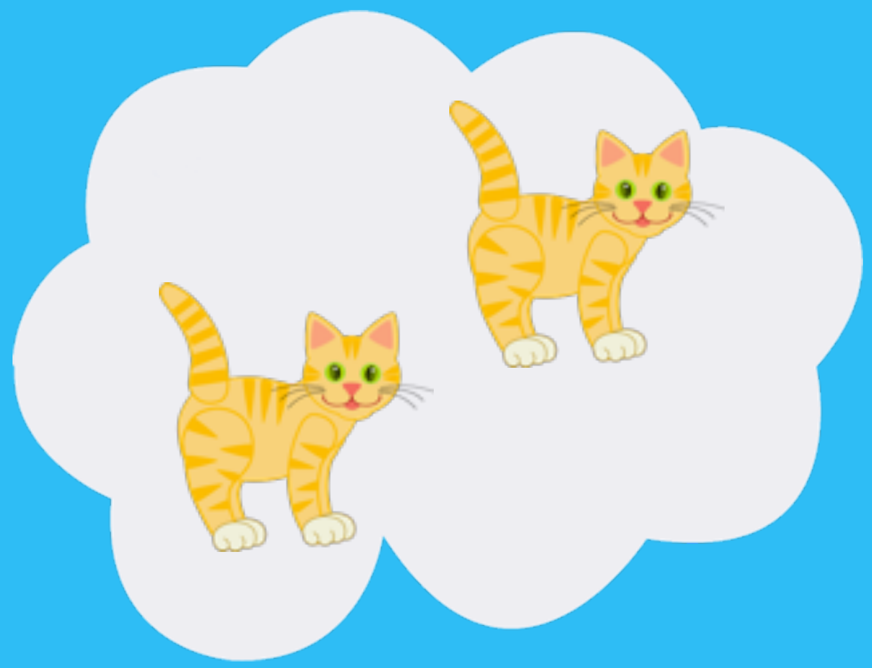
Let’s see!

Math

Example: Are there more cats than dogs?

Instead of counting, we can match members from each set, and if one set doesn't have a match, then it's larger.





Movie

Example: Are there more cats than dogs?

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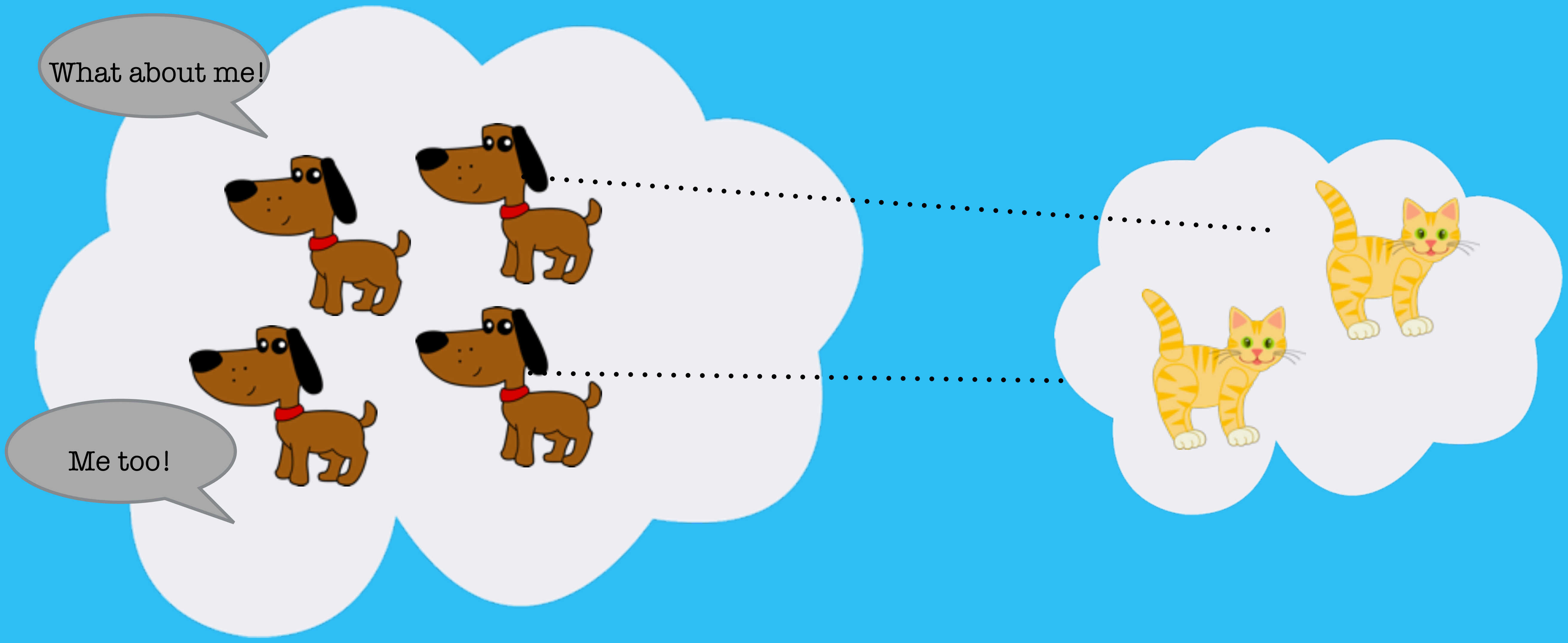


Movie

Thus, there are more dogs than cats, but we didn't even need to count them all to know.

We can do the same with infinities to see which ones are larger than other! We only need to find a mapping or pairing of sets such that they don't all match to see if one is larger. Recall the movie again...

There are infinite numbers between 0 and 1. There's .1 and .12 and .112 and an infinite collection of others. Of course, there is a bigger infinite set of numbers between 0 and 2, or between 0 and a million.

Movie

We'll use a very similar example to show that some infinities are bigger than others.

Consider 2 sets:

- whole numbers - a number w/out fraction
- real numbers - decimal numbers than are not complex

Is there a matching between these two infinite sets?

Movie

Whole Numbers

1, 2, 3, 4, 5, 6, 7, 8, 9...

Real Numbers

3.14159625..., 2, e,...

Movie

The answer is **FALSE**.

To prove this, we only must show that there is a number in the Real Number set that's not in the Whole Number set.

That's easy though…right? Can you prove it though?

Movie

To prove, we'll construct a unique number that's not in the whole number set. Without loss of generality, let's consider this set. Can we account for all of the decimal numbers?

We'll show that NO, it's not possible to count all of the decimal numbers.

.111111...

.000000...

.101010...

.110011...

Movie

In our first set, the set on the right and left have the same size. Now we can construct an element where nothing on the left side can map to it.

Sequence 1	.111111...
Sequence 2	.000000...
Sequence 3	.101010...
Sequence 4	.110011...

Movie

.111111...

.000000...

.101010...

.110011...

New decimal: .1

Movie

.111111...

.000000...

.101010...

.110011...

New decimal: .10

Movie

.111111...

.000000...

.101010...

.110011...

New decimal: .1010

Movie

.111111...

.000000...

.101010...

.110011...

New decimal: .1010

Invert the new decimal to get: .0101

This new decimal is not on our initial list. Consider repeating this until the infinite position. We've made an element that a whole number cannot map to because our first set had already mapped the infinite set of whole numbers already!

Movie

Sequence 1	.111111...
Sequence 2	.000000...
Sequence 3	.101010...
Sequence 4	.110011...
...	...
?	.0101

....

(no mapping)

Because there is no mapping, this shows that the size of the sets are different.

Math

We have proven that the infinite set of whole numbers is smaller than the infinite set of real numbers. This is an example of one infinity being bigger than another infinity.

$$\infty \quad < \quad \boldsymbol{\infty}$$

Summary

Size, not value

Show mapping to prove sizes are same/different

Prove different sizes by finding more elements in one set

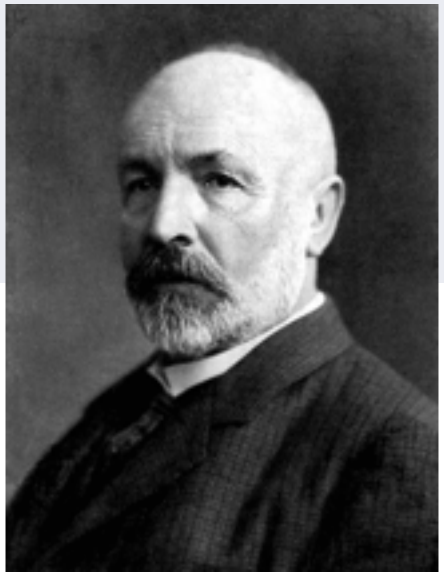
Conclude relative size of infinities

Summary

"rational numbers are like the stars in the sky and the irrational numbers are the black behind the stars"

MathBook

mathbook



Georg Cantor is contemplating the meaning of infinite sets

Wall Info Photos Boxes

Basic Information

Sex:	Male
Birthday:	March 3, 1845
Hometown:	Saint Petersburg, Russian Empire
Relationship Status:	Married to Vally Guttmann
Religious Views:	Lutheran

View photos of Georg (5)
Send Georg a message
Poke message

Information

Networks:
Washington D.C.
Birthday:
March 3, 1845
Religion:
Lutheran
Hometown:
Saint Petersburg, Russian Empire

Friends

Richard Dedekind

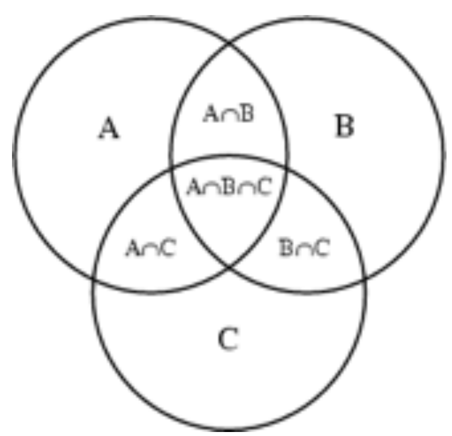
Gösta Mittag-Leffler

Write something…

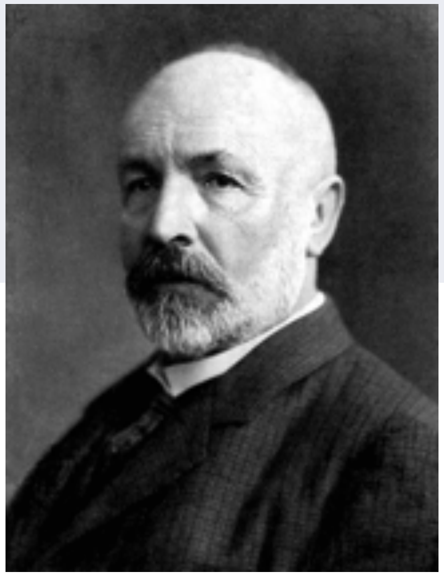
The essence of mathematics lies in its freedom.

Share

Georg Cantor created set theory



Georg Cantor started working at University of Halle at age 34 facebook Wall Photos Flair Boxes Georg Cantor Logout



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Created “Cantor's diagonal argument”

$$
\begin{aligned}
s_1 &= (\underline{0}, 0, 0, 0, 0, 0, 0, \ldots)\\
s_2 &= (1, \underline{1}, 1, 1, 1, 1, 1, \ldots)\\
s_3 &= (0, 1, \underline{0}, 1, 0, 1, 0, \ldots)\\
s_4 &= (1, 0, 1, \underline{0}, 1, 0, 1, \ldots)\\
s_5 &= (1, 1, 0, 1, \underline{0}, 1, 1, \ldots)\\
s_6 &= (0, 0, 1, 1, 0, \underline{1}, 1, \ldots)\\
s_7 &= (1, 0, 0, 0, 1, 0, \underline{0}, \ldots)\\
&\ldots\\
s &= (\underline{1}, \underline{0}, \underline{1}, \underline{1}, \underline{1}, \underline{0}, \underline{1}, \ldots)
\end{aligned}
$$

checked into mental institution from depression.

passed away in 1918

Closing

The end?

The end