1. Convert the following six statements into First Order Logic (FOL). Translate the obtained expressions to Conjunctive Normal Forms (CNFs, Steps 1-6 of Lecture 9).

1. You have a dog.
2. Amy buys cat food.
3. Anyone who owns a cat hates anything that chases any cat.
4. Every dog chases some cat.
5. Anyone who buys cat food owns a cat.
6. Someone who hates something owned by another person will not marry that person.

2.Transform the following conclusion into FOL, negate it and convert it to CNF (Steps 1-6 of Lecture 9: Knowledge Representation and Reasoning).

Amy will not marry you.

3. Based on all the previously created CNFs, finalise the conversion to CNF (Steps 7-8 of Lecture 9) and prove the statement from 2.

The FOL expression corresponding to the six statements are:

1. $\exists x\ Dog(x) \wedge Own(You,x)$
2. $Buy(Amy)$
3. $\forall x\ (\exists y(Own(x,y) \wedge Cat(y)) \Rightarrow (\forall z\ (\exists w\ (Cat(w) \wedge Chase(z,w)) \Rightarrow Hate(x,z))))$
4. $\forall x\ Dog(x) \Rightarrow \exists y\ (Cat(y) \wedge Chase(x,y))$
5. $\forall x\ Buy(x) \Rightarrow \exists y\ (Own(x,y) \wedge Cat(y))$
6. $\forall x\ \forall y\ (\exists z(Own(y,z) \wedge Hate(x,z)) \Rightarrow \neg Marry(x,y))$

1. Convert the following six statements into First Order Logic (FOL). Translate the obtained expressions to Conjunctive Normal Forms (CNFs, Steps 1-6 of Lecture 9).

1. **Remove implications:**

7. $\forall x(\neg\exists y(Own(x,y) \wedge Cat(y))) \vee (\forall z(\neg\exists w(Cat(w) \wedge Chase(z,w)) \vee Hate(x,z))))$ (from 3)
8. $\forall x \neg Dog(x) \vee \exists y(Cat(y) \wedge Chase(x,y))$ (from 4)
9. $\forall x \neg Buy(x) \vee \exists y(Own(x,y) \wedge Cat(y))$ (from 5)
10. $\forall x\ \forall y\ (\neg\exists z(Own(y,z) \wedge Hate(x,z))) \vee \neg Marry(x,y))$ (from 6)

2. **Minimise negations:**

11. ∀x∀y(¬Own(x,y) ∨ ¬Cat(y)) ∨ (∀z∀w (¬Cat(w) ∨ ¬Chase(z,w)) ∨ Hate(x,z)) (from 7)
12. ∀x ∀y ∀z(¬Own(y,z) ∨ ¬Hate(x,z)) ∨ ¬Marry(x,y) (from 10)

3. **Standardise variables apart:**

13. ∀x1∀y1(¬Own(x1,y1) ∨ ¬Cat(y1)) ∨ (∀z1∀w1 (¬Cat(w1) ∨ ¬Chase(z1,w1)) ∨ Hate(x1,z1)) (from 11)
14. ∀x2¬Dog(x2)∨∃y2(Cat(y2) ∧ Chase(x2,y2)) (from 8)
15. ∀x3¬Buy(x3)∨∃y3(Own(x3,y3) ∧ Cat(y3)) (from 9)
16. ∀x4∀y4∀z2(¬Own(y4,z2) ∨ ¬Hate(x4,z2)) ∨ ¬Marry(x4,y4) (from 10)

4. **Skolemise existentials:**

17. Dog(D) ∧ Own(YOU,D) (from 1)
18. ∀x2¬Dog(x2)∨(Cat(R(x2)) ∧ Chase(x2,R(x2)) (from 14)
Note: the existential on y2 was in the scope of the universal on x, hence the need for introduction of the function R(x2).

19. ∀x3¬Buy(x3) ∨ (Own(x3,F(x3)) ∧ Cat(F(x3))) (from 15)
Note: the existential on y3 was in the scope of the universal on x, hence the need for introduction of the function.

5. **Drop universals:**

20. ¬Own(x1,y1) ∨ ¬Cat(y1) ∨ ¬Cat(w1) ∨ ¬Chase(z1,w1) ∨ Hate(x1,z1) (from 13)
21. ¬Dog(x2) ∨ (Cat(R(x2)) ∧ Chase(x2,R(x2)) (from 18)
22. ¬Buy(x3) ∨ (Own(x3,F(x3)) ∧ Cat(F(x3))) (from 19)
23. ¬Own(y4,z2) ∨ ¬Hate(x4,z2) ∨ ¬Marry(x4,y4) (from 16)

6. **Convert to CNF:**

24. [¬Dog(x2) ∨ Cat(R(x2))] ∧ [¬Dog(x2) ∨ Chase(x2,R(x2))] (from 21)
25. [¬Buy(x3) ∨ Own(x3,F(x3))] ∧ [¬Buy(x3) ∨ Cat(F(x3))] (from 22)

Final set:
26. Dog(D) ∧ Own(You,D)
27. Buy(Amy)
28. ¬Own(x1,y1) ∨ ¬Cat(y1) ∨ ¬Cat(w1) ∨ ¬Chase(z1,w1) ∨ Hate(x1,z1)
29. [¬Dog(x2) ∨ Cat(R(x2))] ∧ [¬Dog(x2) ∨ Chase(x2,R(x2))]
30. [¬Buy(x3) ∨ Own(x3,F(x3))] ∧ [¬Buy(x3) ∨ Cat(F(x3))]
31. ¬Own(y4,z2) ∨ ¬Hate(x4,z2) ∨ ¬Marry(x4,y4)

2.Transform the following conclusion into FOL, negate it and convert it to CNF (Steps 1-6 of Lecture 9: Knowledge Representation and Reasoning).

Goal: ¬Marry(Amy, You)

Negated goal: ¬¬Marry(Amy, You)

1. **Remove implications:** Nothing to do
2. **Minimise negations:** Marry(Amy, You)
3. **Standardise variables apart:** Nothing to do
4. **Skolemise existentials:** Nothing to do
5. **Drop universals:** Nothing to do
6. **Convert to CNF:** Nothing to do

3. Based on all the previously created CNFs, finalise the conversion to CNF (Steps 7-8 of Lecture 9) and prove the statement from 2.

7-8: Split into disjunctive clauses & standardise variables apart:

1. Dog(D)
2. Own(You, D)
3. Buy(Amy)
4. ¬Own(x1,y1) ∨ ¬Cat(y1) ∨ ¬Cat(w1) ∨ ¬Chase(z1,w1) ∨ Hate(x1,z1)
5. ¬Dog(x2) ∨ Cat(R(x2))
6. ¬Dog(x3) ∨ Chase(x3,R(x3))
7. ¬Buy(x4) ∨ Own(x4,F(x4))
8. ¬Buy(x5) ∨ Cat(F(x5))
9. ¬Own(y2,z2) ∨ ¬Hate(x6,z2) ∨ ¬Marry(x6,y2)
10. Marry(Amy, You)

Resolution proof use the above rule to remove contradiction ie. remove literals with its nagating counterparts

11. ¬Hates(Amy,z2) V ¬Owns(You,z2)) (Resolve 10, 9) Unifier: {Amy/x6, You/y2}
12. ¬Hates(Amy,D) (Resolve 11, 2) Unifier: {D/z2}
13. ¬Own(Amy,y1) V ¬Cat(y1) V ¬Cat(w1) V ¬Chase(D,w1) (Resolve 12,4) Unifier: {Amy/x1, D/z1}
14. Own(Amy, F(Amy)) (Resolve 7,3) Unifier: {Amy/x4}
15. ¬Cat(F(Amy)) V ¬Cat(w1) V ¬Chase(D, w1) (Resolve 14, 13) Unifier: {F(Amy)/y1}
16. Chase(D,R(D)) (Resolve 6,1) Unifier: {D/x3}
17. ¬Cat(F(Amy)) V ¬Cat(R(D)) (Resolve 16, 15) Unifier: {R(D)/w1}

18. Cat(R(D))) (Resolve 5, 1) Unifier: {D/x2} pick the literal with the same function name for unification
19. ¬Cat(F(Amy)) (Resolve 18, 17)
20. Cat(F(Amy)) (Resolve 8, 3) Unifier: {Amy/x5}
21. ∅ (Resolve 20,19)